Computational Coverage of TLG: Displacement*

Glyn Morrill
Universitat Politècnica de Catalunya
Barcelona

Oriol Valentín
Universitat Politècnica de Catalunya
Barcelona

April 14, 2015

Abstract

This paper reports on the coverage of TLG of Morrill (1994) and Moortgat (1997), and on how it has been computer implemented. We computer-analyse examples of displacement: discontinuous idioms, quantification, (medial) relativisation, VP ellipsis, (medial) pied piping, appositive relativisation, parentheticals, gapping, comparative subdeletion, and reflexivisation, and, in the appendix, Dutch verb raising and cross-serial dependency.

Keywords: logical syntax and semantics; parsing as deduction; displacement

1. Introduction

The version of the formalism used is essentially the categorial type logic of Morrill (2014) plus Morrill and Valentín (2014b), and comprises 50 connectives as shown:

	cont. mult.	disc. mult.	add.	qu.	norm. mod.	brack. mod.	exp.	contr. for anaph.
primary	$/$ $\bullet$ $\backslash$ I	$\uparrow$ $\odot$ $\downarrow$ J	$\&$ $\oplus$	$\bigwedge$ $\bigvee$	$\Box$ $\Diamond$	$[\,]^{-1}$ $\langle\rangle$	$!$ $?$	$\mid$
semantically inactive variants	•— —∘ ◐ ∘— —• ◑ W	↑(filled) ↓(open) ◓ ↑(open) ↓(filled) ◒	$\sqcap$ $\sqcup$	$\forall$ $\exists$	■ ♦			
det. synth.	$\triangleleft^{-1}$ $\triangleleft$ $\triangleright^{-1}$ $\triangleright$	ˇ ˆ						except
nondet. synth.	$\div$ $\otimes$	$\Uparrow$ $\circledcirc$ $\Downarrow$						$-$

*Research partially supported by an ICREA Acadèmia to the first author, and, for both authors, by BASMATI MICINN project (TIN2011-27479-C04-03) and grant 2014SGR 890 (MACDA) from AGAUR, Generalitat de Catalunya. We thank two anonymous referees for comments and suggestions.

The heart of the logic is the displacement calculus of Morrill and Valentín (2010) and Morrill et al. (2011) which comprises twin continuous and discontinuous residuated families of connectives having a pure sequent calculus, the tree-based hypersequent calculus, and enjoying Cut-elimination (Valentín, 2012). Other primary connectives are additives, 1st order quantifiers, normal (i.e. distributive) modalities, bracket (i.e. nondistributive) modalities, and the non-linear exponentials, and contraction for anaphora.

We can draw a clear distinction between these primary connectives and the semantically inactive connectives and defined connectives which are abbreviatory and therefore merely for convenience.

There are semantically inactive variants of the continuous and discontinuous multiplicatives, including the words as types predicate W, and semantically inactive variants of the additives, 1st order quantifiers, and normal modalities.

Defined connectives divide into the continuous deterministic (unary) synthetic connectives of projection and injection, and the discontinuous, split and bridge, and the continuous nondeterministic (binary) synthetic connectives of nondirectional division and unordered product, and the discontinuous, nondeterministic extract, infix, and discontinuous product.

Finally there is the negation as failure of ‘except’ (formerly difference), a powerful device for expressing linguistic exceptions (Morrill and Valentín, 2014a).

2. Rules and linguistic applications for primary connectives

1. $$\frac{\Gamma \Rightarrow B\colon \psi \qquad \Delta\langle \overrightarrow{C}\colon z\rangle \Rightarrow D\colon \omega}{\Delta\langle \overrightarrow{C/B}\colon x, \Gamma\rangle \Rightarrow D\colon \omega\{(x\ \psi)/z\}}\ /L \qquad \frac{\Gamma, \overrightarrow{B}\colon y \Rightarrow C\colon \chi}{\Gamma \Rightarrow C/B\colon \lambda y \chi}\ /R$$

2. $$\frac{\Gamma \Rightarrow A\colon \phi \qquad \Delta\langle \overrightarrow{C}\colon z\rangle \Rightarrow D\colon \omega}{\Delta\langle \Gamma, \overrightarrow{A\backslash C}\colon y\rangle \Rightarrow D\colon \omega\{(y\ \phi)/z\}}\ \backslash L \qquad \frac{\overrightarrow{A}\colon x, \Gamma \Rightarrow C\colon \chi}{\Gamma \Rightarrow A\backslash C\colon \lambda x \chi}\ \backslash R$$

3. $$\frac{\Delta\langle \overrightarrow{A}\colon x, \overrightarrow{B}\colon y\rangle \Rightarrow D\colon \omega}{\Delta\langle \overrightarrow{A\bullet B}\colon z\rangle \Rightarrow D\colon \omega\{\pi_1 z/x, \pi_2 z/y\}}\ \bullet L \qquad \frac{\Gamma_1 \Rightarrow A\colon \phi \qquad \Gamma_2 \Rightarrow B\colon \psi}{\Gamma_1, \Gamma_2 \Rightarrow A\bullet B\colon (\phi, \psi)}\ \bullet R$$

4. $$\frac{\Delta\langle \Lambda\rangle \Rightarrow A\colon \phi}{\Delta\langle \overrightarrow{I}\colon x\rangle \Rightarrow A\colon \phi}\ IL \qquad \frac{}{\Lambda \Rightarrow I\colon 0}\ IR$$

Figure 1: Continuous multiplicatives

The continuous multiplicatives of Figure 1, the Lambek connectives, are the basic means of categorial categorization and subcategorization. The directional divisions over, /, and under, \, are exemplified by assignments such as **the**: N/CN for **the man**: N and **sings**: $N\backslash S$ for **John sings**: S, and **loves**: $(N\backslash S)/N$ for **John loves Mary**: S. The continuous product $\bullet$ is exemplified by a ‘small clause’ assignment such as **considers**: $(N\backslash S)/(N\bullet (CN/CN))$ for **John considers Mary socialist**: S.[1] The continuous unit can be used together with additive disjunction to express the optionality of a complement as in **eats**: $(N\backslash S)/(N\oplus I)$

[1]But this makes no different empirical predictions from the more standard type of analysis in CG and G/HPSG which simply treats verbs like consider as taking a noun phrase and an infinitive.

for **John eats fish**: S and **John eats**: S.[2] It can also be used in conjunction with the connective 'except' to prevent the null string being supplied as argument to an intensifier as in **very**: $(CN/CN)/((CN/CN)-I)$ for **very tall man**: CN but $*$**very man**: CN.

$$5. \quad \frac{\Gamma \Rightarrow B\colon \psi \qquad \Delta\langle \overrightarrow{C}\colon z\rangle \Rightarrow D\colon \omega}{\Delta\langle \overrightarrow{C{\uparrow_k}B}\colon x\,|_k\,\Gamma\rangle \Rightarrow D\colon \omega\{(x\ \psi)/z\}}\uparrow_k L \qquad \frac{\Gamma\,|_k\,\overrightarrow{B}\colon y \Rightarrow C\colon \chi}{\Gamma \Rightarrow C{\uparrow_k}B\colon \lambda y\chi}\uparrow_k R$$

$$6. \quad \frac{\Gamma \Rightarrow A\colon \phi \qquad \Delta\langle \overrightarrow{C}\colon z\rangle \Rightarrow D\colon \omega}{\Delta\langle \Gamma\,|_k\,\overrightarrow{A{\downarrow_k}C}\colon y\rangle \Rightarrow D\colon \omega\{(y\ \phi)/z\}}\downarrow_k L \qquad \frac{\overrightarrow{A}\colon x\,|_k\,\Gamma \Rightarrow C\colon \chi}{\Gamma \Rightarrow A{\downarrow_k}C\colon \lambda x\chi}\downarrow_k R$$

$$7. \quad \frac{\Delta\langle \overrightarrow{A}\colon x\,|_k\,\overrightarrow{B}\colon y\rangle \Rightarrow D\colon \omega}{\Delta\langle \overrightarrow{A{\odot_k}B}\colon z\rangle \Rightarrow D\colon \omega\{\pi_1 z/x, \pi_2 z/y\}}\odot_k L \qquad \frac{\Gamma_1 \Rightarrow A\colon \phi \qquad \Gamma_2 \Rightarrow B\colon \psi}{\Gamma_1\,|_k\,\Gamma_2 \Rightarrow A{\odot_k}B}\odot_k R$$

$$8. \quad \frac{\Delta\langle 1\rangle \Rightarrow A\colon \phi}{\Delta\langle \overrightarrow{J}\colon x\rangle \Rightarrow A\colon \phi} JL \qquad \frac{}{1 \Rightarrow J\colon 0} JR$$

Figure 2: Discontinuous multiplicatives

The discontinuous multiplicatives of Figure 2, the displacement connectives, are defined in relation to intercalation. When the value of the k subscript is 1 it may be omitted. Circumfixation, $\uparrow$, is exemplified by a discontinuous idiom assignment **gives**+1+**the**+**cold**+**shoulder**: $(N\backslash S){\uparrow}N$ for **Mary gives John the cold shoulder**: S, and infixation, $\downarrow$, and circumfixation together are exemplified by a quantifier phrase assignment **everyone**: $(S{\uparrow}N){\downarrow}S$ simulating Montague's S14 treatment of quantifying in. Circumfixation and discontinuous product, $\odot$, are illustrated in an assignment to a relative pronoun **that**: $(CN\backslash CN)/((S{\uparrow}N){\odot}I)$ allowing both peripheral and medial extraction: **that John likes**: $CN\backslash CN$ and **that John saw today**: $CN\backslash CN$. Use of the discontinuous product unit, J, in conjunction with except is illustrated in a pronoun assignment **him**: $(((S{\uparrow}N){\uparrow_2}N)-(J{\bullet}((N\backslash S){\uparrow}N))){\downarrow_2}\,(S{\uparrow}N)$ preventing a subject antecedent (Principle B effect).

The additives of Figure 3 have application to polymorphism. For example the additive conjunction & can be used for **rice**: $N\&CN$ as in **rice grows**: S and **the rice grows**: S,[3] and the additive disjunction $\oplus$ can be used for **is**: $(N\backslash S)/(N{\oplus}(CN/CN))$ as in **Bond is 007**: S and **Bond is teetotal**: S.

The quantifiers of Figure 4 have application to features. For example, singular and plural number in **sheep**: $\bigwedge nCNn$ for **the sheep grazes**: S and **the sheep graze**: S. And for a past, present or future tense finite sentence complement: **said**: $(N\backslash S)/\bigvee tSf(t)$ in **John said Mary walked**: S, **John said Mary walks**: S and **John said Mary will walk**: S.

With respect to the normal modalities of Figure 5, the universal has application to intensionality. For example, for a propositional attitude verb **believes**: $\Box((N\backslash S)/\Box S)$ with a modality outermost since the word has a sense, and its sentential complement is an intensional domain, but its subject is not.

The bracket modalities of Figure 6 have application to syntactical domains such as

[2]Note the advantage of this over simply listing intranstive and transitive lexical entries: empirically this latter does not capture the generalisation that in both cases the verb eats combines with a subject to the left, and computationally every lexical ambiguity doubles the lexical insertion search space. Appeal to lexical ambiguity is always available and never interesting, except where there is true ambiguity.

[3]Note the advantage of this approach over assuming an empty determiner: computationally it is not forbidden that there be any number of empty operators in any positions.

9. $$\frac{\Gamma\langle\overrightarrow{A}: x\rangle \Rightarrow C: \chi}{\Gamma\langle\overrightarrow{A\&B}: z\rangle \Rightarrow C: \chi\{\pi_1 z/x\}}\&L_1 \qquad \frac{\Gamma\langle\overrightarrow{B}: y\rangle \Rightarrow C: \chi}{\Gamma\langle\overrightarrow{A\&B}: z\rangle \Rightarrow C: \chi\{\pi_2 z/y\}}\&L_2$$

$$\frac{\Gamma \Rightarrow A: \phi \qquad \Gamma \Rightarrow B: \psi}{\Gamma \Rightarrow A\&B: (\phi, \psi)}\&R$$

10. $$\frac{\Gamma\langle\overrightarrow{A}: x\rangle \Rightarrow C: \chi_1 \qquad \Gamma\langle\overrightarrow{B}: y\rangle \Rightarrow C: \chi_2}{\Gamma\langle\overrightarrow{A\oplus B}: z\rangle \Rightarrow C: z \to x.\chi_1; y.\chi_2}\oplus L$$

$$\frac{\Gamma \Rightarrow A: \phi}{\Gamma \Rightarrow A\oplus B: \iota_1\phi}\oplus R_1 \qquad \frac{\Gamma \Rightarrow B: \psi}{\Gamma \Rightarrow A\oplus B: \iota_2\psi}\oplus R_2$$

Figure 3: Additives

11. $$\frac{\Gamma\langle\overrightarrow{A[t/v]}: x\rangle \Rightarrow B: \psi}{\Gamma\langle\overrightarrow{\bigwedge vA}: z\rangle \Rightarrow B: \psi\{(z\ t)/x\}}\bigwedge L \qquad \frac{\Gamma \Rightarrow A[a/v]: \phi}{\Gamma \Rightarrow \bigwedge vA: \lambda v\phi}\bigwedge R^\dagger$$

12. $$\frac{\Gamma\langle\overrightarrow{A[a/v]}: x\rangle \Rightarrow B: \psi}{\Gamma\langle\overrightarrow{\bigvee vA}: z\rangle \Rightarrow B: \psi\{\pi_2 z/x\}}\bigvee L^\dagger \qquad \frac{\Gamma \Rightarrow A[t/v]: \phi}{\Gamma \Rightarrow \bigvee vA: (t, \phi)}\bigvee R$$

Figure 4: Quantifiers, where † indicates that there is no a in the conclusion

prosodic phrases and extraction islands. For example, **walks**: $\langle\rangle N\backslash S$ for the subject condition, and **before**: $[\,]^{-1}(VP\backslash VP)/VP$ for the adverbial island constraint.

Finally, there are non-linear connectives. The exponentials of Figure 7 have application to sharing. Using the universal exponential, !, for which contraction induces island brackets, we can assign a relative pronoun type **that**: $(CN\backslash CN)/(S/!N)$ allowing parasitic extraction such as **paper that John filed without reading**: CN, where parasitic gaps can appear only in (weak) islands, but can be iterated in subislands, for example, **man who the fact that the friends of admire without praising surprises**.[4]

Using the existential exponential, ?, we can assign a coordinator type **and**: $(?N\backslash N)/N$ allowing iterated coordination as in **John, Bill, Mary and Suzy**: N, or **and**: $(?(S/N)\backslash(S/N))/(S/N)$ for **John likes, Mary dislikes, and Bill hates, London** (iterated right node raising), and so on.

The limited contraction for anaphora, $|$, of Figure 8 also has application to sharing; it can be used for anaphora in an assignment like **it**: $(S\uparrow N)\downarrow(S|N)$ for, e.g., the company$_i$ said it$_i$ **flourished**: S, and it can be used for **such that** relativisation in an assignment **such that**: $(CN\backslash CN)/(S|N)$ for, say, **man such that$_i$ he$_i$ thinks Mary loves him$_i$**: CN.

[4]Morrill (2011b), Chapter 5. In the case that island violations are grammatical, as they are under certain conditions, we assume that the relative pronoun type is not $(CN\backslash CN)/(S/!N)$ but $(CN\backslash CN)/(S/\bigcirc N)$ where $\bigcirc$ is an association and commutation structural modality (Morrill, 1994), Chapter 7. This explains how island violation is possible combinatorially but we leave unanswered the question of how the choice of the relative pronoun type is conditioned by processing factors.

