

# Large-eddy simulation of flow and combustion dynamics in a lean partially premixed swirling combustor

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## Abstract

A lean partially premixed swirling combustor was studied by resolving the complete flow path from the swirl vanes to the chamber outlet with large-eddy simulation (LES). The flow and combustion dynamics for non-reacting and reacting situations was analysed, where the intrinsic effects of swirl vanes and counter flow on the vortex formation, vorticity distribution for non-reacting cases were examined. A modified flame index was introduced to identify the flame regime during the partially premixed combustion. The combustion instability phenomenon was examined by applying Fourier spectra analysis. Several scalar variables were monitored to investigate the combustion dynamics at different operating conditions. The effects of swirl number, Reynolds number, equivalence ratio and nitrogen dilution on combustion dynamics and NO<sub>x</sub> emissions were found to be significant.

*Keywords:* LES, lean partially premixed, swirling flows, combustion dynamics, NO<sub>x</sub>

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## 1. Introduction

Lean premixed (LP) combustion is a promising technology used in gas turbine engines for reducing both NO<sub>x</sub> emissions as well as reaction zone size. To obtain optimal fuel/air mixing at the combustor inlet and avoid combustion instability, such as flame quenching or flashback during LP combustor operation, swirling injectors are typically used in gas turbine systems

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7 to provide the desired fuel/air distribution and produce central toroidal re-  
8 circulation zones (CTRZs), which serve as the dominant flame stabilization  
9 mechanism<sup>1</sup>. However, the streamwise and spanwise vortices breakdown due  
10 to swirl flows give rise to high shear layer instability and turbulence intensity,  
11 which may cause flow motion instability and couple with acoustic waves to  
12 generate combustion oscillation<sup>2</sup>.

13 Many works have been conducted to study the influence of intrinsic  
14 swirling flow on combustion instability. Huang<sup>3</sup> analysed the effect of swirl  
15 number on flow development and combustion dynamics in a lean premixed  
16 combustor. Results indicated that excessive swirl caused the recirculation  
17 zones to move upstream and may result in flame flashback. Fernandes<sup>4</sup> con-  
18 ducted experiments to investigate the swirl effect on flow instability char-  
19 acteristics and found that the instability frequency was determined by the  
20 axial translation of spiral vortex while the angular transportation of the vor-  
21 tex core was the dominant factor when the swirl number exceeded 0.88; the  
22 instability frequency varied parabolically with the swirl number. Sommerer  
23 et al.<sup>5</sup> studied the transition of different combustion regimes using both ex-  
24 perimental and the LES methods in a partially premixed swirling burner and  
25 discovered the critical role of swirl and recirculation zones during flame flash-  
26 back combustion regime. Weigand et al.<sup>6</sup> conducted a detailed experiment  
27 that measured three types of swirl methane flames with different combustion  
28 instability characteristics. Flow field, vortex structure and reaction species  
29 were carefully analysed. Meier<sup>7</sup> analysed the effects of turbulence flow on  
30 the thermal chemistry state and the structures of CH layers in a partially  
31 premixed swirling combustor by applying planar laser-induced fluorescence  
32 (PLIF), laser Doppler velocimetry (LDV) and Raman scattering. Results in-  
33 dicated that swirl flames behaved more like a diffusion flame than a uniformly  
34 premixed flame because of the existence of a spatial gradient in the mixture  
35 fraction and the larger widths of the CH layers compared to premixed flames.

36 Several other works have focused on the effects of different operating con-  
37 ditions on flame dynamics. Huang and Yang<sup>8,9</sup> investigated the effects of  
38 inlet flow conditions on unsteady flame dynamics. Several influencing fac-  
39 tors were identified and analysed. Prakash et al.<sup>10</sup> studied flame dynamics at  
40 different equivalence ratios and developed an efficient method for preventing  
41 lean blow out. Menon et al.<sup>11</sup> simulated a typical GE turbine engine, model  
42 LM6000, with the LES method and investigated the influence of swirl num-  
43 ber and a subgrid model on the mixing and reaction processes. Schluter<sup>12</sup>  
44 focused on developing a combustion oscillation control method and investi-

45 gated the effects of large-scale vortices on combustion dynamics in a swirl  
 46 coaxial combustor. Grinstein<sup>13</sup> examined the effects of combustor confine-  
 47 ment geometry on flame and flow characteristics in a swirl model combustor.  
 48 Duwig<sup>14</sup> detected the flame stabilization factor in a swirl-stabilized combus-  
 49 tor.

50 Although valuable information has been obtained by these studies, the  
 51 flame dynamics and instability mechanism are very complex at lean condi-  
 52 tions considering the existence of swirling flow. While many works have been  
 53 conducted to study such mechanisms under lean premixed operating condi-  
 54 tions, LES investigation for lean partially premixed operating conditions is  
 55 relatively rare. There are still many unresolved issues regarding flame dy-  
 56 namics, NO<sub>X</sub> emissions and the mechanisms of flame/vortex interactions in  
 57 swirl lean partially premixed combustors, which require further investigation.  
 58 In the present work, a swirl-stabilized lean partially-premixed combustor in  
 59 our experiment is simulated using a parallel LES method to examine the in-  
 60 fluence of different operating conditions on flow and flame dynamics together  
 61 with pollution emissions. The theoretical formulation is given in section 2.  
 62 The experimental setup and numerical methods are briefly presented in sec-  
 63 tion 3. Results are discussed in section 4, and a conclusion is given in section  
 64 5.

## 65 2. Theoretical formulation

66 In LES, each variable is decomposed into resolved and subgridded parts  
 67 by a spatial filtering operation, such as  $f = \bar{f} + f''$ , where an over bar – de-  
 68 notes the spatially filtered quantities and double prime '' denotes the subgrid  
 69 scale quantities. The resolved parts are related to the large scale motion  
 70 of turbulent flow and contain the vast majority of turbulence energy, which  
 71 can be directly resolved, while the dissipative subgrid scale structures are  
 72 resolved by simulation. Favre filtering is often employed in consideration of  
 73 compressible flow. The Favre filtered variables can be defined as  $\tilde{f} = \bar{\rho} \bar{f} / \bar{\rho}$ ,  
 74 where the tilde  $\tilde{f}$  denotes the Favre-averaged variables. The governing equa-  
 75 tions of unsteady, reacting, multi-species turbulent flow are described below  
 76 by employing a Favre-averaged filter:

$$77 \quad \frac{\partial \bar{\rho}}{\partial t} + \frac{\partial \bar{\rho} \tilde{u}_j}{\partial x_j} = 0, \quad (1)$$

78

$$\frac{\partial \bar{\rho} \tilde{u}_i}{\partial t} + \frac{\partial}{\partial x_j} (\bar{\rho} \tilde{u}_i \tilde{u}_j - \bar{\tau}_{ij} + \tau_{ij}^{sgs}) = 0, \quad (2)$$

80

$$\frac{\partial \bar{\rho} \tilde{E}}{\partial t} + \frac{\partial}{\partial x_j} (\bar{\rho} \tilde{E} \tilde{u}_j + \bar{q}_j - \tilde{u}_i \bar{\tau}_{ij} + H_i^{sgs} - \sigma_i^{sgs}) = \bar{Q}^c, \quad (3)$$

82

$$\frac{\partial \bar{\rho} \tilde{Y}_m}{\partial t} + \frac{\partial}{\partial x_j} (\bar{\rho} \tilde{u}_j \tilde{Y}_m + \bar{\rho} \tilde{V}_{j,m} \tilde{Y}_m + \phi_{i,m}^{sgs} - \theta_{i,m}^{sgs}) = \bar{\rho}_m^c, \quad (4)$$

84

where  $m = 1, \dots, N$ . In the above equations, the superscript  $sgs$  denotes the unsolved subgrid terms that have to be modelled for equations to be solved. The subgrid-scale stress tensor is closed by the eddy viscosity hypothesis as:

$$\tau_{ij}^{sgs} = -2\bar{\rho}\nu_t(\tilde{S}_{ij} - \frac{1}{3}\tilde{S}_{kk}\delta_{ij}) + \frac{2}{3}\bar{\rho}k^{sgs}\delta_{ij}, \quad (5)$$

88

where  $\nu_t$  is given by  $\nu_t = C_\nu k^{sgs1/2} \bar{\Delta}$ , and the strain rate is defined as  $\tilde{S}_{ij} = 1/2(\partial \tilde{u}_i / \partial x_j + \partial \tilde{u}_j / \partial x_i)$ . To obtain  $\nu_t$ , the subgrid kinetic energy  $k^{sgs}$  is resolved by the following governing equation, as proposed by Menon<sup>15</sup>:

$$\frac{\partial \bar{\rho} k^{sgs}}{\partial t} + \frac{\partial \bar{\rho} \tilde{u}_j k^{sgs}}{\partial x_j} = P^{sgs} - D^{sgs} + \frac{\partial}{\partial x_j} (\bar{\rho} \frac{\nu_t}{Pr_t} \frac{\partial k^{sgs}}{\partial x_j}), \quad (6)$$

