AN ADAPTIVE DESIGN OF AN ALL-ZERO SPECTRAL ESTIMATOR

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ABSTRACT

An adaptive method to design a spectral estimator for signals showing an effectively lag-limited autocorrelation function is introduced. This procedure constitutes a parametric alternative to classical windowing and overlapping (WOSA) non-parametric methods of spectral estimation.

The algorithm provides high quality results and it implies a low computational load; these facts justify an important position for the proposed method among both classical methods based on the periodogram and modern all-pole methods in cases in which the signal under analysis verifies the mentioned condition of having an effectively finite duration autocorrelation function.

INTRODUCTION

Given a finite set of samples of a zero mean, stationary stochastic process realization, $\{x(k)\}$, $k = 0,\ldots, N-1$, an estimator of the power spectral density of the process is the Fourier transform

$$\hat{S}_{xx}(\omega) = \sum_{l=-\infty}^{\infty} \hat{r}_{xx}(l) \exp(-jl\omega) \qquad (1)$$

of the sequence $\{\hat{r}_{xx}(l)\}$, an estimator of the process autocorrelation function $\{r_{xx}(l)\}$. The main problem in this estimation consist on processing $\{x(k)\}$, $k = 0,\ldots, N-1$, to obtain $\{\hat{r}_{xx}(l)\}$.

$\hat{S}_{xx}(\omega)$ must be a non-negative function for all ω , since it estimates a power spectral density. To satisfy this, it is necessary a form

$$\hat{r}_{xx}(l) = \sum_{m=-\infty}^{\infty} b(m + l)\, b(m) \qquad (2)$$

for the autocorrelation estimator.

Different methods of spectral estimation start from (2). The most widely originally used, based on the periodogram, is called Weighted Overlapped-Segment Averaging (WOSA) method {1} {2}; it introduces

$$b(m) = w(m)\, x(m) \qquad (3)$$

where $\{w(m)\}$, $m = 0,\ldots,M$, is a data window. This window can be applied with lengths less than N, forming $\{\hat{r}_{xx}(l)\}$ by averaging the different functions obtained through (2).

The methods that basically follow the above steps are considered non-parametric methods: they do not assume any previous knowledge about the class of process being analyzed. Nevertheless, we must remark that they force a finite duration $\{\hat{r}_{xx}(l)\}$.

Other procedures, generally known as parametric methods, assume that $\{b(m)\}$ is the impulse response of a linear system driven by a zero mean, unit variance white noise sequence $\{n(k)\}$. This point of view allows to relate directly the spectral estimation problem with the linear systems theory. In these cases, (2) can be interpreted as the autocorrelation of the output from the linear system

$$\hat{x}(k) = \sum_{m=-\infty}^{\infty} b(m)\; n(k - m) \qquad (4)$$

$$\hat{r}_{xx}(l) = r_{\hat{x}\hat{x}}(l) = E\left[\hat{x}(k + l)\; \hat{x}(k)\right] = \sum_{m=-\infty}^{\infty} b(m + l)\, b(m) \qquad (5)$$

These parametric methods are classified in three main groups in accordance with the structure assumed for the linear system:

* Moving Average (MA) methods, when we consider a FIR filter;
* Autoregressive (AR) methods, when we select a pure (all-pole) IIR filter;

* Autoregressive-Moving Average (ARMA) methods, when we work with a general IIR (having zeros and poles) filter.

MEM methods {3} and all those based on designing a linear predictor for the signal under analysis can be included in the second group. There are a great number of ARMA procedures, among which we can show up the recently published contributions of Cadzow {4} , Kaveh {5} and Kinkel et alia {6} .

MA (with non-parametric) methods seemed to be destined to become obsolete, given the surprising results obtained with the other methods. But they have returned to be an important alternative, because they have proved to offer high quality spectral estimators for signals that can be considered of effectively duration-limited autocorrelation functions.

From the above, it can be concluded that, in fact, a WOSA method implies an MA structure for the signal, and, thus, we must expect similar results following these spectral estimation methods.

This work presents a procedure to compute adaptively $\{b(m)\}$ for MA sequences that offers remarkably good results in cases in which $\{\hat{r}_{xx}(l)\}$ can be considered of effectively finite duration. The algorithm is competitive with (in general, it overcomes) WOSA methods on simulated MA sequences.