13. $$\frac{\Gamma\langle\overrightarrow{A}: x\rangle \Rightarrow B\colon\psi}{\Gamma\langle\overrightarrow{\Box A}: z\rangle \Rightarrow B\colon\psi\{^{\vee}z/x\}}\Box L \qquad \frac{\Box/\blacksquare\Gamma \Rightarrow A\colon\phi}{\Box/\blacksquare\Gamma \Rightarrow \Box A\colon{}^{\wedge}\phi}\Box R$$

14. $$\frac{\Box/\blacksquare\Gamma\langle\overrightarrow{A}: x\rangle \Rightarrow \Diamond/\blacklozenge B\colon\psi}{\Box\Gamma\langle\overrightarrow{\Diamond A}: z\rangle \Rightarrow \Diamond/\blacklozenge B\colon\psi\{^{\cup}z/x\}}\Diamond L \qquad \frac{\Gamma \Rightarrow A\colon\phi}{\Gamma \Rightarrow \Diamond A\colon{}^{\cap}\phi}\Diamond R$$

Figure 5: Normal modalities, where $\Box/\blacksquare\Gamma$ signifies a structure all the types of which have main connective $\Box$ or $\blacksquare$

15. $$\frac{\Delta\langle\overrightarrow{A}: x\rangle \Rightarrow B\colon\psi}{\Delta\langle[\overrightarrow{[\,]^{-1}A}: x]\rangle \Rightarrow B\colon\psi}[\,]^{-1}L \qquad \frac{[\Gamma] \Rightarrow A\colon\phi}{\Gamma \Rightarrow [\,]^{-1}A\colon\phi}[\,]^{-1}R$$

16. $$\frac{\Delta\langle[\overrightarrow{A}: x]\rangle \Rightarrow B\colon\psi}{\Delta\langle\overrightarrow{\langle\rangle A}: x\rangle \Rightarrow B\colon\psi}\langle\rangle L \qquad \frac{\Gamma \Rightarrow A\colon\phi}{[\Gamma] \Rightarrow \langle\rangle A\colon\phi}\langle\rangle R$$

Figure 6: Bracket modalites

3. Implementation

A computational lexicon and parser integrates the grammatical features of the previous section, and of the remaining connectives, which defines a fragment including:

- the PTQ examples of Dowty et al. (1981), Chapter 7;
- the discontinuity examples of Morrill et al. (2011);
- relativisation, including islands and parasitic gaps;
- constituent coordination, non-constituent coordination, coordination of 'unlike' types, ATBE, and a unitary lexical type analyses of simplex and complex gapping.

The implementation is CatLog2, a categorial parser/theorem-prover comprising 6000 lines of Prolog using backward chaining proof-search in the tree-based hypersequent calculus (Morrill, 2011a), and the focusing of Andreoli (Andreoli, 1992) rather than normalisation as in CatLog (Morrill, 2012). In addition to focusing, the implementation exploits the count-invariance of van Benthem (1991) and Valentín et al. (2013). In this paper we present the second item in the list above.

4. Displacement examples

In this section we analyse the displacement examples of the article Morrill et al. (2011) presenting the displacement calculus.[5] The first example, however, is modified in view of Morrill and Valentín (2014b). It is a discontinuous idiom (we include the indexation of CatLog, which contains the numeration of the source, within the example displays):

(1) (tdc(43)) **[mary]+gave+the+man+the+cold+shoulder** : Sf

[5]Note how in the input to CatLog brackets mark islands: single brackets for weak islands such as subjects and double brackets for strong islands such as relative clauses and coordinate structures (Morrill, 2011b), Chapter 5.

$$17.\qquad \frac{\Gamma\langle A\colon x\rangle \Rightarrow B\colon\psi}{\Gamma\langle !A\colon x\rangle \Rightarrow B\colon\psi}\,!L \qquad \frac{!A_1\colon x_1,\ldots,!A_n\colon x_n \Rightarrow A\colon\phi}{!A_1\colon x_1,\ldots,!A_n\colon x_n \Rightarrow !A\colon\phi}\,!R$$

$$\frac{\Delta\langle !A\colon x,\Gamma\rangle \Rightarrow B\colon\psi}{\Delta\langle \Gamma, !A\colon x\rangle \Rightarrow B\colon\psi}\,!P \qquad \frac{\Delta\langle \Gamma, !A\colon x\rangle \Rightarrow B\colon\psi}{\Delta\langle !A\colon x,\Gamma\rangle \Rightarrow B\colon\psi}\,!P$$

$$\frac{\Delta\langle !A_0\colon x_0,\ldots,!A_n\colon x_n,[!A_0\colon y_0,\ldots,!A_n\colon y_0,\Gamma]\rangle \Rightarrow B\colon\psi}{\Delta\langle !A_0\colon x_0,\ldots,!A_n\colon x_n,\Gamma\rangle \Rightarrow B\colon\psi\{x_0/y_0,\ldots,x_n/y_n\}}\,!C$$

$$18.\qquad \frac{\Gamma \Rightarrow A\colon\phi}{\Gamma \Rightarrow ?A\colon[\phi]}\,?R \qquad \frac{\Gamma \Rightarrow A\colon\phi \qquad \Delta \Rightarrow ?A\colon\psi}{\Gamma,\Delta \Rightarrow ?A\colon[\phi|\psi]}\,?E$$

Figure 7: Exponentials

$$19.\qquad \frac{\Gamma \Rightarrow A\colon\phi \qquad \Delta\langle\overrightarrow{A}\colon x;\overrightarrow{B}\colon y\rangle \Rightarrow D\colon\omega}{\Delta\langle\Gamma;\overrightarrow{B|A}\colon z\rangle \Rightarrow D\colon\omega\{\phi/x,(z\ \phi)/y\}}\,|L$$

$$\frac{\Gamma\langle\overrightarrow{B_0}\colon y_0;\ldots;\overrightarrow{B_n}\colon y_n\rangle \Rightarrow D\colon\omega}{\Gamma\langle\overrightarrow{B_0|A}\colon z_0;\ldots;\overrightarrow{B_n|A}\colon z_n\rangle \Rightarrow D|A\colon\lambda x\omega\{(z_0\ x)/y_0,\ldots,(z_n\ x)/y_n\}}\,|R$$

Figure 8: Limited contraction for anaphora

Lexical lookup yields:

(2) $[\blacksquare Nt(s(f)) : m], \Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/(\exists aNa \RIGHTcircle W[the, cold, shoulder])) :$
$\hat{}\lambda A\lambda B(Past\ ((\check{}shun\ A)\ B)), \blacksquare\forall n(Nt(n)/CNn) : \iota, \Box CNs(m) : man,$
$\blacksquare W[the, cold, shoulder] : 0 \ \Rightarrow\ Sf$

There is the derivation:

$$\frac{}{\boxed{CNs(m)} \Rightarrow CNs(m)}$$

$$\frac{\boxed{CNs(m)} \Rightarrow CNs(m)}{\boxed{\Box CNs(m)} \Rightarrow CNs(m)}\,\Box L \qquad \frac{}{\boxed{Nt(s(m))} \Rightarrow Nt(s(m))}$$

$$\frac{\boxed{\Box CNs(m)} \Rightarrow CNs(m) \qquad \boxed{Nt(s(m))} \Rightarrow Nt(s(m))}{\boxed{Nt(s(m))/CNs(m)}, \Box CNs(m) \Rightarrow Nt(s(m))}\,/L$$

$$\frac{\boxed{Nt(s(m))/CNs(m)}, \Box CNs(m) \Rightarrow Nt(s(m))}{\boxed{\forall n(Nt(n)/CNn)}, \Box CNs(m) \Rightarrow Nt(s(m))}\,\forall L$$

$$\frac{\boxed{\forall n(Nt(n)/CNn)}, \Box CNs(m) \Rightarrow Nt(s(m))}{\boxed{\blacksquare\forall n(Nt(n)/CNn)}, \Box CNs(m) \Rightarrow Nt(s(m))}\,\blacksquare L$$

$$\frac{\boxed{\blacksquare\forall n(Nt(n)/CNn)}, \Box CNs(m) \Rightarrow Nt(s(m))}{\blacksquare\forall n(Nt(n)/CNn), \Box CNs(m) \Rightarrow \boxed{\exists aNa}}\,\exists R$$

$$\frac{}{\boxed{W[the, cold, shoulder]} \Rightarrow W[the, cold, shoulder]}$$

$$\frac{\boxed{W[the, cold, shoulder]} \Rightarrow W[the, cold, shoulder]}{\boxed{\blacksquare W[the, cold, shoulder]} \Rightarrow W[the, cold, shoulder]}\,\blacksquare L$$

$$\frac{\blacksquare\forall n(Nt(n)/CNn), \Box CNs(m) \Rightarrow \boxed{\exists aNa} \qquad \boxed{\blacksquare W[the, cold, shoulder]} \Rightarrow W[the, cold, shoulder]}{\blacksquare\forall n(Nt(n)/CNn), \Box CNs(m), \blacksquare W[the, cold, shoulder] \Rightarrow \boxed{\exists aNa \RIGHTcircle W[the, cold, shoulder]}}\,\RIGHTcircle R$$

$$\frac{}{\boxed{Nt(s(f))} \Rightarrow Nt(s(f))}$$

$$\frac{\boxed{Nt(s(f))} \Rightarrow Nt(s(f))}{\boxed{\blacksquare Nt(s(f))} \Rightarrow Nt(s(f))}\,\blacksquare L$$

$$\frac{\boxed{\blacksquare Nt(s(f))} \Rightarrow Nt(s(f))}{\blacksquare Nt(s(f)) \Rightarrow \boxed{\exists gNt(s(g))}}\,\exists R$$

$$\frac{\blacksquare Nt(s(f)) \Rightarrow \boxed{\exists gNt(s(g))}}{[\blacksquare Nt(s(f))] \Rightarrow \boxed{\langle\rangle\exists gNt(s(g))}}\,\langle\rangle R \qquad \frac{}{\boxed{Sf} \Rightarrow Sf}$$

$$\frac{[\blacksquare Nt(s(f))] \Rightarrow \boxed{\langle\rangle\exists gNt(s(g))} \qquad \boxed{Sf} \Rightarrow Sf}{[\blacksquare Nt(s(f))], \boxed{\langle\rangle\exists gNt(s(g))\backslash Sf} \Rightarrow Sf}\,\backslash L$$

$$\frac{\blacksquare\forall n(Nt(n)/CNn), \Box CNs(m), \blacksquare W[the, cold, shoulder] \Rightarrow \boxed{\exists aNa \RIGHTcircle W[the, cold, shoulder]} \qquad [\blacksquare Nt(s(f))], \boxed{\langle\rangle\exists gNt(s(g))\backslash Sf} \Rightarrow Sf}{[\blacksquare Nt(s(f))], \boxed{(\langle\rangle\exists gNt(s(g))\backslash Sf)/(\exists aNa \RIGHTcircle W[the, cold, shoulder])}, \blacksquare\forall n(Nt(n)/CNn), \Box CNs(m), \blacksquare W[the, cold, shoulder] \Rightarrow Sf}\,/L$$

$$\frac{[\blacksquare Nt(s(f))], \boxed{(\langle\rangle\exists gNt(s(g))\backslash Sf)/(\exists aNa \RIGHTcircle W[the, cold, shoulder])}, \blacksquare\forall n(Nt(n)/CNn), \Box CNs(m), \blacksquare W[the, cold, shoulder] \Rightarrow Sf}{[\blacksquare Nt(s(f))], \boxed{\Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/(\exists aNa \RIGHTcircle W[the, cold, shoulder]))}, \blacksquare\forall n(Nt(n)/CNn), \Box CNs(m), \blacksquare W[the, cold, shoulder] \Rightarrow Sf}\,\Box L$$

This delivers semantics:

(3) $(Past\ ((\check{}shun\ (\iota\ \check{}man))\ m))$

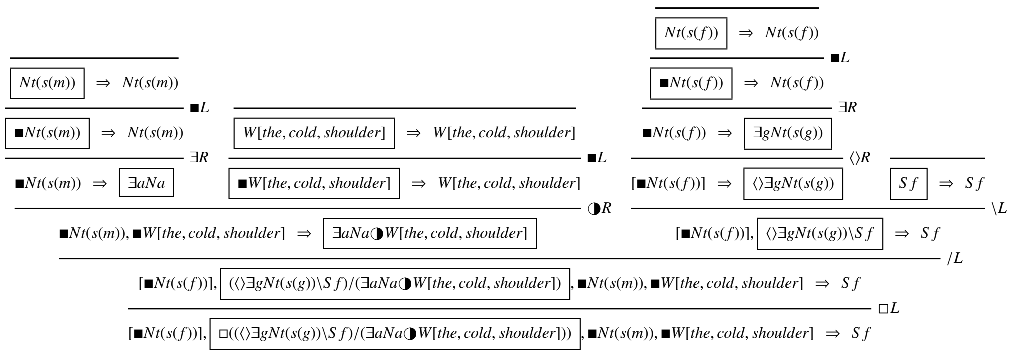
Similarly:

(4) (tdc(4343)) **[mary]+gave+john+the+cold+shoulder** : Sf

Lexical lookup yields:

(5) $[\blacksquare Nt(s(f)) : m], \Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/(\exists aNa\circledcirc W[the, cold, shoulder])) :$
$\hat{}\lambda A\lambda B(Past\ ((\check{}shun\ A)\ B)), \blacksquare Nt(s(m)) : j, \blacksquare W[the, cold, shoulder] : 0 \ \Rightarrow\ Sf$

There is the derivation:



This delivers semantics:

(6) $(Past\ ((\check{}shun\ j)\ m))$

The next example has medial quantification:

(7) (tdc(47)) **[john]+gave+every+book+to+mary** : Sf

Lexical lookup yields:

(8) $[\blacksquare Nt(s(m)) : j], \Box((\langle\rangle\exists aNa\backslash Sf)/(\exists bNb\bullet PPto)) : \hat{}\lambda A\lambda B(Past\ (((\check{}give\ \pi_2 A)\ \pi_1 A)\ B)),$
$\blacksquare\forall g(\forall f((Sf\uparrow Nt(s(g)))\downarrow Sf)/CNs(g)) : \lambda C\lambda D\forall E[(C\ E) \rightarrow (D\ E)], \Box CNs(n) : book,$
$\blacksquare((PPto/\exists aNa)\sqcap\forall n((\langle\rangle Nn\backslash Si)/(\langle\rangle Nn\backslash Sb))) : \lambda FF, \blacksquare Nt(s(f)) : m \ \Rightarrow\ Sf$

There is the derivation:

This delivers semantics:

(9) $\forall C[(\check{~}book\ C) \rightarrow (Past\ (((\check{~}give\ m)\ C)\ j))]$

The following example has subordinate clause existential quantification, exhibiting de re/de dicto ambiguity:

(10) (tdc(50)) **[mary]+thinks+[someone]+left** : $S f$

Lexical lookup yields:

(11) $[\blacksquare Nt(s(f)) : m], \Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/(CPthat\sqcup\Box Sf)) : \hat{}\lambda A\lambda B(Pres\ ((\check{}think\ A)\ B)),$
$[\Box\forall f((Sf\uparrow\blacksquare\forall gNt(g))\downarrow Sf) : \hat{}\lambda C\exists D[(\check{}person\ D) \wedge (C\ D)]], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf) :$
$\hat{}\lambda E(Pres\ (\check{}leave\ E)) \Rightarrow Sf$