92

where the production term  $P^{sgs}$  and the dissipation rate term  $D^{sgs}$  are closed by  $-\tau_{ij}^{sgs} \partial \tilde{u}_i / \partial x_j$ ,  $C_\epsilon \bar{\rho} k^{sgs3/2} / \bar{\Delta}$  respectively. The model constants  $C_\nu$  and  $C_\epsilon$  are chosen to be 0.067 and 0.916, respectively, according to Calhoon<sup>16</sup>. The remaining unclosed subgrid terms are modelled by gradient assumption method. The filtered viscous stress tensor is closed by:

$$\tau_{ij} = \mu(\partial u_i / \partial x_j + \partial u_j / \partial x_i) - \frac{2}{3}\mu(\partial u_k / \partial x_k)\delta_{ij}, \quad (7)$$

98

The filtered diffusion velocities in the filtered governing equations of the turbulent flow are obtained using Fick's law  $V_{j,m} = (-D_m/Y_m)(\partial Y_m / \partial x_j)$ . The reaction source term in the energy equation is given by:

$$\bar{Q}^c = \sum_r [\sum_m (a_{mr} - b_{mr})(\Delta h_f^0)_m] \dot{\omega}_r, \quad (8)$$

101



102 Pressure is determined by the equation of state for a perfect gas mixture. The  
 103 detail about the numerical scheme could refer to Zheng<sup>17</sup> .

104 The eddy dissipation concept (EDC) model could reflect the interaction  
 105 between turbulence and chemistry with low computational expense<sup>18</sup> . Thus,  
 106 this model is employed in the present work to describe the flame chemistry in  
 107 partially premixed conditions. The EDC combustion model was based on the  
 108 assumptions that the filtered flow field is composed of surroundings(0) and  
 109 fine structures (\*) and that dissipation of turbulent kinetic energy as well as  
 110 chemical reactions only take place in fine structures. The volume fraction of  
 111 fine structure in each cell  $\gamma^*$  and the mean residence time of reactive mixture  
 112 within the fine structure  $\tau^*$  are obtained by:

$$113 \quad \gamma^* = \text{Re}_\Delta^{-3/4} \quad , \quad (9)$$

$$114 \quad \tau^* = 1.23 \sqrt{\frac{\Delta \mu}{\bar{\rho} (k^{sgs})^{1.5}}} \quad , \quad (10)$$

116 following Fureby's approach<sup>19</sup> based on energy cascade model, where  $\text{Re}_\Delta =$   
 117  $u' \Delta / \nu$  is the filtered scale Reynolds number. The fluctuating velocity is ap-  
 118 proximated by  $u' = \sqrt{2k^{sgs}/3}$ . The reaction term in  $m$ th species conservation  
 119 equation is described as:

$$120 \quad \frac{\bar{\rho}(Y_m^* - Y_m^0)}{\tau^*} = \dot{\omega}_m(\bar{\rho}, Y_m^*) \quad , \quad (11)$$

121  $Y_m^0$  and  $Y_m^*$  represent  $m$ th species mass fraction in the surroundings and fine  
 122 structures, As each filtered species mass fraction can be described as:

$$123 \quad \tilde{Y}_m = \gamma^* Y_m^* + (1 - \gamma^*) Y_m^0 \quad , \quad (12)$$

124 thus, the following equation can be derived:

$$125 \quad \bar{\rho}(Y_m^* - \tilde{Y}_m) = (1 - \gamma^*) \tau^* \dot{\omega}_m(\bar{\rho}, Y_m^*) \quad , \quad (13)$$

126 In the present study, the fine structure volume fraction  $\gamma^*$  is so small com-  
 127 pared to the cell size that it could be set to zero, implying that the reaction  
 128 takes place in a minimal region of each cell.

129 **3. Experimental and numerical setup**

130 The schematic diagram of the experimental combustor is shown in Fig.1.  
 131 The combustor consists of a 52-degree, eight spiral swirl-vaned air inlet with  
 132 a swirl number 0.9 and five column fuel injectors located downstream of the  
 133 vanes, followed by an annular chamber. Air and fuel are injected in opposite  
 134 directions and premixed inside a short passage before combustion takes place  
 in the chamber.

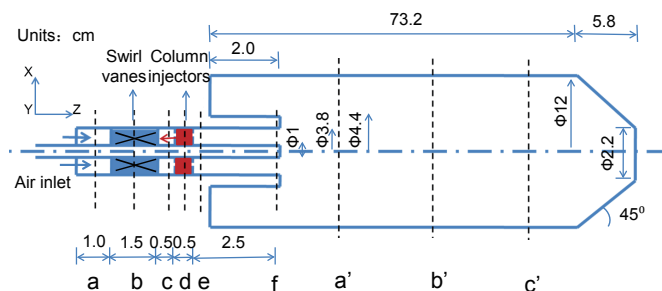


Figure 1: Schematic diagram of experimental combustor.

135

136 An empirical three-step reduced methane reaction mechanism is employed  
 137 in the present study<sup>20</sup>, which is obtained by a least-squares regression empirical  
 138 fitting method. Details are shown in Tab.1. Experiments conducted  
 139 by Griebel<sup>21</sup> are used to validate the reduced reaction mechanism. Comparative  
 140 results of the flame front and axial velocity field between the numerical  
 141 simulation conducted by FLUENT and experiment are shown in Fig.2. Good  
 142 agreement is obtained. Besides, comparison of the computational results of  
 143 the outlet temperature at different equivalence ratios between the two chemical  
 144 mechanisms using COSLAB with the laminar free-flame propagation  
 145 model indicates that the reduced mechanism shows good correlation with  
 146 the Berkeley's GRI-mech 3.0 methane chemical mechanism. These results  
 147 are shown in Fig.3, which demonstrates that the three-step reduced mechanism  
 148 can be applied with reasonable accuracy to the study of natural gas  
 combustion in our cases.

149 A five-step NO global mechanism for a lean premixed combustor is employed  
 150 to investigate NO<sub>x</sub> emissions under different operating conditions<sup>20</sup>.  
 151 The flame NO formation by Zeldovich as well as nitrous oxide mechanisms  
 152 are described in step 1, while step 2 shows the prompt and NNH mechanisms.  
 153 According to the analysis focusing on the relative importance of each step  
 154 on NO<sub>x</sub> formation using COSILAB conducted by Zheng<sup>17</sup>, step 1 and 2  
 155

156 contribute more than 50% of total  $\text{NO}_x$  emission, which means that reduc-  
 157 ing the reaction rates in steps 1 and 2 is crucial to control  $\text{NO}_x$  formation.  
 158 The thermal  $\text{NO}$  formation rate through H-atom and O-atom attack on  $\text{N}_2\text{O}$   
 159 mechanisms is shown in steps 3 and 4, respectively. A small portion of flame  
 160  $\text{NO}$  emission through the prompt and NNH mechanisms just downstream of the  
 161 flame is represented in step 5. Details are shown in Tab.2. Comparative  
 162 results of  $\text{NO}$  emissions between the numerical simulation using COSLAB  
 163 and experimental measurements<sup>21</sup> are shown in Fig.4, which demonstrate  
 that the reduced mechanism has reasonable accuracy.

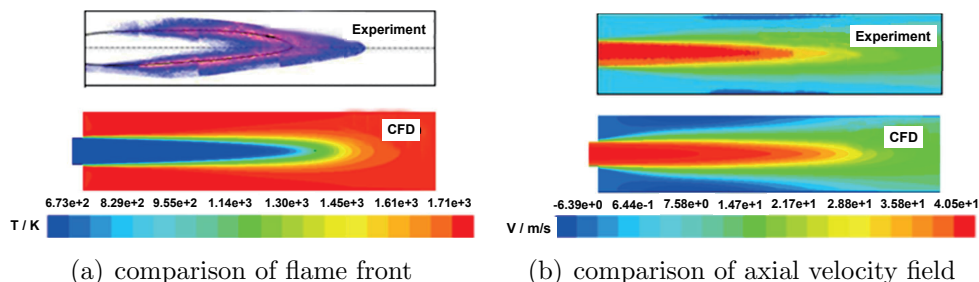


Figure 2: Comparative results of the flame front and axial velocity between the experiment and numerical simulation with reduced mechanism.

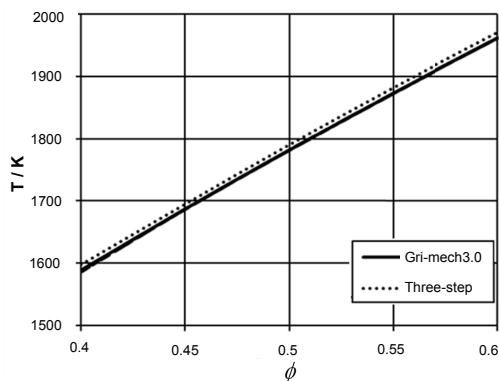


Figure 3: Comparison of the outlet temperature between three-step reduced methane mechanism and GRI-mech 3.0 reaction mechanism.

