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ON THE DESIGN OF BANDPASS AND BANDSTOP DIGITAL FILTERS FROM LOWPASS ANALOG PROTOTYPES

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Abstract

Presently, the transformation of lowpass analog prototypes constitutes an useful method of design for recursive digital filters. Although the involved bilinear and frequency band transformations are well known, the best criterion to choose the parameters of the transformations has not yet been investigated. This paper deals with the determination of such parameters in order to require the lowest order for the analog prototype.

I.- INTRODUCTION

The approximation theory for analog lowpass filters constitutes a well established topic [1]. This fact supports the great popularity attained by the digital filter design from analog lowpass prototypes. As it is well known, this approach needs a way to digitize the continous-time prototype and, when we are concerned with the design of bandpass or bandstop filters, a frequency transformation, which can be implemented both in the analog (reactance transformation [1]) and the digital (allpass transformation [2]) domains. Thus, there are two approaches for designing recursive bandpass or bandstop digital filters; these approaches are summarized in Figure 1.

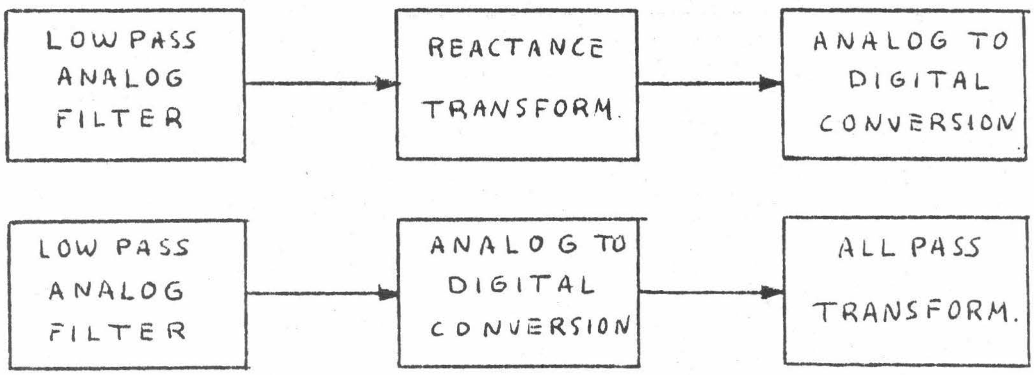


Figure 1.- Equivalent design approaches from an analog lowpass prototype.

The bilinear transformation [3] is the most widely used tool to obtain digital approximations from analog ones. It can be shown that, when the bilinear transformation is applied to digitize the analog filter, both approaches in Figure 1 become equivalent. The entire transformation from an analog lowpass prototype to a bandpass digital filter is [3]:

$$\lambda = \frac{1 - 2\cos w_0 T\, z^{-1} + z^{-2}}{1 - z^{-2}} \qquad (1)$$

λ being the complex analog frequency and z the digital variable. When designing a bandstop filter, the whole transformation is [3]:

$$\lambda = \frac{1 - z^{-2}}{1 - 2 \cos w_0 T \, z^{-1} + z^{-2}} \qquad (2)$$

In this paper, we are concerned with the determination of the parameter w_0T. Although both transformations (1) and (2) have been studied, the values of w_0T allowing to arrange the digital filter specifications by transforming a lowpass analog prototype of the lowest possible order, have not been investigated. We introduce a value of w_0T that attains this goal.

2.-STATING THE BANDPASS DESIGN

Let us consider the design of a bandpass digital filter, whose loss specifications are shown symbolically in Figure 2. These imply that for the lower stopband ($|w| < w_{a1}$) the attenuation should be greater than α_{a1}, for the upper stopband ($|w| > w_{a2}$) greater than α_{a2}, and for the passband ($w_{p1} < |w| < w_{p2}$) less than α_p.

The transformation (1) means the following frequency relation:

$$\Omega = \frac{\cos w_0 T - \cos wT}{\sin wT} \qquad (3)$$

It is clear that w_0 should verify

$$w_{a1} < w_0 < w_{a2} \qquad (4)$$

since it constitutes the central frequency of the digital filter passband.