There is the de re derivation:

$$\boxed{Nt(s(A))} \Rightarrow Nt(s(A))$$
$$\forall L$$
$$\boxed{\forall gNt(g)} \Rightarrow Nt(s(A))$$
$$\blacksquare L$$
$$\boxed{\blacksquare\forall gNt(g)} \Rightarrow Nt(s(A))$$
$$\exists R$$
$$\blacksquare\forall gNt(g) \Rightarrow \boxed{\exists gNt(s(g))}$$
$$\langle\rangle R$$
$$[\blacksquare\forall gNt(g)] \Rightarrow \boxed{\langle\rangle\exists gNt(s(g))} \qquad \boxed{Sf} \Rightarrow Sf$$
$$\backslash L$$
$$[\blacksquare\forall gNt(g)], \boxed{\langle\rangle\exists gNt(s(g))\backslash Sf} \Rightarrow Sf$$
$$\Box L$$
$$[\blacksquare\forall gNt(g)], \boxed{\Box(\langle\rangle\exists gNt(s(g))\backslash Sf)} \Rightarrow Sf$$
$$\Box R$$
$$[\blacksquare\forall gNt(g)], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf) \Rightarrow \Box Sf$$
$$\sqcup R$$
$$[\blacksquare\forall gNt(g)], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf) \Rightarrow \boxed{CPthat\sqcup\Box Sf}$$

$$\boxed{Nt(s(f))} \Rightarrow Nt(s(f))$$
$$\blacksquare L$$
$$\boxed{\blacksquare Nt(s(f))} \Rightarrow Nt(s(f))$$
$$\exists R$$
$$\blacksquare Nt(s(f)) \Rightarrow \boxed{\exists gNt(s(g))}$$
$$\langle\rangle R$$
$$[\blacksquare Nt(s(f))] \Rightarrow \boxed{\langle\rangle\exists gNt(s(g))} \qquad \boxed{Sf} \Rightarrow Sf$$
$$\backslash L$$
$$[\blacksquare Nt(s(f))], \boxed{\langle\rangle\exists gNt(s(g))\backslash Sf} \Rightarrow Sf$$

$$/L$$
$$[\blacksquare Nt(s(f))], \boxed{(\langle\rangle\exists gNt(s(g))\backslash Sf)/(CPthat\sqcup\Box Sf)}, [\blacksquare\forall gNt(g)], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf) \Rightarrow Sf$$
$$\Box L$$
$$[\blacksquare Nt(s(f))], \boxed{\Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/(CPthat\sqcup\Box Sf))}, [\blacksquare\forall gNt(g)], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf) \Rightarrow Sf$$
$$\uparrow R$$
$$[\blacksquare Nt(s(f))], \Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/(CPthat\sqcup\Box Sf)), [1], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf) \Rightarrow Sf\uparrow\blacksquare\forall gNt(g) \qquad \boxed{Sf} \Rightarrow Sf$$
$$\downarrow L$$
$$[\blacksquare Nt(s(f))], \Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/(CPthat\sqcup\Box Sf)), [\boxed{(Sf\uparrow\blacksquare\forall gNt(g))\downarrow Sf}], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf) \Rightarrow Sf$$
$$\forall L$$
$$[\blacksquare Nt(s(f))], \Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/(CPthat\sqcup\Box Sf)), [\boxed{\forall f((Sf\uparrow\blacksquare\forall gNt(g))\downarrow Sf)}], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf) \Rightarrow Sf$$
$$\Box L$$
$$[\blacksquare Nt(s(f))], \Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/(CPthat\sqcup\Box Sf)), [\boxed{\Box\forall f((Sf\uparrow\blacksquare\forall gNt(g))\downarrow Sf)}], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf) \Rightarrow Sf$$

This delivers semantics:

(12) $\exists B[(\check{}person\ B) \wedge (Pres\ ((\check{}think\ \hat{}(Pres\ (\check{}leave\ B)))\ m))]$

And the de dicto derivation:

$$Nt(s(A)) \Rightarrow Nt(s(A))$$
$$\forall L$$
$$\forall gNt(g) \Rightarrow Nt(s(A))$$
$$\blacksquare L$$
$$\blacksquare\forall gNt(g) \Rightarrow Nt(s(A))$$
$$\exists R$$
$$\blacksquare\forall gNt(g) \Rightarrow \exists gNt(s(g))$$
$$\langle\rangle R$$
$$[\blacksquare\forall gNt(g)] \Rightarrow \langle\rangle\exists gNt(s(g)) \qquad Sf \Rightarrow Sf$$
$$\backslash L$$
$$[\blacksquare\forall gNt(g)], \langle\rangle\exists gNt(s(g))\backslash Sf \Rightarrow Sf$$
$$\Box L$$
$$[\blacksquare\forall gNt(g)], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf) \Rightarrow Sf$$
$$\uparrow R$$
$$[1], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf) \Rightarrow Sf{\uparrow}\blacksquare\forall gNt(g) \qquad Sf \Rightarrow Sf$$
$$\downarrow L$$
$$[(Sf{\uparrow}\blacksquare\forall gNt(g)){\downarrow}Sf], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf) \Rightarrow Sf$$
$$\forall L$$
$$[\forall f((Sf{\uparrow}\blacksquare\forall gNt(g)){\downarrow}Sf)], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf) \Rightarrow Sf$$
$$\Box L$$
$$[\Box\forall f((Sf{\uparrow}\blacksquare\forall gNt(g)){\downarrow}Sf)], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf) \Rightarrow Sf$$
$$\Box R$$
$$[\Box\forall f((Sf{\uparrow}\blacksquare\forall gNt(g)){\downarrow}Sf)], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf) \Rightarrow \Box Sf$$
$$\sqcup R$$
$$[\Box\forall f((Sf{\uparrow}\blacksquare\forall gNt(g)){\downarrow}Sf)], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf) \Rightarrow CPthat\sqcup\Box Sf$$

$$Nt(s(f)) \Rightarrow Nt(s(f))$$
$$\blacksquare L$$
$$\blacksquare Nt(s(f)) \Rightarrow Nt(s(f))$$
$$\exists R$$
$$\blacksquare Nt(s(f)) \Rightarrow \exists gNt(s(g))$$
$$\langle\rangle R$$
$$[\blacksquare Nt(s(f))] \Rightarrow \langle\rangle\exists gNt(s(g)) \qquad Sf \Rightarrow Sf$$
$$\backslash L$$
$$[\blacksquare Nt(s(f))], \langle\rangle\exists gNt(s(g))\backslash Sf \Rightarrow Sf$$

$$/L$$
$$[\blacksquare Nt(s(f))], (\langle\rangle\exists gNt(s(g))\backslash Sf)/(CPthat\sqcup\Box Sf), [\Box\forall f((Sf{\uparrow}\blacksquare\forall gNt(g)){\downarrow}Sf)], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf) \Rightarrow Sf$$
$$\Box L$$
$$[\blacksquare Nt(s(f))], \Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/(CPthat\sqcup\Box Sf)), [\Box\forall f((Sf{\uparrow}\blacksquare\forall gNt(g)){\downarrow}Sf)], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf) \Rightarrow Sf$$

This delivers semantics:

(13) $(Pres\ ((\check{}think\ \hat{}\exists D[(\check{}person\ D) \wedge (Pres\ (\check{}leave\ D))])\ m))$

The next example exhibits classical quantifier scope ambiguity:

(14) (tdc(53)) **[everyone]+loves+someone** : Sf

There is the subject wide scope reading (cf. everyone loves their (respective) mother) and the object wide scope reading (cf. everyone loves (one and) the (same) queen). Lexical lookup yields:

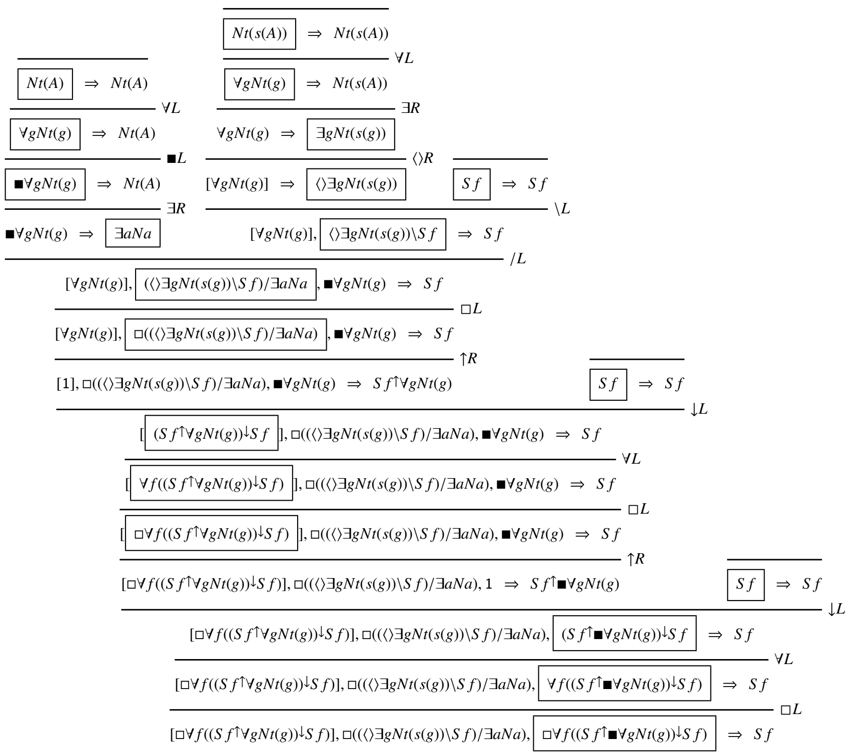
(15) $[\Box\forall f((Sf{\uparrow}\forall gNt(g)){\downarrow}Sf) : \hat{}\lambda A\forall B[(\check{}person\ B) \to (A\ B)]], \Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa) : \hat{}\lambda C\lambda D(Pres\ ((\check{}love\ C)\ D)), \Box\forall f((Sf{\uparrow}\blacksquare\forall gNt(g)){\downarrow}Sf) : \hat{}\lambda E\exists F[(\check{}person\ F) \wedge (E\ F)] \Rightarrow Sf$

There is the subject wide scope derivation as follows in which the subject quantifier is processed closest to the root:

This delivers semantics:

(16) $\forall B[(\check{}person\ B) \rightarrow \exists E[(\check{}person\ E) \wedge (Pres\ ((\check{}love\ E)\ B))]]$

And there is the object wide scope derivation as follows in which the object quantifier is processed closest to the root:



This delivers the semantics:

(17) $\exists B[(\check{}person\ B) \wedge \forall E[(\check{}person\ E) \rightarrow (Pres\ ((\check{}love\ B)\ E))]]$

The next example is of medial relativisation:

(18) (tdc(54)) **dog**+[[**that**+[**mary**]+**saw**+**today**]] : $CNs(n)$

But it is not analysed as in Morrill et al. (2011) but as in Morrill (2011b), Chapter 5. Note the double brackets for the strong island relative clause. The lexical lookup yields:

(19) $\Box CNs(n) : dog, [[\blacksquare\forall n([]^{-1}[]^{-1}(CNn\backslash CNn)/\blacksquare((\langle\rangle Nt(n)\sqcap!\blacksquare Nt(n))\backslash Sf)) :$
$\lambda A\lambda B\lambda C[(B\ C) \wedge (A\ C)], [\blacksquare Nt(s(f)) : m], \Box((\langle\rangle\exists aNa\backslash Sf)/(\exists aNa\oplus CPthat)) :$
$\hat{}\lambda D\lambda E(Past\ ((D \rightarrow F.(\check{}seee\ F); G.(\check{}seet\ G))\ E)), \Box\forall a\forall f((\langle\rangle Na\backslash Sf)\backslash(\langle\rangle Na\backslash Sf)) :$
$\hat{}\lambda H\lambda I(\check{}today\ (H\ I))]]\ \Rightarrow\ CNs(n)$

There is the derivation in Figure 9. This delivers semantics:

(20) $\lambda C[(\check{}dog\ C) \wedge (\check{}today\ (Past\ ((\check{}seee\ C)\ m)))]$

The next example is of VP ellipsis:

(21) (tdc(58a)) [**john**]+**slept**+**before**+[**mary**]+**did** : Sf

(22) $[\blacksquare Nt(s(m)) : j], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf) : \hat{}\lambda A(Past\ (\check{}sleep\ A)), \blacksquare(\forall a\forall f((\langle\rangle Na\backslash Sf)\backslash(\langle\rangle Na\backslash Sf))/Sf) :$
$\lambda B\lambda C\lambda D((before\ B)\ (C\ D)), [\blacksquare Nt(s(f)) : m], \blacksquare\forall a\forall g\forall b\forall h((((\langle\rangle Na\backslash Sg)\uparrow(\langle\rangle Nb\backslash Sh))/(\exists c\langle\rangle Nc\backslash Sf))\backslash$
$((\langle\rangle Na\backslash Sg)\uparrow(\langle\rangle Nb\backslash Sh))) : \lambda E\lambda F((E\ F)\ F)\ \Rightarrow\ Sf$

There is the derivation in Figure 10. This delivers the semantics:

(23) $((before\ (Past\ (\check{}sleep\ m)))\ (Past\ (\check{}sleep\ j)))$

In tdc(64) there is medial pied-piping:

(24) (tdc(64)) **mountain**+[[**the**+**painting**+**of**+**which**+**by**+**cezanne**+[**john**]+**sold**+
for+**tenmilliondollars**]] : $CNs(n)$

Lexical lookup yields:

(25) $\Box CNs(n) : mountain, [[\blacksquare\forall n(Nt(n)/CNn) : \iota, \Box(CNs(n)/PPof) : \hat{}\lambda A((\check{}of\ A)\ \check{}painting),$
$\Box((\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa)) : \hat{}(\check{}of, \lambda BB), \blacksquare\forall n\forall m((Nt(n)\uparrow Nt(m))\downarrow$
$([]^{-1}[]^{-1}(CNm\backslash CNm)/\blacksquare((\langle\rangle Nt(n)\sqcap!\blacksquare Nt(n))\backslash Sf))) : \lambda C\lambda D\lambda E\lambda F[(E\ F) \wedge (D\ (C\ F))],$
$\Box(\forall n(CNn\backslash CNn)/\exists aNa) : \hat{}\lambda G\lambda H((\check{}by\ G)\ H), \blacksquare Nt(s(m)) : c, [\blacksquare Nt(s(m)) : j],$
$\Box((\langle\rangle\exists aNa\backslash Sf)/(\exists bNb\bullet PPfor)) : \hat{}\lambda I\lambda J(Past\ (((\check{}sell\ \pi_2 I)\ \pi_1 I)\ J)),$
$\blacksquare(PPfor/\exists aNa) : \lambda KK, \Box Nt(s(n)) : tenmilliondollars]]\ \Rightarrow\ CNs(n)$

There is the derivation:

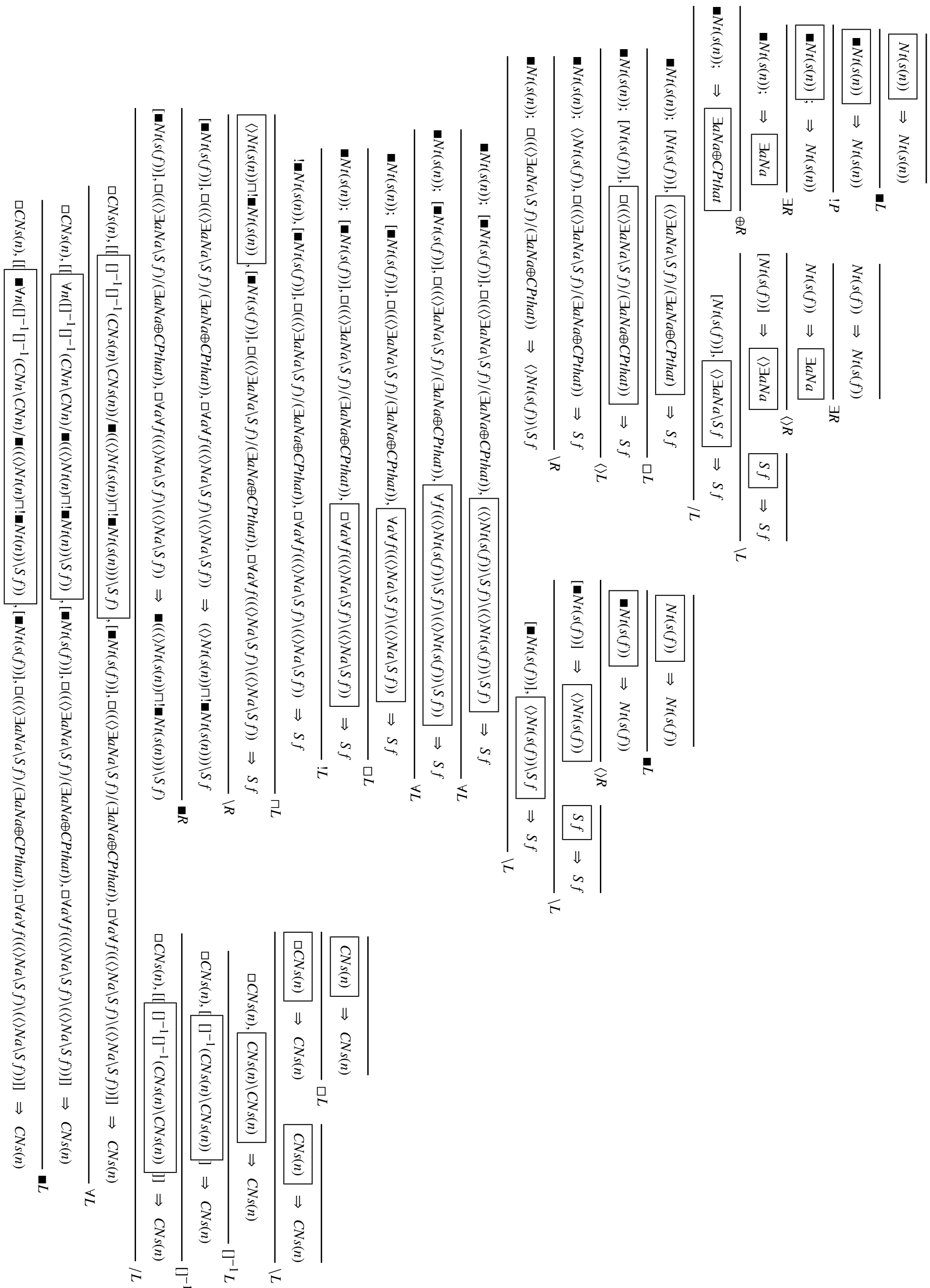


Figure 9: Derivation of **dog which Mary saw today**

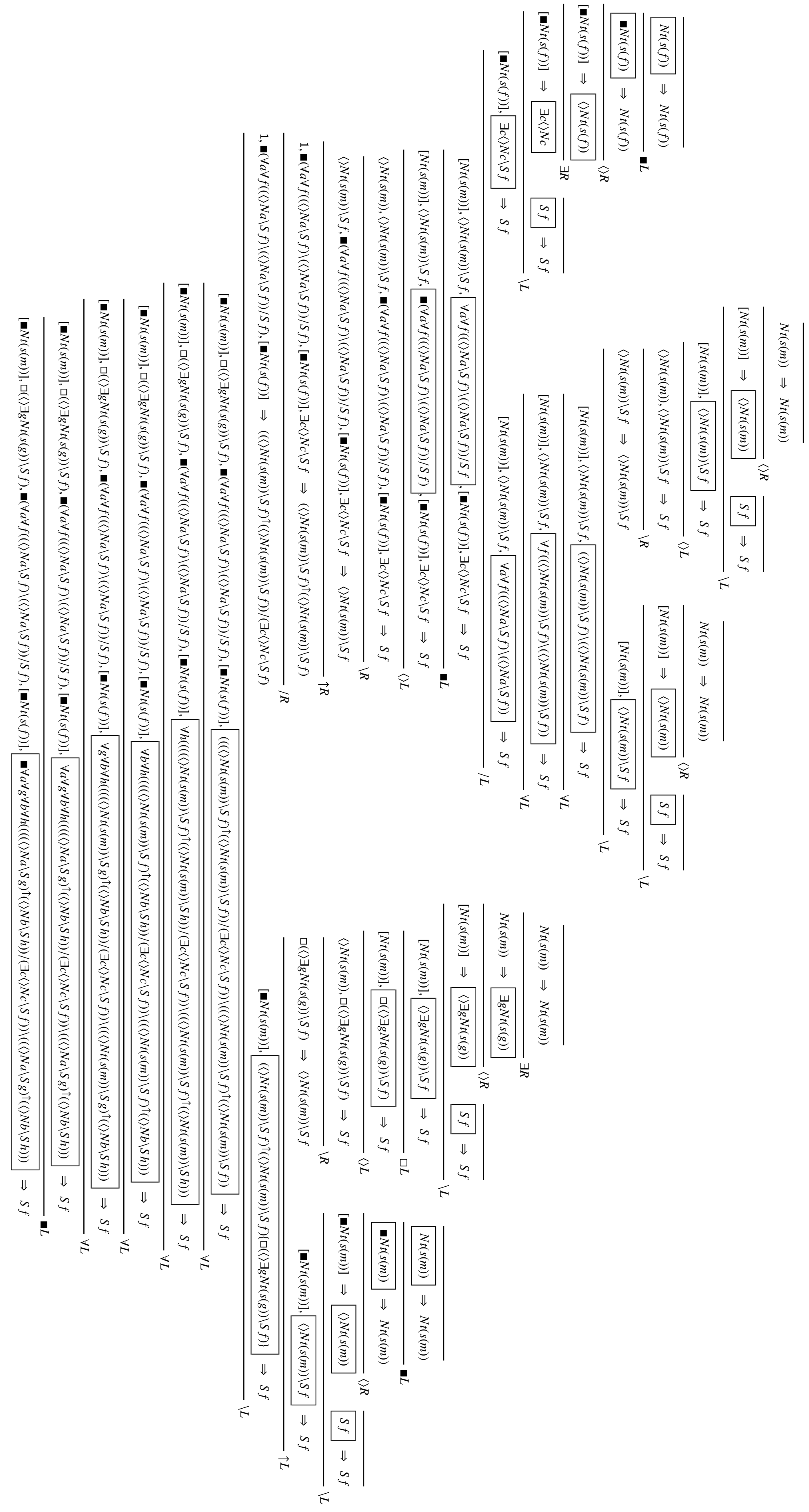


Figure 10: Derivation of **John slept before Mary did**

$$\dfrac{}{Nt(s(n)) \Rightarrow Nt(s(n))}$$

$$\dfrac{Nt(s(n)) \Rightarrow Nt(s(n))}{Nt(s(n)) \Rightarrow \boxed{\exists aNa}}\,\exists R \qquad \dfrac{}{\boxed{PPof} \Rightarrow PPof}$$

$$\dfrac{Nt(s(n)) \Rightarrow \boxed{\exists aNa} \qquad \boxed{PPof} \Rightarrow PPof}{\boxed{PPof/\exists aNa}, Nt(s(n)) \Rightarrow PPof}\,/L$$

$$\dfrac{\boxed{PPof/\exists aNa}, Nt(s(n)) \Rightarrow PPof}{\boxed{(\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa)}, Nt(s(n)) \Rightarrow PPof}\,\&L$$

$$\dfrac{\boxed{(\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa)}, Nt(s(n)) \Rightarrow PPof}{\boxed{\square((\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa))}, Nt(s(n)) \Rightarrow PPof}\,\square L \qquad \dfrac{}{\boxed{CNs(n)} \Rightarrow CNs(n)}$$

$$\dfrac{\boxed{\square((\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa))}, Nt(s(n)) \Rightarrow PPof \qquad \boxed{CNs(n)} \Rightarrow CNs(n)}{\boxed{CNs(n)/PPof}, \square((\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa)), Nt(s(n)) \Rightarrow CNs(n)}\,/L$$

$$\dfrac{\boxed{CNs(n)/PPof}, \square((\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa)), Nt(s(n)) \Rightarrow CNs(n)}{\boxed{\square(CNs(n)/PPof)}, \square((\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa)), Nt(s(n)) \Rightarrow CNs(n)}\,\square L \qquad \dfrac{}{\boxed{CNs(n)} \Rightarrow CNs(n)}$$

$$\dfrac{\boxed{\square(CNs(n)/PPof)}, \square((\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa)), Nt(s(n)) \Rightarrow CNs(n) \qquad \boxed{CNs(n)} \Rightarrow CNs(n)}{\square(CNs(n)/PPof), \square((\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa)), Nt(s(n)), \boxed{CNs(n)\backslash CNs(n)} \Rightarrow CNs(n)}\,\backslash L$$

$$\dfrac{\square(CNs(n)/PPof), \square((\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa)), Nt(s(n)), \boxed{CNs(n)\backslash CNs(n)} \Rightarrow CNs(n)}{\square(CNs(n)/PPof), \square((\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa)), Nt(s(n)), \boxed{\forall n(CNn\backslash CNn)} \Rightarrow CNs(n)}\,\forall L$$

$$\dfrac{}{\boxed{Nt(s(m))} \Rightarrow Nt(s(m))} \qquad \dfrac{\boxed{Nt(s(m))} \Rightarrow Nt(s(m))}{\boxed{\blacksquare Nt(s(m))} \Rightarrow Nt(s(m))}\,\blacksquare L \qquad \dfrac{\boxed{\blacksquare Nt(s(m))} \Rightarrow Nt(s(m))}{\blacksquare Nt(s(m)) \Rightarrow \boxed{\exists aNa}}\,\exists R$$

$$\dfrac{\blacksquare Nt(s(m)) \Rightarrow \boxed{\exists aNa} \qquad \square(CNs(n)/PPof), \square((\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa)), Nt(s(n)), \boxed{\forall n(CNn\backslash CNn)} \Rightarrow CNs(n)}{\square(CNs(n)/PPof), \square((\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa)), Nt(s(n)), \boxed{\forall n(CNn\backslash CNn)/\exists aNa}, \blacksquare Nt(s(m)) \Rightarrow CNs(n)}\,/L$$

$$\dfrac{\square(CNs(n)/PPof), \square((\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa)), Nt(s(n)), \boxed{\forall n(CNn\backslash CNn)/\exists aNa}, \blacksquare Nt(s(m)) \Rightarrow CNs(n)}{\square(CNs(n)/PPof), \square((\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa)), Nt(s(n)), \boxed{\square(\forall n(CNn\backslash CNn)/\exists aNa)}, \blacksquare Nt(s(m)) \Rightarrow CNs(n)}\,\square L \qquad \dfrac{}{\boxed{Nt(s(n))} \Rightarrow Nt(s(n))}$$

$$\dfrac{\square(CNs(n)/PPof), \square((\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa)), Nt(s(n)), \boxed{\square(\forall n(CNn\backslash CNn)/\exists aNa)}, \blacksquare Nt(s(m)) \Rightarrow CNs(n) \qquad \boxed{Nt(s(n))} \Rightarrow Nt(s(n))}{\boxed{Nt(s(n))/CNs(n)}, \square(CNs(n)/PPof), \square((\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa)), Nt(s(n)), \square(\forall n(CNn\backslash CNn)/\exists aNa), \blacksquare Nt(s(m)) \Rightarrow Nt(s(n))}\,/L$$

$$\dfrac{\boxed{Nt(s(n))/CNs(n)}, \square(CNs(n)/PPof), \square((\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa)), Nt(s(n)), \square(\forall n(CNn\backslash CNn)/\exists aNa), \blacksquare Nt(s(m)) \Rightarrow Nt(s(n))}{\boxed{\forall n(Nt(n)/CNn)}, \square(CNs(n)/PPof), \square((\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa)), Nt(s(n)), \square(\forall n(CNn\backslash CNn)/\exists aNa), \blacksquare Nt(s(m)) \Rightarrow Nt(s(n))}\,\forall L$$

$$\dfrac{\boxed{\forall n(Nt(n)/CNn)}, \square(CNs(n)/PPof), \square((\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa)), Nt(s(n)), \square(\forall n(CNn\backslash CNn)/\exists aNa), \blacksquare Nt(s(m)) \Rightarrow Nt(s(n))}{\boxed{\blacksquare\forall n(Nt(n)/CNn)}, \square(CNs(n)/PPof), \square((\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa)), Nt(s(n)), \square(\forall n(CNn\backslash CNn)/\exists aNa), \blacksquare Nt(s(m)) \Rightarrow Nt(s(n))}\,\blacksquare L$$

$$\dfrac{\boxed{\blacksquare\forall n(Nt(n)/CNn)}, \square(CNs(n)/PPof), \square((\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa)), Nt(s(n)), \square(\forall n(CNn\backslash CNn)/\exists aNa), \blacksquare Nt(s(m)) \Rightarrow Nt(s(n))}{\blacksquare\forall n(Nt(n)/CNn), \square(CNs(n)/PPof), \square((\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa)), 1, \square(\forall n(CNn\backslash CNn)/\exists aNa), \blacksquare Nt(s(m)) \Rightarrow Nt(s(n))\uparrow Nt(s(n))}\,\uparrow R$$