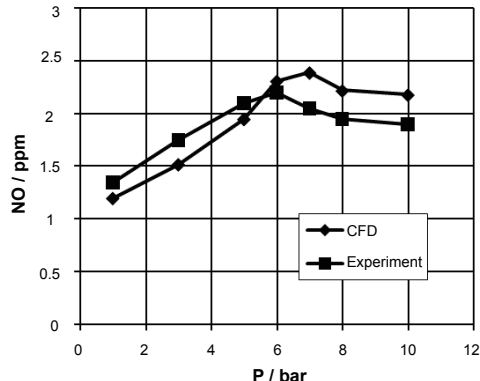


Figure 4: Comparative results of outlet  $\text{NO}$  emissions between experiment and numerical simulation with five-step  $\text{NO}$  global mechanism.

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Table 1: Three step global mechanism for methane

Step	Equation
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1	$\text{CH}_4 + 1.5\text{O}_2 \rightarrow \text{CO} + 2\text{H}_2\text{O}$
Reaction Rate	$10^{13.354-0.004628p} \exp(-(21932 + 269.4p)/T) \times [\text{CH}_4]^{1.3-0.01148p} [\text{O}_2]^{0.01426} [\text{CO}]^{0.1987}$
2	$\text{CO} + 0.5\text{O}_2 \rightarrow \text{CO}_2$
Reaction Rate	$10^{14.338+0.1091p} \exp(-(22398 + 75.1p)/T) \times [\text{CO}]^{1.359-0.0109p} [\text{H}_2\text{O}]^{0.0912+0.0909p} [\text{O}_2]^{0.891+0.0127p}$
3	$\text{CO}_2 \rightarrow \text{CO} + 0.5\text{O}_2$
Reaction Rate	$10^{15.814-0.0716p} \exp(-(64925.8 - 334.31p)/T) \times [\text{CO}_2]$

Table 2: Five step global mechanism for NO formation

Step	Equation	Mechanism
1	$\text{N}_2 + \text{O}_2 \Rightarrow 2\text{NO}$	Zeldovich and nitrous oxide mechanisms
Reaction Rate	$10^{14.122+0.0376p} \exp(-(46748 + 126.6p)/T) \times [\text{CO}]^{0.8888-0.0006p} [\text{O}_2]^{1.1805+0.0344p}$	
2	$\text{N}_2 + \text{O}_2 \Rightarrow 2\text{NO}$	prompt and NNH mechanisms
Reaction Rate	$10^{29.8327-4.7822 \log p} \exp(-(61265 + 704.7p)/T) \times [\text{CO}]^{2.7911-0.0488p} [\text{O}_2]^{2.4613}$	
3	$\text{N}_2 + \text{O}_2 \Rightarrow 2\text{NO}$	H-atom attack on N2O mechanism
Reaction Rate	$10^{14.592} \exp(-69158/T) [\text{N}_2][\text{H}_2\text{O}]^{0.5} \times [\text{O}_2]^{0.25} T^{-0.7}$	
4	$\text{N}_2 + \text{O}_2 \Rightarrow 2\text{NO}$	O-atom attack on N2O mechanism
Reaction Rate	$10^{10.317} \exp(-52861/T) \times [\text{N}_2][\text{O}_2]$	
5	$\text{N}_2 + \text{O}_2 \Rightarrow 2\text{NO}$	prompt and NNH mechanisms
Reaction Rate	$10^{14.967} \exp(-68899/T) \times [\text{N}_2][\text{O}_2]^{0.5} T^{-0.5}$	

165 The computational fluid dynamics modelling program is based on KIVA-4  
166 code, in which the Arbitrary Lagrangian Eulerian (ALE) numerical scheme is  
167 employed based on the finite volume method. Each cycle is divided into three  
168 phases. In phases 1 and 2, the grid vertices move with the fluid velocity, and  
169 convection is absent across cell boundaries; the Lagrangian calculation is then  
170 performed. Mass and energy source terms resulting from the chemical reac-  
171 tion are computed in phase 1, while diffusion terms and other source terms  
172 in the governing equations are obtained in phase 2 using a modified SIMPLE  
173 (Semi-Implicit Method for Pressure-Linked Equation) method. In phase 3,  
174 the flow field is rezoned onto the new computational mesh and then frozen,  
175 and convection terms are calculated by means of a second-order accuracy  
176 quasi-second-order upwind scheme based on an explicit method. Courant  
177 stability restrictions are absent as Lagrangian computation is largely implicit  
178 and Eulerian calculation can be subcycled. According to code validation and  
179 grid dependence test conducted by Zheng<sup>17</sup>, 1.3 million cells in an unstruc-  
180 tured mesh with a grid resolution of 0.1 cm is applied in the present work.  
181 Velocity-inlet and outflow conditions are used along with no-slip, adiabatic  
182 walls. The platform with Intel Nehalem 2.26GHz CPU in Lancaster Univer-  
183 sity and the HPC platform with Intel Xeon 5670 in Tsinghua University are  
184 used to complete the computation.

## 185 4. Results and discussion

### 186 4.1. Non-reacting cases

187 Contour plots of the vorticity field corresponding to different cross-sections  
188 from axial locations  $a$  to  $f$  marked in Fig.1 are shown in Fig.5. A confined  
189 high vorticity area is observed in Fig.5(a) close to the walls due to strong  
190 shear stresses caused by the velocity difference between the walls and the  
191 ambient fluid at the air inlet. As the flow and vortices shift downstream and  
192 pass through the swirl vanes, the shear and stretch rates increase rapidly  
193 due to swirl effects, which contribute to more frequent vortex production  
194 and breakdown as well as vortex shedding. The vorticity field corresponding  
195 to the swirler location is shown in Fig.5(b). Counter flow between the swirling  
196 air and injected fuel make the shear stress increase further, and as a result,  
197 more widely intense vortex distributions are observed in Fig.5(c). However,  
198 high vorticity zones deviate from the central body walls, which indicate the  
199 centrifugal force effect induced by swirl flow. Because the installation angle  
200 of the column fuel injectors are different from those of the swirl vanes, the  
201 position discrepancy effect will collaborate with the swirl effect to enhance  
202 fuel/air mixing and result in an asymmetrical distribution of vorticity. This  
203 asymmetry is shown in Fig.5(d) and becomes distinct in Fig.5(e). When the  
204 fuel/air mixture moves down to the dump plane, also referred as location  
205  $f$ , the asymmetrical distribution of high vorticity has nearly vanished, and  
206 the deviation distance is further increased, which can be speculated from  
207 Fig.5(f).

208 Contour plots of the vorticity field and the corresponding streamlines at  
209 different axial locations  $a'$  to  $c'$  are shown in Fig.6. As the mixture moves  
210 into the chamber, the flow scale expands suddenly, and the vorticity magni-  
211 tude decreases due to the vortices extending according to the conservation  
212 of angular momentum. Small portion of high vorticity distribution can be  
213 observed in Fig.6(a) at axial location  $a'$ , but is less evident in Fig.6(b) and  
214 Fig.6(c). The damped tangential velocity leads to the radial pressure gradi-  
215 ent under the effects of centrifugal force. A low-pressure core emerges and  
216 aligns with the axial axis at a downstream location next to the dump plane,  
217 which results in a concentrated distribution of spanwise vortices around the  
218 symmetry axis, which can be easily seen in Fig.6(d) and Fig.6(e). However,  
219 from the streamwise of location  $c'$  shown in Fig.6(f), the core is propelled  
220 away from the centreline and expands downstream helicoidally due to vor-  
221 tex core precession; spiral vortices breakdown gives rise to more stagnation

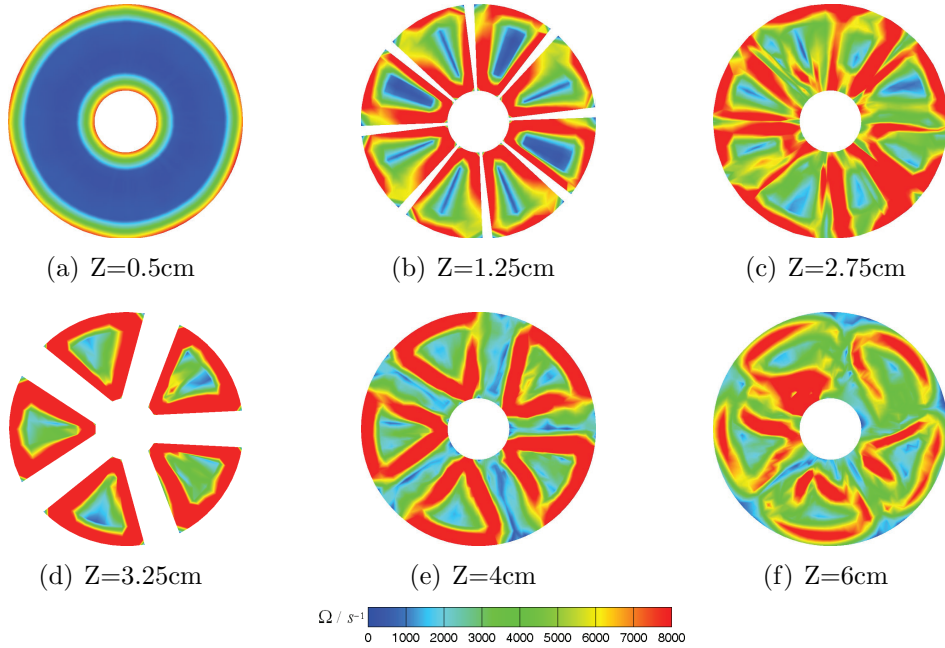


Figure 5: Contour plots of vorticity field corresponding to different axial locations.

