Optimum Allocation of Distributed Generation in Multi-feeder Systems Using Long Term Evaluation and Assuming Voltage-Dependent Loads

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Abstract

The analysis of actual distribution systems with penetration of distributed generation requires powerful tools with capabilities that until very recently were not available in distribution software tools; for instance, probabilistic and time mode simulations. This paper presents the work made by the authors to expand some procedures previously implemented for using OpenDSS, a freely available software tool for distribution system studies, when it is driven as a COM DLL from MATLAB using a parallel computing environment. The paper details the application of parallel computing to the allocation of distributed generation for optimum reduction of energy losses in a multi-feeder distribution system when the system is evaluated during a long period (e.g., the target is to minimize energy losses for periods longer than one year) and voltage-dependent load models are used. The long term evaluation is carried out by assuming that the connection of the generation units is sequential, and using a divide and conquer approach to speed up calculations. The main goals are to check the viability of a Monte Carlo method in some studies for which parallel computing can be advantageously applied and propose a procedure for quasi-optimum allocation of photovoltaic generation in a multi-feeder distribution system.

Keywords: Distribution System, Distributed Generation, Long Term Evaluation, Loss Minimization, Monte Carlo Method, Parallel Computing.

1. Introduction

Distributed generation can support voltage, reduce losses, provide backup power, improve local power quality and reliability, provide ancillary services, and defer distribution system upgrade [1]-[3]. Modelling of renewable generation raises several challenges to distribution load flow calculations since capabilities for representing intermittent generators, voltage-control equipment, or multi-phase unbalanced systems are required. In addition, the study of systems with intermittent non-dispatchable resources will usually require a probabilistic approach and calculations performed over an arbitrary time period that may range from minutes to years. Load representation is another important issue since voltage-dependent loads with random variation must also be accounted for. These issues complicate the study since software tools have to combine new analysis capabilities with a high number of models for representing various generation technologies, besides the conventional distribution system components, and include capabilities for time mode calculations [4].

The optimum allocation of distributed generation can be seen from two different perspectives:

- From the independent producer's point of view the goal is to optimize the benefit. Although the utility will usually impose some constraints to the generation units to be connected to its system (e.g. a maximum rated power), it can be assumed by default that the units can be connected to any system node. Therefore, the optimization approach will be in general a feasibility study whose main goal is to check the viability of the installation and select the most economical size (irrespectively of the location) and, in case of dispatchable units, the control strategy that will maximize the benefit; see for instance [5], [6].
- From the utility's point of view the goal is to maximize the positive impact of distributed generation (e.g., voltage support, energy losses, investment deferring) and minimize or avoid

those aspects that can negatively affect the system performance (e.g. miscoordination of protective devices, overvoltages during low load periods); see for instance [7].

Several surveys of the methods proposed for optimum allocation of distributed generation have been presented in the literature; see [8] and [9]. This work is related to the second perspective; namely, the optimum allocation of photovoltaic (PV) generation aimed at minimizing distribution system energy losses taking into account some constraints (e.g., there is a maximum voltage that should not be exceeded; there is a thermal limit for each system line section).

A previous paper by the authors presented a procedure for optimum allocation of PV generation units aimed at minimizing the yearly energy losses of a distribution system; the procedure was based on the Monte Carlo method and the utilization of a multicore computing environment (e.g., multicore installation) [10]. The work used OpenDSS as a COM DLL driven from MATLAB [11].

The present study is aimed at estimating the optimum allocation of PV generation using again a distributed computing environment but considering a long term evaluation (i.e., 10 or more years). That is, the goal is to estimate the rated power and location of PV generators whose connection will minimize distribution system energy losses for a certain time period. In addition, it is assumed that the connection of PV generation units will be sequential and the loads can be voltage-dependent.

Some information is required when considering time-varying load and intermittent generation; namely, the variation of load and photovoltaic generation over the evaluation period. Different algorithms were previously implemented by the authors in MATLAB to obtain node load and PV generation yearly curves for their application with OpenDSS, see references [12] and [13].

Some studies (e.g. feasibility studies) rely on evaluation periods as long as 20 years or more; in such cases time-varying curves must be yearly updated taking into account randomness (for both load and generation shapes) and the forecasted load variation. For this purpose the algorithms previously developed by the authors and presented in [12] and [13] have been modified in order to create system curves adequate for long term evaluation. By default, it is assumed that PV generation units will inject only active power.

The paper has been organized as follows. The main characteristics of the system tested in this work are presented in Section 2, while Section 3 introduces the Monte Carlo approaches applied in this study. Section 4 explores the application of the approaches based on the conventional Monte Carlo method and the refined method proposed by the authors in [10] to the simultaneous connection of one or more units considering that the target is to minimize the energy losses of the test system during one year. The conclusions of that study are used in Section 5, which presents the application of the Monte Carlo method to the optimum allocation of PV generation units using a long term evaluation and assuming a sequential (i.e. non-simultaneous) connection of the generators. Section 6 proposes an alternative methodology for optimum allocation of distributed PV generators based on a “divide and conquer” approach [14], assuming again a sequential connection of PV generation units. The main results and some consequences of the study presented in this paper are discussed in Section 7. The main conclusions of the paper are summarized in Section 8.

2. Test System

Figure 1 shows the diagram of the test system. It is a three-phase overhead system serving three feeders with different topologies and load characteristics. The system is based on IEEE test feeders [15], and has been created on purpose for this study. The model includes a simplified representation of the high-voltage system. Some of the main characteristics of the substation transformer are given below:

- High-voltage rating: 230 kV
- Low-voltage rating: 4.16 kV
- Rated power of substation transformer: 10000 kVA.

Table I presents basic information for the three feeders of the test system. Table II summarizes the information used to obtain PV generation curves. The studies have been carried out using the PV generator model presented in [12]. For the present study a PV generator is connected to the system through a step-up interconnection transformer, see Figure 2.

The guidelines for building the test system model in OpenDSS are those previously proposed and applied in [10], [16], or [17].

Table I – Test System Information

Feeder	Total active power (kW)	Total reactive power (kvar)	Number of loads	Lengths (km)
A	1700	930.72	51	26.36
B	2045	992.69	31	18.52
C	2290	899.38	19	11.81

Table II – Summary of Solar Resources

Average monthly clearness index	0.51, 0.56, 0.56, 0.55, 0.54, 0.57, 0.60, 0.57, 0.55, 0.50, 0.48, 0.49
Slope angle of the panel	35º
Normal operating cell temperature	45ºC
Average monthly daily minimum temperatures	6.77, 6.97, 8.39, 10.0, 13.3, 17.1, 19.9, 20.5, 18.2, 15.3, 10.7, 8.18
Average monthly daily maximum temperatures	12.2, 12.7, 14.5, 16.1, 19.5, 23.5, 26.3, 26.4, 23.7, 20.2, 15.7, 13.2