I. SPECTRAL ESTIMATION OF MA SEQUENCES

Assuming a causal FIR filter having an impulse response $\{b(m)\}$, $m = 0,\ldots,M$, with $b(0) = 1$, we can write

$$x(k) = n(k) + \sum_{m=1}^{M} b(m)\, n(k-m) \tag{6}$$

We will center our interest in establishing the $\{b(m)\}$, $m = 1,\ldots,M$, in an adaptive form, computing $\{\hat{b}(m, k)\}$, $m = 1,\ldots,M$, $k = 0,\ldots,N-1$, by using the pseudoinversion sequence $\{e(k)\}$, $k = 0,\ldots, N-1$, of the "approximating" system that obeys to the equations

$$\hat{x}(k) = \sum_{m=1}^{M} \hat{b}(m, k)\, e(k-m) \tag{7a}$$

$$e(k) = x(k) - \hat{x}(k) \tag{7b}$$

as shown in Fig. 1. Note that $E[x(k)] = 0$ implies $E[\hat{x}(k)] = E[e(k)] = 0$. When k is great enough, $\{e(k)\}$ will approach a white sequence with variance σ^2 (as $n(k)$); then, we can use a spectral estimator

$$\hat{S}_{xx}(\omega) = \sigma^2 \left| 1 + \sum_{m=1}^{M} \hat{b}(m, N-1) \exp(-jm\omega) \right|^2 \tag{8}$$

processing all the available data.