222 points and vortices numbers with different sizes, which is consistent with  
 223 the conclusions of Wang et al.<sup>22</sup>. As the frequency of vortex core precession  
 224 increases with increased Reynolds number, many vortices with high vorticity  
 225 next to the combustor walls are observed with high Reynolds number,  
 226 leading to increased tangential velocity due to the squeezing effects against  
 227 the wall. This can be inferred from Fig.7(c), where high velocity magnitudes  
 228 appear around the chamber wall compared to Fig.7(a) and Fig.7(b).

#### 229 4.2. Reacting cases

230 The following section investigates the influence of swirl number, Reynolds  
 231 number, equivalence ratio and nitrogen dilution on combustion dynamics and  
 232 NOx emissions under the standard operating condition of  $Re = 6.3 \times 10^4$ ,  
 233  $\varphi = 0.65$ ,  $p = 3bar$  and fuel composition given by  $CH_4 : N_2 = 4 : 1$ .

##### 234 4.2.1. Swirl number effects

235 In gas turbine engines, the bubble and helix modes are the most popular  
 236 type of vortex breakdown due to Kelvin-Helmholtz (K-H) instability in the  
 237 axial and azimuthal shear layers. Helix mode usually takes place on the outer

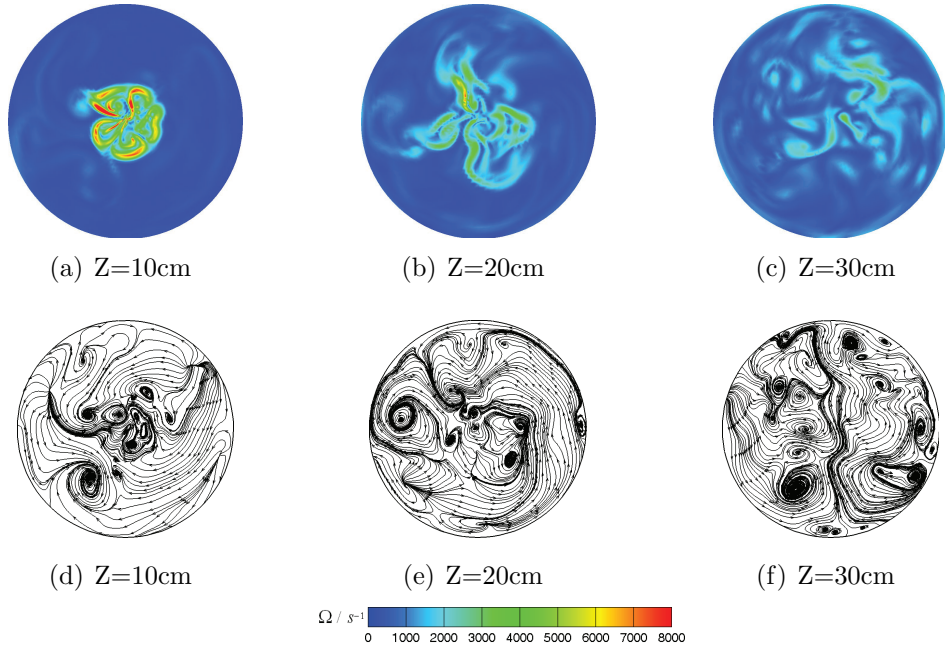


Figure 6: Contour plots of vorticity field and corresponding streamlines at different axial locations.

238 edge of the shear layers and behaves like large scale spiral vortices rotating in  
 239 the combustor and then crushing into several fine scale vortices; bubble mode  
 240 often occurs on the edge of shear layers and behaves as small scale vortices  
 241 shed along the axial direction. The iso-surface of vorticity ( $\Omega = 5000$ ) at  
 242 locations of  $r < 0.01m$  and  $r > 0.01m$  with different swirl numbers are  
 243 shown in Fig.8, where  $r$  is the approximate location of the shear layer. More  
 244 fine-scale vortices shed from the large-scale vortices and the initial breakdown  
 245 position moves upstream with increased swirl number, which can be inferred  
 246 from Fig.8(a) and Fig.8(b). Obvious small-scale spiral vortices breakdown  
 247 can be observed in Fig.8(c). However, as the swirl number increases to 1.2,  
 248 as shown in Fig.8(d), such phenomena become inconspicuous, which can be  
 249 interpreted as the disturbance arising from bubble-type vortex breakdown  
 250 developing upstream to the flame root and weakening the helix instability  
 251 with an increased swirl number.

252 To analyse the flame regimes in a partially premixed combustion process,

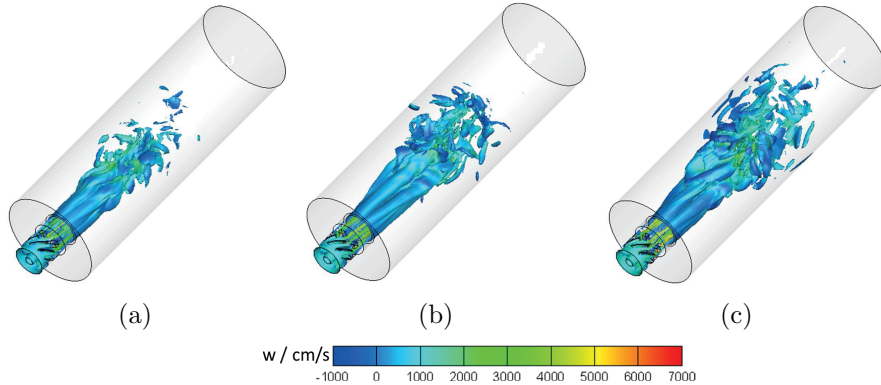


Figure 7: Iso-surfaces of vorticity field flooded with axial velocity with different Re numbers. (a)  $Re = 5.3 \times 10^4$ ; (b)  $Re = 6.3 \times 10^4$ ; (c)  $Re = 7.4 \times 10^4$ .

253 a modified flame index is applied as following definition:

$$254 \quad MFI = \frac{\nabla Y_{\text{fuel}} \cdot \nabla Y_{\text{O}_2}}{|\nabla Y_{\text{fuel}} \cdot \nabla Y_{\text{O}_2}|}, \quad (14)$$

255 where  $Y_{\text{fuel}}$  and  $Y_{\text{O}_2}$  represent the fuel and oxidizer mass fractions, respec-  
 256 tively. When  $MFI > 0$ , the combustion is occurring in a premixed regime;  
 257 the diffusion regime is confirmed when  $MFI < 0$ . The flame index distri-  
 258 butions with different swirl numbers are shown in Fig.9. where the flame  
 259 is characterized by an iso-surface with  $T = 1800\text{K}$ . The entire flame struc-  
 260 ture is shown in the centre of the figure, while the diffusion and premixed  
 261 flames are exhibited at left and right positions, respectively. Proportion of  
 262 premixed combustion increases with higher swirl numbers relating to intense  
 263 bubble vortex breakdown and entrainment effects in the recirculation zones.  
 264 The premixed region is also shown to move upstream when the swirl number  
 265 increases further, which can be explained by improved mixing between the  
 266 fuel and oxidizer ahead of the combustion taking place in the upstream re-  
 267 gion. Here, the unmixedness parameter<sup>23</sup> is introduced to identify the effect  
 268 of swirl number on the mixing process. The parameter is defined as:

$$269 \quad U = Y_f'' / \overline{Y_f} (1 - \overline{Y_f}), \quad (15)$$

270 where  $\overline{Y_f}$  and  $Y_f''$  represent the time-averaged fuel mass fraction and its fluc-  
 271 tuation over the mean value, respectively. A near zero value of  $U$  indicates  
 272 better mixing has been achieved. By collecting valid data along the flow



273 field, a probability density function (PDF) of  $U$  is obtained:

$$274 \quad P(U) = \sum_{i=1}^N [\text{H}(U_i - U) - \text{H}(U_i - U - \Delta U)] / N \Delta U, \quad (16)$$

275 where  $H$  is Heaviside function,  $\Delta U$  represents the selected interval of un-  
 276 mixedness,  $N = U / \Delta U$ . The PDF of  $U$  at a position downstream of the  
 277 swirl vanes with different swirl number is shown in Fig.10. The narrower  
 278 distribution of the PDF near zero with an increased swirl number indicates  
 279 better mixing between the fuel and oxidizers, which results in an increased  
 280 ratio of premixed combustion along with a reduction of temperature and  
 $\text{NO}_x$  emissions.