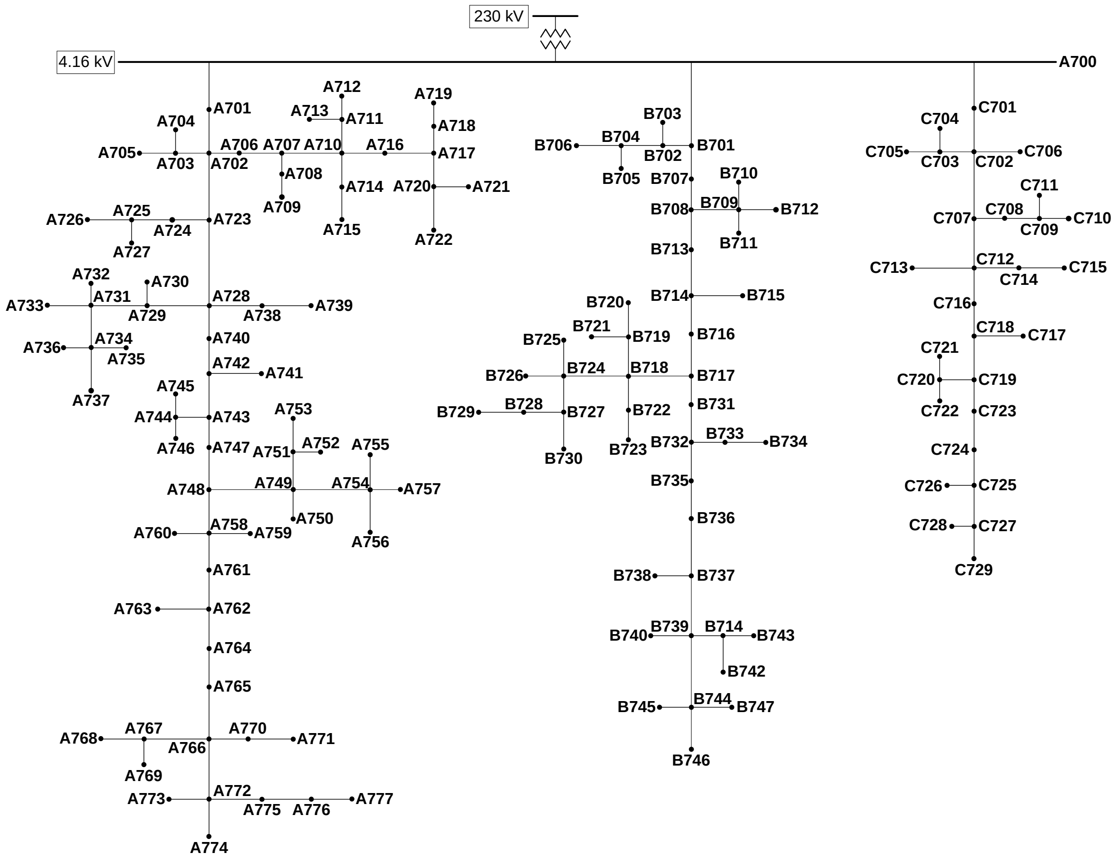


Figure 1. Test system configuration.

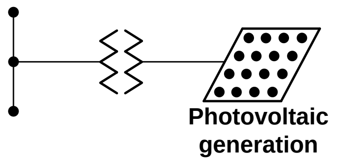


Figure 2. Configuration of a PV generator.

3. Monte Carlo Procedure for Optimum Allocation of Generation Units

The procedure has been implemented taking into account certain rules when choosing locations and sizes for the PV generators:

- The generators will only be connected to MV nodes, but taking into account that a PV generator is connected to the LV terminals of the interconnection transformer and the whole PV generator model includes the transformer, see Figure 2.

- The user can specify a list of nodes to which PV generators cannot be connected.
- The rated power of the interconnection transformer is chosen once the rated power of the PV plant has been selected; it is rounded in steps of 50 kVA. By default, the short circuit impedance is 6%.

The general procedure for short term (one year) evaluations may be summarized as follows (see also [10]):

1. Estimate the locations (feeders and nodes) and rated power values of generation units. The locations to which PV generators will be connected are determined by generating as many random values as units to be allocated and using a uniform distribution. Beforehand, the user has to fix the maximum generation power that can be connected to each node. This value is derived from the maximum thermal limit of the line sections connected to the selected node.
   This step is carried out as follows:
   - When only one generator is to be allocated, the maximum rated power will depend on the selected location. Every time a system node is randomly selected, the maximum rated power for that node is determined according to the following steps: (i) calculate the maximum non-coincident active power (i.e. the active power value that results from adding the active rated power of all load nodes) for the feeder where the node is located; (ii) check the maximum power that can be carried by the line sections connected to the selected node; (iii) compare the previous power values and choose the minimum one; this will be the maximum rated power that a generation unit can be assigned during the Monte Carlo execution. Then generate a random number uniformly distributed between 0 and 1; the rated power value is the result of multiplying this random number by the above maximum rated power value.
   - When two or more units are to be simultaneously connected to the same feeder, the following changes are introduced in the procedure:
     - ✓ Generate an independent uniformly-distributed random value for the initial rated power of each generation unit using the maximum rated power fixed for every node as the upper endpoint of each uniform distribution.
     - ✓ Compare the maximum rated powers for all the chosen locations and choose the maximum value, P_{DG_MAX}.
     - ✓ Generate the penetration factor as a random number uniformly distributed between 0 and 1. Multiply the maximum value found in the previous step by this penetration factor; the result will be the overall rated power of distributed generators.

       $$\sum_{i=1}^{NG} P_{DG\_i} = pf \cdot P_{DG\_MAX} \tag{1}$$

       where NG is the number of generation units under evaluation, P_{DG_i} is the rated power of unit i, pf is the penetration factor, and P_{DG_MAX} is the maximum DG rated power found in the previous step.
     - ✓ Calculate the scale factor from the initial rated powers as follows:

       $$sf = \frac{pf \cdot P_{DG\_MAX}}{\sum_{i=1}^{NG} P_{DGinit\_i}} \tag{2}$$

       where sf is the scale factor and P_{DGinit_i} is the initial rated power for unit i (obtained in the first step).
     - ✓ Obtain generator rated powers by scaling initial rated powers

       $$P_{DG\_i} = sf \cdot P_{DGI\_i} \tag{3}$$

   Note that the order in which random values for locations and rated powers are generated matters: first, the location nodes; afterwards, the rated powers.
2. Perform the load flow calculation. Neglect the case if one of the following conditions is satisfied: (i) the voltage at one node exceeds the fixed maximum value; (ii) the current through one or more system sections is above the thermal limit.

3. Stop the procedure when the specified number of runs or samples (according to the terminology of the Monte Carlo method) is reached.

The procedure summarized above has been implemented in MATLAB, which is used to calculate the random variables and control the execution of OpenDSS [10]. The implementation of the procedure for any number of cores when using parallel computing is based on the library developed by M. Buehren [18]. The multicore installation used in this work has 60 cores. For more details about the installation see [10] and [15].

A refinement of the Monte Carlo method aimed at reducing the simulation time was proposed in [10]. A similar refinement has been implemented in this work, although some minor changes were needed with respect to the approach proposed in [10] due to the configuration of the new test system.

4. Short Term Evaluation

The evaluation period for this first study will be one year. The study is performed without including substation transformer losses in the optimization process. The number of runs/samples is always chosen as a multiple of the number of core nodes (60 cores in this work) to obtain a homogeneously distributed burden among processing units.

Table III provides some information about the system behavior before connecting distributed generation and considering three different load models (constant power model, constant impedance model, ZIP model).

Table III – Short Term Evaluation (1 year) – Operating Conditions without Distributed Generation

		Constant power load model	Constant impedance load model	ZIP load model
Feeder A	Energy supplied from substation (kWh)	7322657.4	7371150.4	7346639.2
	Energy losses (kWh)	62924.8	62645.1	62768.9
	Energy losses (%)	0.8593	0.8498	0.8543
Feeder B	Energy supplied from substation (kWh)	5766675.6	5763902.0	5764512.9
	Energy losses (kWh)	56683.4	55593.9	56102.2
	Energy losses (%)	0.9829	0.9645	0.9732
Feeder C	Energy supplied from substation (kWh)	11741836.9	11615827.8	11676053.9
	Energy losses (kWh)	153289.3	148248.6	150643.1
	Energy losses (%)	1.3054	1.2762	1.2901
Total feeders	Energy supplied from substation (kWh)	24831170.0	24750880.3	24787206.1
	Energy losses (kWh)	272897.6	266487.6	269514.2
	Energy losses (%)	1.0990	1.0766	1.0873

The ZIP load model is defined as a combination of constant power, constant current and constant impedance load models [19]. When this model is used, weighting factors are assigned to specify active and reactive powers for each of these three components, being the sum of the weighting factors equal to unity for both active and reactive powers. For the present study each load component has been assigned a weighting factor equal to 1/3 for both active and reactive powers; this means that 1/3 of the load behaves respectively as a constant power, constant current and constant impedance load. For more details about component fractions and power factors of common actual loads see [19].

Table IV presents the results obtained after applying the procedure with different number of units and runs, and assuming the load behaves according to the constant power model.

This first study will be used to fix the number of runs of the Monte Carlo method when one, two or four generation units are to be connected. The required number of runs depends on the variance of the target variable, the energy losses measured at the substation terminals, and the desired accuracy [20]. Here the convergence of the method is checked by comparing the energy losses. It is assumed the method has converged when the variations of the energy losses are within a margin of 1%.