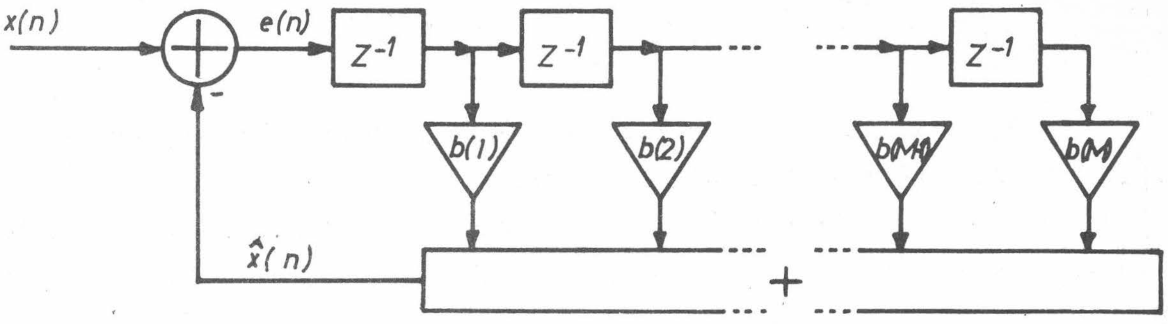

Fig. 1

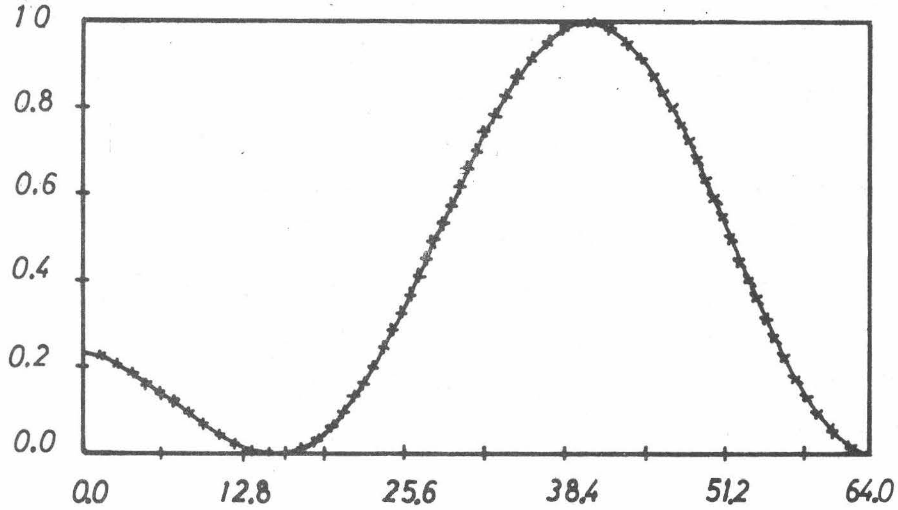

Fig. 2

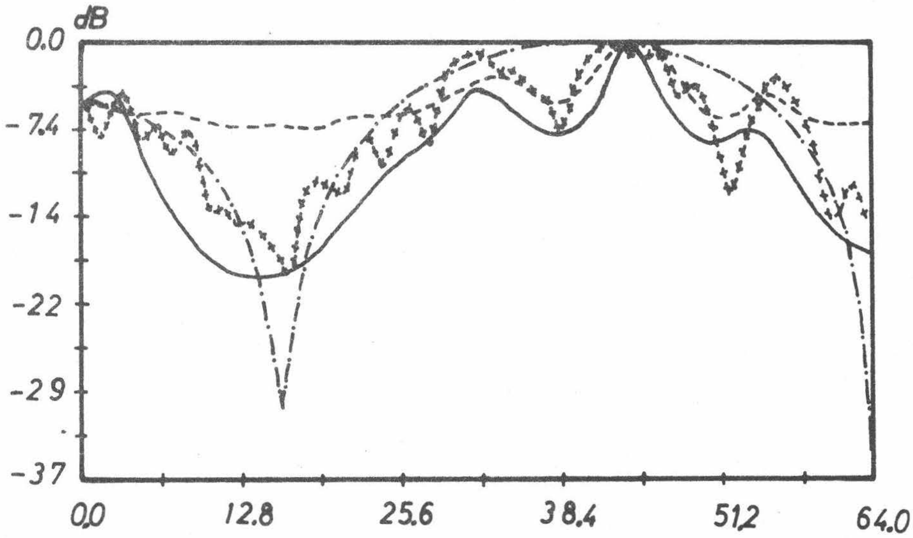

Fig. 3 ——— MEM
- - - - - - - Widrow
+ + + + + + + + WOSA
-·-·-·-·- Proposed method (order 3)

II. THE PROPOSED ALGORITHM TO COMPUTE THE FILTER COEFFICIENTS

We propose the recursive algorithm

$$\hat{b}(m, k + 1) = \hat{b}(m, k) + \mu(m, k)\ e(k) \tag{9}$$

under the condition of minimizing the coefficient errors

$$\tilde{b}(m, k) = b(m) - \hat{b}(m, k) \tag{10}$$

at each step. (9) and (10) imply

$$\tilde{b}(m, k + 1) = \tilde{b}(m, k) - \mu(m, k)\ e(k) \tag{11}$$

and a zero mean for the coefficient errors; and considering that an optimum design must obey to the Orthogonality Principle (i.e., the error $\tilde{b}(m, k + 1)$ must be orthogonal to the data $e(k)$), (11) implies

$$\mu(m, k) = \frac{E\left[\tilde{b}(m, k)\ e(k)\right]}{E\left[e^2(k)\right]} \tag{12}$$

We will discuss the computation of (12) in the two next Sections.

III. COMPUTATION OF $\mu(m, k)$ TO UPDATE THE FILTER COEFFICIENTS

From (6) and (7a, b), we can write

$$e(k) = n(k) + \sum_{m=1}^{M} \left[b(m)\ n(k - m) - \hat{b}(m, k)\ e(k - m) \right] \tag{13}$$

Since $e(k - m)$ will approach $n(k - m)$ in running the algorithm, we can use to compute (12) the approximation to (13)

$$e(k) \simeq n(k) + \sum_{m=1}^{M} \left[b(m) - \hat{b}(m, k) \right] e(k - m) = n(k) + \sum_{m=1}^{M} \tilde{b}(m, k)\ e(k - m) \tag{14}$$

Now, we will assume:

* independence (then, orthogonality) between $n(k)$ and $\tilde{b}(m, k)$, all m and k;

* independence (then, orthogonality) between $\tilde{b}(l, k)$ and $\tilde{b}(m, k)$, all k, l and m with $l \neq m$;

* orthogonality between $e(k - m)$ and $\tilde{b}(l, k)$, all k, l and m. This is a reasonable hypothesis, because $\tilde{b}(l, k)$ must be orthogonal to the data in recursively estimating $\hat{b}(l, k)$, and $\hat{b}(m, k)$ can be recursively written in function of the $\{e(k - m)\}$, $m \geq 1$.

Under these assumptions, multiplying both sides of (14) by $\tilde{b}(m, k)$

and taking expectations, we arrive at

$$E\left[\tilde{b}(m, k)\ e(k)\right] \simeq E\left[\tilde{b}^2(m, k)\ e(k - m)\right] \quad (15)$$

and we will approximate it by

$$E\left[\tilde{b}(m, k)\ e(k)\right] \simeq S^2(m, k)\ e(k - m) \quad (16)$$

where we have introduced $S^2(m, k) = E\left[\tilde{b}^2(m, k)\right]$ and considered that $\{e(k - m)\}$, $m \geqslant 1$, are available data.