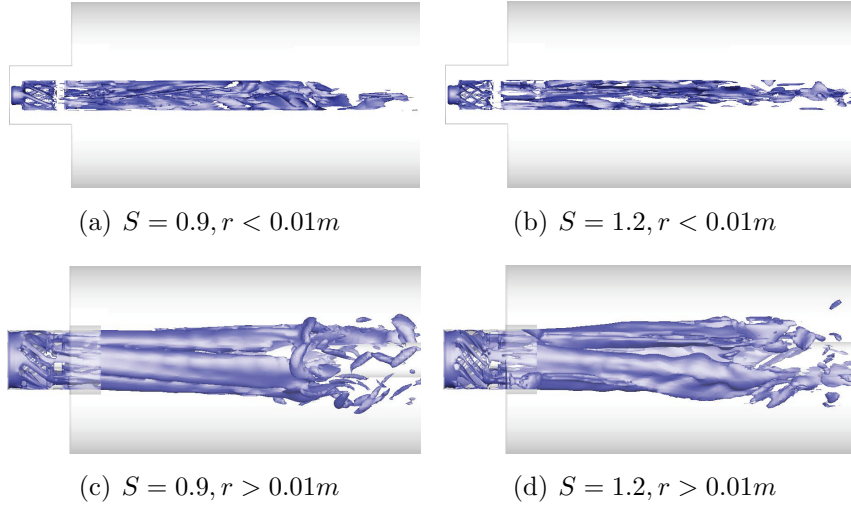


Figure 8: Iso-surfaces of vorticity ( $\Omega = 5000$ ) with different swirl numbers,  $r$  is the approximate location of the shear layer.

281

282 In order to quantitatively analyse the partially premixed combustion,  
 283 the proportion of premixed combustion (PPC) is defined as:

$$284 \quad \text{PPC} = \frac{\int_{\text{MFI} > 0} dV}{\int_{\text{MFI} \neq 0} dV}, \quad (17)$$

285 Fig.11 shows the proportion of premixed combustion and  $\text{NO}_x$  emissions with  
 286 different swirl numbers. The proportion of premixed regime only accounts

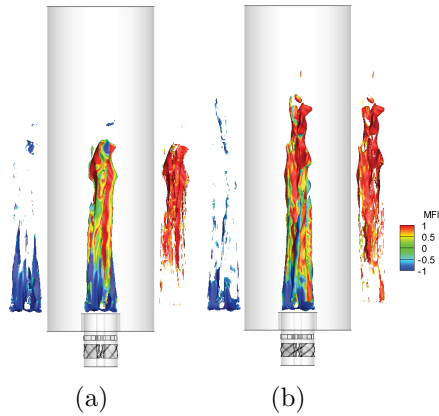


Figure 9: Modified flame index distribution with different swirl numbers.(a) $S = 0.9$ ;(b) $S = 1.2$ .

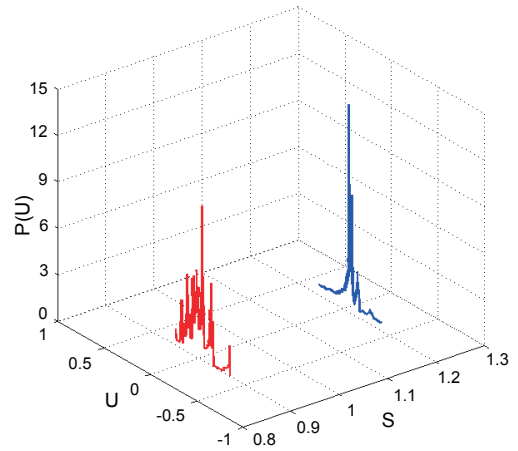


Figure 10: PDF distribution of unmixedness with different swirl numbers(axial location c).

287 for 20% at most during the partially premixed combustion, which indicates  
 288 that partially premixed combustion occurring more like diffusion flame at  
 289 lean intense swirling conditions. This is consistent with the conclusion of  
 290 Meier<sup>7</sup>. PPC increases when the swirl number increases from 0.9 to 1.2 due  
 291 to better mixing between the fuel and oxidizers. In lean premixed combustion,  
 292 NO formation reaction rates of step 1 and 2 reduce significantly; this  
 293 normally results in low NO<sub>x</sub> emissions. However, the reduction degree of  
 294 NO<sub>x</sub> formation is not so distinct compared to the increase of the swirl number,  
 295 which can be interpreted as a large swirl number contributing to better  
 296 mixing between the fuel and oxidizers while also leading to a long retention  
 297 time of reaction products arising from intense entrainment, which facilitates  
 298 the formation of NO. Thus, an appropriate swirl number is critical for gas  
 299 turbine chamber design to achieve superior mixing and minimal pollution  
 300 emissions.

301 For the purpose of studying the frequency and amplitude of instability  
 302 during combustion with the Fourier spectra analysis method, several monitoring  
 303 points are set along the flow field to record the time variation of the thermal  
 304 variables. A typical point located in the shear layer at midstream  
 305  $Z=15$  cm is chosen for the sake of capturing both shear layer instability and  
 306 vortex breakdown instability. A fast Fourier transform (FFT) of the temperature  
 307 trace for different swirl numbers is shown in Fig.12, where the dominant  
 308 frequency indicates shear layer instability and harmonic waves indicate vor-

309 tex breakdown instability. The swirl number does not show much influence  
 310 on the dominant frequency, and all flames' temperature experience a dom-  
 311 inant frequency of approximately 995 Hz, as derived from the shear layer  
 312 instability. However, the amplitude corresponding to the main frequency is  
 313 shown to increase with increasing swirl number, which can be explained as  
 314 an intense velocity fluctuation with increased swirl number. More harmonic  
 315 waves also show that vortex breakdown instability appears when the swirl  
 number increases.

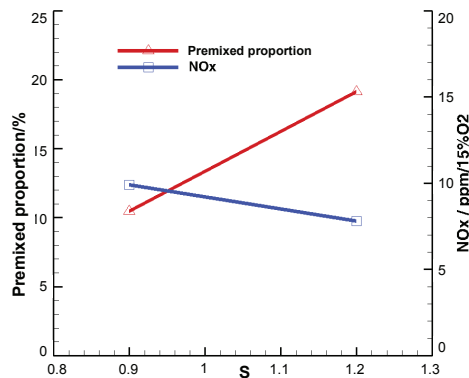


Figure 11: Proportion of premixed combustion and NO<sub>x</sub> emission with different swirl numbers.

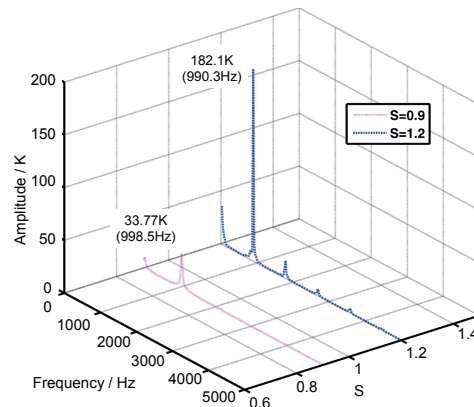


Figure 12: FFT analysis of temperature with different swirl numbers.

316

#### 317 4.2.2. Reynolds number effects

318 The CRZs characterized by the dotted lines with different Reynolds numbers in the reacting cases are shown in Fig.13. The size of the CRZ is shown  
 319 to increase due to larger pressure drop induced by increased axial momen-  
 320 tum, and the vortex structures become more symmetrical with increased  
 321 Reynolds numbers; this is consistent with the observation in non-reacting  
 322 cases. However, the size increases beyond that of corresponding cases with-  
 323 out reactions under the same Reynolds number due to thermal expansion  
 324 effects arising from the chemical reaction. A larger low-speed region appears  
 325 at larger Reynolds number, which improves flame stability during partially  
 326 premixed combustion.  
 327

328 From the analysis of non-reacting cases, high Reynolds numbers con-  
 329 tribute to more frequent vortex breakdown and consequently better degrees  
 330 of mixing between the fuel and oxidizers. Fig.14 shows the PDF distribution

331 of the unmixedness parameter just downstream of the swirl vanes with dif-  
 332 ferent Reynolds numbers. The better degree of mixing between the fuel and  
 333 oxidizers can be concluded from the narrower distribution of the PDF near  
 334 zero with increased Reynolds number. Due to the better mixing between  
 335 the fuel and oxidizers, the length of the diffusion flame is decreased while  
 336 the premixed flame is slightly increased with increased Reynolds numbers;  
 337 this can be seen in the modified flame index distribution shown in Fig.15,  
 338 where the flame surface is still characterized by an iso-surface of temper-  
 339 ature  $T = 1800\text{K}$ . Moreover, being different from the effect of the swirl  
 340 number on  $\text{NO}_x$  formation, large Reynolds numbers shorten the retention  
 341 time of hot reaction products, which indicates that  $\text{NO}_x$  emissions can be  
 342 suppressed effectively. As the frequency and amplitude of the shear layer  
 343 and vortex breakdown instabilities increase with Reynolds number, the en-  
 344 tire flame becomes more wrinkled, and necking effects can be seen with the  
 largest Reynolds number case shown in Fig.15(c).