The results presented in Table IV show that for any number of PV generation units the resulting energy losses are rather similar with 1560, 2400 or 3120 runs, although the combinations of rated powers are different from each other. At the end, the differences between the energy losses are within the desired 1% margin. This behavior proves that only small variations in the energy losses are achieved when the evaluation point is close to the actual optimum solution and suggests that the ultimate goal could be reaching a quasi-optimum result if a significant reduction in the simulation time

Table IV – Short Term Evaluation (1 year) – Optimum Allocation of PV Generation Units (Constant Power)

Optimum Allocation of One PV Generation Unit				
Runs		1560	2400	3120
Unit 1	Node	C724	C723	C723
	Power (kW)	1045.5	1236.2	1277.1
Energy losses (kWh)		256694.4	256486.0	256460.9
Optimum Allocation of Two PV Generation Units				
Runs		9600	14400	19200
Unit 1	Node	A770	A772	A773
	Power (kW)	196.3	246.5	279.9
Unit 2	Node	C719	C719	C724
	Power (kW)	1244.3	1088.2	1188.1
Energy losses (kWh)		253445.5	253714.9	253105.8
Optimum Allocation of Four PV Generation Units				
Runs		33600	50400	67200
Unit 1	Node	A766	A769	A766
	Power (kW)	343.0	360.3	284.6
Unit 2	Node	B740	B714	B721
	Power (kW)	195.79	233.9	34.75
Unit 3	Node	B747	B736	B744
	Power (kW)	292.2	366.0	320.4
Unit 4	Node	C723	C719	C725
	Power (kW)	1109.8	1140.0	1110.5
Energy losses (kWh)		250175.5	250491.9	249938.3

can be achieved. This is one of the principles on which the refined method proposed in [10] is founded: if a new combination of location nodes and rated powers is close to an already simulated combination, this new run is discarded. The Euclidean distance between solutions defined in [10] is also applied here. As in the previous work, this Euclidean distance is again 5%.

Table V summarizes the results upon the application of both the conventional and the refined Monte Carlo methods to optimum allocation of one, two, and four generation units with a different number of runs and different load models. Note that when two or more units are to be allocated the results have been ordered, beginning with the largest rated power.

Several conclusions are derived from the results shown in this table. As expected, the more units to be connected, the larger the energy loss reduction. It is also evident that although the results with different load models exhibit variations of the optimum rated power values and connection nodes, and they depend on the number of runs, the energy loss values resulting from combinations with the same number of units are very similar and differences between results from different models are rather low. Finally, the differences between the rated power values obtained with the two approaches when four units are to be allocated suggest that more runs are required.

From the resulting node locations one can conclude that, if the feeders are divided into three zones with a similar number of nodes and power demand (e.g. top, middle and bottom zones), then the generation units should be in general located at the bottom zones of the three feeders; only a few results correspond to nodes located at middle zones (e.g. when the load is represented by a constant power model and four units are to be allocated, only one unit should not be located at the bottom).

Table VI provides a summary of the main results derived from the three load models when using the conventional Monte Carlo method. Note that in most scenarios the highest rated power generation corresponds to the ZIP model but the maximum energy loss reduction is achieved when assuming constant power load models. As for the Monte Carlo approaches, it can be concluded that, except when only one unit is to be allocated, not much reduction of the simulation time has been achieved (actually less than 20%) with the refined method. In addition, it is very obvious that only when using a multicore environment the simulation times are affordable: although the test system is not large, more than four hours of computing time are required when four units are to be allocated.

One can deduce from the results presented in Tables V and VI that when one unit is allocated the energy loss reduction is no more than 6.0%, but when two units are connected the energy loss reduction increases not much more than 1% and a similar increment is achieved when passing from two to four units. In case of a sequential connection (see Sections 5 and 6), this means that it is with the first unit with which the largest energy loss reduction is achieved, and that unit should be

Table V – Short Term Evaluation (1 year) – Comparison of Simulation Results - 60 Cores

Scenario		Original method	Refined method
Constant Power Load Model			
Scenario		Original method	Refined method
One PV generator Runs = 3120	Node	C723	C723
	Rated power (kW)	1277.1	1211.1
	Energy losses (kWh)	256460.9	256671.6
	Simulation time (s)	783.8	276.2
Two PV generators Runs = 19200	Nodes	C724 - A773	C724 - A765
	Rated powers (kW)	1188.1 - 279.9	1364.4 - 429.5
	Energy losses (kWh)	253105.8	253387.8
	Simulation time (s)	4818.9	4501.9
Four PV generators Runs = 67200	Nodes	C725 - B744 - A766 - B721	C723 - B731 - A762 - B742
	Rated powers (kW)	1110.5 - 320.4 - 284.6 - 34.7	1291.4 - 454.6 - 383.8- 195.5
	Energy losses (kWh)	249938.3	249663.1
	Simulation time (s)	17009.6	15762.6
Constant Impedance Load Model			
Scenario		Original method	Refined method
One PV generator Runs = 3120	Node	C723	C724
	Rated power (kW)	1182.2	1141.1
	Energy losses (kWh)	252166.2	252239.5
	Simulation time (s)	733.4	267.6
Two PV generators Runs = 19200	Nodes	C723 - A767	C723 - A772
	Rated powers (kW)	1089.2 - 200.2	1161.5 - 299.7
	Energy losses (kWh)	249436.9	249003.6
	Simulation time (s)	4488.1	4277.8
Four PV generators Runs = 67200	Nodes	C724 - C719 - A763 - B747	C720 - C725 - B744 - A773
	Rated powers (kW)	747.4 - 378.3 - 326.8 - 191.9	595.3 - 427.5 - 333.4 - 140.3
	Energy losses (kWh)	247096.5	245883.5
	Simulation time (s)	16074.7	15141.1
ZIP Load Model			
Scenario		Original method	Refined method
One PV generator Runs = 3120	Node	C723	C723
	Rated power (kW)	1232.9	1318.2
	Energy losses (kWh)	254228.5	254298.3
	Simulation time (s)	820.8	297.3
Two PV generators Runs = 19200	Nodes	C719 - A767	C723 - A765
	Rated powers (kW)	1140.0 - 282.7	1349.3 - 381.0
	Energy losses (kWh)	250820.0	251098.9
	Simulation time (s)	5013.8	4678.8
Four PV generators Runs = 67200	Nodes	C723 - B747 - A775 - A756	C724 - B731 - C728 - A759
	Rated powers (kW)	1140.3 - 289.1 - 228.4 - 120.9	624.6 - 598.3 - 460.7 - 282.4
	Energy losses (kWh)	247948.0	248701.0
	Simulation time (s)	17912.8	17268.3

connected to the feeder with the largest potential energy loss reduction (feeder C in this study). However, the second and subsequent units should be connected taking into account not only the potential reduction that can be achieved in each feeder but also the number of units already connected to each feeder (as discussed in Section 6).

Since the substation works as a link between the system feeders and substation losses are not accounted for in the optimization target, one may wonder whether the procedure to optimally allocate PV generators can be carried out feeder by feeder. This will be discussed and analyzed in the next sections; the main goal of this section was to explore the application of the Monte Carlo approaches and obtain some conclusions that will be applied to long term evaluations and the sequential (non-simultaneous) connection of PV generation units.

5. Long Term Evaluation

Some long term studies, such as feasibility studies, are carried out considering a period that can reach up to 20 or more years [5], [6]. This section presents the results derived from a long term study aimed at estimating the optimum size and location of PV generators when the target is to minimize the

Table VI – Short Term Evaluation (1 year) – Operating Conditions with Distributed Generation
Conventional Monte Carlo Method

Number of Units	Results	Constant power load model	Constant impedance load model	ZIP load model
1 Unit	Energy supplied from substation (kWh)	22705909.5	22833076.5	22762590.4
	Total generation (kW)	1277.1	1182.2	1232.9
	Energy losses (kWh)	256460.9	252166.2	254228.5
	Energy loss reduction (kWh)	16436.7	14321.4	15285.7
	Energy loss reduction (%)	6.02	5.37	5.67
2 Units	Energy supplied from substation (kWh)	22387324.6	22657444.4	22448908.9
	Total generation (kW)	1468.1	1289.5	1422.7
	Energy loss (kWh)	253105.8	249436.9	250820.0
	Energy loss reduction (kWh)	19791.8	17050.7	18694.2
	Energy loss reduction (%)	7.25	6.39	6.93
4 Units	Energy supplied from substation (kWh)	21918119.1	22080626.7	21864456.0
	Total generation (kW)	1750.3	1644.5	1778.8
	Energy loss (kWh)	249938.3	247096.5	247948.0
	Energy loss reduction (kWh)	22959.3	19391.1	21566.2
	Energy loss reduction (%)	8.41	7.27	8.00

energy losses and the generation units are sequentially connected; that is, they are not simultaneously connected, as in the previous section.