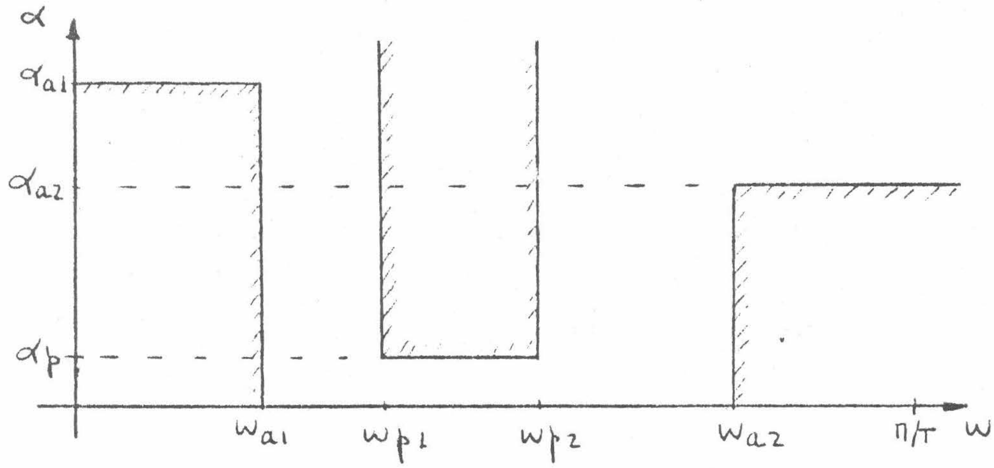


Figure 2.- Bandpass digital filter specifications.

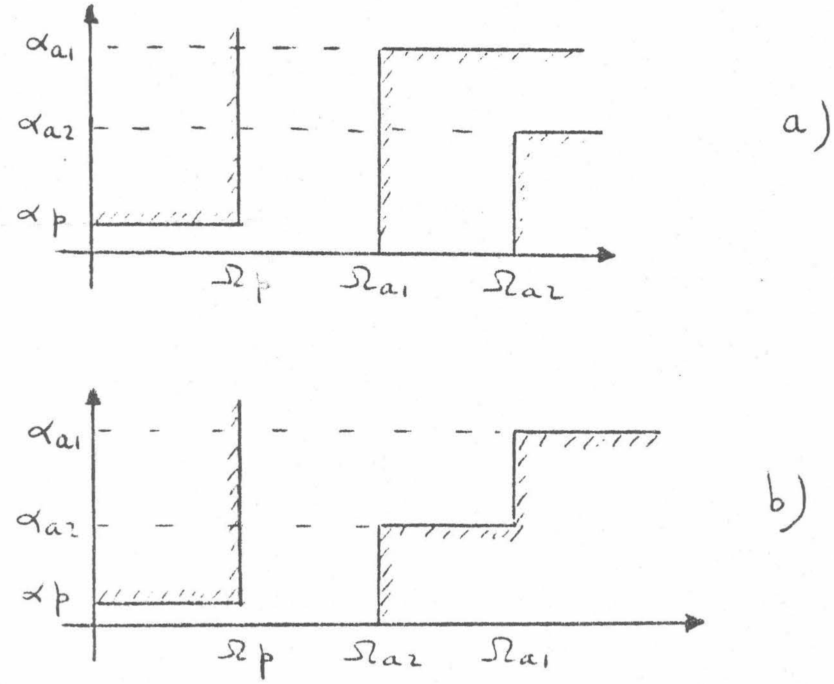


Figure 3.- Analog prototype specifications:

a) if $\Omega_{a1} \leqslant \Omega_{a2}$

b) if $\Omega_{a1} > \Omega_{a2}$

In order to ease the later discussion, it is convenient to substitute the sine and cosine by the equivalent expresions in function of the tangent of the half angle, because this function is monotonic for $|wT| \leqslant \pi$. Thus, we obtain

$$\Omega = \frac{1}{2(1 + tg^2 w_o T/2)} \; \frac{tg^2 wT/2 - tg^2 w_o T/2}{tg\ wT/2} \qquad (5)$$