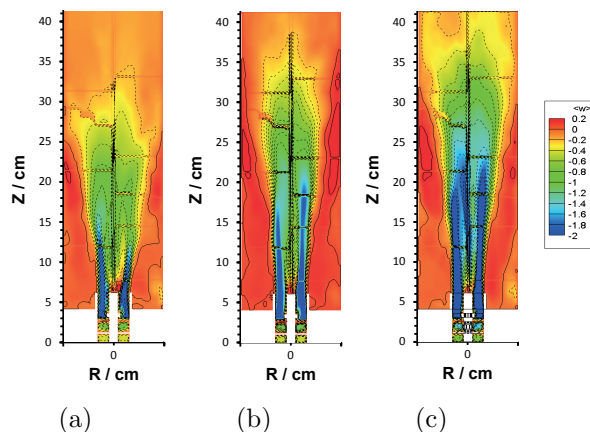


Figure 13: The CRZs characterized by the dotted lines with different Re numbers.(a) $\text{Re} = 5.3 \times 10^4$ ;(b) $\text{Re} = 6.3 \times 10^4$ ;(c) $\text{Re} = 7.4 \times 10^4$ .

345

#### 346 4.2.3. Equivalence ratio effects

347 Lean premixed combustion appears to be a promising technology for  
 348 pollution reduction in gas turbine engines. In the present study, lean pre-  
 349 mixed combustion is achieved by adjusting the inlet flow rate. The contour  
 350 plots of the time-averaged temperature with different equivalence ratios at  
 351  $\text{Re} = 6.3 \times 10^4$ ,  $p = 3\text{bar}$  and composition given by  $\text{CH}_4 : \text{N}_2 = 4 : 1$  are

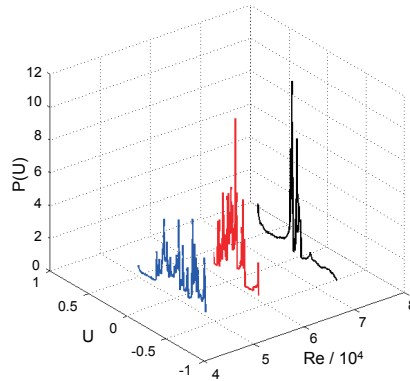


Figure 14: PDF distribution of unmixedness with different Re numbers(axial location c).

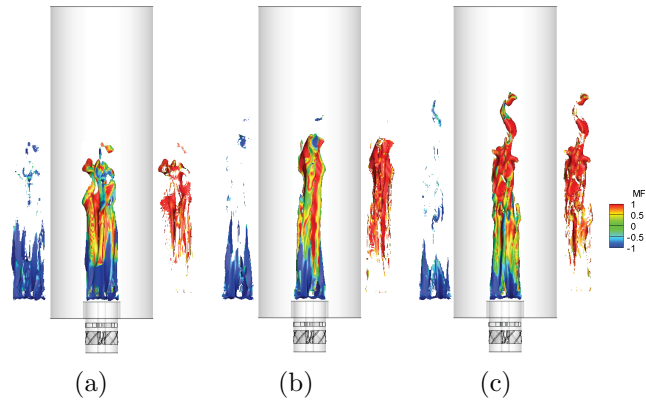


Figure 15: Modified flame index distribution with different Re numbers.(a) $Re = 5.3 \times 10^4$ ;(b) $Re = 6.3 \times 10^4$ ;(c) $Re = 7.4 \times 10^4$ .

352 shown in Fig.16. When the equivalence ratio approaches the lean flammability  
 353 of 0.45 for methane combustion, the high temperature region and flame  
 354 length dramatically decrease due to the chemical reaction of fuel and excess  
 355 air, which is crucial for emission reductions of thermal  $NO_x$ . However, lean  
 356 combustion tends to motivate combustion instabilities, such as flame flash-  
 357 back or blow off. From the instantaneous temperature distribution along  
 358 the centreline shown in Fig.17, large fluctuations of temperature emerge at  
 359 an equivalence ratio of 0.5 near the flame root at approximately  $Z=10$ cm  
 360 compared to other cases. A decrease of equivalence ratio is consistent with  
 361 a decrease in the velocity of fuel injection, and due to the counter flow di-  
 362 rections of the fuel and swirl air, the shear stress and concentration gradient

363 is also decreased. Mixing is thus reduced compared to high ratio cases. Pre-  
364 mixed combustion proportions during partially premixed combustion process  
365 significantly reduce when the equivalence ratio approaches the lean flamma-  
366 bility of 0.45 for methane combustion. When the mixture flows into the  
367 chamber and ignites, many local positions are still at the equivalence ratio  
368 below the lean flammability or the combustion reaction does not take place.  
369 Such a phenomenon leads to temperature nonuniformity near the flame root.

370 Fig.18 shows the proportion of premixed combustion and  $\text{NO}_x$  emissions  
371 at different equivalence ratios. According to the above analysis, a smaller  
372 equivalence ratio lowers the proportion of premixed combustion but also re-  
373 duces  $\text{NO}_x$  formation. Fig.19 shows the iso-surfaces of NO mass fractions at  
374 different equivalence ratios, in which active regions of NO formation move up-  
375 stream with decreased equivalence ratios, which give rise to shorter reaction  
376 zones and relatively small high temperature flame surfaces. Consequently,  
377 thermal  $\text{NO}_x$  reduces significantly. The iso-surface of  $Y_{\text{NO}} = 1 \times 10^{-6}$  shown  
378 in Fig.19(c) has also entered into the vortex breakdown region downstream  
379 of the chamber, and intense turbulent pulsation along with stirring effects  
380 will enhance  $\text{NO}_x$  formation.

381 The FFT of the flame temperature trace at different equivalence ratios is  
382 shown in Fig.20. The main frequency and amplitude of the shear layer insta-  
383 bility vary unapparently when the equivalence ratio reduces from 0.8 to 0.65;  
384 harmonic waves corresponding to vortex breakdown instability also shows  
385 the same tendency. However, as the ratio approaches the lean flammability,  
386 the main frequency and amplitude of the shear layer instability increase sud-  
387 denly, and the fluctuation of temperature increases from 33.45 K to 80.01  
388 K. Many harmonic waves appear around the main frequency, indicating the  
389 complexity of the flame instability at lean ratio conditions.

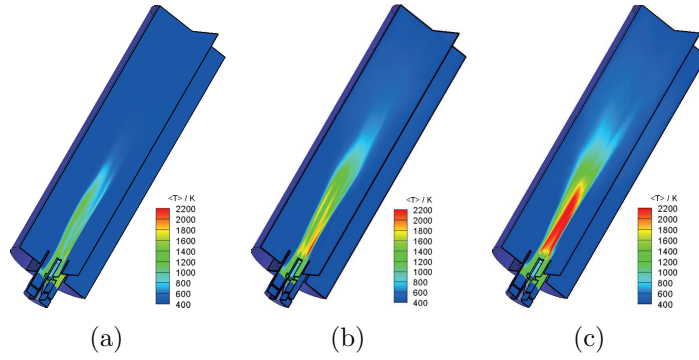


Figure 16: Contour plots of time-averaged temperature at different equivalence ratios. (a)  $\phi = 0.5$ ; (b)  $\phi = 0.65$ ; (c)  $\phi = 0.8$ .

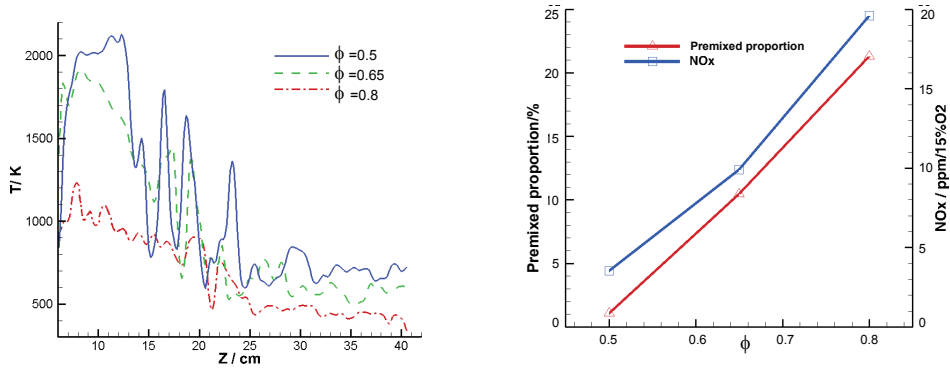


Figure 17: Instantaneous temperature along the centerline at different equivalence ratios. Figure 18: Proportion of premixed combustion and NO<sub>x</sub> emissions with different equivalence ratios.