The Monte Carlo procedure applied to the previous studies has been adapted for long term evaluations. Some important aspects of the new procedure are summarized below:

1. System curves (load shapes, solar irradiance and panel temperature curves) are updated for every year of the simulation period. In the case of node loads this is carried out taking into account the expected yearly variation. For solar irradiance and PV panel temperature, the implemented algorithm is applied by introducing some randomness that guarantees that the yearly curves are different every year.
2. The rules for the determination of locations and rated powers are initially (i.e. when the first unit is to be allocated in each feeder) the same that were used for the short term evaluation. Once some generation has been located at one feeder, the procedure for the second and subsequent units will exhibit some differences: (i) the connection node cannot be any of the nodes selected for previously located units; (ii) the maximum power of the new unit can be the value that results after subtracting the rated power of the already allocated units from the maximum non-coincident power of the corresponding feeder loads, but taking into account that the rated power of the selected unit cannot exceed the maximum power that can be connected to the new selected node.

Two additional aspects are to be considered for this study: the expected yearly variation of loads and the time at which each PV generator would be connected to the distribution system. Figure 3 shows the variation assumed for all loads of a given feeder during the evaluation period. Note that negative variations have also been assumed. In the short term evaluation presented in the previous section, the connection of the generation units (when more than one unit had to be allocated) was simultaneous; that is, all units had to be connected at the beginning of the year. As already mentioned, in the study presented in this section, the connection of units is not simultaneous but sequential; Table VII shows the year of connection for each PV generation unit. By default, it is assumed that the generators are connected at the beginning of the year.

To understand the way in which the new study has been carried out, it is worth mentioning that:

- Up to 8 PV generation units will be connected to the three feeders of the test system; see Table VII.
- The optimization period, every time a PV generation unit is added to the system, is 10 years. That is, the target upon which the locations and rated powers of the units are selected is the minimization of energy losses during 10 years. According to this, if a unit is connected at the beginning of year 6, the energy losses to be accounted for are those produced from the beginning of year 6 until the end of year 15 (see Table VII).

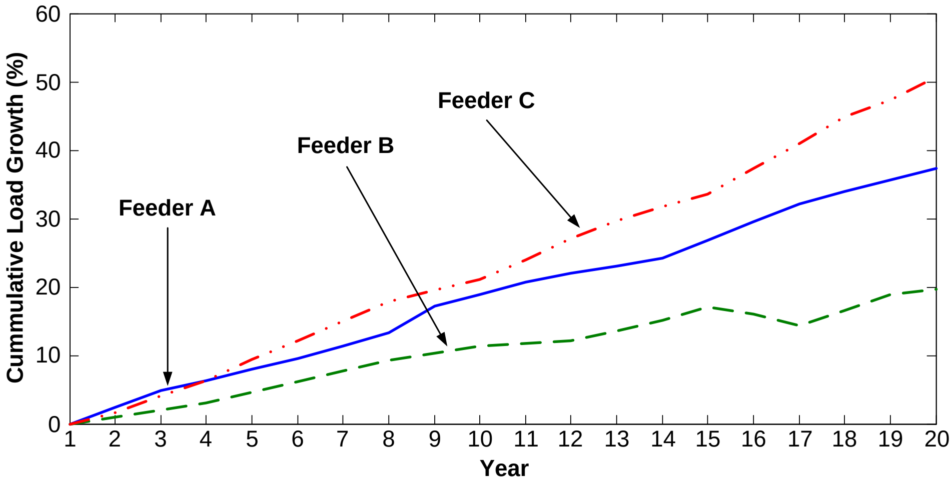


Figure 3. Load growth.

Table VII – Scenario for Long Term Evaluation

PV Unit	Year																
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	•	X	X	X	X	X	X	X	X	X	–	–	–	–	–	–	–
2		•	X	X	X	X	X	X	X	X	X	–	–	–	–	–	–
3			•	X	X	X	X	X	X	X	X	X	–	–	–	–	–
4				•	X	X	X	X	X	X	X	X	X	–	–	–	–
5					•	X	X	X	X	X	X	X	X	X	–	–	–
6						•	X	X	X	X	X	X	X	X	X	–	–
7							•	X	X	X	X	X	X	X	X	X	–
8								•	X	X	X	X	X	X	X	X	X

Notes: • = Year of connection, X = Considered for optimization; – = Not considered for optimization

Table VIII gives the energy losses that would correspond to the entire period of evaluation (i.e., 17 years) without distributed generation and considering the three types of load models.

Table IX shows the results obtained upon the application of both the conventional Monte Carlo and the refined method during the period of evaluation (17 years). The number of runs to be used every year PV generation is connected will be based on the experience obtained from the previous study. As shown in Tables IV and V, when one generator is to be connected, 3120 runs will suffice. Table X summarizes the main results corresponding to each load model.

The maximum coincident active power measured at the substation terminals for each load model during the period of study is as follows: (1) constant power model = 5938.1 kW, (2) constant impedance model = 5547.7 kW; (3) ZIP model = 5727.1 kW. This means that after connecting eight generation units, the total rated power of all the generation units is less than 60% of the maximum coincident power of the system at the end of the period. In addition, one can observe that the maximum rated power of a generation unit can exceed 1400 kW with some load models. Such high value can be accepted according to the interconnection policies adopted by many utilities. However, it can exceed the limit adopted by other utilities, see for instance [21]; in such case, the procedure should be modified to include that limit in calculations.

Figure 4 shows the rated power of the generation unit to be connected each year and the corresponding feeder to which it must be connected when using the conventional Monte Carlo method. The plots also show the cumulative reduction of energy losses, not the yearly reduction.

As expected, the largest value of the rated powers corresponds to the first unit to be allocated; that is, the rated power of the unit to be connected at the beginning of year 1 is larger than the rated power of any unit to be connected in subsequent years, irrespectively of the load model. However, due to the yearly load variation, when a unit is to be connected to a feeder in which other generation units have been previously connected, the optimum rated power of the new PV generation unit will not always be smaller than any other PV generator in operation because the energy losses to be compensated for a certain 10-year term could be larger than for a previous term; for instance, rated powers to be connected at the beginning of year 5 are larger than those to be connected at the beginning of year 4. As with the short term evaluation, the highest reduction of energy losses corresponds now to the constant power load model, while the lowest generation is required when the load is represented as constant impedance.

Observe that when comparing the results presented in Table IX with those presented in Table V, the resulting rated power values are different; this is basically due to the fact that in Table IX the location and rated power of the PV generation unit have been derived from considering a long term evaluation (i.e. 10 years), which can justify larger values.

The total power to be allocated is similar with the three load models: differences between load models are less than 2% with either the conventional Monte Carlo method or the refined one (see Table X). As for the reduction of energy losses, the resulting values are different for each load model, but differences are again not too large. In addition, the resulting energy losses are basically the same with the two Monte Carlo approaches. This supports the conclusion that a quasi-optimum solution can be reached by considering different combinations of locations and rated powers because the optimum reduction of energy losses is not very sensitive with respect to rated powers and locations of generators.

The behavior of the energy loss reduction deserves some special attention. Figure 4 shows the energy loss reduction obtained at the end of a given year considering all the energy losses produced from the beginning of the period (i.e. year 1). One can note that the total reduction at the end of the studied period is not too large; less than 11%. There are several causes that justify this quantity.

Table VIII – Long Term Evaluation (17 years) – Energy Losses without Distributed Generation

	Constant power load model	Constant impedance load model	ZIP load model
Feeder A	1498994.5 kWh	1457234.6 kWh	1476761.4 kWh
Feeder B	1180259.1 kWh	1135125.5 kWh	1156116.3 kWh
Feeder C	3786100.3 kWh	3543363.1 kWh	3656742.1 kWh
Total system	6465354.0 kWh	6135723.3 kWh	6289620.0 kWh

