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A REAL TIME SIMULATION MODEL FOR LOGNORMAL FADING CHANNELS

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In this paper we present a mathematical model for the simulation of lognormal fading channels. A modeling technique and its validation by computer simulation are described with special emphasis on the aspects related to the spectrum shaping. The obtained results have shown that it is possible to control the fading depth and the fading rate with a good enough reliability in their usual variation ranges.

INTRODUCTION

Simulation plays an important role in computer-aided analysis and design of Communications and Radar systems. In recent years, propagation models with long-term fading characterized by random processes with log-normal probability density functions (pdf) have been developed and are currently being used in a variety of applications.

The generation of complex correlated processes with lognormal statistics is not trivial. In fact, many recently appeared papers [1],[2],[3] are focussed in this subject.

In [1] a canonical model which carries out the fading spectrum shaping by means of a linear operator on a batch block of uncorrelated samples is presented. Besides the difficulties associated to find the linear operator to get the appropriate correlation function, its main drawback relies on the fact that it do not process sequentially, thus making difficult its interoperation with commercially available simulators which are usually based on a sequential processing of data.

The model presented in [3] makes use of a deterministic approach in order to get Rayleigh and log-normal statistics. In case that only the second one were required, the main drawback is that the output process is real (not coherent). On the other hand it is likely less reliable as far as the statistics on the extreme values is concerned and neither is straightforward the control of the frequency spread.

MODEL OF THE MULTIPLICATIVE PROCESS

In order to overcome the problems associated to the above mentioned approaches, we propose a model constituted by a filter, and a complex exponential non-linear device as it is shown in figure 1.

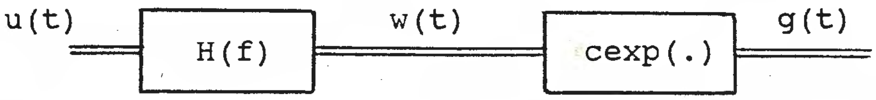

Figure 1. Functional block diagram of the model

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The variable u(t) is a complex white Gaussian process with the real and imaginary components mutually uncorrelated. The autocorrelation function of g(t) will be determined by the combined effect of the filter and the non-linear memoryless transformation. This non-linear function has been choosen because it transforms a pair of jointly distributed Gaussian variables into a complex one having log-normal envelope.

It is also straigthforward to show that the pdf of the phase will be Gaussian. But this would not be worrying assuming that the phase standard deviation is big enough compared with π. In fact the phase must be understood modulo-2π. Therefore folding the phase distribution at every 2π interval we get a quasi uniform characteristic inside the interval $(-\pi,\pi)$. In turn, this will be one of the assumptions to be validated.

The frequency spread of g(t), and therefore the fading rate, can be controlled by specifying a shape for the output spectrum and allowing the control on its characteristic parameters (bandwidth,...). As it was mentioned earlier, the output spectrum will be conformed by the combined effect of the filter transfer function and the non-linear device. In fact the mean and the autocorrelation of g(t) can be written as follows [1]

$$E\{g(t)\} = \exp((R_{xx}(0)-R_{yy}(0))/2)\ (\cos(R_{xy}(0)+ j\sin(R_{yx}(0))) \tag{1}$$

$$R_{gg}(t) = c\exp(R_{xx}(t)+R_{yy}(t)+R_{xx}(0)-R_{yy}(0) - j(R_{xy}(t)-R_{yx}(t))) \tag{2}$$

Concerning the average power of the output process it must be noted that it could be controlled indistinctly at the input or at the output of the complex exponential. The control at the output is like if we were adding a dc term to the Gaussian process w(t). In this case the pdf of the lognormal process becomes spread out as much more output power is required. Therefore the probability that the output envelope is below a given fading depth may be very low even for a great output power.On the contrary, the control at the input makes the pdf more concentrated near the origin, allowing a more flexible control of the probability of exceeding a given fading depth with moderate output powers. As a consequence we choosed this second approach although its inherent drawbacks which will be pointed out later.

