CONSTANT FALSE ALARM RATE CONTROL FOR FREQUENCY HOPPING SPREAD SPECTRUM SYSTEMS *

Ramón Agustí, Juan J. Olmos

E.T.S.I.Telecomunicación. Apdo. 30.002, 08080 Barcelona, Spain

SUMMARY

A control scheme suited for setting the false alarm rate to a fixed value in Frequency-Hopping Spread-Spectrum Communications is proposed. The scheme performance is determined in the presence of a partial band interference. For this purpose, both analytical and computer simulation techniques have been used with a good agreement in the obtained results.

1. INTRODUCTION

There are certain situations in Frequency-Hopping (FH) Spread-Spectrum Communications that call for the receiver to have a constant false alarm rate (CFAR) regardless the channel activity. That could be the case in the FH acquisition process [1] or in the so-called FH-MFSK multiple access system [2]. Here, in particular, the maximum number of simultaneous users per Hz the system can hold up depends to a high degree upon the decision threshold setting [3]. The convinience of having a CFAR has also been proved in the presence of sinusoidal tone interferers [4].

In this paper a CFAR control scheme suited to work properly in the presence of a partial band intereference is introduced and its performance assessed.

2. INTERFERENCE MODELLING

It is a common practice in FH communications to model the interference as a two level gaussian noise with noise power spectral density N_I as it is shown in Fig. 1. Then, the normalized square envelope of the detected interference, I, has the probability density function (pdf):

$$f_I(u) = \begin{cases} \exp(-u) & I = \dfrac{R^2}{2\alpha B_T N_I} \geq 0 \\ 0 & \text{otherwise} \end{cases} \qquad (1)$$

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where R is the detected envelope and α is the transmission bandwidth (B_T) fraction ocuppied by the interference. No receiver noise is assumed. This model is quite accurate when dealing with jammers. The same exponential pdf for the interference is also encountered in FH multiple access systems. That is the case for the FH-MFSK mobile radio system with control power [5] or even for a rather conventional FH multiple access with the interference to desired power ratio, γ, characterized by the pdf [6]: $f_\gamma(u)=\eta\exp(-\eta u)$. This expression, that includes (1) as a particular case, will then be retained in our analysis.

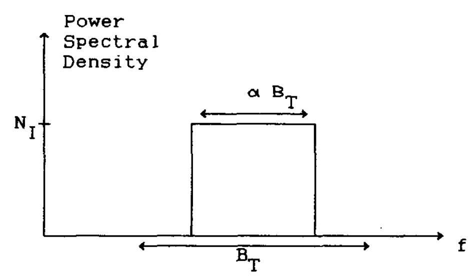

Figure 1: Two level partial band noise interference

3.CFAR CONTROL SCHEME

The proposed CFAR control scheme is shown in Fig. 2. The false alarm probability to be held constant is f_0. F_1 and F_2 are low pass filters, G is an amplifier gain, r(t) is the interference signal present at the receiver input, x(t) is the sample and hold output, F(t) is the F_2 output, and u(t) is the controlled threshold setting. So, being $T_h^{-1}=R_h$ the hopping rate, in the sampling instants $t_K=KT_h$ (k integer) we have $x(t_K)=x_K=1$, when $r(t_K)\geq u(t_K)=u_K$ and $x_K=0$ otherwise

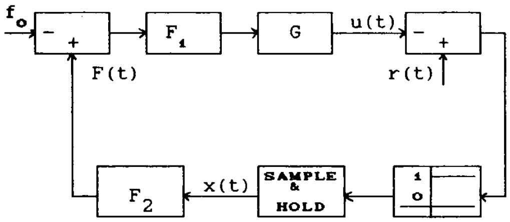

Figure 2: CFAR scheme

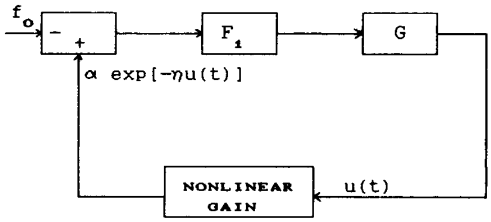

Figure 3: Equivalent model of the CFAR scheme

Hence, it is expected that the loop behaviour tends to drive F(t) close to its equilibrium value: f_0. A deeper understanding of the circuit is achieved by noting that F_2 works out the average of the x_K samples provided it is chosen as a sufficiently narrow low pass filter. That is, for a fixed $u(t)=u_0$ and a stationary interference activity, we have

$$P_{FA} = \text{Prob}\left\{x_K=1\right\} \cong F(t_K) = F_K \qquad (2)$$

being P_{FA} the false alarm probability. The equality would apply for a zero-bandwidth F_2. Specifically if the pdf of the random variable $r(t_K)$ is

$$f_r(z) = \alpha\eta\exp(-\eta z) + (1-\alpha)\delta(z), \qquad (3)$$

then

$$P_{FA} = \alpha\eta\int_{u_0}^{\infty} \exp(-\eta r)\,dr = \alpha\exp(-\eta u_0) \qquad (4)$$

and the CFAR scheme can be modeled as Fig. 3 shows. Here, we have u(t) instead of a fixed u_0 value threshold, hovever, (4) can still be used if we assume the overall time response of the loop to be much grater than that corresponding to F_2. The equation governing the loop behaviour will then be

$$u(t) = G\left\{\alpha\exp[-\eta u(t)]-f_0\right\}*f_1(t) \qquad (5)$$

where $f_1(t)$ is the impulse of F_1 and (*) denotes convolution. A suitable F_1 filter that turns (5) into a Bernoulli's equation is an integrator. That is, (5) becomes