Table IX – Long Term Evaluation (17 years) – Comparison of Simulation Results

Generation unit		Constant power load model	Constant impedance load model	ZIP load model
		Conventional - Refined method	Conventional - Refined method	Conventional - Refined method
Unit 1	Node	C723 - C724	C724 - C724	C724 - C724
	Rated power (kW)	1415.2 - 1334.4	1228.4 - 1277.7	1414.1 - 1301.4
Unit 2	Node	A766 - A765	A767 - A766	A765 - A766
	Rated power (kW)	375.2 - 415.6	373.7 - 414.4	367.8 - 344.1
Unit 3	Node	B739 – B739	B735 - B736	B737 - B739
	Rated power (kW)	501.9 - 505.5	556.2 - 473.9	521.9 - 511.6
Unit 4	Node	C729 - C715	C715 - C709	C711 - C709
	Rated power (kW)	279.2 - 288.3	195.8 - 321.7	227.7 - 328.1
Unit 5	Node	C709 - C709	C709 - C715	C715 - C715
	Rated power (kW)	237.4 - 245.6	325.1 - 193.2	144.7 - 203.7
Unit 6	Node	C715 - C729	C729 - C729	C722 - C729
	Rated power (kW)	163.2 - 156.7	142.0 - 154.3	191.1 - 229.8
Unit 7	Node	A754 - B727	A749 - B726	A749 - A749
	Rated power (kW)	236.6 - 184.3	272.8 - 104.5	187.0 - 143.0
Unit 8	Node	B724 - C722	C722 - C722	B724 - B727
	Rated power (kW)	185.0 - 237.2	187.5 - 141.3	136.4 - 143.9
Energy losses (kWh)		5766679.8 - 5777333.0	5547082.9 - 5551270.4	5660782.4 - 5659358.8
Simulation time (s)		65365.1 - 21390.1	63091.8 - 20131.5	70679.4 - 22565.9

Table X – Long Term Evaluation (17 years) – Summary of Main Results

Load Model	Total Generation (kW) (Conventional – Refined)	Cumulative Energy Loss Reduction (%) (Conventional – Refined)
Constant power	3393.9 - 3368.1	10.80 - 10.64
Constant impedance	3281.7 - 3081.5	9.59 - 9.52
ZIP	3191.1 - 3206.0	9.99 - 10.02

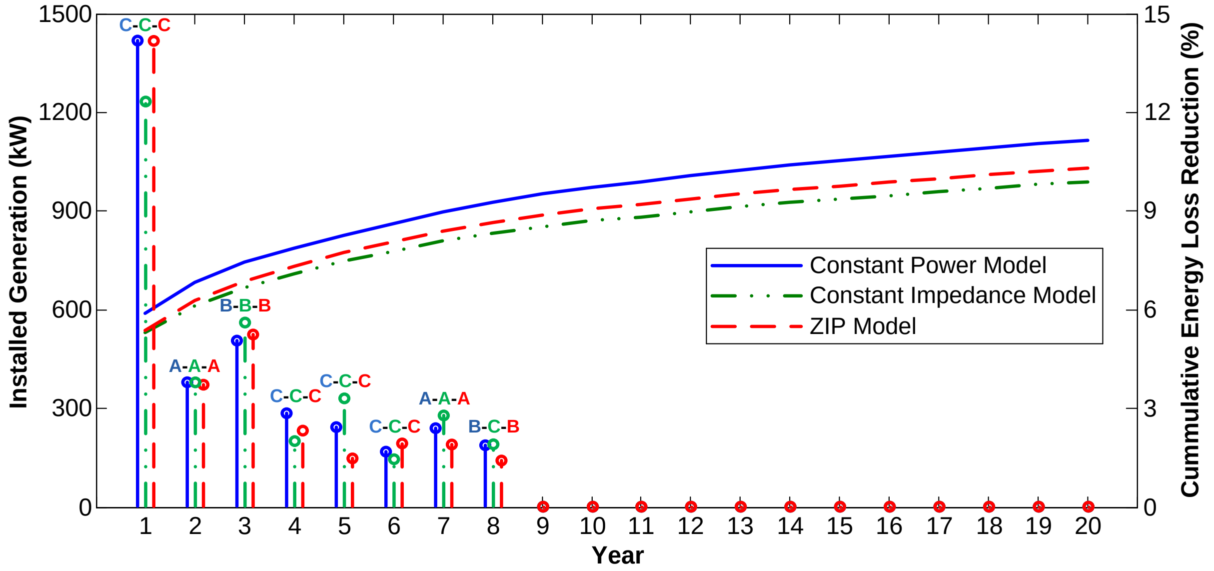


Figure 4. Sequential connection of optimum rated PV generators – Conventional Monte Carlo method.

Consider the reduction achieved during the first year (about 6%); the main reasons for this result might be the following:

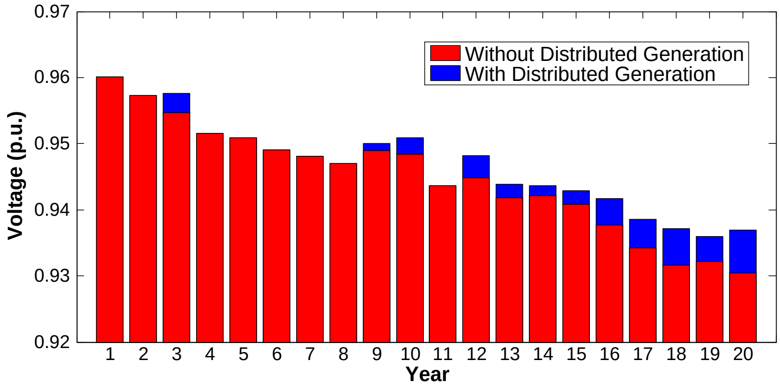
- only losses produced in Feeder C, to which the first unit is to be connected, are compensated by the PV generator (although the substation transformer couples the three feeders and loads can be voltage dependent, the effect of the coupling is very small on the other two feeders);
- since PV generators inject only active power, energy losses caused by the reactive component of load currents are not compensated;
- the connected PV generators will not inject power during many hours of the day and the injected power during other hours will be below or far below the maximum value they can inject;
- since the optimization period is 10 years and the load at the feeder to which the generator will be connected will be higher at the end of this period, the selected rated power will be such that the unit will overcompensate active load during the first years and undercompensate during the last years.

One can also observe that the reduction of energy losses continues after the last PV generation unit has been connected. Remember that the last unit is connected at the beginning of the 8th year, and its rated power is selected to minimize losses until the end of the 17th year. However, one can also observe that the cumulative energy loss reduction continues increasing after the optimization period (i.e. after year 17). The condition to obtain this result is proved in the Appendix of this paper and can be summarized as follows (see Appendix for definitions): the cumulative factor at the end of one year will be larger than at the end of the previous year if the energy loss reduction factor corresponding to that year is larger than the cumulative factor at the end of the previous year. This means that one year after the optimization period (i.e. end of year 18) the cumulative energy loss reduction factor will exceed the cumulative energy loss reduction factor at the end of the optimization period (i.e. end of year 17) if the energy loss reduction factor corresponding to year 18 exceeds the cumulative energy loss reduction factor at the end of year 17. At the end of the optimization period the cumulative energy loss reduction factor with a constant power load model is less than 11% (see Figure 4) while the energy loss reduction factor during the subsequent years is above 11.5%; therefore, if no more generation units are connected, the trend will continue until the year at which the cumulative energy loss reduction factor exceeds the energy loss reduction factor corresponding to the subsequent year.

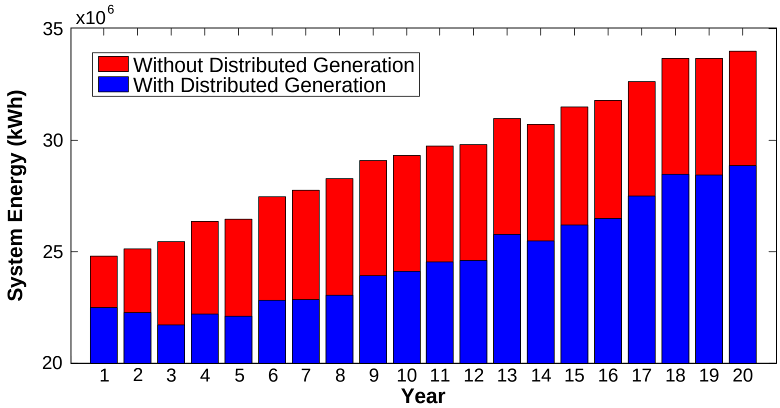
Obviously, the reduction of energy losses could be larger if the number of units to be connected was higher. The study presented in this section is not aimed at estimating the number of units that could achieve the maximum reduction of energy losses.

From the simulation times required in the case studies presented in Table IX it is evident that a significant reduction in the CPU time can be achieved by using the refined approach only when units are connected one by one. This is an expected result from the study previously presented in Section 4.

