AN ALTERNATIVE APPROACH TO IMPLEMENT A RECURSIVE INTERPOLATION

Miguel A. Lagunas-Hernández*, José B. Mariño-Acebal**, Climent Nadeu-Camprubí* and Aníbal R. Figueiras-Vidal**.

* ETSI Telecomunicación, Jorge Girona Salgado s/n, Barcelona 34, Spain.
**ETSI Telecomunicación, Cdad. Universitaria, Madrid 3, Spain.

ABSTRACT

An alternative procedure to decompose a recursive interpolation in a polyphase network that avoids numerical problems of previous methods is introduced. Symmetrization of the obtained polyphase network to further reduce the number of multiplications per second is also considered.

The design of an interpolator filter for a 60-channel transmultiplexer is considered as an illustrative example.

INTRODUCTION

Te use to interpolate and decimate {1} of recursive filters was considered for the first time by Bellanger et alia {2} , introducing the polyphase network. This scheme allows a saving of about 50% in the number of necessary multiplications to change the sampling rate with respect to the direct application of a recursive filter.

If the ratio between the original and final sampling frequencies is an integer number P and we associate z^{-1} with the highest frequency, it will be possible to write the filter transfer function H(z) in the form

$$H(z) = \sum_{i=0}^{P-1} z^{-i} H_i(z^P) \tag{1}$$

where

$$H_i(z^P) = N_i(z^P)/D(z^P), \quad i = 0,\dots,P-1 \tag{2}$$

This is the polyphase representation for the recursive filter H(z). Combining (1) and (2), we obtain

$$H(z) = N(z) / D(z^P) \tag{3}$$

where

$$N(z) = \sum_{i=0}^{P-1} z^{-i} N_i(z^P) \tag{4}$$

Then, if N(z), nonrecursive part of H(z), has a symmetrical impulse response, we will obtain an additional 50% saving in the number of multiplications {1} {3} .

We introduce in this paper an alternative approach to the polyphase design of a recursive filter forcing a symmetrical N(z).

I. OBTAINING THE POLYPHASE NETWORK

The following identity

$$1/(1 - p_j z^{-1}) = \sum_{i=0}^{P-1} p_j^i z^{-i} / (1 - p_j^P z^{-P}) \tag{5}$$

is used to derive the polyphase network from the recursive H(z); applying it to the expression of H(z) in function of its poles and corresponding residues,

$$H(z) = A_0 + \sum_{j=1}^{K} A_j /(1 - p_j z^{-1}) \tag{6}$$

where K is the order of H(z), we can write

$$H(z) = A_0 + \sum_{j=1}^{K} A_j \left[\sum_{i=0}^{P-1} z^{-i} p_j^i \right] / (1 - p_j^P z^{-P}) =$$

$$= A_0 + \sum_{i=0}^{P-1} z^{-i} \left[\sum_{j=1}^{K} A_j p_j^i / (1 - p_j^P z^{-P}) \right] =$$

$$= A_0 + \sum_{j=1}^{K} A_j/(1 - p_j^P z^{-P}) + \sum_{i=1}^{P-1} z^{-i} \left[\sum_{j=1}^{K} A_j p_j^i/(1 - p_j^P z^{-P}) \right] \quad (7)$$

Identifying (7) and (1),

$$H_0(z^P) = A_0 + \sum_{j=1}^{K} A_j/(1 - p_j^P z^{-P}) = N_0(z^P)/D(z^P) \quad (8a)$$

$$H_i(z^P) = \sum_{j=1}^{K} A_j / (1 - p_j^P z^{-P}) = N_i(z^P) / D(z^P) \quad (8b)$$

(8a, b) are an intersting alternative to the formulas proposed by Bellanger et alia {2}

$$D(z^P) = \prod_{j=1}^{K} (1 - p_j^P z^{-P}) \quad (9a)$$

$$N(z) = \sum_{i=0}^{P-1} z^{-i} N_i(z^P) = A \prod_{j=1}^{K} \left[(1 - z_j z^{-1}) \sum_{i=0}^{P-1} p_j^i z^{-i} \right] \quad (9b)$$

where z_j are the zeros of $H(z)$. From a computational point of view, the relative advantages of (8a, b) are that the number of operations to calculate each $H_i(z^P)$ is independent of P, and that each $H_i(z^P)$ can be obtained independently of the rest. These properties imply for the proposed method a lower sensitivity to computational errors in obtaining each $H_i(z^P)$.

II. OBTAINING THE SYMMETRY

The output of the interpolating polyphase network can be obtained according to the following operations

$$X_1(z^P) = X(z^P) / D(z^P) \quad (10a)$$

$$Y(z) = \sum_{i=0}^{P-1} z^{-i} N_i(z) X_1(z^P) = N(z) X_1(z^P) \quad (10b)$$

We can see in (9a) that the order of $D(z^P)$ in z^P is identical to the order of H(z) in z, K; and (9b) says us that the order of N(z) is KP. Therefore

$$N_{ms} = Kf_m + (KP + 1) f_m = [K(P + 1) + 1] f_m \quad (11)$$

where f_m is the sampling frequency of the sequence to be interpolated, is the number of multiplications per second required to interpolate.

Once we have obtained each $N_i(z^P)$ with (8a, b), we can form $N(z)$, that, in a general case, has not a symmetrical impulse response; but, when P is great enough, we can force this symmetry introducing an approximation.