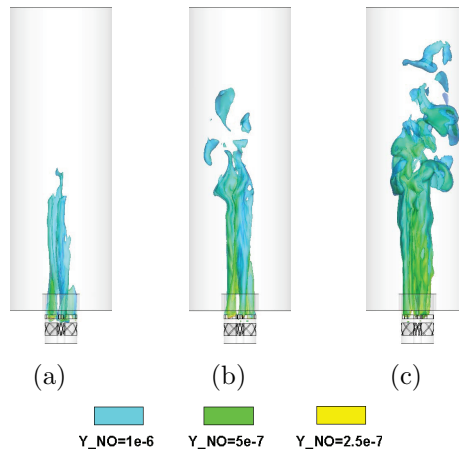


Figure 19: Iso-surface of NO mass fraction with different equivalence ratios. (a)  $\phi = 0.5$ ; (b)  $\phi = 0.65$ ; (c)  $\phi = 0.8$ .

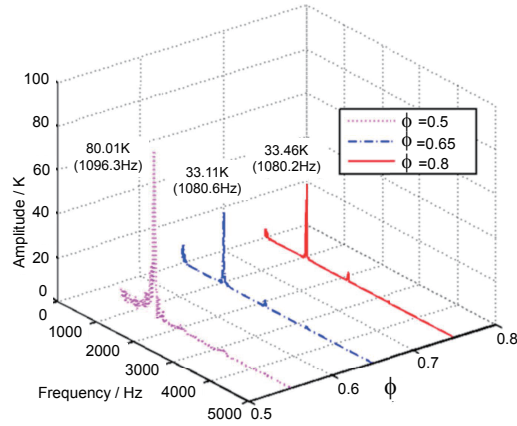


Figure 20: FFT analysis of temperature at different equivalence ratios.

390 *4.2.4. Nitrogen dilution effect*

391 During gas turbine combustion processes, inert gas is usually applied to  
 392 dilute the natural gas to reduce pollution emissions. In the present work,  
 393 nitrogen gas is used to form three mixed gases ( $\text{CH}_4 : \text{N}_2 = 1 : 0, 4 : 1, 2 : 1$ ),  
 394 which are then examined to analyse the effect of nitrogen dilution on combus-  
 395 tion dynamics. The time-averaged temperature and flame length are reduced  
 396 with the increased proportion of  $\text{N}_2$ , as shown in Fig.21. The flame temper-  
 397 ature is decreased proportionally upstream of combustor with an increased  
 398 nitrogen volume fraction and rather than a uniform temperature distribu-  
 399 tion being obtained downstream; this indicates that nitrogen dilution can  
 400 dramatically reduce thermal  $\text{NO}_x$  emissions.

401 Due to the effects of  $\text{N}_2$  dilution, heat release is decreased compared to  
 402 pure methane gas and consequently the central recirculation zone is reduced  
 403 due to lower thermal expansion. The high molecular weight of  $\text{N}_2$  will tend  
 404 to weaken the swirl flow and azimuthal momentum, which results in a lower  
 405 frequency of vortex breakdown. Such phenomena can be inferred from Fig.22,  
 406 which shows fewer spanwise vortices. Because poor mixing is achieved with  
 407 an increased  $\text{N}_2$  percentage, the premixed combustion regime is decreased  
 408 during the operating process, but lower thermal  $\text{NO}_x$  emissions are obtained  
 409 due to a lower flame temperature and smaller reaction zones. The proportion  
 410 of the premixed combustion regime and thermal  $\text{NO}_x$  emissions with different  
 411  $\text{N}_2$  content is shown in Fig.23.

412 The FFT of the flame temperature trace with different nitrogen contents  
 413 is shown in Fig.24. The fluctuation of temperature decreases from 33.11 K



414 to 16.41 K with increased  $N_2$  content, which can be explained by the weak-  
 415 ened effect of swirl movement caused by the larger molecular weight of  $N_2$ .  
 416 However, the amplitude increases to 62.57 K when the  $N_2$  content content  
 417 is further increased, and the dilution effects make the mixture approach the  
 418 lean flammability of methane combustion. As a result, combustion insta-  
 419 bility arises and becomes more severe when the amount of inert gas further  
 420 increases.

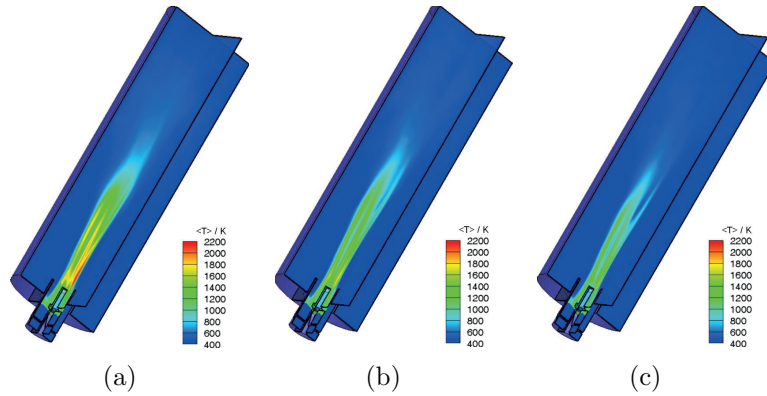


Figure 21: Contour plots of time-averaged temperature with different  $N_2$  contents.(a) $CH_4 : N_2 = 1 : 0$ ; (b) $CH_4 : N_2 = 4 : 1$ ; (c) $CH_4 : N_2 = 2 : 1$ .

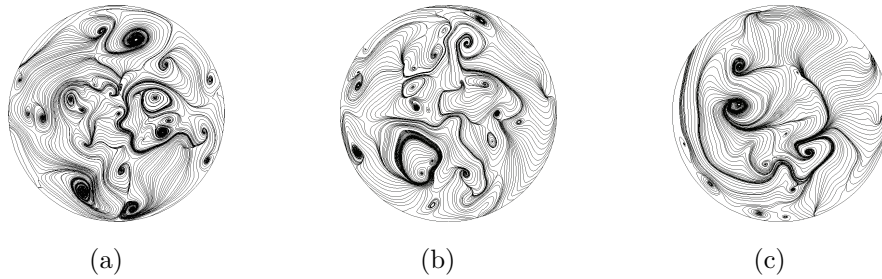


Figure 22: Spanwise vortices structure with different  $N_2$  contents.(a) $CH_4 : N_2 = 1 : 0$ ; (b) $CH_4 : N_2 = 4 : 1$ ; (c) $CH_4 : N_2 = 2 : 1$ .

## 421 5. Conclusion

422 An experimental lean partially premixed swirling combustor is studied  
 423 using the large-eddy simulation method for both non-reacting and reacting

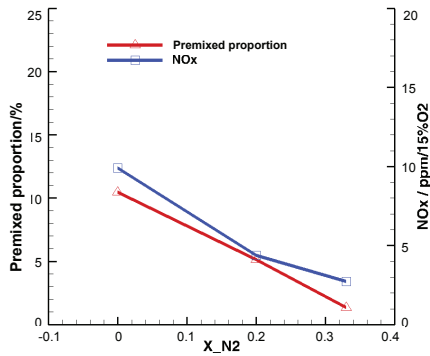


Figure 23: The proportion of premixed combustion regime and thermal  $\text{NO}_x$  emissions with different  $\text{N}_2$  contents.

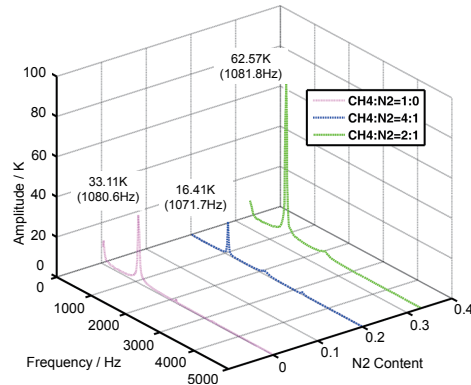


Figure 24: FFT analysis of temperature with different  $\text{N}_2$  contents.

424 cases under different operating conditions. In the non-reacting cases, the  
 425 intrinsic effects of swirl vanes and counter flow on flow dynamics are investi-  
 426 gated by analysing the vorticity magnitude and streamlines at different axial  
 427 positions. The influence of Reynolds number on vortex structures and vortex  
 428 breakdown shows that a high Reynolds number contributes to more frequent  
 429 vortex breakdown and the appearance of high vorticity near the combustor  
 430 walls.

431 In the reacting cases, the influence of swirl number, Reynolds number,  
 432 equivalence ratio and nitrogen dilution on combustion dynamics and  $\text{NO}_x$   
 433 emissions are examined. The conversion between bubble- and helix-type  
 434 vortex breakdown is shown to be directly related to the swirl number. The  
 435 proportion of the premixed flame regime is shown to increase with swirl num-  
 436 ber and with shear layer instability and vortex breakdown instability. The  
 437 length of the diffusion flame is shown to decrease while the premixed flame is  
 438 slightly increased with increased Reynolds number. As the equivalence ratio  
 439 approaches the lean flammability, the main frequency and amplitude of the  
 440 shear layer instability increased suddenly, and the fluctuation of temperature  
 441 increases from 33.45 K to 80.01 K. Many harmonic waves are also shown to  
 442 appear near the main frequency. Flame instability at a lean ratio is very  
 443 complex. The flame lengths and heat release are shown to decrease with  
 444 increased  $\text{N}_2$  content. Due to the larger molecular weight of  $\text{N}_2$ , additional  
 445 inert gas tends to restrain the swirl flow, which decreases the size of recircu-  
 446 lation zones and the frequency of vortex breakdown. The proportion of the  
 447 premixed flame regime and  $\text{NO}_x$  emissions are also reduced with higher  $\text{N}_2$

448 content. But the combustion instability becomes severe with the increased  
449 amount of inert gas.