①

$$\dfrac{}{\boxed{Nt(s(n))} \Rightarrow Nt(s(n))} \qquad \dfrac{\boxed{Nt(s(n))} \Rightarrow Nt(s(n))}{\boxed{\blacksquare Nt(s(n))} \Rightarrow Nt(s(n))}\,\blacksquare L \qquad \dfrac{\boxed{\blacksquare Nt(s(n))} \Rightarrow Nt(s(n))}{\boxed{\blacksquare Nt(s(n))}; \Rightarrow Nt(s(n))}\,!P \qquad \dfrac{\boxed{\blacksquare Nt(s(n))}; \Rightarrow Nt(s(n))}{\blacksquare Nt(s(n)); \Rightarrow \boxed{\exists bNb}}\,\exists R$$

$$\dfrac{}{\boxed{Nt(s(n))} \Rightarrow Nt(s(n))} \qquad \dfrac{\boxed{Nt(s(n))} \Rightarrow Nt(s(n))}{\boxed{\square Nt(s(n))} \Rightarrow Nt(s(n))}\,\square L \qquad \dfrac{\boxed{\square Nt(s(n))} \Rightarrow Nt(s(n))}{\square Nt(s(n)) \Rightarrow \boxed{\exists aNa}}\,\exists R \qquad \dfrac{}{\boxed{PPfor} \Rightarrow PPfor}$$

$$\dfrac{\square Nt(s(n)) \Rightarrow \boxed{\exists aNa} \qquad \boxed{PPfor} \Rightarrow PPfor}{\boxed{PPfor/\exists aNa}, \square Nt(s(n)) \Rightarrow PPfor}\,/L \qquad \dfrac{\boxed{PPfor/\exists aNa}, \square Nt(s(n)) \Rightarrow PPfor}{\boxed{\blacksquare(PPfor/\exists aNa)}, \square Nt(s(n)) \Rightarrow PPfor}\,\blacksquare L$$

$$\dfrac{\blacksquare Nt(s(n)); \Rightarrow \boxed{\exists bNb} \qquad \boxed{\blacksquare(PPfor/\exists aNa)}, \square Nt(s(n)) \Rightarrow PPfor}{\blacksquare Nt(s(n));\ \blacksquare(PPfor/\exists aNa), \square Nt(s(n)) \Rightarrow \boxed{\exists bNb\bullet PPfor}}\,\bullet R$$

$$\dfrac{}{\boxed{Nt(s(m))} \Rightarrow Nt(s(m))} \qquad \dfrac{\boxed{Nt(s(m))} \Rightarrow Nt(s(m))}{\boxed{\blacksquare Nt(s(m))} \Rightarrow Nt(s(m))}\,\blacksquare L \qquad \dfrac{\boxed{\blacksquare Nt(s(m))} \Rightarrow Nt(s(m))}{\blacksquare Nt(s(m)) \Rightarrow \boxed{\exists aNa}}\,\exists R \qquad \dfrac{\blacksquare Nt(s(m)) \Rightarrow \boxed{\exists aNa}}{[\blacksquare Nt(s(m))] \Rightarrow \boxed{\langle\rangle\exists aNa}}\,\langle\rangle R$$

$$\dfrac{[\blacksquare Nt(s(m))] \Rightarrow \boxed{\langle\rangle\exists aNa} \qquad \boxed{Sf} \Rightarrow Sf}{[\blacksquare Nt(s(m))], \boxed{\langle\rangle\exists aNa\backslash Sf} \Rightarrow Sf}\,\backslash L$$

$$\dfrac{\blacksquare Nt(s(n));\ \blacksquare(PPfor/\exists aNa), \square Nt(s(n)) \Rightarrow \boxed{\exists bNb\bullet PPfor} \qquad [\blacksquare Nt(s(m))], \boxed{\langle\rangle\exists aNa\backslash Sf} \Rightarrow Sf}{\blacksquare Nt(s(n));\ [\blacksquare Nt(s(m))], \boxed{(\langle\rangle\exists aNa\backslash Sf)/(\exists bNb\bullet PPfor)}, \blacksquare(PPfor/\exists aNa), \square Nt(s(n)) \Rightarrow Sf}\,/L$$

$$\dfrac{\blacksquare Nt(s(n));\ [\blacksquare Nt(s(m))], \boxed{(\langle\rangle\exists aNa\backslash Sf)/(\exists bNb\bullet PPfor)}, \blacksquare(PPfor/\exists aNa), \square Nt(s(n)) \Rightarrow Sf}{\blacksquare Nt(s(n));\ [\blacksquare Nt(s(m))], \boxed{\square((\langle\rangle\exists aNa\backslash Sf)/(\exists bNb\bullet PPfor))}, \blacksquare(PPfor/\exists aNa), \square Nt(s(n)) \Rightarrow Sf}\,\square L$$

$$\dfrac{\blacksquare Nt(s(n));\ [\blacksquare Nt(s(m))], \boxed{\square((\langle\rangle\exists aNa\backslash Sf)/(\exists bNb\bullet PPfor))}, \blacksquare(PPfor/\exists aNa), \square Nt(s(n)) \Rightarrow Sf}{!\blacksquare Nt(s(n)), [\blacksquare Nt(s(m))], \square((\langle\rangle\exists aNa\backslash Sf)/(\exists bNb\bullet PPfor)), \blacksquare(PPfor/\exists aNa), \square Nt(s(n)) \Rightarrow Sf}\,!L$$

$$\dfrac{!\blacksquare Nt(s(n)), [\blacksquare Nt(s(m))], \square((\langle\rangle\exists aNa\backslash Sf)/(\exists bNb\bullet PPfor)), \blacksquare(PPfor/\exists aNa), \square Nt(s(n)) \Rightarrow Sf}{\boxed{\langle\rangle Nt(s(n))\sqcap !\blacksquare Nt(s(n))}, [\blacksquare Nt(s(m))], \square((\langle\rangle\exists aNa\backslash Sf)/(\exists bNb\bullet PPfor)), \blacksquare(PPfor/\exists aNa), \square Nt(s(n)) \Rightarrow Sf}\,\sqcap L$$

$$\dfrac{\boxed{\langle\rangle Nt(s(n))\sqcap !\blacksquare Nt(s(n))}, [\blacksquare Nt(s(m))], \square((\langle\rangle\exists aNa\backslash Sf)/(\exists bNb\bullet PPfor)), \blacksquare(PPfor/\exists aNa), \square Nt(s(n)) \Rightarrow Sf}{[\blacksquare Nt(s(m))], \square((\langle\rangle\exists aNa\backslash Sf)/(\exists bNb\bullet PPfor)), \blacksquare(PPfor/\exists aNa), \square Nt(s(n)) \Rightarrow (\langle\rangle Nt(s(n))\sqcap !\blacksquare Nt(s(n)))\backslash Sf}\,\backslash R$$

$$\dfrac{[\blacksquare Nt(s(m))], \square((\langle\rangle\exists aNa\backslash Sf)/(\exists bNb\bullet PPfor)), \blacksquare(PPfor/\exists aNa), \square Nt(s(n)) \Rightarrow (\langle\rangle Nt(s(n))\sqcap !\blacksquare Nt(s(n)))\backslash Sf}{[\blacksquare Nt(s(m))], \square((\langle\rangle\exists aNa\backslash Sf)/(\exists bNb\bullet PPfor)), \blacksquare(PPfor/\exists aNa), \square Nt(s(n)) \Rightarrow \blacksquare((\langle\rangle Nt(s(n))\sqcap !\blacksquare Nt(s(n)))\backslash Sf)}\,\blacksquare R$$

②

$$\boxed{CNs(n)} \Rightarrow CNs(n)$$

$$\frac{\boxed{CNs(n)} \Rightarrow CNs(n)}{\boxed{\Box CNs(n)} \Rightarrow CNs(n)}\ \Box L \qquad \boxed{CNs(n)} \Rightarrow CNs(n)$$

$$\Box CNs(n), \boxed{CNs(n)\backslash CNs(n)} \Rightarrow CNs(n) \quad \backslash L$$

$$\Box CNs(n), [\boxed{[]^{-1}(CNs(n)\backslash CNs(n))}] \Rightarrow CNs(n) \quad []^{-1}L$$

$$\Box CNs(n), [[\boxed{[]^{-1}[]^{-1}(CNs(n)\backslash CNs(n))}]] \Rightarrow CNs(n) \quad []^{-1}L$$

①　②

$$\Box CNs(n), [[\boxed{[]^{-1}[]^{-1}(CNs(n)\backslash CNs(n))/\blacksquare((\langle\rangle Nt(s(n))\sqcap !\blacksquare Nt(s(n)))\backslash Sf)}, [\blacksquare Nt(s(m))], \Box((\langle\rangle \exists aNa\backslash Sf)/(\exists bNb\bullet PPfor)), \blacksquare(PPfor/\exists aNa), \Box Nt(s(n))]] \Rightarrow CNs(n) \quad /L$$

$$\Box CNs(n), [[\blacksquare\forall n(Nt(n)/CNn), \Box(CNs(n)/PPof), \Box((\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa)), \boxed{(Nt(s(n))\uparrow Nt(s(n)))\downarrow([]^{-1}[]^{-1}(CNs(n)\backslash CNs(n))/\blacksquare((\langle\rangle Nt(s(n))\sqcap !\blacksquare Nt(s(n)))\backslash Sf))}, \Box(\forall n(CNn\backslash CNn)/\exists aNa), \blacksquare Nt(s(m)), [\blacksquare Nt(s(m))], \Box((\langle\rangle \exists aNa\backslash Sf)/(\exists bNb\bullet PPfor)), \blacksquare(PPfor/\exists aNa), \Box Nt(s(n))]] \Rightarrow CNs(n) \quad \downarrow L$$

$$\Box CNs(n), [[\blacksquare\forall n(Nt(n)/CNn), \Box(CNs(n)/PPof), \Box((\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa)), \boxed{\forall m((Nt(s(n))\uparrow Nt(m))\downarrow([]^{-1}[]^{-1}(CNm\backslash CNm)/\blacksquare((\langle\rangle Nt(s(n))\sqcap !\blacksquare Nt(s(n)))\backslash Sf)))}, \Box(\forall n(CNn\backslash CNn)/\exists aNa), \blacksquare Nt(s(m)), [\blacksquare Nt(s(m))], \Box((\langle\rangle \exists aNa\backslash Sf)/(\exists bNb\bullet PPfor)), \blacksquare(PPfor/\exists aNa), \Box Nt(s(n))]] \Rightarrow CNs(n) \quad \forall L$$

$$\Box CNs(n), [[\blacksquare\forall n(Nt(n)/CNn), \Box(CNs(n)/PPof), \Box((\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa)), \boxed{\forall n\forall m((Nt(n)\uparrow Nt(m))\downarrow([]^{-1}[]^{-1}(CNm\backslash CNm)/\blacksquare((\langle\rangle Nt(n)\sqcap !\blacksquare Nt(n))\backslash Sf)))}, \Box(\forall n(CNn\backslash CNn)/\exists aNa), \blacksquare Nt(s(m)), [\blacksquare Nt(s(m))], \Box((\langle\rangle \exists aNa\backslash Sf)/(\exists bNb\bullet PPfor)), \blacksquare(PPfor/\exists aNa), \Box Nt(s(n))]] \Rightarrow CNs(n) \quad \forall L$$

$$\Box CNs(n), [[\blacksquare\forall n(Nt(n)/CNn), \Box(CNs(n)/PPof), \Box((\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa)), \boxed{\blacksquare\forall n\forall m((Nt(n)\uparrow Nt(m))\downarrow([]^{-1}[]^{-1}(CNm\backslash CNm)/\blacksquare((\langle\rangle Nt(n)\sqcap !\blacksquare Nt(n))\backslash Sf)))}, \Box(\forall n(CNn\backslash CNn)/\exists aNa), \blacksquare Nt(s(m)), [\blacksquare Nt(s(m))], \Box((\langle\rangle \exists aNa\backslash Sf)/(\exists bNb\bullet PPfor)), \blacksquare(PPfor/\exists aNa), \Box Nt(s(n))]] \Rightarrow CNs(n) \quad \blacksquare L$$

This delivers semantics:

(26) $\lambda D[(\check{}mountain\ D) \wedge (Past\ (((\check{}sell\ \check{}tenmilliondollars)\ (\iota\ ((\check{}by\ c)\ ((\check{}of\ D)\ \check{}painting))))\ j))]$

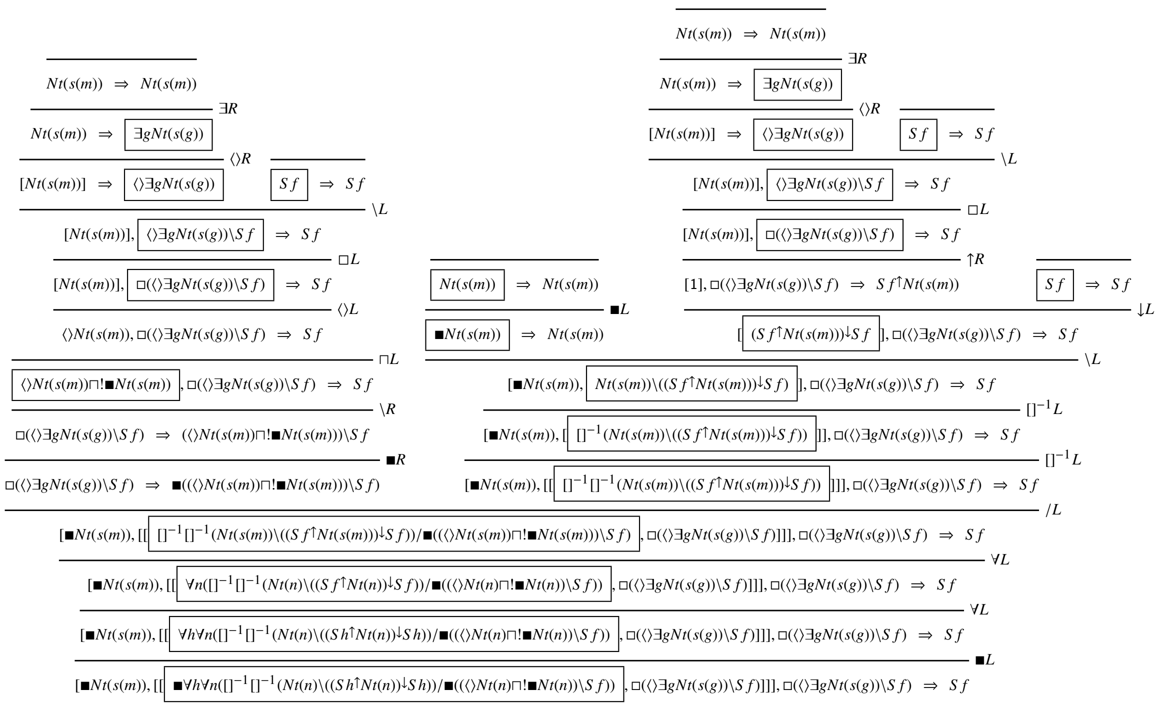
In appositive relativisation a relative clause, marked off by a prosodic phrase, modifies a full noun phrase:

(27) (tdc(67)) **[john**+[[**who**+**jogs**]]]+**sneezed** : Sf

The lexical lookup yields:

(28) $[\blacksquare Nt(s(m)) : j, [[\blacksquare\forall h\forall n([]^{-1}[]^{-1}(Nt(n)\backslash((Sh{\uparrow}Nt(n)){\downarrow}Sh))/\blacksquare((\langle\rangle Nt(n)\sqcap!\blacksquare Nt(n))\backslash Sf)) :$
$\lambda A\lambda B\lambda C[(A\ B)\wedge(C\ B)], \square(\langle\rangle\exists gNt(s(g))\backslash Sf) : \hat{}\lambda D(Pres\ (\check{}jog\ D))]]], \square(\langle\rangle\exists gNt(s(g))\backslash Sf) :$
$\hat{}\lambda E(Past\ (\check{}sneeze\ E)) \Rightarrow Sf$

There is the derivation:



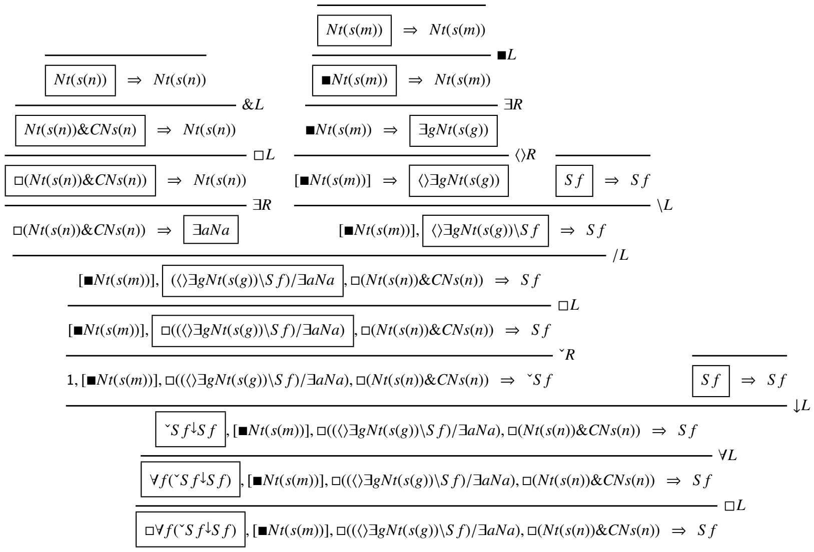
This delivers semantics:

(29) $[(Pres\ (\check{}jog\ j)) \wedge (Past\ (\check{}sneeze\ j))]$

There follow four placements of a parenthetical adverbial within a sentence all derived from the same types and all assigning the same semantics. First:

(30) (tdc(70a)) **fortunately**+[**john**]+**has**+**perseverance** : Sf

(31) $\square\forall f(\check{}Sf{\downarrow}Sf) : fortunately, [\blacksquare Nt(s(m)) : j], \square(((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa) :$
$\hat{}\lambda A\lambda B(Pres\ ((\check{}have\ A)\ B)), \square(Nt(s(n))\&CNs(n)) : \hat{}((gen\ \check{}perseverance), \check{}perseverance)$
$\Rightarrow Sf$

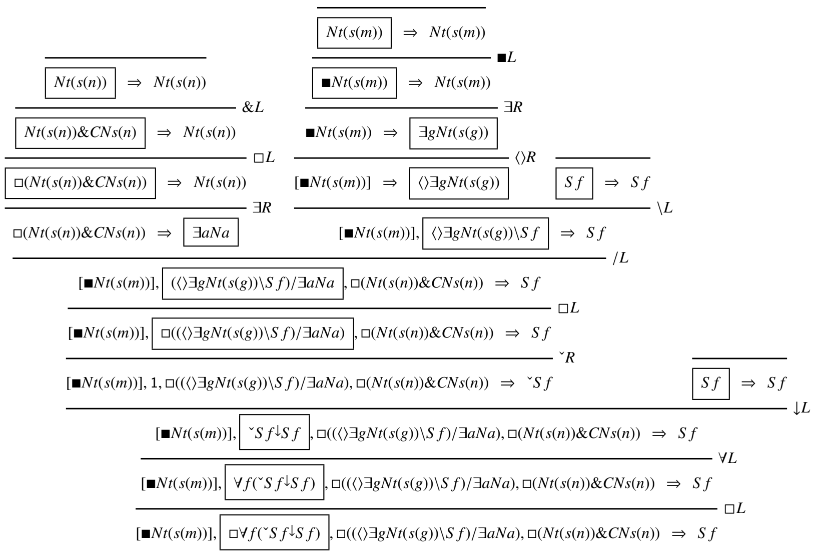


(32) $(\check{}fortunately\ (Pres\ ((\check{}have\ (gen\ \check{}perseverance))\ j)))$

Second:

(33) (tdc(70b)) **[john]+fortunately+has+perseverance** : Sf

(34) $[\blacksquare Nt(s(m)) : j], \Box\forall f(\check{}Sf{\downarrow}Sf) : fortunately, \Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa) :$
$\hat{}\lambda A\lambda B(Pres\ ((\check{}have\ A)\ B)), \Box(Nt(s(n))\&CNs(n)) : \hat{}((gen\ \check{}perseverance), \check{}perseverance)$
$\Rightarrow\ Sf$



(35) $(\check{}fortunately\ (Pres\ ((\check{}have\ (gen\ \check{}perseverance))\ j)))$

Third:

(36) (tdc(70c)) **[john]+has+fortunately+perseverance** : Sf

(37) $[\blacksquare Nt(s(m)) : j], \Box(((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa) : \hat{}\lambda A\lambda B(Pres\ ((\check{}have\ A)\ B)), \Box\forall f(\check{}Sf{\downarrow}Sf) : fortunately, \Box(Nt(s(n))\&CNs(n)) : \hat{}((gen\ \check{}perseverance), \check{}perseverance)\ \Rightarrow\ Sf$

$$\overline{\boxed{Nt(s(n))} \Rightarrow Nt(s(n))}$$
$$\boxed{Nt(s(n))\&CNs(n)} \Rightarrow Nt(s(n)) \quad \&L$$
$$\boxed{\Box(Nt(s(n))\&CNs(n))} \Rightarrow Nt(s(n)) \quad \Box L$$
$$\Box(Nt(s(n))\&CNs(n)) \Rightarrow \boxed{\exists aNa} \quad \exists R$$

$$\overline{\boxed{Nt(s(m))} \Rightarrow Nt(s(m))}$$
$$\boxed{\blacksquare Nt(s(m))} \Rightarrow Nt(s(m)) \quad \blacksquare L$$
$$\blacksquare Nt(s(m)) \Rightarrow \boxed{\exists gNt(s(g))} \quad \exists R$$
$$[\blacksquare Nt(s(m))] \Rightarrow \boxed{\langle\rangle\exists gNt(s(g))} \quad \langle\rangle R \qquad \overline{\boxed{Sf} \Rightarrow Sf}$$
$$[\blacksquare Nt(s(m))], \boxed{\langle\rangle\exists gNt(s(g))\backslash Sf} \Rightarrow Sf \quad \backslash L$$

$$[\blacksquare Nt(s(m))], \boxed{(\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa}, \Box(Nt(s(n))\&CNs(n)) \Rightarrow Sf \quad /L$$
$$[\blacksquare Nt(s(m))], \boxed{\Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa)}, \Box(Nt(s(n))\&CNs(n)) \Rightarrow Sf \quad \Box L$$
$$[\blacksquare Nt(s(m))], \Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa), 1, \Box(Nt(s(n))\&CNs(n)) \Rightarrow \check{}Sf \quad \check{}R \qquad \overline{\boxed{Sf} \Rightarrow Sf}$$
$$[\blacksquare Nt(s(m))], \Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa), \boxed{\check{}Sf{\downarrow}Sf}, \Box(Nt(s(n))\&CNs(n)) \Rightarrow Sf \quad {\downarrow}L$$
$$[\blacksquare Nt(s(m))], \Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa), \boxed{\forall f(\check{}Sf{\downarrow}Sf)}, \Box(Nt(s(n))\&CNs(n)) \Rightarrow Sf \quad \forall L$$
$$[\blacksquare Nt(s(m))], \Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa), \boxed{\Box\forall f(\check{}Sf{\downarrow}Sf)}, \Box(Nt(s(n))\&CNs(n)) \Rightarrow Sf \quad \Box L$$

(38) $(\check{}fortunately\ (Pres\ ((\check{}have\ (gen\ \check{}perseverance))\ j)))$

And fourth:

(39) (tdc(70d)) **[john]+has+perseverance+fortunately** : Sf

(40) $[\blacksquare Nt(s(m)) : j], \Box(((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa) : \hat{}\lambda A\lambda B(Pres\ ((\check{}have\ A)\ B)), \Box(Nt(s(n))\&CNs(n)) : \hat{}((gen\ \check{}perseverance), \check{}perseverance), \Box\forall f(\check{}Sf{\downarrow}Sf) : fortunately\ \Rightarrow\ Sf$

$$\overline{\boxed{Nt(s(n))} \Rightarrow Nt(s(n))}$$
$$\boxed{Nt(s(n))\&CNs(n)} \Rightarrow Nt(s(n)) \quad \&L$$
$$\boxed{\Box(Nt(s(n))\&CNs(n))} \Rightarrow Nt(s(n)) \quad \Box L$$
$$\Box(Nt(s(n))\&CNs(n)) \Rightarrow \boxed{\exists aNa} \quad \exists R$$

$$\overline{\boxed{Nt(s(m))} \Rightarrow Nt(s(m))}$$
$$\boxed{\blacksquare Nt(s(m))} \Rightarrow Nt(s(m)) \quad \blacksquare L$$
$$\blacksquare Nt(s(m)) \Rightarrow \boxed{\exists gNt(s(g))} \quad \exists R$$
$$[\blacksquare Nt(s(m))] \Rightarrow \boxed{\langle\rangle\exists gNt(s(g))} \quad \langle\rangle R \qquad \overline{\boxed{Sf} \Rightarrow Sf}$$
$$[\blacksquare Nt(s(m))], \boxed{\langle\rangle\exists gNt(s(g))\backslash Sf} \Rightarrow Sf \quad \backslash L$$

$$[\blacksquare Nt(s(m))], \boxed{(\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa}, \Box(Nt(s(n))\&CNs(n)) \Rightarrow Sf \quad /L$$
$$[\blacksquare Nt(s(m))], \boxed{\Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa)}, \Box(Nt(s(n))\&CNs(n)) \Rightarrow Sf \quad \Box L$$
$$[\blacksquare Nt(s(m))], \Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa), \Box(Nt(s(n))\&CNs(n)), 1 \Rightarrow \check{}Sf \quad \check{}R \qquad \overline{\boxed{Sf} \Rightarrow Sf}$$
$$[\blacksquare Nt(s(m))], \Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa), \Box(Nt(s(n))\&CNs(n)), \boxed{\check{}Sf{\downarrow}Sf} \Rightarrow Sf \quad {\downarrow}L$$
$$[\blacksquare Nt(s(m))], \Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa), \Box(Nt(s(n))\&CNs(n)), \boxed{\forall f(\check{}Sf{\downarrow}Sf)} \Rightarrow Sf \quad \forall L$$
$$[\blacksquare Nt(s(m))], \Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa), \Box(Nt(s(n))\&CNs(n)), \boxed{\Box\forall f(\check{}Sf{\downarrow}Sf)} \Rightarrow Sf \quad \Box L$$

(41) $(\check{}fortunately\ (Pres\ ((\check{}have\ (gen\ \check{}perseverance))\ j)))$

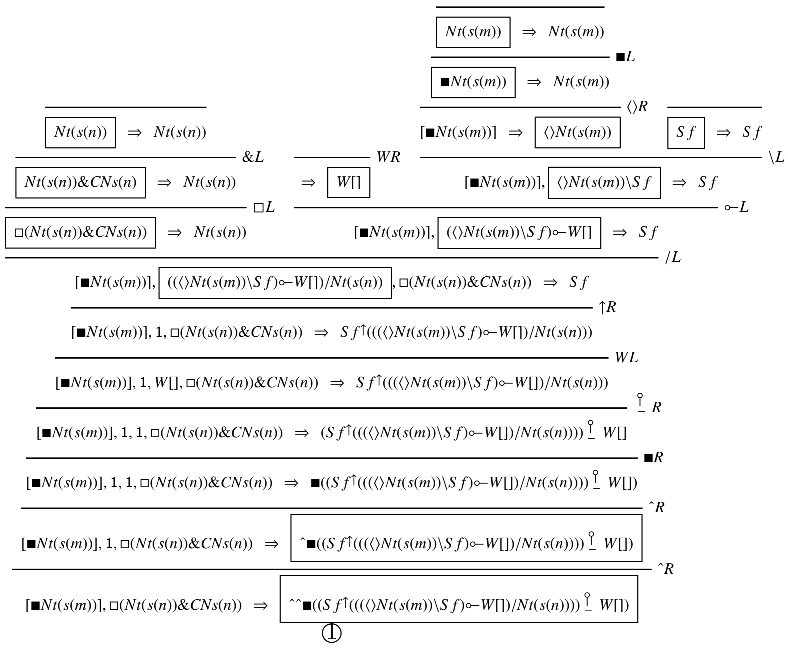
The next example is gapping:

(42) (tdc(73)) [[[**john**]+**studies**+**logic**+**and**+[**charles**]+**phonetics**]] : Sf

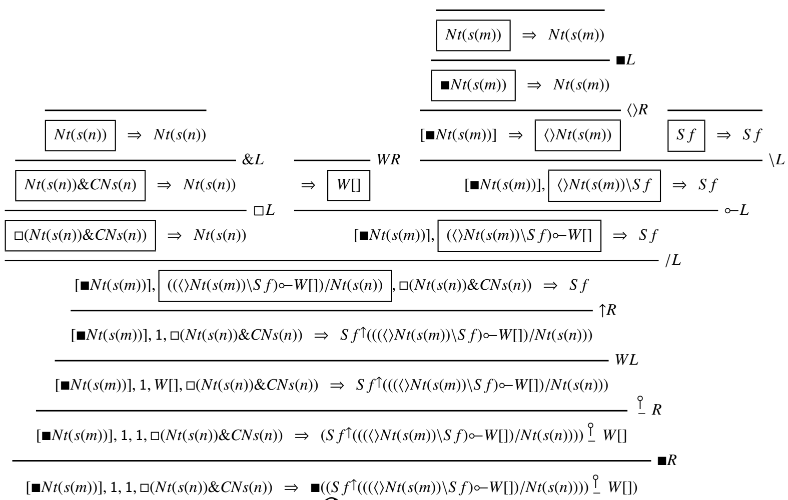
(Note the double brackets for the coordinate structure strong island.) The treatment of the example, however, is modified according to Morrill and Valentín (2014b) in view of the observations of Kubota and Levine (2012). Lexical lookup yields:

(43) $[[[\blacksquare Nt(s(m)) : j], \square((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa) : \hat{}\lambda A\lambda B(Pres\ ((\check{}study\ A)\ B)),$ $\square(Nt(s(n))\&CNs(n)) : \hat{}((gen\ \check{}logic), \check{}logic), \blacksquare\forall w\forall a\forall b\forall f((\blacksquare((Sf\uparrow(((\langle\rangle Na\backslash Sf)\circ\!\!-Ww)/Nb))\,\overset{\circ}{\underset{-}{\uparrow}}\,Ww)\backslash[]^{-1}[]^{-1}((Sf\uparrow(((\langle\rangle Na\backslash Sf)\circ\!\!-Ww)/Nb))\,\overset{\circ}{\underset{-}{\uparrow}}\,Ww))/\hat{}\hat{}\blacksquare((Sf\uparrow(((\langle\rangle Na\backslash Sf)\circ\!\!-Ww)/Nb))\,\overset{\circ}{\underset{-}{\uparrow}}\,Ww)) : \lambda C\lambda D\lambda E[(D\ E) \wedge (C\ E)], [\blacksquare Nt(s(m)) : c], \square(Nt(s(n))\&CNs(n)) :$ $\hat{}((gen\ \check{}phonetics), \check{}phonetics)]] \Rightarrow Sf$

There is the derivation:



①



②

$$Nt(s(m)) \Rightarrow Nt(s(m))$$

$$\exists R: \quad Nt(s(m)) \Rightarrow \boxed{\exists g Nt(s(g))}$$

$$\langle\rangle R: \quad [Nt(s(m))] \Rightarrow \boxed{\langle\rangle \exists g Nt(s(g))}$$

$$\boxed{Sf} \Rightarrow Sf$$

$$\backslash L: \quad [Nt(s(m))], \boxed{\langle\rangle \exists g Nt(s(g)) \backslash Sf} \Rightarrow Sf$$

$$Nt(s(n)) \Rightarrow Nt(s(n))$$

$$\exists R: \quad Nt(s(n)) \Rightarrow \boxed{\exists a Na}$$

$$/L: \quad [Nt(s(m))], \boxed{(\langle\rangle \exists g Nt(s(g)) \backslash Sf)/\exists a Na}, Nt(s(n)) \Rightarrow Sf$$

$$\square L: \quad [Nt(s(m))], \boxed{\square((\langle\rangle \exists g Nt(s(g)) \backslash Sf)/\exists a Na)}, Nt(s(n)) \Rightarrow Sf$$

$$\langle\rangle L: \quad \langle\rangle Nt(s(m)), \square((\langle\rangle \exists g Nt(s(g)) \backslash Sf)/\exists a Na), Nt(s(n)) \Rightarrow Sf$$

$$\backslash R: \quad \square((\langle\rangle \exists g Nt(s(g)) \backslash Sf)/\exists a Na), Nt(s(n)) \Rightarrow \langle\rangle Nt(s(m)) \backslash Sf$$

$$WL: \quad \square((\langle\rangle \exists g Nt(s(g)) \backslash Sf)/\exists a Na), Nt(s(n)), W[\,] \Rightarrow \langle\rangle Nt(s(m)) \backslash Sf$$

$$\multimap R: \quad \square((\langle\rangle \exists g Nt(s(g)) \backslash Sf)/\exists a Na), Nt(s(n)) \Rightarrow (\langle\rangle Nt(s(m)) \backslash Sf) \multimap W[\,]$$

$$/R: \quad \square((\langle\rangle \exists g Nt(s(g)) \backslash Sf)/\exists a Na) \Rightarrow ((\langle\rangle Nt(s(m)) \backslash Sf) \multimap W[\,])/Nt(s(n))$$

$$\boxed{Sf} \Rightarrow Sf$$

$$\uparrow L: \quad \boxed{Sf{\uparrow}(((\langle\rangle Nt(s(m)) \backslash Sf) \multimap W[\,])/Nt(s(n)))\{\square((\langle\rangle \exists g Nt(s(g)) \backslash Sf)/\exists a Na)\}} \Rightarrow Sf$$

$$WR: \quad \Rightarrow \boxed{W[\,]}$$

$$\overset{\circ}{\_}\, L: \quad \boxed{(Sf{\uparrow}(((\langle\rangle Nt(s(m)) \backslash Sf) \multimap W[\,])/Nt(s(n)))) \overset{\circ}{\_}\, W[\,]\{\square((\langle\rangle \exists g Nt(s(g)) \backslash Sf)/\exists a Na) :\}} \Rightarrow Sf$$

$$[\,]^{-1} L: \quad [\boxed{[\,]^{-1}((Sf{\uparrow}(((\langle\rangle Nt(s(m)) \backslash Sf) \multimap W[\,])/Nt(s(n)))) \overset{\circ}{\_}\, W[\,]\{\square((\langle\rangle \exists g Nt(s(g)) \backslash Sf)/\exists a Na) :\}}] \Rightarrow Sf$$

$$[\,]^{-1} L: \quad [[\boxed{[\,]^{-1}[\,]^{-1}((Sf{\uparrow}(((\langle\rangle Nt(s(m)) \backslash Sf) \multimap W[\,])/Nt(s(n)))) \overset{\circ}{\_}\, W[\,]\{\square((\langle\rangle \exists g Nt(s(g)) \backslash Sf)/\exists a Na) :\}}]] \Rightarrow Sf$$