When there are only amplitude specifications for $H(e^{j\omega})$, we can think in substituting the numerator of $H(z)$ for another numerator, $N'(z)$, that approximates the amplitude of the frequency response of $N(z)$ but having a linear phase. This possibility can be used when the corresponding approximation maintains the interpolation filter under sepecifications and this do not require an order for $N'(z)$ that compensates the reduction in the multiplications per second offered by the symmetry of $N'(z)$.

The above possibility can be illustrated by means of the following practical example: the design of an interpolator to include in a 60-channel T D M - F D M transmultiplexer (proposed by Bonnerot et alia {4}). Fig. 1 shows the attenuation constraints for this filter. The final sampling rate is 512 KHz; the sampling frequency ratio is $P = 128$.

An elliptic filter having an order 7 verifies this constraint; this filter attenuates the input by 70 dB on the stopband. We can obtain $D(z^P)$ and $N(z)$ by means of formulas (9a, b); the corresponding $N(z)$ has an impulsional response with $897 = 7 \cdot 128 + 1$ nonzero samples. We select a length 796 for the impulse response of $N'(z)$, to obtain a common order for each $N'_i(z)$ in

$$N'(z) = \sum_{i=0}^{P-1} z^{-i} N'_i(z) \tag{12}$$

a condition that has been incorporated in the design of {4} . To introduce $N'(z)$,

* we discard the last (negligible) sample in the impulse response of $N(z)$;
* by means of a DFT, we compute the amplitude of the frequency response of the truncated sequence;
* we associate a linear phase to this amplitude response;
* we derive $N'(z)$ by means of an inverse DFT.

The resulting polyphase network maintains a ripple of 0.5 dB on the passband and a maximum attenuation of 66 dB. on the stopband, when we quantize its coefficients with 20 bits.

The previous method is, in fact, a frequency sampling design. Applying optimum procedures {5} will allow better results. But the result offered by the very simple frequency sampling is good enough to illustrate the potential advantages of substituting $N(z)$ for a $N'(z)$ having the same frequency response amplitude and a linear phase.

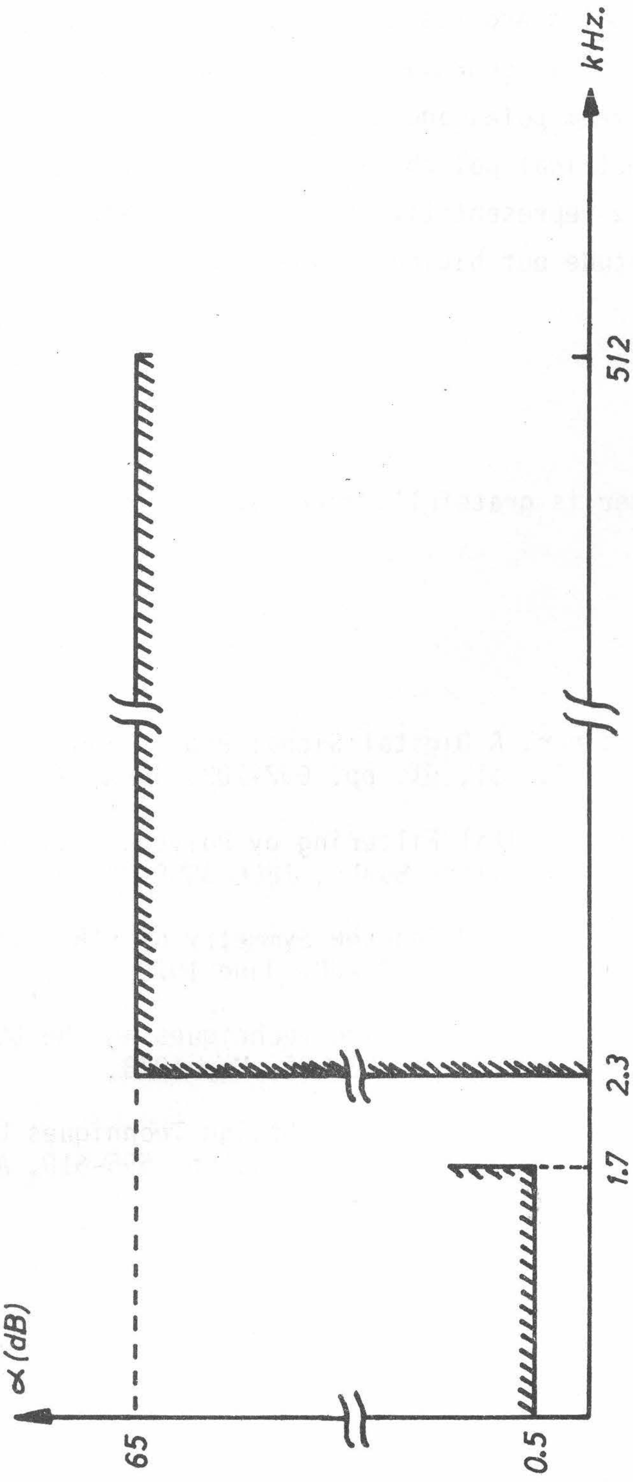

Fig. 1

CONCLUSIONS

To start from poles and residues of a transfer function to design a polyphase representation of a recursive filter seems to be computationally more efficient than to start from poles and zeros.

To obtain a symmetrical polyphase network, a substitution of the numerator of the obtained polyphase representation by a new numerator with identical frequency response amplitude but having a linear phase can be an interesting possibility.

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