CONSTANT FALSE ALARM RATE FOR A RADAR DATA FUSION CENTER WITH N PARALLEL DISTRIBUTED CELL AVERAGING RECEIVERS

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ABSTRACT

In this paper the performance of a fusion data center for a network of N radar receivers is analyzed. The total probabilities of detection and false alarm are obtained using a rank fusion rule. A constant false alarm rate operation is proposed and has been analyzed for Rayleigh targets and interference. For a given probability of fused false alarm an iterative method to calculate the threshold of detection of each receiver and the best rank is presented, with allows the maximization of the probability of detection.

INTRODUCTION

In this paper, we propose a fusion algorithm based on a rank fusion rule, that is: supposing that we have N adaptative threshold detectors, the Data Fusion Center determines the presence of a target in the tested cell if K of the N detectors have made the same decision (rule of rank K with $1 \le K \le N$). For N receivers this represents all the possible cases between the rule OR (K=1) and the AND rule (K=N). The proposed system is adaptative and optimun for each interference and target model under consideration, since for a fixed probability of false alarm in the Fusion Center, each receiver threshold Ti can be calculated permitting the user to choosed the rank K that maximizes the probability of detection in the Fusion Center (Pdt). The system has been analyzed for Rayleigh targets and interference and with N different CA-CFAR distributed detectors (with a diffe rent number of estimation cells Mi).

METHOD EVALUATION

Figure |1| presents the problem in consideration in which a radar observation scenario that generally includes targets and clutter is shown. The scenario is observed by N spatially distributed detectors, which in principle, can be complete systems (N radars), or N receivers of a multistatic system. These detectors can be different but their characteristics must be perfectly known by the Control and Management Center.

Under the binary hypothesis of detection:
H0 : Target signal absent.
H1 : Target signal present.
We can define the probabilities of detection and false alarm for each detector (Pd_i and Pfa_i) as:
Pd_i = P(H1/H1) in detector "i".
Pfa_i = P(H1/H0) in detector "i".

In general, we will work with N different receivers in such a way that in the Fusion Center we will be provided with a decision vector D:

$$D = (d1,\ d2,\ d3,\ \ldots\ldots,\ dN) \qquad (1)$$

where d1, d2, etc. represent the decisions of target presence or absence of each receiver:

di = 1 : Receiver "i" decides "presence of target".
di = 0 : Receiver "i" decides "absence of target".

The decision algorithm that we propose to develop as a decision rule R(D) in the Fusion Center is the following:

$$R(D) = \begin{cases} 1 \text{ (presence of target) if } \sum_{i=i}^{N} di \ge K \\ 0 \text{ (absence of target) if } \sum_{i=i}^{N} di < K \end{cases} \qquad (2)$$

The probabilities of detection and false alarm in the Fusion Center, can be expresed as a function of the probabilities of each detector as the joint probability of N independent events (expressions (1) and (2) in |2| |3|):

$$Pdt = \sum_{D} R(D) \cdot \prod_{S0} (1 - Pd_i) \cdot \prod_{S1} Pd_i \qquad (3)$$

$$Pfat = \sum_{D} R(D) \cdot \prod_{S0} (1 - Pfa_i) \cdot \prod_{S1} Pfa_i \qquad (4)$$

where:
$\sum_D$: The summation of all possible combinations of decisions.
S1: The detector group that has decided the presence of target.
S0: The detector group that has decided the absence of target.
R(D): The Decision Rule.

The rule can be generalized for the case in which one wants to weigh the decisions of each receiver, since the receivers or the communication channels that connect them Fusion Center will have in general different qualities. For this case we will use a vector A in which its components represent the weights of every receivers-communication-chanel pair according to its performance.

$$A = (a1,\ a2,\ a3,\ \ldots.\ aN) \qquad (5)$$

where ai is the normalized weight factor ($0 \le ai \le$) representative of the quality of the receiver-channel "i" performance.