This relation, applied to the specifications in Figure 2, imply for the lowpass analog prototype the requeriments shown in Figure 3, where

$$\Omega_p = \max\ (\Omega_{p1}, \Omega_{p2}) \qquad (6)$$

with

$$\Omega_{p1} = \frac{1}{2(1 + tg^2 w_o T/2)} \; \frac{tg^2 w_o T/2 - tg^2 w_{p1} T/2}{tg\ w_{p1} T/2} \qquad (7.a)$$

$$\Omega_{p2} = \frac{1}{2(1 + tg^2 w\ T/2)} \; \frac{tg^2 w_{p2} T/2 - tg^2 w_o T/2}{tg\ w_{p2} T/2} \qquad (7.b)$$

being the transformed frequencies of w_{p1} and w_{p2}, respectively, and

$$\Omega_{a1} = \frac{1}{2(1+tg^2 w_o T/2)} \; \frac{tg^2 w_o T/2 - tg^2 w_{a1} T/2}{tg\, w_{a1} T/2} \qquad (8.a)$$

$$\Omega_{a2} = \frac{1}{2(1+tg^2 w_o T/2)} \; \frac{tg^2 w_{a2} T/2 - tg^2 w_o T/2}{tg\, w_{a2} T/2} \qquad (8.b)$$

being the transformed ones of w_{a1} and w_{a2}.

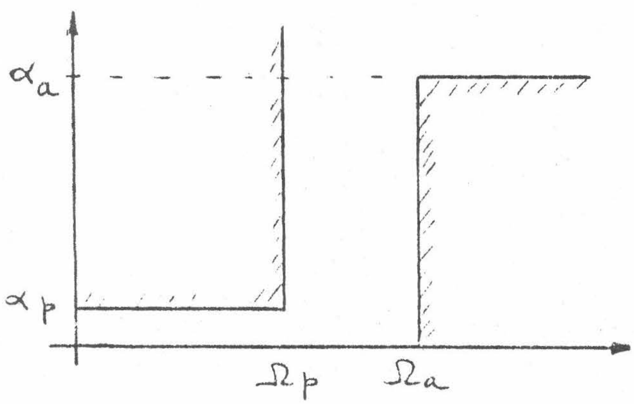
From the expresions (7) and (8), it is obvious that the analog prototype requirements are function of $w_o T$; therefore, the order necessary for the prototype (and, hence, for the digital filter) rests on this parameter.

In the situation shown in Figure 3.a, the prototype specification can be simplified to the usual one shown in Figure 4, with $\alpha_a = \alpha_{a1}$ and $\Omega_a = \Omega_{a1}$; in this case, the suitable prototype order is inmediately achieved [4]. The situation of Figure 3.b does not represent a similar case; however, it is easly understood that, for any case, the prototype order must be the greatest of those required for the following stopband specifications:

Case 1: $\alpha_a = \alpha_{a1}$, $\Omega_a = \Omega_{a1}$ (9.a)

Case 2: $\alpha_a = \alpha_{a2}$, $\Omega_a = \Omega_{a2}$ (9.b)

As it is well known, to meet the requirements of Figure 4, the analog lowpass approximations require an order that is only function of the selectivity and discrimination parameters, defined in the same Figure 4. In both cases specified by (9), only the selectivity parameter Ks depends on $w_o T$; therefore, to determine the best possible value for $w_o T$, we have to analyze such dependence.



$$Ks = \frac{\Omega_p}{\Omega_a}$$

$$Kd = \left(\frac{10^{\alpha_p/10} - 1}{10^{\alpha_a/10} - 1}\right)^{1/2}$$

Figure 4.- Lowpass specifications in function of the selectivity Ks and discrimination Kd parameters.