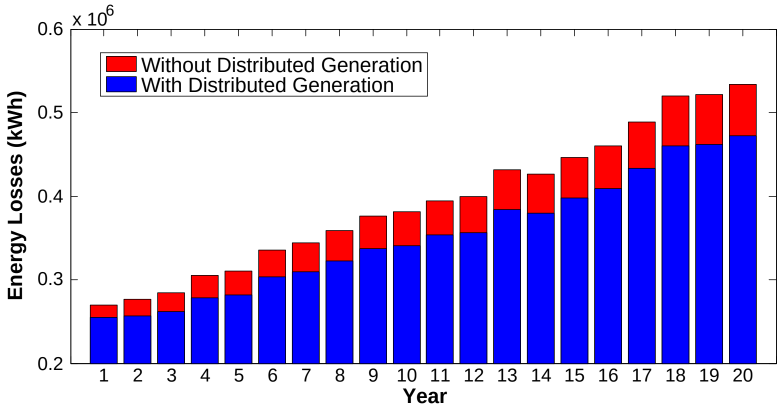
Figure 5 shows some results derived from this study with the ZIP model only. It is obvious from these results that DG has a positive impact on the node voltages (Figure 5a shows how the voltages are increased after distributed generation is connected), and important reductions of the energy required from the HV system and in the distribution test system losses are achieved. For instance, Figure 5b shows how the energy required from the HV system is reduced in about 5000 MWh during the last years of the evaluation period.



a) Impact of DG on the minimum voltage values



b) Energy required from the HV system (measured at the secondary substation terminals)



c) Energy losses (without considering substation losses)

Figure 5. Simulation results – Conventional Monte Carlo method - ZIP Model.

Some interesting conclusions can also be derived from the results presented in these plots. For instance, randomness in load and generation values is evident from the minimum voltage values obtained for every year: although the loads are larger at the end of the 9th year than at the end of the 8th year in all feeders and no generation is connected after the 8th year, the impact on the minimum voltage obtained during the 9th year is larger. This is due to a (random) coincidence of large load and low generation at the time the lowest voltage was achieved for the 8th year. On the other hand, one

can conclude that, in spite of that randomness, after the last unit has been connected, the energy not required from the HV system remains basically constant during the last years; see plots of Figures 5b and 5c. Note also that the negative load variation assumed for some year is also reflected in Figure 5.
It is important to keep in mind that with the time step used in this study (i.e. 1 hour) some relevant information and behavior is missed. For instance, the actual impact of a sudden cloudy scenario cannot be analyzed since the time frame required for such a study should be in the range of one minute. In addition, the test system does not include voltage regulators that could solve undervoltage conditions caused by a sudden reduction of the PV generation. It is also important to remember that the load variation when steps of one minute or less are used will not be as smooth as assumed in this study (for which one-hour step is used); see for instance [22] and [23].

6. A New Approach for Optimum Allocation of PV Generation

A. A Heuristic Methodology

The following conclusions are derived from the results presented in the previous section:

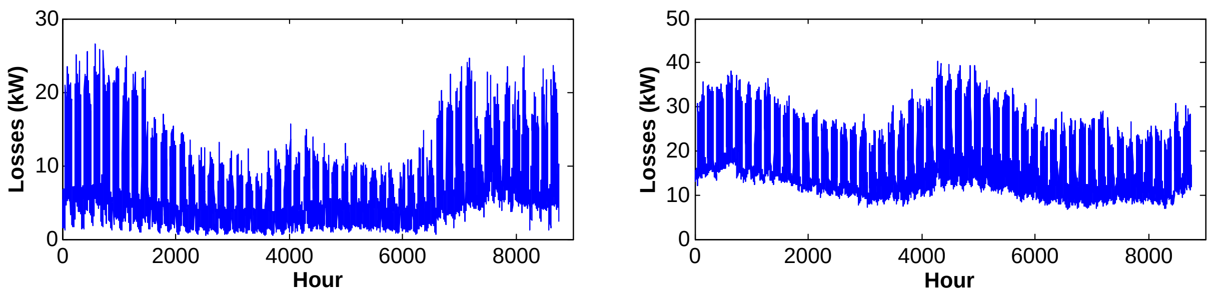
- The first three generation units are connected to the three feeders (one per feeder) and the order in which they are connected goes from the feeder with the highest initial energy losses (Feeder C) to the feeder with the lowest initial energy losses (Feeder B). Note that this connection order (C-A-B) does not correspond to the order that results from feeder loads (the highest load is in Feeder C and the lowest one is in Feeder A, so this order is then C-B-A), see Table VIII. From the fourth year on, the feeders to which optimal units are to be connected depend on the remaining energy losses and the number of units already connected to each feeder; both quantities can vary with the load model. See below for more details.
- The resulting energy losses at the end of the evaluation period (i.e. end of year 17) are similar with all load models.

This suggests that the optimum allocation might be carried out using a different procedure that could be summarized as follows: (1) every time a generation unit has to be allocated, determine the feeder in which the highest energy loss reduction can be achieved for the period of study (in this work the period is 10 years); (2) proceed to estimate the optimum location and rated power by scanning only the feeder to which the unit will be connected.
Note that if the Monte Carlo method is going to be applied considering only one feeder, the number of runs/samples could be much lower than when all the feeders are included in the node selection, and the simulation time can be significantly reduced by using the refined method.
An important aspect to be considered is that the selection of the feeder to which the new optimum generation unit will be connected cannot be based only on the energy losses that would have resulted before connecting the generation unit during the period of evaluation (e.g. 10 years). As detected from the results presented in Table V, the maximum energy loss reduction that can be achieved for a given feeder after connecting an optimum generator depends not only on the expected energy losses for that feeder but also on the number of generation units already connected to the feeder.
Table XI illustrates this behavior. The table provides the additional maximum percentage of energy loss reduction that can be achieved during one year in each feeder (not in the entire system) with the three load models every time a new generation unit is connected. The table shows the factors that result for the first four units; the study was actually expanded to consider that up to five units could be connected to each feeder.
The reduction percentages have been calculated with respect to the energy losses of the corresponding feeder. According to the table, the maximum reduction corresponds always to the first unit. Note that the additional reduction that can be achieved with subsequent units is rather small; in addition, the reduction factors depend on the feeder and the load model, and for every feeder all of them decrease as the number of units connected to the feeder increases.
Note also that the maximum energy loss reduction factor that can be achieved with the first unit is different for every feeder. This is a consequence of loads and losses profiles in every feeder. As an example, Figure 6 compares the distribution of losses for Feeders B and C during the first year; before connecting any unit. One can observe that if the PV generation is going to be larger during summer

time (as one has to assume for a system located at the northern hemisphere), then the impact will be comparatively higher in Feeder C than in Feeder B.

Table XI – Short Term Evaluation (1 year) – Additional Reduction of Energy Losses

Constant Power Load Model						
Unit	Feeder A		Feeder B		Feeder C	
	Losses (kWh)	Reduction (%)	Losses (kWh)	Reduction (%)	Losses (kWh)	Reduction (%)
0	62924.80	-----	56683.40	-----	153289.30	-----
1	58955.75	6.31	53269.15	6.02	136934.08	10.67
2	58667.95	0.49	52783.95	0.91	135737.67	0.87
3	58596.23	0.12	52661.60	0.23	134896.12	0.62
4	58535.42	0.10	52595.28	0.13	134330.47	0.42
Constant Impedance Load Model						
Unit	Feeder A		Feeder B		Feeder C	
	Losses (kWh)	Reduction (%)	Losses (kWh)	Losses (kWh)	Reduction (%)	Losses (kWh)
0	62645.10	-----	55593.90		148248.60	
1	59028.84	5.77	52484.16	5.59	133951.72	9.64
2	58685.15	0.58	52133.23	0.67	133065.36	0.66
3	58506.99	0.30	51958.05	0.34	132350.99	0.54
4	58402.53	0.18	51854.18	0.20	131772.04	0.44
ZIP Load Model						
Unit	Feeder A		Feeder B		Feeder C	
	Losses (kWh)	Reduction (%)	Losses (kWh)	Reduction (%)	Losses (kWh)	Reduction (%)
0	62768.90	-----	56102.20	-----	150643.10	-----
1	59013.21	5.98	52812.49	5.86	135429.76	10.10
2	58579.31	0.74	52333.56	0.97	134610.63	0.60
3	58454.01	0.21	52108.37	0.43	133921.26	0.51
4	58328.39	0.21	51951.19	0.30	133531.09	0.29



Feeder B | Feeder C

Figure 6. Power losses – First year.

The following comments are aimed at illustrating some consequences of these results. Consider, for instance, Feeder C; according to Table XI, the first unit can cause an energy loss reduction of up to 10.67% if the constant power load model is used, but the reductions achieved with subsequent units are always below 1%. The selection of the feeder to which the first unit will be connected must be made taking into account not only the energy losses on that feeder before connecting the unit but also the potential reduction of energy losses that the unit can achieve (according to Table XI). In fact this rule applies every time a unit is to be connected, and this means that the feeder to which the next unit will be connected is not necessarily that with the highest energy losses, but that in which the next unit will cause the highest reduction of losses.