②

①

$$\backslash L: \quad [[[\blacksquare Nt(s(m))], \square((\langle\rangle \exists g Nt(s(g)) \backslash Sf)/\exists a Na), \square(Nt(s(n))\& CNs(n)), \boxed{\blacksquare((Sf{\uparrow}(((\langle\rangle Nt(s(m)) \backslash Sf) \multimap W[\,])/Nt(s(n)))) \overset{\circ}{\_}\, W[\,]) \backslash [\,]^{-1}[\,]^{-1}((Sf{\uparrow}(((\langle\rangle Nt(s(m)) \backslash Sf) \multimap W[\,])/Nt(s(n)))) \overset{\circ}{\_}\, W[\,])}]] \Rightarrow Sf$$

$$/L: \quad [[[\blacksquare Nt(s(m))], \square((\langle\rangle \exists g Nt(s(g)) \backslash Sf)/\exists a Na), \square(Nt(s(n))\& CNs(n)), \boxed{(\blacksquare((Sf{\uparrow}(((\langle\rangle Nt(s(m)) \backslash Sf) \multimap W[\,])/Nt(s(n)))) \overset{\circ}{\_}\, W[\,]) \backslash [\,]^{-1}[\,]^{-1}((Sf{\uparrow}(((\langle\rangle Nt(s(m)) \backslash Sf) \multimap W[\,])/Nt(s(n)))) \overset{\circ}{\_}\, W[\,]))/^{\wedge\wedge}\blacksquare((Sf{\uparrow}(((\langle\rangle Nt(s(m)) \backslash Sf) \multimap W[\,])/Nt(s(n)))) \overset{\circ}{\_}\, W[\,])}, [\blacksquare Nt(s(m))], \square(Nt(s(n))\& CNs(n))]] \Rightarrow Sf$$

$$\forall L: \quad [[[\blacksquare Nt(s(m))], \square((\langle\rangle \exists g Nt(s(g)) \backslash Sf)/\exists a Na), \square(Nt(s(n))\& CNs(n)), \boxed{\forall f((\blacksquare((Sf{\uparrow}(((\langle\rangle Nt(s(m)) \backslash Sf) \multimap W[\,])/Nt(s(n)))) \overset{\circ}{\_}\, W[\,]) \backslash [\,]^{-1}[\,]^{-1}((Sf{\uparrow}(((\langle\rangle Nt(s(m)) \backslash Sf) \multimap W[\,])/Nt(s(n)))) \overset{\circ}{\_}\, W[\,]))/^{\wedge\wedge}\blacksquare((Sf{\uparrow}(((\langle\rangle Nt(s(m)) \backslash Sf) \multimap W[\,])/Nt(s(n)))) \overset{\circ}{\_}\, W[\,])}, [\blacksquare Nt(s(m))], \square(Nt(s(n))\& CNs(n))]] \Rightarrow Sf$$

$$\forall L: \quad [[[\blacksquare Nt(s(m))], \square((\langle\rangle \exists g Nt(s(g)) \backslash Sf)/\exists a Na), \square(Nt(s(n))\& CNs(n)), \boxed{\forall b \forall f((\blacksquare((Sf{\uparrow}(((\langle\rangle Nt(s(m)) \backslash Sf) \multimap W[\,])/Nb)) \overset{\circ}{\_}\, W[\,]) \backslash [\,]^{-1}[\,]^{-1}((Sf{\uparrow}(((\langle\rangle Nt(s(m)) \backslash Sf) \multimap W[\,])/Nb)) \overset{\circ}{\_}\, W[\,]))/^{\wedge\wedge}\blacksquare((Sf{\uparrow}(((\langle\rangle Nt(s(m)) \backslash Sf) \multimap W[\,])/Nb)) \overset{\circ}{\_}\, W[\,])}, [\blacksquare Nt(s(m))], \square(Nt(s(n))\& CNs(n))]] \Rightarrow Sf$$

$$\forall L: \quad [[[\blacksquare Nt(s(m))], \square((\langle\rangle \exists g Nt(s(g)) \backslash Sf)/\exists a Na), \square(Nt(s(n))\& CNs(n)), \boxed{\forall a \forall b \forall f((\blacksquare((Sf{\uparrow}(((\langle\rangle Na \backslash Sf) \multimap W[\,])/Nb)) \overset{\circ}{\_}\, W[\,]) \backslash [\,]^{-1}[\,]^{-1}((Sf{\uparrow}(((\langle\rangle Na \backslash Sf) \multimap W[\,])/Nb)) \overset{\circ}{\_}\, W[\,]))/^{\wedge\wedge}\blacksquare((Sf{\uparrow}(((\langle\rangle Na \backslash Sf) \multimap W[\,])/Nb)) \overset{\circ}{\_}\, W[\,])}, [\blacksquare Nt(s(m))], \square(Nt(s(n))\& CNs(n))]] \Rightarrow Sf$$

$$\forall L: \quad [[[\blacksquare Nt(s(m))], \square((\langle\rangle \exists g Nt(s(g)) \backslash Sf)/\exists a Na), \square(Nt(s(n))\& CNs(n)), \boxed{\forall w \forall a \forall b \forall f((\blacksquare((Sf{\uparrow}(((\langle\rangle Na \backslash Sf) \multimap Ww)/Nb)) \overset{\circ}{\_}\, Ww) \backslash [\,]^{-1}[\,]^{-1}((Sf{\uparrow}(((\langle\rangle Na \backslash Sf) \multimap Ww)/Nb)) \overset{\circ}{\_}\, Ww))/^{\wedge\wedge}\blacksquare((Sf{\uparrow}(((\langle\rangle Na \backslash Sf) \multimap Ww)/Nb)) \overset{\circ}{\_}\, Ww))}, [\blacksquare Nt(s(m))], \square(Nt(s(n))\& CNs(n))]] \Rightarrow Sf$$

$$\blacksquare L: \quad [[[\blacksquare Nt(s(m))], \square((\langle\rangle \exists g Nt(s(g)) \backslash Sf)/\exists a Na), \square(Nt(s(n))\& CNs(n)), \boxed{\blacksquare \forall w \forall a \forall b \forall f((\blacksquare((Sf{\uparrow}(((\langle\rangle Na \backslash Sf) \multimap Ww)/Nb)) \overset{\circ}{\_}\, Ww) \backslash [\,]^{-1}[\,]^{-1}((Sf{\uparrow}(((\langle\rangle Na \backslash Sf) \multimap Ww)/Nb)) \overset{\circ}{\_}\, Ww))/^{\wedge\wedge}\blacksquare((Sf{\uparrow}(((\langle\rangle Na \backslash Sf) \multimap Ww)/Nb)) \overset{\circ}{\_}\, Ww))}, [\blacksquare Nt(s(m))], \square(Nt(s(n))\& CNs(n))]] \Rightarrow Sf$$

This delivers semantics:

(44) $[(Pres\ ((\check{}study\ (gen\ \check{}logic))\ j)) \wedge (Pres\ ((\check{}study\ (gen\ \check{}phonetics))\ c))]$

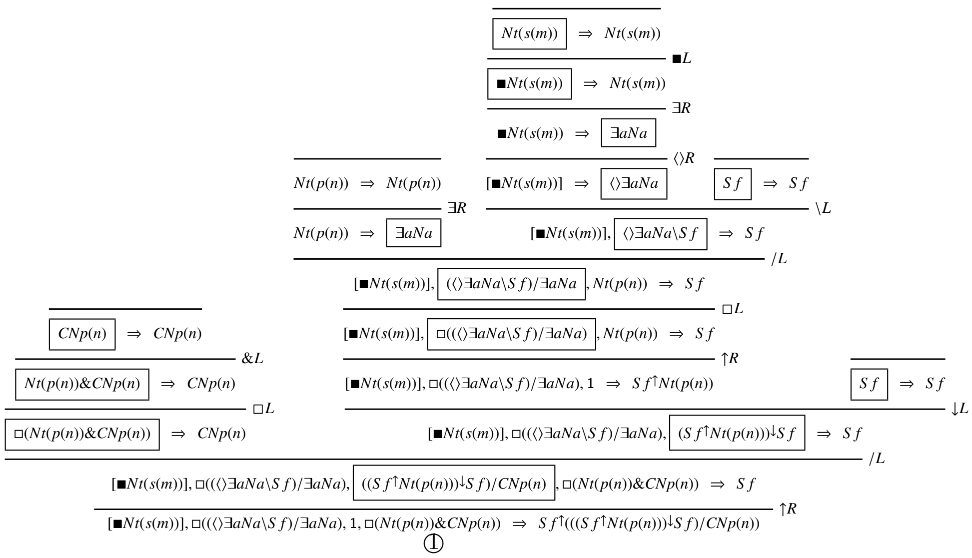
Example tdc(75) contains comparative subdeletion:

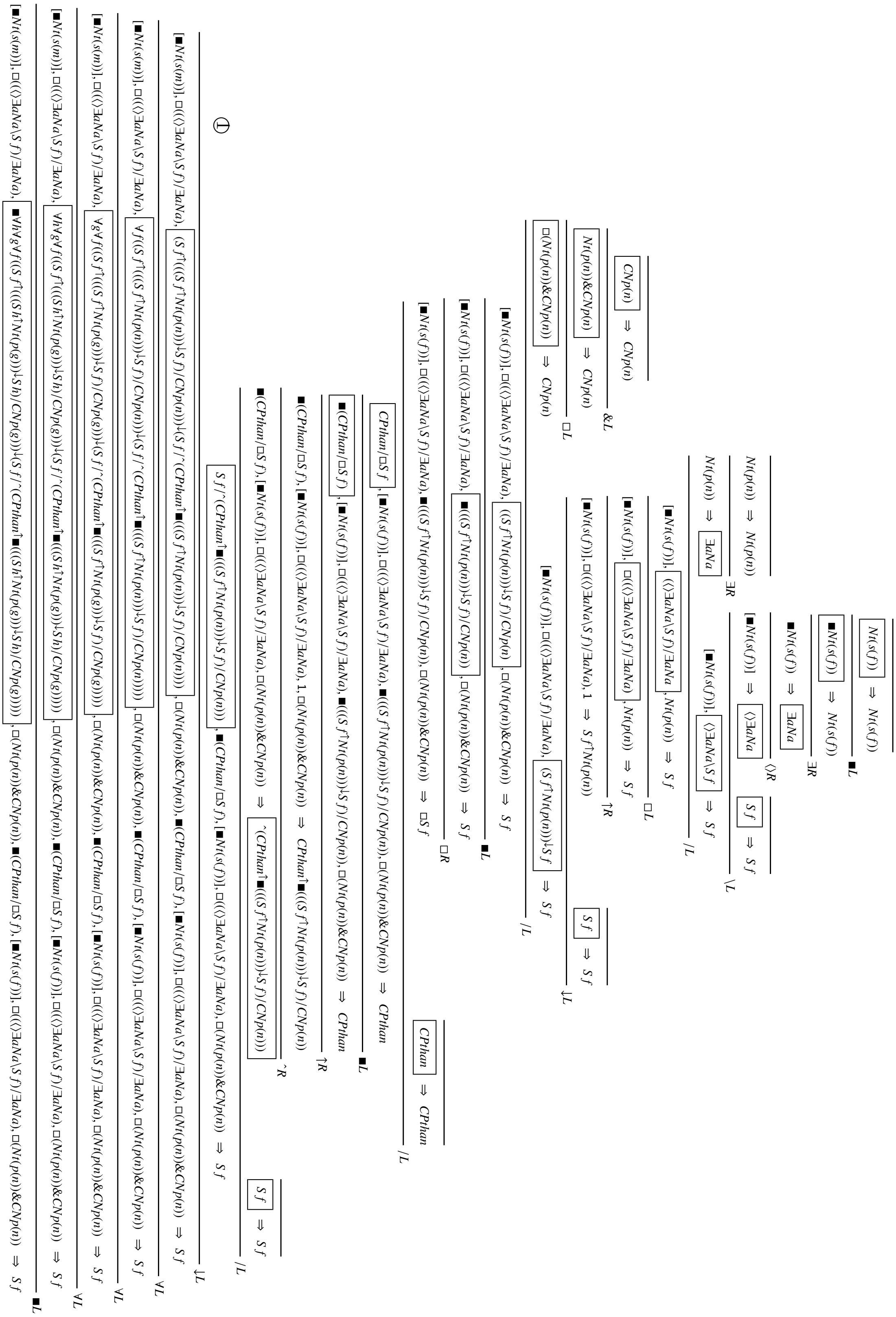
(45) (tdc(75)) **[john]+ate+more+donuts+than+[mary]+bought+bagels** : Sf

Lexical lookup yields:

(46) $[\blacksquare Nt(s(m)) : j], \Box((\langle\rangle\exists aNa\backslash Sf)/\exists aNa) : \hat{}\lambda A\lambda B(Past\ ((\check{}eat\ A)\ B)),$
$\blacksquare\forall h\forall g\forall f((Sf{\uparrow}(((Sh{\uparrow}Nt(p(g))){\downarrow}Sh)/CNp(g))){\downarrow}(Sf/\hat{}(CPthan{\uparrow}\blacksquare(((Sh{\uparrow}Nt(p(g))){\downarrow}Sh)/CNp(g))))) :$
$\lambda C\lambda D[|\lambda E(C\ \lambda F\lambda G[(F\ E)\wedge(G\ E)])| > |\lambda H\check{}(D\ \lambda I\lambda J[(I\ H)\wedge(J\ H)])|], \Box(Nt(p(n))\&CNp(n)) :$
$\hat{}((gen\ \check{}donuts), \check{}donuts), \blacksquare(CPthan/\Box Sf) : \lambda KK, [\blacksquare Nt(s(f)) : m], \Box((\langle\rangle\exists aNa\backslash Sf)/\exists aNa) :$
$\hat{}\lambda L\lambda M(Past\ ((\check{}buy\ L)\ M)), \Box(Nt(p(n))\&CNp(n)) : \hat{}((gen\ \check{}bagels), \check{}bagels) \Rightarrow Sf$