3.- THE MAIN RESULT

The main resul in this paper is stated in the following:

Lemma: In both cases 1 and 2 the lowest value for the selectivity parameter is achieved with

$$tg^2 w_o T/2 = tg\, w_{p1} T/2 \quad tg\, w_{p2} T/2 \qquad (10)$$

Proof.- Let us consider case 1, and assume that

$$tg^2 w_o T/2 \leqslant tg\, w_{p1} T/2 \quad tg\, w_{p2} T/2 \qquad (11)$$

This condition implies that $\Omega_{p1} \leqslant \Omega_{p2}$, as easly checked from (7); thus, according to (6), (7.b) and (8.a) we can write

$$Ks = \frac{\Omega_{p2}}{\Omega_{a1}} = \frac{tg^2 w_{p2} T/2 - tg^2 w_o T/2}{tg^2 w_o T/2 - tg^2 w_{a1} T/2} \; \frac{tg\, w_{a1} T/2}{tg\, w_{p2} t/2}$$

Ks achieves its minimum value, when $tg^2 w_o T/2$ is maximun; then, according to the condition (11), Ks is minimum for $tg^2 w_o T/2$ given by (10).

On the contrary, if we assume that

$$tg^2 w_o T/2 \geqslant tg\, w_{p1} T/2 \quad tg\, w_{p2} T/2 \qquad (12)$$

it is easy to prove in (7) that $\Omega_{p1} \geqslant \Omega_{p2}$; then, (6), (7.a) and (8.a) lead to

$$Ks = \frac{\Omega_{p1}}{\Omega_{a1}} = \frac{tg^2 w_o T/2 - tg^2 w_{p1} T/2}{tg^2 w_o T/2 - tg^2 w_{a1} T/2} \; \frac{tg\, w_{a1} T/2}{tg\, w_{p1} T/2}$$

For $tg^2 w_o T/2$ verifying (12), it is inmediate that

$$\frac{d\, Ks}{d\, tg^2 w_o T/2} > 0$$

Accordingly, $tg^2 w_o T/2$ defined in (10) provides the lowest value for Ks. This result proves the lemma for Case 1. A similar analysis allows to get the same result for Case 2; thus, the lemma is proved.

Theorem 1.— The value for $w_o T$ expresed in (10), allows us to meet the specifications in Figure 2 with the lowest possible order.

Proof.- It is known [4] that, with the discrimination parameter held fixed, the less is the selectivity the less is the order required for a lowpass analog filter. Thus, according to

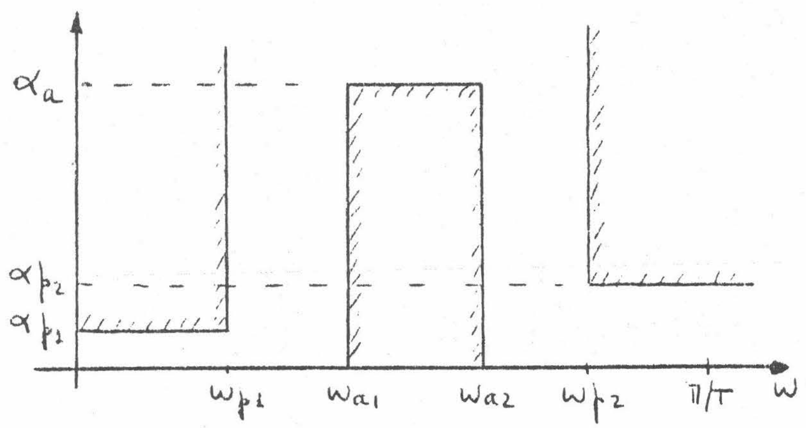


Figure 5.- Bandstop digital filter specifications.

Lemma, the value for $tg^2 w_o T/2$ provided in (10) guarentees the lowest possible order for the prototype and, hence, for the digital filter.

So far we have analized the bandpass design; a similar study for the bandstop case leads to the following result:

Theorem 2.- The value given by

$$tg^2 w_o T/2 = tg\ w_{a1} T/2 \quad tg\ w_{a2} T/2$$

for the parameter $w_o T$ of the transformation (2) allows to meet the specifications in Figure 5 with the lowest possible order.

4.- CONCLUSION

- We have established a value for the parameter $w_o T$ of the transformations (1) and (2) that guarantees, to meet the filter specifications, the minimun possible required order for the analog prototype. However, it does not imply that we have to choose necessarily such value for attainning this lowest order; given that the order is an integer, values for $w_o T$ close to that providing the minimum selectivity parameter can also define analog prototype specifications which can be matched with the lowest possible order. Thus, the introduced theorems provide useful criteria rather than rigid conclusions for choosing $w_o T$.

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