The reduction factors provided in Table XI will be used to select the feeder to which the optimum generation units will be connected using the following heuristic approach:

- When the first unit has to be allocated, the feeder to be selected is that in which the largest energy loss reduction during the period of study (i.e. 10 years) can be achieved.
- When one or more generation units have already been allocated in the distribution system, the

selection of the next feeder to which a unit will be connected is made by estimating the potential reduction of energy losses that the next unit can cause. The reduction percentages that a new unit can cause have been estimated from the previous studies and are as shown in Table XII.

Table XII – Long Term Evaluation – Percentage Reduction of Energy Losses (%)

Feeder	Constant Power					Constant Impedance					ZIP				
	U1	U2	U3	U4	U5	U1	U2	U3	U4	U5	U1	U2	U3	U4	U5
A	6.5	0.50	0.15	0.10	0.05	6.0	0.60	0.30	0.20	0.10	6.0	0.75	0.20	0.20	0.19
B	6.0	0.90	0.25	0.15	0.05	5.5	0.70	0.35	0.20	0.10	6.0	1.0	0.45	0.30	0.05
C	10.5	0.85	0.6	0.4	0.1	9.5	0.65	0.55	0.45	0.15	10.0	0.60	0.50	0.30	0.25

Note than the quantities specified in Table XII are slightly different from those presented in Table XI. Since the quantities specified in Table XI were estimated from a short term study, they cannot be very accurate for a long term evaluation; therefore, they have been rounded. Since the values shown in the table are only used to rank the feeders to which the subsequent generation units will be connected they do not have to be very accurate. It is obvious than the percentages can only provide good results for long term evaluations if the profiles of losses in each feeder exhibit a similar yearly pattern during the entire evaluation period, as in this study.

To better understand the way in which this procedure must be applied, assume that the first unit has already been allocated. According to all results previously presented, it should have been connected to Feeder C, since it is in this feeder where the largest energy loss reduction can be achieved. For each load model, the location of the second unit will be then selected from the maximum potential energy loss reduction that will result from using the percentages shown in Table XII corresponding to Unit 2 for Feeder C and Unit 1 for feeders A and B.

Table XIII shows the results obtained when the new methodology and the two Monte Carlo methods are applied. The Monte Carlo runs every time a new unit had to be allocated were 1560, irrespectively of the selected feeder. It is evident that, although there are some differences with respect to the results presented in Table IX, the patterns of connections are basically the same and the energy losses at the end of the period are very similar for all the load models and the two Monte Carlo methods. However, the simulation time required with the new methodology is now three times shorter when the refined method is applied. Table XIV summarizes the main results corresponding to each load model.

Table XIII – Long Term Evaluation (17 years) – Comparison of Simulation Results with Alternative (Divide and Conquer) Procedure

Generation unit		Constant power load model	Constant impedance load model	ZIP load model
		Conventional - Refined method	Conventional - Refined method	Conventional - Refined method
Unit 1	Node	C724 - C724	C723 - C723	C724 - C723
	Rated power (kW)	1366.6 - 1363.7	1321.9 - 1298.8	1330.2 - 1341.4
Unit 2	Node	A766 - A765	A766 - A766	A766 - A766
	Rated power (kW)	423.5 - 379.3	380.5 - 434.7	376.8 - 277.8
Unit 3	Node	B737 - B737	B739 - B739	B739 - B739
	Rated power (kW)	502.2 - 462.6	457.4 - 469.9	410.7 - 424.7
Unit 4	Node	C715 - C711	C728 - C729	C715 - C728
	Rated power (kW)	230.4 - 325.9	280.4 - 290.1	182.1 - 291.7
Unit 5	Node	C711 - C715	C711 - C715	C711 - C715
	Rated power (kW)	273.3 - 223.8	229.2 - 143.4	276.3 - 191.5
Unit 6	Node	C729 - C729	C715 - C711	B724 - B719
	Rated power (kW)	185.5 - 108.4	137.9 - 206.5	277.7 - 188.6
Unit 7	Node	B724 - B726	A749 - A757	A749 - A758
	Rated power (kW)	230.0 - 146.6	194.7 - 93.7	192.7 - 188.1
Unit 8	Node	A754 - A748	B727 - B727	C729 - C711
	Rated power (kW)	188.4 - 231.5	189.9 - 100.4	232.1 - 202.2
Energy losses (kWh)		5767106.5 - 5779733.9	5542878.1 - 5554953.7	5647596.8 - 5654882.4
Simulation time (s)		32865.5 - 7290.9	30942.7 - 7067.5	34387.4 - 7916.3

Table XIV – Long Term Evaluation (17 years) – Summary of Main Results with Alternative Procedure

Load Model	Total Generation (kW) (Conventional – Refined)	Cumulative Energy Loss Reduction (%) (Conventional – Refined)
Constant power	3400.2 - 3242.2	10.79 - 10.60
Constant impedance	3192.3 - 3037.9	9.66 - 9.46
ZIP	3278.9 - 3106.4	10.20 - 10.09

As mentioned above, this is not a rigorous procedure because, among other aspects, feeders are actually coupled through the substation transformer, so the loss reduction achieved after connecting a generation unit will impact on the substation terminal voltage. However, one can confidently assume that the calculations will provide accurate enough results when the differences between the estimated energy losses in different feeders are rather large.

In addition, it is evident that the margin for reducing losses in a given feeder after two units have been connected to that feeder will be very small, and estimating the correct feeder to which the next unit has to be connected will be more difficult after two units have been connected to any feeder.

Although the procedure makes the approach dependent of the system under study (i.e. system configuration and feeder load patterns), the results presented in the previous sections support the fact that the reduction of energy losses is not very sensitive to changes of locations and rated powers once the solution is not far from the optimum. That is, a quasi-optimum solution can provide very similar results to that corresponding to the optimum, and in turn it can be obtained by significantly reducing the computation effort.

The proposed divide and conquer approach for long term evaluations can be summarized as follows:

1. Estimate the energy loss reduction factors for every feeder of the system using a one-year evaluation, as shown in Table XI.
2. Estimate the feeders to which the first and subsequent units will be connected using a table similar to Table XII.
3. Obtain the energy loss reduction for the evaluation period (10 years in this study) every time a generation unit is to be connected.

B. Some Remarks

- It is worth insisting on the fact that the selection of the feeder to which the next unit has to be connected results from the combination of the energy losses that will be produced in each feeder during a term (i.e. 10 years) before a unit is connected and the potential reduction of energy losses that such unit will cause in every feeder. This selection can be accurately made assuming there is a low coincidence between load and energy losses profiles corresponding to all feeders; see Figure 6. If the reduction factors shown in Table XII are selected from the experience corresponding to only the first year, as in this study, this is still true if the profiles do not significantly change during the evaluation period and the coincidence between feeders remains low.
- Given the information supposedly available for the test system (see Figure 3), larger energy loss reduction might be achieved by applying a different optimization procedure. According to Figure 3, the load variation can be (and has been) estimated for a period of 20 years; therefore, the optimization could be expanded during the entire period for each PV generation unit, beginning from the moment at which any unit is to be connected to the system. The way in which the information shown in Figure 3 is used in this work assumes that the utility can accurately predict load variations for a period of 10 years, so every time a generation unit is to be connected the optimization target is to minimize energy losses for the next 10 years.

7. Discussion

- The results obtained with all the procedures detailed in previous sections are according to what other similar studies have predicted. For instance, the allocation of capacitor banks for optimum reduction of power losses has been studied in the literature [24]. The theoretical solution for feeder loads uniformly distributed shows that the differences between allocating one or more optimum-size capacitor banks is rather small; this basically means that if capacitor banks were sequentially connected, the first one would be much larger than any other bank subsequently connected, and the reduction of losses achieved with the second or later-connected banks would be small [24]. The optimum allocation of generation units that supply only active power in feeders with uniformly load is the same that for capacitor banks; that is, formulas to obtain the optimum values are the same after swapping active and reactive powers [25].