The decision rule R(D), (2), can now be expressed as:

$$R(D) = \begin{cases} 1 \text{ (target present) if } A \cdot D^T \ge Z \\ 0 \text{ (target absent) if } A \cdot D^T < Z \end{cases} \qquad (6)$$

where $A.D^T$ is the scalar product of the weighting vector "A", (5), and the decision vector D, (1), with Z being a real number such that $0 \le Z \le N$. The rule continues to be a rank rule and is nothing more than a generalization of the expression given in (2), since effectively, expression (6) becomes expression (2) when all the weighting factors ai = 1. We still have a rank rule K if we round Z by its nearest integer.

Once the decision rule's rank K or Z is fixed, expressions (3) and (4) which indicate the probabilities of detection and false alarm in the Fusion Center can be developed as a function of K and N:

$$Pdt_{kN} = \sum_{i=k}^{N} \left\{ \left[\sum_{p=0}^{i-k} (-1)^p \binom{i}{p} \right] \cdot \left[\sum_{q=1}^{N-i+1} Pd_q \left(\sum_{r=q+1}^{N-i+2} Pd_r \left(\sum_{s=r+1}^{N-i+3} Pd_s (\ldots) \cdot \right) \right) \right] \right\} \tag{7}$$

$$Pfat_{kN} = \sum_{i=k}^{N} \left\{ \left[\sum_{p=0}^{i-k} (-1)^p \binom{i}{p} \right] \cdot \left[\sum_{q=1}^{N-i+1} Pfa_q \left(\sum_{r=q+1}^{N-i+2} Pfa_r \left(\sum_{s=r+1}^{N-i+3} Pfa_s (\ldots) \cdot \right) \right) \right] \right\} \tag{8}$$

expressions obtained recursively where:

- Pdt_{KN} and $Pfat_{KN}$ are the total probabilities of detection and false alarm for a K rank rule applied to N receivers.
- Pd_j and Pfa_j are the probabilities of detection and false alarm for the "j" receiver ("j" = "q" or "r" or "s" etc.)

Expressions (7) and (8) represent the Data Fusion Center probabilities of Detection and False Alarm, for a K rank fusion rule and N receivers. That is, the Fusion Center decides the presence or absence of a target based on at least K decisions of target presence of the N receivers.

For N Constant False Alarm Ratio (CFAR) distributed receivers or detectors, and given the target and clutter statistics, the Pd_i and Pfa_i (Probabilities of Detection and False Alarm of each receiver) can be written in general as

$$Pd_i = f(Ti,Mi,SNRi) \tag{9}$$

$$Pfa_i = f(Ti,Mi) \tag{10}$$

where:

- Ti is the scale factor of threhold detection.
- Mi number of estimation cells of the clutter level.
- SNRi is the signal-to-noise ratio.

In the case of Cell Averaring CA-CFAR receivers the expressions (9) and (10) become:

$$Pd_i = \frac{(1+Si)^{Mi}}{(1+Si+Ti)^{Mi}} \tag{11}$$

$$Pfa_i = \frac{1}{(1+Ti)^{Mi}} \tag{12}$$

$$Ti = \frac{Ci}{Mi} \tag{13}$$

where:

- Si is the signal-to-noise ratio in the detector "i".
- Mi is the total number of estimation cells of the detector "i".
- Ti is the scale factor of threshold detection of the detector "i".
- Ci "i" detector threshold value.

For a set of N CA-CFAR receivers and sustituting the expressions (11) to (13) into expressions (7) and (8) we obtain a non linear equation.

$$Pdt = Pdt_{KN}(T1,\ T2,\ T3,\ \ldots,TN) \tag{14}$$

that we want to maximize under the restriction imposed by another function Pfat:

$$Pfat = Pfat_{KN}(T1,T2,T3,\ \ldots,TN) \tag{15}$$

To solve this, an objetive function is defined |1|

$$J(T1,T2,..,TN) = Pdt_{KN}(T1,T2,\ldots,TN) + \beta \cdot Pfat_{KN}(T1,T2,\ldots,TN) \tag{16}$$

where:

- β is the Lagrange Multiplier.
- μ is the desired value for Pfat (the probability of false alarm in the Data Fusion Center).
- Pdt_{KN} and $Pfat_{KN}$ are the probabilities of detection and false alarm in the Data Fusion Center for a fusion rule of rank K with N distributed receivers.