450 As the future design of gas turbine combustor is driven by higher power-  
451 densities, lower NO<sub>x</sub> emissions, and excellent combustion stability, lean pre-  
452 mixed (LP) combustion can prove to be a promising technology for reducing  
453 pollutant emissions and improving fuel efficiency. The conclusions obtained  
454 for flame dynamics and instability mechanism under lean partially premixed  
455 combustion regime with intense swirl flow would help to provide essential in-  
456 formation for the design of gas turbine combustor systems, as most practical  
457 combustors operate with lean partially premixed mode of combustion.

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- 463 [1] A. K. Gupta, D. G. Lilley, N. Syred, Swirl flows, Tunbridge Wells, Kent,  
464 England, Abacus Press, 1984, 488 p. 1.
- 465 [2] Y. Huang, V. Yang, Dynamics and stability of lean-premixed swirl-  
466 stabilized combustion, Progress in Energy and Combustion Science  
467 35 (4) (2009) 293–364.
- 468 [3] Y. Huang, V. Yang, Effect of swirl on combustion dynamics in a lean-  
469 premixed swirl-stabilized combustor, Proceedings of the Combustion In-  
470 stitute 30 (2) (2005) 1775–1782.
- 471 [4] E. Fernandes, M. Heitor, S. Shtork, An analysis of unsteady highly  
472 turbulent swirling flow in a model vortex combustor, Experiments in  
473 Fluids 40 (2) (2006) 177–187.
- 474 [5] Y. Sommerer, D. Galley, T. Poinso, S. Ducruix, F. Lacas, D. Veynante,  
475 et al., Large eddy simulation and experimental study of flashback and  
476 blow-off in a lean partially premixed swirled burner, Journal of Turbu-  
477 lence 5 (37) (2004) 1–3.
- 478 [6] P. Weigand, W. Meier, X. Duan, W. Stricker, M. Aigner, Investigations  
479 of swirl flames in a gas turbine model combustor: I. flow field, structures,

- 480 temperature, and species distributions, *Combustion and flame* 144 (1)  
481 (2006) 205–224.
- 482 [7] W. Meier, X. Duan, P. Weigand, Reaction zone structures and mixing  
483 characteristics of partially premixed swirling ch<sub>4</sub>/air flames in a gas  
484 turbine model combustor, *Proceedings of the combustion Institute* 30 (1)  
485 (2005) 835–842.
- 486 [8] Y. Huang, V. Yang, Bifurcation of flame structure in a lean-premixed  
487 swirl-stabilized combustor: transition from stable to unstable flame,  
488 *Combustion and Flame* 136 (3) (2004) 383–389.
- 489 [9] Y. Huang, H.-G. Sung, S.-Y. Hsieh, V. Yang, Large-eddy simulation  
490 of combustion dynamics of lean-premixed swirl-stabilized combustor,  
491 *Journal of Propulsion and Power* 19 (5) (2003) 782–794.
- 492 [10] S. Prakash, S. Nair, T. Muruganandam, Y. Neumeier, T. Lieuwen, J. M.  
493 Seitzman, B. T. Zinn, Acoustic based rapid blowout mitigation in a swirl  
494 stabilized combustor, in: *ASME Turbo Expo 2005: Power for Land, Sea,  
495 and Air*, American Society of Mechanical Engineers, 2005, pp. 443–451.
- 496 [11] S. Menon, W. Kim, C. Stone, B. Sekar, Large-eddy simulation of fuel-  
497 air mixing and chemical reactions in swirling flow combustor, *AIAA* 99  
498 3440.
- 499 [12] J. Schluter, Static control of combustion oscillations by coaxial flows: a  
500 large-eddy-simulations investigation, *Journal of Propulsion and Power*  
501 20 (3) (2004) 460–467.
- 502 [13] F. Grinstein, C. Fureby, Les studies of the flow in a swirl gas combustor,  
503 *Proceedings of the combustion institute* 30 (2) (2005) 1791–1798.
- 504 [14] C. Duwig, L. Fuchs, Study of flame stabilization in a swirling combustor  
505 using a new flamelet formulation, *Combustion science and technology*  
506 177 (8) (2005) 1485–1510.
- 507 [15] S. Menon, P.-K. Yeung, W.-W. Kim, Effect of subgrid models on the  
508 computed interscale energy transfer in isotropic turbulence, *Computers  
509 & fluids* 25 (2) (1996) 165–180.

- 510 [16] W. H. Calhoun Jr, On subgrid combustion modeling for large-eddy sim-  
511 ulations, Ph.D. thesis, Georgia Institute of Technology (1996).
- 512 [17] Y. Zheng, M. Zhu, D. M. Martinez, X. Jiang, Large-eddy simulation of  
513 mixing and combustion in a premixed swirling combustor with synthesis  
514 gases, *Computers & Fluids* 88 (2013) 702–714.
- 515 [18] B. F. Magnussen, The Eddy Dissipation Concept for Turbulent Com-  
516 bustion Modelling: Its Physical and Practical Implications, SINTEF,  
517 1990.
- 518 [19] C. Fureby, A comparative study of flamelet and finite rate chemistry les  
519 for a swirl stabilized flame, *Journal of Engineering for Gas Turbines and*  
520 *Power* 134 (4) (2012) 041503.
- 521 [20] I. V. Novosselov, P. C. Malte, Development and application of an eight-  
522 step global mechanism for cfd and crn simulations of lean-premixed com-  
523 bustors, *Journal of Engineering for Gas Turbines and Power* 130 (2)  
524 (2008) 021502.
- 525 [21] P. Griebel, P. Siewert, P. Jansohn, Flame characteristics of turbulent  
526 lean premixed methane/air flames at high pressure: Turbulent flame  
527 speed and flame brush thickness, *Proceedings of the Combustion Insti-  
528 tute* 31 (2) (2007) 3083–3090.
- 529 [22] S. Wang, V. Yang, G. Hsiao, S.-Y. Hsieh, H. C. Mongia, Large-eddy  
530 simulations of gas-turbine swirl injector flow dynamics, *Journal of Fluid  
531 Mechanics* 583 (2007) 99–122.
- 532 [23] F. Biagioli, F. Güthe, Effect of pressure and fuel–air unmixedness on nox  
533 emissions from industrial gas turbine burners, *Combustion and Flame*  
534 151 (1) (2007) 274–288.

## 535 **Nomenclature**

- 536  $(\Delta h_f^0)_m$  standard heat of formation ( $kJ/mol$ )
- 537  $\Delta$  filtered scale
- 538  $\delta_{ij}$  kronecker delta

539	$\dot{\omega}_r$	chemical reaction rate
540	$\gamma^*$	volume fraction of fine structure
541	$\mu$	molecular viscosity coefficient ( $N \cdot s / m^2$ )
542	$\nu_t$	the eddy viscosity
543	$\Omega$	vorticity magnitude ( $s^{-1}$ )
544	$\phi_{i,m}^{sgs}$	species mass flux
545	$\rho$	density ( $Kg/m^3$ )
546	Pr	Prandtl number
547	$\sigma_i^{sgs}$	unresolved viscous work
548	$\tau$	stress tensor
549	$\tau^*$	residence time of reactive mixture
550	$\tau_{ij}^{sgs}$	subgrid-scale stress tensor
551	$\theta_{i,m}^{sgs}$	diffusive mass flux
552	$\varphi$	equivalence ratio
553	$a_{mr}, b_{mr}$	Chemical reaction equation coefficient
554	$D_m$	$m$ th species diffusion coefficient
555	$dV$	volume increment
556	$E$	system energy ( $kJ$ )
557	$H_i^{sgs}$	subgrid heat flux
558	$i, j$	coordinate index
559	$m$	component index
560	$MFI$	modified flame index
561	$N$	total number of species

562	$p$	pressure ( $Pa$ )
563	$PPC$	proportion of premixed combustion
564	$q$	heat flux ( $J/m^2 \cdot s$ )
565	$Re$	Reynolds number
566	$S$	swirl number
567	$S_{ij}$	strain rate tensor
568	$T$	temperature (K)
569	$t$	time ( $s$ )
570	$U$	unmixedness parameter
571	$u$	velocity ( $m \cdot s^{-1}$ )
572	$u'$	fluctuating velocity ( $m \cdot s^{-1}$ )
573	$x, y, z$	Cartesian coordinate
574	$Y$	species mass fraction
575	$\dot{\rho}^c$	mass reaction source term