- It is important to emphasize that the procedures applied in this work are just heuristic approaches in which some rules could be fixed in a different manner. For instance, all procedures have been implemented considering limits for the rated power of the generation units; see, for instance, the description of the procedure presented at the beginning of Section 6, in which the maximum power of the new unit can be the value that results after subtracting the rated power of the already allocated units from the maximum non-coincident power of the corresponding feeder loads. This rule is aimed at matching maximum active generation and maximum active load; however, since load and generation peaks do not occur at the same time this cannot avoid some specific situations (i.e. power flow reversal across the substation). A different rule could be aimed at avoiding such power flow reversal or a power flow value that could exceed the substation rated power.
- In a long term study, in which loads will generally increase with time; the percentage of energy loss reduction can increase due to the load variation. This justifies the not-so-small sizes of the second and subsequent units connected to a given feeder in Tables IX and XIII.
- If generation units are sequentially connected, the energy loss reduction that can be achieved for any feeder might be negligible after the third optimum unit has been connected if the load variation with time is very small. Actually, the connection of four or more units at a given feeder can only be justified due to the load increment.
- In a deregulated market in which independent producers connect their generation units to the distribution system without considering their impact on energy losses, the pattern of connections will usually be very different from that obtained in this paper. According to the present study and with the target of minimum losses, the first unit would leave small room to the subsequent units, which should significantly reduce their sizes. This means that such optimum scenario is not too realistic. However, the study can have some usefulness to utilities since it provides an estimation of the optimum reduction of energy losses, information that can be used by utilities to find out how far from that optimum operation their systems are running.
- Approaches different to that presented here can also be considered and could be easily implemented. For instance, the calendar of connection of PV units could be different (with longer or shorter intervals between the connection of subsequent units) and a different (either shorter or longer) optimization period be used. However, it is doubtful that an accurate estimation of load profiles could always be made for periods longer than 10 years. In addition, remember that the long term study has been carried out by assuming that the test system configuration remains untouched, something that could be untrue in many actual distribution systems.
- Given that a quasi-minimum value of the energy losses can be achieved with several combinations of rated powers, the target could include not only the minimization of energy losses but the minimization of the total rated power generation. In addition, a minimum value of node voltages could be fixed. Remember that, according Figure 5, some node voltage will reach a value below 0.95 pu after connecting the optimum PV generators.
- The study could consider, in addition to the minimization of the total rated power generation, the minimization of PV generation costs. At the time this work is written, those costs are significantly falling, due among other reasons to the number of PV plants that are being connected to the grid elsewhere. This makes difficult any realistic analysis that included generation costs in the target. However, a simple approach that used a sensitivity analysis of the PV generation costs could be very useful; that is, the target would be the optimum allocation of PV generation units aimed at minimizing long term energy losses considering the minimization of PV generation costs and

different decreasing rates of generation costs.

8. Conclusion

This work has presented the results obtained when applying a Monte Carlo method to the optimum allocation of distributed generation on a multi-feeder distribution system considering a long term evaluation (i.e. 10 or more years) and using different voltage-dependent load models.

From the obtained results, it is evident that the application of a conventional Monte Carlo method to very large distribution systems (i.e. with several thousands of nodes) to which several dozens of DG units are to be connected might not be carried out in an affordable time even if a large multicore installation (i.e. with several hundreds of cores) was used. This can be very obvious when a long term evaluation is carried out. However, the results show that the applied method is valid for finding quasi-optimum solutions since the values resulting from consecutive runs produce rather small variations in the energy losses (and in their reduction) once the solution is closed to the optimum one. This somehow justify the application of the refined Monte Carlo method, although the experience derived from this work proves that the achievements in simulation time reductions are significant only when one generator is to be allocated.

The paper has also proved that an alternative divide and conquer procedure can be considered with large multi-feeder systems. A quasi optimum solution can be obtained without having to include the entire distribution system in the optimization procedure: if generators are to be allocated one by one during a given period, the study can be carried out by analyzing only the feeder in which the highest energy loss reduction can be achieved; then the refined approach can be advantageously used to significantly reduce the simulation time.

The main results derived from the application of different load models prove that the total power of the PV generation to be allocated exhibits some dependency with respect the load model, but the differences between the resulting energy losses derived from different load models are not too large.

The allocation of distributed generators in deregulated systems is not made following the approach applied in this paper with which the optimum reduction of energy losses is achieved by connecting the largest generation unit at the beginning of the period with little room for more energy loss reduction in subsequent years. The usefulness of this study is in the insight it provides about the impact that the connection of PV generation can have on the system energy losses; that is, utilities can obtain from this or similar study important information about the maximum loss reduction they should expect. For instance, values shown in Table XIV give an estimation of the optimum energy loss reduction a utility should expect with a given number of generation units. As for this result (i.e. the reduction is below 12%), a provisional conclusion could be that the impact is not too significant. However, it is important to keep in mind that only losses due to the active power load can be decreased with the approach assumed in this study and energy losses cannot be reduced during many hours if only PV generation is connected to the system. Of course, a larger loss reduction can be achieved by also compensating reactive power.

Appendix

Assume the energy losses caused without and with distributed generation during a given year i are ΔE_i and $\Delta E'_i$, respectively The energy loss reduction factor for that year is defined as follows:

$$r_i = \frac{\Delta E_i - \Delta E'_i}{\Delta E_i} \qquad \text{(A1)}$$

This expression can be rewritten to obtain

$$r_i = 1 - \frac{\Delta E'_i}{\Delta E_i} = 1 - f_i \qquad \text{(A2)}$$

where f_i is the ratio between energy losses with and without distributed generation, respectively, for year i.

Note that once a generation unit has been connected the energy losses during any period subsequent to the connection are smaller than they would be without the generation unit. Since the first unit in this study is connected at the beginning of year 1, this means that $1 > f_i > 0$ for any year *i*.

The cumulative energy loss reduction factor for a period of *n* years is defined as:

$$R_n = \frac{\sum_{i=1}^{n} \Delta E_i - \sum_{i=1}^{n} \Delta E'_i}{\sum_{i=1}^{n} \Delta E_i} \qquad \text{(A3)}$$

For an optimization period of *n* years (in Table VII is 17 years), the goal is to determine the condition that has to be fulfilled to obtain a cumulative energy loss reduction factor at year (*n*+1) larger than that at year *n*. That is, the condition to obtain is $R_{n+1} > R_n$.

This can be easily determined by using the following result. Given two fractions (*a*/*b*) and (*x*/*y*), in which *a*, *b*, *x* and *y* are real and positive numbers, if follows that:

$$\begin{aligned} \frac{a+x}{b+y} &> \frac{a}{b} \quad \text{if} \quad \frac{a}{b} < \frac{x}{y} \\ \frac{a+x}{b+y} &< \frac{a}{b} \quad \text{if} \quad \frac{a}{b} > \frac{x}{y} \end{aligned} \qquad \text{(A4)}$$

Equation (A3) can be rewritten as follows:

$$R_n = 1 - \frac{\sum_{i=1}^{n} \Delta E'_i}{\sum_{i=1}^{n} \Delta E_i} = 1 - F_n \qquad \text{(A5)}$$

where

$$F_n = \frac{\sum_{i=1}^{n} \Delta E'_i}{\sum_{i=1}^{n} \Delta E_i} \qquad \text{(A6)}$$

The condition to be obtained ($R_{n+1} > R_n$) will then occur if

$$F_{n+1} < F_n \qquad \text{(A7)}$$

The factor *F* at the end of year (*n* + 1) can be written as:

$$F_{n+1} = \frac{\sum_{i=1}^{n+1} \Delta E'_i}{\sum_{i=1}^{n+1} \Delta E_i} = \frac{\sum_{i=1}^{n} \Delta E'_i + \Delta E'_{n+1}}{\sum_{i=1}^{n} \Delta E_i + \Delta E_{n+1}} \qquad \text{(A8)}$$

According to second inequality of (A4), condition (A7) will be fulfilled if

$$\frac{\sum_{i=1}^{n} \Delta E'_i}{\sum_{i=1}^{n} \Delta E_i} > \frac{\Delta E'_{n+1}}{\Delta E_{n+1}} \qquad \text{(A9)}$$

which can be rewritten as $F_n > f_{n+1}$.

This result is equivalent to the following one:

$$1 - F_n = R_n < r_{n+1} = 1 - f_{n+1} \qquad \text{(A10)}$$

This result means that the value of the cumulative energy loss reduction factor corresponding to the end of a given year, R_{n+1}, will exceed the value corresponding to the end of the previous year, R_n, if the energy loss reduction factor of year *n*+1, r_{n+1}, exceeds the cumulative energy loss reduction factor at the end of the previous year, R_n; that is, the cumulative factor will continue increasing while the energy loss reduction factor of one year is larger than the cumulative factor at the end of the previous year.

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