The non-linear system of equations to be solved is given below:

$$\frac{\partial J(T1,T2,\ldots,TN,\beta)}{\partial T1} = 0$$

$$\frac{\partial J(T1,T2,\ldots,TN,\beta)}{\partial T2} = 0$$

$$\vdots$$

$$\frac{\partial J(T1,T2,\ldots,TN,\beta)}{\partial TN} = 0$$

$$Pfat_{KN}(T1,T2,\ldots,TN) = \mu$$

which is a completely determined system of N+1 equations and N+1 unknowns, those corresponding to T1,T2, T3, ..., TN and β. As can be seen, the system of equations is strongly non-linear, but fortunately equations (7) and (8) are recursive and allow a numerical treatment on a computer varying K and N. We must solve the system for each rank K from K=1 to K=N and observe for each Pfat which rank K gives the maximun detection probability Pdt in the Fusion Center.

To solve the non-linear system expressed in (17), an iterative method is needed. In this work, the Newton Raphson method has been used, which can be generalized to a system of N equations and N unknowns |4|. In our case this method is applied to the following matrix expression obtaining approximate solutions that converge to the optimum values for each Ti.

$$|Ti,\beta|^T_{z+1} = |Ti,\beta|^T_z + |\partial F/\partial(Ti,\beta)|^{-1} \cdot |F|^T \tag{18}$$

$$|Ti,\beta|_z = |T1_z, T2_z, \ldots, TN_z, \beta_z| \tag{19}$$

$$|Ti,\beta|_{z+1} = |T1_{z+1}, T2_{z+1}, \ldots, TN_{z+1}, \beta_{z+1}| \tag{20}$$

$$|F| = \frac{\partial J(T1,T2,..\beta)}{\partial T1}, \frac{\partial J(T1,T2,..\beta)}{\partial T2} \ \ldots \ \ldots \frac{\partial J(T1,T2,\ldots,\beta)}{\partial TN}, \ (Pfat-\mu) \tag{21}$$

The principal drawback of this method is that it needs an initial solution near the exact one to converge. To find and initial solution, the bisection method is used; a method that can only be applied to non-linear quations of only one unknown. Nevertheles, with the assumption that all the receivers are identical, equation (15) reduces to a function of only one variable directly solvable by the bisection method, but to initialize the system (18) is necessary to determine βo and which receiver is assumed to be equal to the rest.

For any group of different receivers, the use of only one initial solution $T1_0 = T2_0 = ... = TN_0$ does not force necessarily the convergence of the system of optimization (18). The system is convergent only in the case of completely alike receivers. The computer program developed to resolve system (18) obtains the initial solution by applying the bisection method in a more sophisticated manner. Fos the calculation of the initial solution, the program assumes that all the receivers are equal to the first, and by applying the bisection method, the value of $T1_0$ is found. Secondly, all the receivers are assumed equal to the second one and again the bisection method is applied to find $T2_0$. Continuing in this manner, we obtain the initial solution vector $|T1_0, T2_0, T3_0, ..., TN_0|$.

The determination of βo is done after the calculation of the different values that should be taken by β so that each of the system equations (20) is executed giving to each Ti its initial value Ti_0, then taking for βo the average value of the βi calculations in the same way, as shown:

$$\beta o = \frac{1}{N}\sum_{i=1}^{N} - \frac{\dfrac{\partial PdtkN(T1,T2,....,TN)_o}{\partial Ti}}{\dfrac{\partial PfatkN(T1,T2,....,TN)_o}{\partial Ti}} \qquad (22)$$

The results of the application of this recursive method allows the determination for each group of different receivers CA-CFAR, and for each constant level of false alarm in the fusion center (Pfat=μ), to know which rank K maximizes the Detection Probability in the Fusion Center (Pdt).