$$\dot{y}-Gf_0\eta y = -G\alpha\eta y^2 \qquad (6)$$

where the dot denotes time derivative and $y=\exp[-\eta u(t)]$. A same form of equation is also encountered in the modelling of AGC circuits [7]. In our case, the stationary solution with $\alpha=\alpha(t)$ is found by solving (6) as

$$F(t) = f_0\cdot\tau_1\cdot\frac{\alpha(t)}{\exp(-t/\tau_1)*\alpha(t)} \qquad (7)$$

where $\tau_1=1/Gf_0\eta$ is the loop time constant. By defining $\alpha(t)=\alpha_0+\Delta\cos 2\pi\alpha_m t$, a dynamical interference model can be introduced. In this case from (7) it turns out that $F(t)=f_0$, provided that $2\pi\alpha_m \ll \tau_1^{-1}$. In order for F_2, chosen here to be a RC filter, not to modify the CFAR loop response, its time constant, τ_2, must be much lower than τ_1. Hence, with $\tau_2=NT_h$,

$$G = \delta\cdot\frac{R_H}{N}\,\frac{1}{f_0\eta} \qquad (8)$$

has to be verified for $\delta \ll 1$.

Different conditions regarding the N value selection are also considered. So, once α_m is fixed, a minimum N should ensure that $\alpha(t)$ can hardly change during NT_h so that F_2 can work out the average operation correctly. Hence $2\pi\alpha_m \ll (NT_h)^{-1}$, that is a less restrictive condition than $2\pi\alpha_m \ll \tau_1^{-1}$. On the other hand a N finite value causes F(t) to deviate from f_0. This dispersion can be calculated through the r.m.s. value

$$\phi = \sqrt{E\left\{|F(t)-f_0|^2\right\}} \qquad (9)$$

where E denotes the mean operator. In the absence of feedback control and assuming a steady state situation, we would have

$$x(t) = \sum_{n=-\infty}^{\infty} a_n \text{rect}_{T_h}(t-nT_h) \qquad (10)$$

where $a_n \varepsilon (o,1)$ and $\text{rect}_{T_h}(t)$ is the rectangular function of duration T_h.

The x(t) signal can be better formulated as

$$x(t) = \sum_{n=-\infty}^{\infty} (a_n-b_0)\text{rect}_{T_h}(t-nT_h)+b_0 = \psi(t)+f_0 \qquad (11)$$

where $b_0=E(a_n)=f_0$ and $\psi(t)$ denotes the term causing F(t) to deviate from f_0. Then

f_0	0,1		0,3		0,5	
N	100	1000	100	1000	100	1000
ϕ	$2,13\ 10^{-2}$	$6,7\ 10^{-3}$	$3,17\ 10^{-2}$	$1,04\ 10^{-2}$	$3,52\ 10^{-2}$	$1,13\ 10^{-2}$

Table 1 : Some representative dispersion values of the proposed CFAR scheme for $\eta=1$.

$$\phi^2 = \int_{-\infty}^{\infty} G_\psi(f)\,|H_2(f)|^2 df \qquad (12)$$

where $G_\psi(f)$ is the power spectral density of $\psi(t)$ and $H_2(f)$ is the transfer function of the F_2 filter. In particular

$$G_\psi(f) = E\left\{(a_n - f_0)^2\right\} T_h \left[\frac{\sin(\pi f T_h)}{\pi f T_h}\right]^2 \qquad (13)$$

and

$$|H_2(f)| = \frac{1}{1+2\pi f \tau_2}\ , \qquad (14)$$

where $H_2(f)$ is the transfer function of the F_2 filter. Given that $E(a_n)=f_0$, it results form (12), (13) and (14) that

$$\phi^2 \cong 4,84\ \frac{f_0(1-f_0)}{N}\ . \qquad (15)$$

Actually the presence of a feedback control in the CFAR scheme tends to decrease ϕ, therefore a suitable N can be chosen from

$$\phi < 2,2\ .\sqrt{\frac{f_0(1-f_0)}{N}}\ . \qquad (16)$$

4. RESULTS

The non-linear nature of the proposed CFAR scheme prevents us from doing an exact analysis to find ϕ, so a simulation has been carried out to confirm all about the aformentioned expected performance. For this purpose we have used

$$u_K = u_{K-1} + (F_K - f_0)\ \frac{G}{R_h} \qquad (17)$$

where G is determined from (8) with $\delta=0,1$. Analogously the F_2 behaviour is simulated by

$$F_K = \frac{1}{N}\ x_K + \exp(-N) F_{K-1}. \qquad (18)$$

Table 1 shows the ϕ values obtained in some representative cases. It is worthwile to note that these values as well as those obtained from multiple simulations satisfy a scaled version of expresion (12), that is

$$\phi \cong 0,71\ \sqrt{\frac{f_0(1-f_0)}{N}}\ . \qquad (19)$$

Moreover, as forseen from (8) and (16), no ϕ variations shows up as result of considering different α_m values. In that sense, it is concluded that for all the cases analized with $\Delta=0,1$, the degradation with regard to the stationary case has only been noticeable when $2\pi\alpha_m > 0,01(NT_h)^{-1}$. No ϕ variations have also been apreciated for reasonable η values ($0,4<\eta<1,4$). Since G depends on η, it is concluded that such a dependency has at most a second order effect.

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