SPECTRUM SHAPING OF g(t)

First of all we assume that the cross-correlation functions between the real and the imaginary components of the process at the input of the non-linear function are both equal to zero and also that, for the same process, the autocorrelation of the imaginary component is proportional to the autocorrelation of the real part. In the following paragraphs we note by L the proportionality factor. This is

$$R_{xy}(t) = R_{yx}(t) = 0 \tag{3}$$

$$R_{yy}(t) = L\,R_{xx}(t) \tag{4}$$

Then (1) and (2) become

$$E\{g(t)\} = \exp(((1-L)R_{xx}(0))/2) \tag{5}$$

$$R_{gg}(t) = \exp((1+L)R_{xx}(t) + (1-L)R_{xx}(0)) \tag{6}$$

and the output power will be

$$P = R_{gg}(0) = \exp(2R_{xx}(0)) \tag{7}$$

Also under the above mentioned hypothesis is easy to show that the squared magnitude of the transfer function H(f) is proportional to the Fourier transform of the following expression

$$-(1-L)R_{xx}(0)/(1+L) + \ln (R_{gg}(t) \ / \ (1+L) \tag{8}$$

from which we can realize that it is not possible to guarantee the existence of a solution for any output spectrum (autocorrelation) shape.

Proposed approach

Comming back to the meaning of expression (8) it is clear that if the output spectrum is Gaussian the squared magnitude of H(f) adopts a closed expression, thus simplifying the involved mathematics. In this context it must be kept in mind that our final goal is not so much to get a predefined spectrum but to control its characteristic parameters.

Assuming that the output spectrum is Gaussian and using (8) it can be seen that the autocorrelation of w(t) should be an inverted parabola centered around the origin. This is of course a non feasible autocorrelation function. Therefore an approach must be used. In the following two paragraphs we summarize it.

First of all, the parabola is truncated from the point where it is zero (t_o) as it is shown in figure 2. In this case, the autocorrelation of the lognormal process becomes Gaussian inside the interval ($-t_o,t_o$) and flat outside of it. Moreover, the flat term is inversely proportional to the exponential of the factor L. Therefore, making L as much great as possible the fitting of the output spectrum to a Gaussian one is improved.

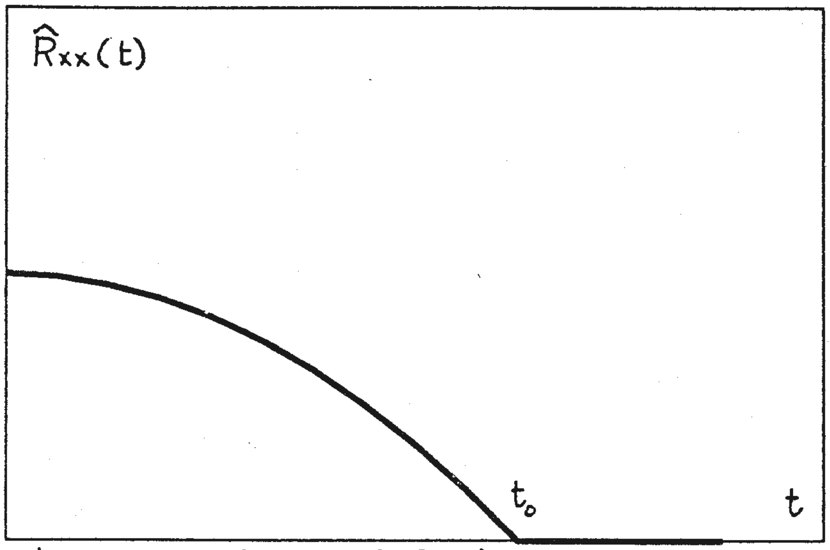

Figure 2. The parabola is truncated from the point where it is zero.

On the other hand, to simplify the filter synthesis, the truncated parabola is approached with a Gaussian function (figure 3).