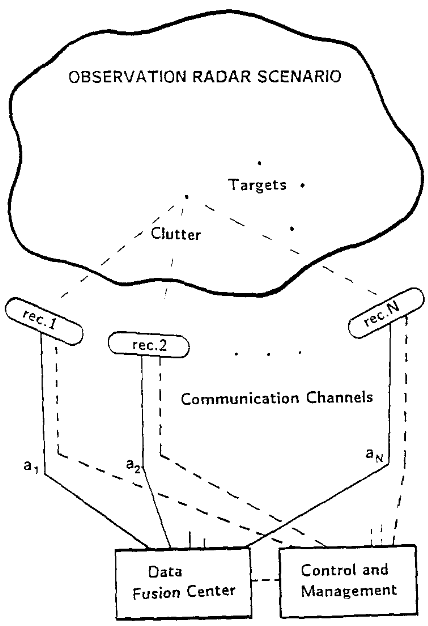

Figure 1

The solution of the system of equations (17), for Rayleigh targets and interference with CA-CFAR detectors according to the method proposed has been performed on various receiver combinations, of which some of the results obtained are given below in detail:

Fig. 2 shows the Pdt as a function of SNRi for a set of three identical receivers with M=8 and with the Pfat fixed at 0.0001, it is shown that for SNRi less than 20 dB the OR rule (K=1) is not the best one, the best is K=2.

Fig. 3 shows the same than fig. 2 for a set of 5 identical CA CFAR receivers of M=32 for a Pfat fixed at 0.00001, in this case the rule that maximizes the Pdt for SNRi less than 32 dB is again the rule K=2, the rule OR (K=1) is worse yhan K=3 for SNRi less than 10 dB.

Fig. 4 and fig. 5 show the Pdt as a function of Pfat, for two sets of different CA-CFAR receivers, also in these cases the OR rule is not the best one.

Figures 6 and 7 show the performance of other sets of CA-CFAR receivers with a more realistic number of estimation cells M, the curves show that if we want to maximize the Pdt under the restriction imposed by a constant Pfat, there is always a rule K that maximizes it and in general is K ≠ 1.

Fig. 8 shows the scale threshold factor T for a set of three receivers as a function of Pfat.

REFERENCES

|1| A.Farina & G.Galati."Curso de Procesado de Señal y Datos Radar". C.E.M. MADRID 25-29 April 1988.

|2| M.Barkat and P.K.Varshney. " Cell-Averaging CFAR Detection with distributed Radars and Data Fucion". Intern. Conf. on Radar. LONDON 87. 19-21 oct.pp.165-69

|3| M.Barkat and P.K.Varshney "Decentralized CFAR Signal Detection" IEEE-AES,Vol.AES-25, nº2, March-89, pp.141 149.

|4| K.Arbenz & A.Wohlhauser."Advanced Mathematics for Practicing Engineers". Artech House 1986.

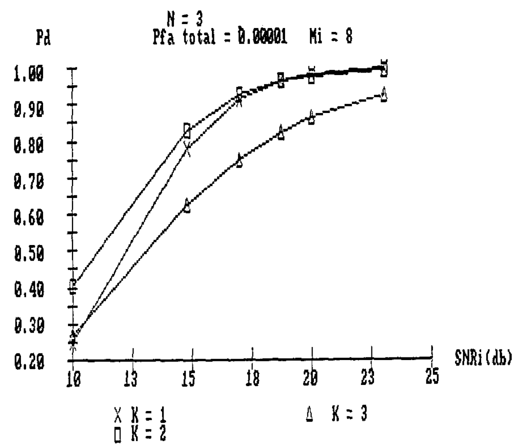

Figure 2

Pdt as a function of SNRi for Three identical receivers with M = 8 estimation cells.