Now, we will compute $E\left[e^2(k)\right]$ to complete the calculation of (12). Taking expectations of the squares of both terms in (14), and approximating

$$E\left[\tilde{b}(m, k)\ e(k-m)\ \tilde{b}(1, k)\ e(k-1)\right] \simeq E\left[\tilde{b}(m, k)\ \tilde{b}(1, k)\right] e(k-m)\ e(k-1) \quad (17)$$

and considering the above specified orthogonalities, we arrive at

$$E\left[e^2(k)\right] = E\left[n^2(k)\right] + \sum_{m=1}^{M} S^2(m, k)\ e^2(k - m) \quad (18)$$

IV. RECURSIVE EVALUATION OF $S^2(m, k)$

From recursive equation (15) we can derive a recursive form for $S^2(m,k)$

$$S^2(m, k+1) \simeq S^2(m,k) + \mu^2(m,k)\ E\left[e^2(k)\right] - 2\mu(m,k)\ E\left[\tilde{b}(m, k)\ e(k)\right] \quad (19)$$

and considering expression (12) for $\mu(m, k)$, we can rewrite (19) in the form

$$S^2(m, k + 1) \simeq S^2(m, k) - \mu^2(m, k)\ E\left[e^2(k)\right] \quad (20)$$

Equation (20) completes the recursive computation of $\mu(m, k + 1)$, and, hence, the proposed algorithm.

V. FINAL ALGORITNM

In conclusion, we can establish the following algorithm:

Initialize $\{\hat{b}(m, 0)\}$, $\{S^2(m, 0)\}$ $(m = 1,\dots,M)$; $k = 0$; introduce L (for σ^2)

1. $\hat{x}(k) = \sum_{m=1}^{M} \hat{b}(m, k)\ e(k - m)$

2. $e(k) = x(k) - \hat{x}(k)$

3. $E\left[e^2(k)\right] = L + \sum_{m=1}^{M} S^2(m, k)\ e^2(k - m)$

4. $\mu(m, k) = S^2(m, k)\, e(k - m) / E\left[e^2(k)\right]$ $(m = 1,\ldots,M)$

5. $\hat{b}(m, k + 1) = \hat{b}(m, k) + \mu(m, k)\, e(k)$ $(m = 1,\ldots,M)$

6. $S^2(m, k + 1) = S^2(m, k) - \mu^2(m, k)\, E\left[e^2(k)\right]$ $(m = 1,\ldots,M)$

7, $k = k + 1$

8. If $k + N$, go to 10

9. Go to 1

10. $\hat{S}_{xx}(\omega) = L \left| 1 + \sum_{m=1}^{M} \hat{b}(m, N - 1) \exp(-jm\omega) \right|^2$

End

This procedure offers a computational load that, in our opinion, is of the order of those corresponding to other similar methods.

We must comment two aspects of the previous algorithm. First: the value assumed for $E\left[n^2(k)\right]$, L, has not a remarkable importance for the quality of the obtained spectral estimation. Our experiences indicate that $L = 1$ is an adequate selection when the dynamic range of the signal is also about 1. Second: the (positive) initial values of $\{S^2(m, 0)\}$ $(m = 1,\ldots,M)$ slightly affect to the convergence time of the algorithm.

VI. A SIMULATION EXAMPLE

To check the efficiency of the above method, we have simulated some MA signals and applied them the algorithm. We show here results corresponding to one of the simulated sequences: that obtained through a system having a transfer function

$$B(z) = 1 - 0.41\, z^{-1} - 0.41\, z^{-2} + z^{-3} \qquad (21)$$

Fig. 2 shows the exact corresponding spectrum. The results of MEM {3} and Widrow {7} spectral estimations, having 10 correlation points the first and order 20 the second, are shown in Fig. 3. Fig. 3 shows also the result obtained using a WOSA with a rectangular time weighting and using the proposed method.

Comparison of Figs. 2 and 3 allows us to conclude that it is basic to know the kind of sequence being analyzed: MEM and Widrow methods offer worse results than WOSA and the proposed algorithm because they are all-pole schemes. It can be also seen the excellent finding of the zero that is obtained applying the introduced method. We must also remark that there are not important differences in the results when we consider different orders of the proposed algorithm.

CONCLUSIONS

We have presented a spectral estimation procedure that is related with the classical non-parametric methods. It is an adaptive algorithm, having all the advantages that this character represents for non-stationary signals, and, in our opinion, it offers a low computational load. This implies an advantageous position for this new method with respect to both classical methods based on the periodogram and modern all-pole methods in cases in which the signal has an effectively lag-limited autocorrelation function.

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