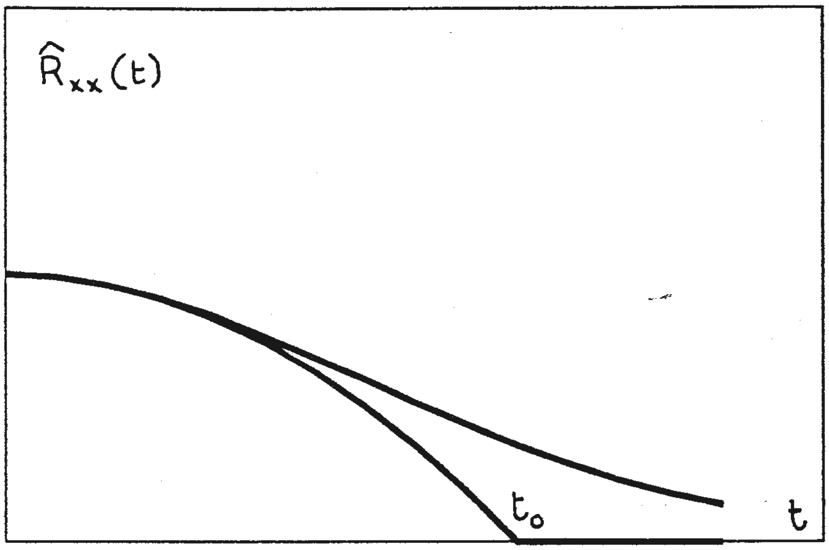

Figure 3. The truncated parabola is fitted to a Gaussian function.

In order to fit the Gaussian function to the parabola, the second one is equated to the first two terms of the series expansion of the Gaussian function. Therefore, the filter of the model will be Gaussian with a variance proportional to the width of the output spectrum and also inversely proportional to the factor L. This means that L can not be made arbitrarily high in order to avoid that this filter bandwidth becomes much lower than the simulation sampling frequency.

MODEL VALIDATION

In order to validate the model and implicitly to determine the range of values for the factor L we proceeded to simulate the system represented in figure 1.

Filter synthesis

In order to determine the filter recurrency equation its magnitude was expanded in series of Laguerre polynomials. In fact, previous work had revealed that this gives a better precission than other series expansions, like Taylor or Tchebyshev, for the same filter order.

In the present case we choosed an order ten expansion (five cascaded cells of order two) which leads to a Gaussian transfer function in a dynamic margin of about 80 dB.

Summary and interpretation of results

From the tests carried out with different algorithms for the evaluation of the factor L, the following can be stated:

> The discrepancies between the required and the estimated output power are not minimized increasing the number of simulated samples.
>
> The output spectrum is Gaussian only for small values of the output power.

Keeping in mind that we had seen analytically that it was possible to reach the model objectives, we attributed the above mentioned discrepancies to the possible statistical errors in the noise source with which we feed the model. Therefore, an exhaustive analysis of such noise source was done and it revealed that in fact the cross-correlation functions are not identically zero and also that they show a residual mean value. This statistical errors are relatively small and in most applications involving linear transforms can be neglected. But in our case we use the complex exponential which magnifies them and therefore it is reasonable to expect errors in the output power and asymetries in the output spectrum. As the development of a new noise source was out of the scope of this research, we could only reduce the errors by means of a careful evaluation of the filter noise equivalent bandwidth.

On the other hand, the factor L adopts a minimum value around 20 when the output power reaches the maximum (100 power units (pu) is a fair upper bound). As a consequence the approach in this case is not good enough yielding to a narrower output spectrum.

In spite of the discrepancies observed in the spectrum shape, the relative errors measured in the frequency spread are not greater than 10% and 20% when the output power is little than 20 pu and 100 pu respectively.

Finally, the probability density function of the phase (modulo-2π) is practically uniform.

CONCLUSIONS

This model and the associated implementation approach overcomes the drawbacks of other existing ones. In fact, it allows the sequential (real-time) generation of a coherent lognormal / uniform random process intended to simulate long-term fading channels. The fading frequency spread can be controlled with relative errors not exceeding the 20%. Moreover, the probability of a fading depth of 30 dB can be made as high as 10^{-2}.

The model limitations which have been pointed out throughout the paper come mainly from its sensitivity to the statistical errors of the white noise source. However, in the context of the analysis of transmission systems under long-term fading, they do not constitute any obstacle for its use.

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