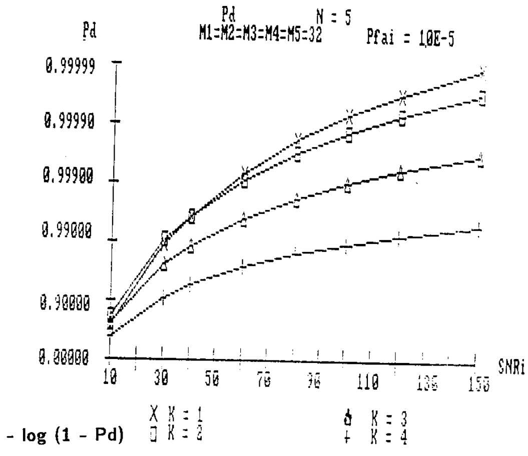

Figure 3

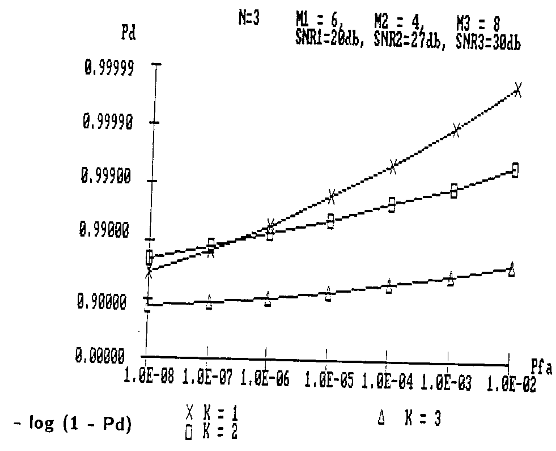

Figure 4

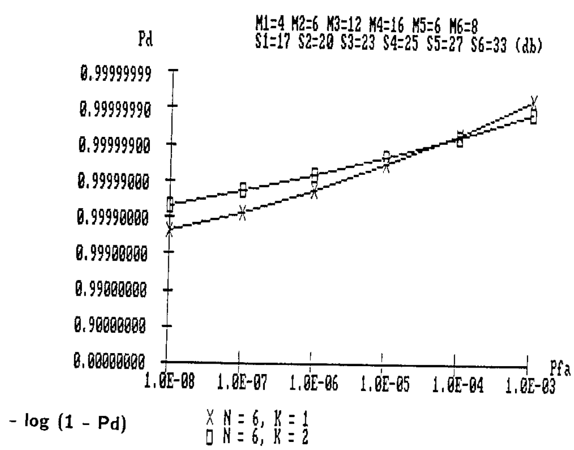

Figure 5

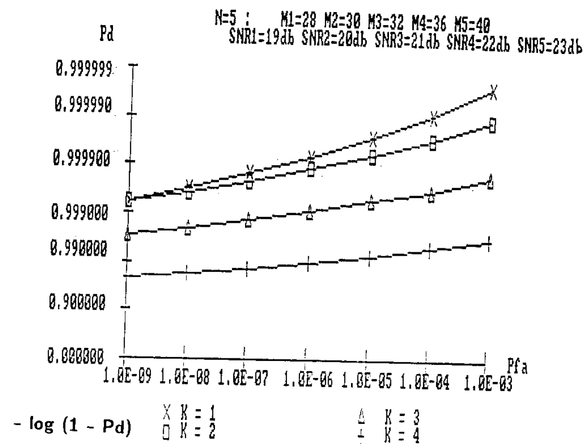

Figure 6

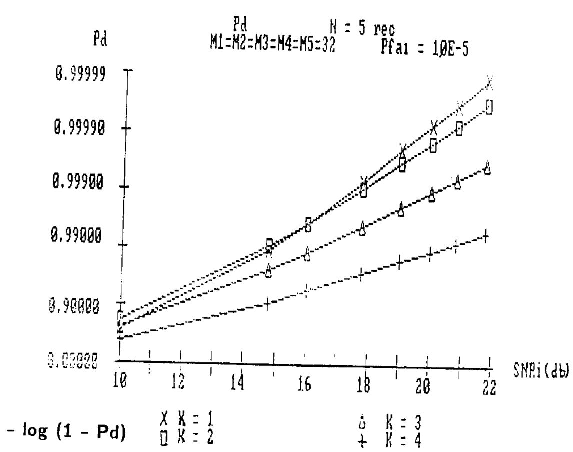

Figure 7

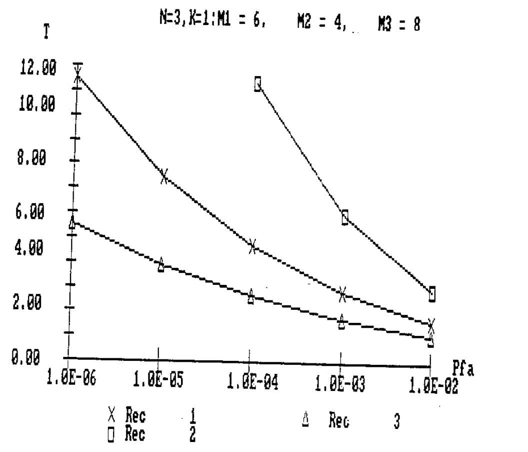

Figure 8