There is the derivation:





This delivers semantics:

(47) $[|\lambda C[(\check{}\,\boldsymbol{donuts}\ C) \wedge (\mathit{Past}\ ((\check{}\,\boldsymbol{eat}\ C)\ \boldsymbol{j}))]| > |\lambda F[(\check{}\,\boldsymbol{bagels}\ F) \wedge (\mathit{Past}\ ((\check{}\,\boldsymbol{buy}\ F)\ \boldsymbol{m}))]|]$

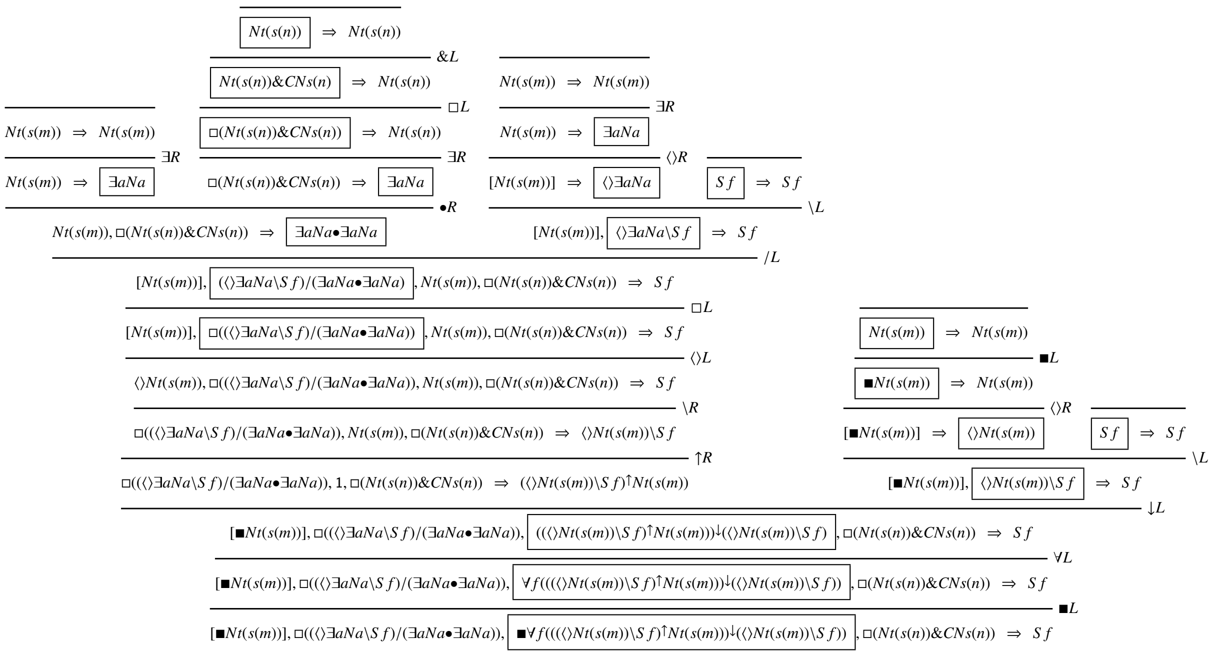
Finally, there is the medial reflexivisation:

(48) (tdc(86a)) **[john]+bought+himself+coffee** : Sf

The lexical lookup yields:

(49) $[\blacksquare Nt(s(m)) : j], \square((\langle\rangle\exists aNa\backslash Sf)/(\exists aNa\bullet\exists aNa)) : \hat{}\lambda A\lambda B(Past\ (((\check{}buy\ \pi_1 A)\ \pi_2 A)\ B)),$
$\blacksquare\forall f(((\langle\rangle Nt(s(m))\backslash Sf)\uparrow Nt(s(m)))\downarrow(\langle\rangle Nt(s(m))\backslash Sf)) : \lambda C\lambda D((C\ D)\ D), \square(Nt(s(n))\& CNs(n)) :$
$\hat{}((gen\ \check{}coffee), \check{}coffee)\ \Rightarrow\ Sf$

There is the derivation:



This delivers semantics:

(50) $(Past\ (((\check{}buy\ j)\ (gen\ \check{}coffee))\ j))$

5. Conclusions

It is our view that computational implementation of wide and deep empirical analysis is not only a desideratum, but also necessary, to verify the correct interaction of the myriad factors of language. With this paper we aim to set the pace on this methodology and we invite others to follow.

References

Andreoli, J. M. 1992. Logic programming with focusing in linear logic. Journal of Logic and Computation 2(3):297–347.

Dowty, David R., Robert E. Wall, and Stanley Peters. 1981. Introduction to Montague Semantics, vol. 11 of Synthese Language Library. Dordrecht: D. Reidel.

Kubota, Yusuke and Robert Levine. 2012. Gapping as like-category coordination. In D. Bechet and A. Dikovsky, eds., Logical Aspects of Computational Linguistics, vol.

7351 of Lecture Notes in Computer Science, 135–150. Springer Berlin Heidelberg. ISBN 978-3-642-31261-8.

Moortgat, Michael. 1997. Categorial Type Logics. In J. van Benthem and A. ter Meulen, eds., Handbook of Logic and Language, 93–177. Amsterdam and Cambridge, Massachusetts: Elsevier Science B.V. and the MIT Press.

Morrill, Glyn. 2011a. Logic Programming of the Displacement Calculus. In S. Pogodalla and J.-P. Prost, eds., Proceedings of Logical Aspects of Computational Linguistics 2011, LACL'11, Montpellier, No. LNAI 6736 in Springer Lecture Notes in AI, 175–189. Berlin: Springer.

Morrill, Glyn. 2012. CatLog: A Categorial Parser/Theorem-Prover. In LACL 2012 System Demonstrations, Logical Aspects of Computational Linguistics 2012, 13–16. Nantes.

Morrill, Glyn. 2014. A Categorial Type Logic. In M. M. Claudia Casadio, Bob Coeke and P. Scott, eds., Categories and Types in Logic, Language and Physics: Essays Dedicated to Jim Lambek on the Occasion of His 90th Birthday, vol. 8222 of LNCS, FoLLI Publications in Logic, Language and Information, 331–352. Berlin: Springer.

Morrill, Glyn and Oriol Valentín. 2010. Displacement Calculus. Linguistic Analysis 36(1–4):167–192. Special issue Festschrift for Joachim Lambek, http://arxiv.org/abs/1004.4181.

Morrill, Glyn and Oriol Valentín. 2014a. Displacement Logic for Anaphora. Journal of Computing and System Science 80:390–409. http://dx.doi.org/10.1016/j.jcss.2013.05.006.

Morrill, Glyn and Oriol Valentín. 2014b. Semantically Inactive Multiplicatives and Words as Types. In N. Asher and S. Soloviev, eds., Proceedings of Logical Aspects of Computational Linguistics, LACL'14, Toulouse, No. 8535 in LNCS, FoLLI Publications on Logic, Language and Information, 149–162. Berlin: Springer.

Morrill, Glyn, Oriol Valentín, and Mario Fadda. 2011. The Displacement Calculus. Journal of Logic, Language and Information 20(1):1–48. Doi 10.1007/s10849-010-9129-2.

Morrill, Glyn V. 1994. Type Logical Grammar: Categorial Logic of Signs. Dordrecht: Kluwer Academic Publishers.

Morrill, Glyn V. 2011b. Categorial Grammar: Logical Syntax, Semantics, and Processing. New York and Oxford: Oxford University Press.

Valentín, Oriol. 2012. Theory of Discontinuous Lambek Calculus. Ph.D. thesis, Universitat Autònoma de Barcelona, Barcelona.

Valentín, Oriol, Daniel Serret, and Glyn Morrill. 2013. A Count Invariant for Lambek Calculus with Additives and Bracket Modalities. In G. Morrill and M.-J. Nederhof, eds., Proceedings of Formal Grammar 2012 and 2013, vol. 8036 of Springer LNCS, FoLLI Publications in Logic, Language and Information, 263–276. Berlin: Springer.

van Benthem, J. 1991. Language in Action: Categories, Lambdas, and Dynamic Logic. No. 130 in Studies in Logic and the Foundations of Mathematics. Amsterdam: North-Holland. Revised student edition printed in 1995 by the MIT Press.

Appendix: CatLog output for Dutch verb raising and cross-serial dependency

kan : $(NA\backslash Si){\downarrow}(NA\backslash Sf) : \lambda B\lambda C((isable\ (B\ C))\ C)$
las : $NA\backslash(Nt(s(B))\backslash Sf) : read$
wil : $(NA\backslash Si){\downarrow}(NA\backslash Sf) : \lambda B\lambda C((want\ (B\ C))\ C)$
kan : $Q/{}^{\wedge}(Sf{\uparrow}((NA\backslash Si){\downarrow}(NA\backslash Sf))) : \lambda B(B\ \lambda C\lambda D((isable\ (C\ D))\ D))$
las : $Q/{}^{\wedge}(Sf{\uparrow}(NA\backslash(Nt(s(B))\backslash Sf))) : \lambda C(C\ read)$
wil : $Q/{}^{\wedge}(Sf{\uparrow}((NA\backslash Si){\downarrow}(NA\backslash Sf))) : \lambda B(B\ \lambda C\lambda D((want\ (C\ D))\ D))$
alles : $(SA{\uparrow}Nt(s(n))){\downarrow}SA : \lambda B\forall C[(thing\ C) \to (B\ C)]$
boeken : $Np(n) : books$
cecilia : $Nt(s(f)) : c$
de : $Nt(s(A))/CNA : the$
helpen : $\rhd^{-1}((NA\backslash Si){\downarrow}(NB\backslash(NA\backslash Si))) : \lambda C\lambda D((help\ (C\ D))\ D)$
henk : $Nt(s(m)) : h$
jan : $Nt(s(m)) : j$
kunnen : $\rhd^{-1}((NA\backslash Si){\downarrow}(NA\backslash Si)) : \lambda B\lambda C((isable\ (B\ C))\ C)$
lezen : $\rhd^{-1}(NA\backslash(NB\backslash Si)) : read$
nijlpaarden : $CNp(n) : hippos$
voeren : $\rhd^{-1}(NA\backslash(NB\backslash Si)) : feed$
zag : $(Nt(s(A))\backslash Si){\downarrow}(NB\backslash(Nt(s(A))\backslash Sf)) : \lambda C\lambda D((saw\ (C\ D))\ D)$

(d(1)) **jan+boeken+las** : Sf

$Nt(s(m)) : j, Np(n) : books, NA\backslash(Nt(s(B))\backslash Sf) : read \Rightarrow Sf$

$$\dfrac{\dfrac{}{Np(n) \Rightarrow Np(n)} \qquad \dfrac{\dfrac{}{Nt(s(m)) \Rightarrow Nt(s(m))} \quad \dfrac{}{\boxed{Sf} \Rightarrow Sf}}{Nt(s(m)), \boxed{Nt(s(m))\backslash Sf} \Rightarrow Sf}\backslash L}{Nt(s(m)), Np(n), \boxed{Np(n)\backslash(Nt(s(m))\backslash Sf)} \Rightarrow Sf}\backslash L$$

$((read\ books)\ j)$

$Nt(s(m)) : j, Np(n) : books, Q/{}^{\wedge}(Sf{\uparrow}(NA\backslash(Nt(s(B))\backslash Sf))) : \lambda C(C\ read) \Rightarrow Sf$

(d(2)) **jan+boeken+kan+lezen** : Sf

$Nt(s(m)) : j, Np(n) : books, (NA\backslash Si){\downarrow}(NA\backslash Sf) : \lambda B\lambda C((isable\ (B\ C))\ C), \rhd^{-1}(ND\backslash(NE\backslash Si)) : read \Rightarrow Sf$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{Np(n) \Rightarrow Np(n)} \qquad \dfrac{\dfrac{}{Nt(s(m)) \Rightarrow Nt(s(m))} \quad \dfrac{}{\boxed{Si\{1\}} \Rightarrow Si}}{Nt(s(m)), \boxed{Nt(s(m))\backslash Si\{1\}} \Rightarrow Si}\backslash L}{Nt(s(m)), Np(n), \boxed{Np(n)\backslash(Nt(s(m))\backslash Si)\{1\}} \Rightarrow Si}\backslash L}{Nt(s(m)), Np(n), 1, \boxed{\rhd^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si))} \Rightarrow Si}\rhd^{-1}L}{Np(n), 1, \rhd^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si)) \Rightarrow Nt(s(m))\backslash Si}\backslash R \qquad \dfrac{\dfrac{}{Nt(s(m)) \Rightarrow Nt(s(m))} \quad \dfrac{}{\boxed{Sf} \Rightarrow Sf}}{Nt(s(m)), \boxed{Nt(s(m))\backslash Sf} \Rightarrow Sf}\backslash L}{Nt(s(m)), Np(n), \boxed{(Nt(s(m))\backslash Si){\downarrow}(Nt(s(m))\backslash Sf)}, \rhd^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si)) \Rightarrow Sf}{\downarrow}L$$

$((isable\ ((read\ books)\ j))\ j)$

$Nt(s(m)) : j, Np(n) : books, Q/{}^{\wedge}(Sf{\uparrow}((NA\backslash Si){\downarrow}(NA\backslash Sf))) : \lambda B(B\ \lambda C\lambda D((isable\ (C\ D))\ D)), \rhd^{-1}(NE\backslash(NF\backslash Si)) : read \Rightarrow Sf$

(d(3)) **jan+boeken+wil+kunnen+lezen** : Sf

$Nt(s(m)) : j, Np(n) : books, (NA\backslash Si){\downarrow}(NA\backslash Sf) : \lambda B\lambda C((want\ (B\ C))\ C), \rhd^{-1}((ND\backslash Si){\downarrow}(ND\backslash Si)) : \lambda E\lambda F((isable\ (E\ F))\ F), \rhd^{-1}(NG\backslash(NH\backslash Si)) : read \Rightarrow Sf$

$$Nt(s(m)) \Rightarrow Nt(s(m)) \qquad \boxed{Si\{1\}} \Rightarrow Si$$
$$\backslash L:\quad Nt(s(m)), \boxed{Nt(s(m))\backslash Si\{1\}} \Rightarrow Si \qquad Np(n) \Rightarrow Np(n)$$
$$\backslash L:\quad Nt(s(m)), Np(n), \boxed{Np(n)\backslash(Nt(s(m))\backslash Si)\{1\}} \Rightarrow Si$$
$$\rhd^{-1}L:\quad Nt(s(m)), Np(n), 1, \boxed{\rhd^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si))} \Rightarrow Si$$
$$\backslash R:\quad Np(n), 1, \rhd^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si)) \Rightarrow Nt(s(m))\backslash Si$$
$$Nt(s(m)) \Rightarrow Nt(s(m)) \qquad \boxed{Si\{1\}} \Rightarrow Si \qquad \backslash L:\quad Nt(s(m)), \boxed{Nt(s(m))\backslash Si\{1\}} \Rightarrow Si$$
$$\downarrow L:\quad Nt(s(m)), Np(n), \boxed{(Nt(s(m))\backslash Si)\downarrow(Nt(s(m))\backslash Si)\{1\}}, \rhd^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si)) \Rightarrow Si$$
$$\rhd^{-1}L:\quad Nt(s(m)), Np(n), 1, \boxed{\rhd^{-1}((Nt(s(m))\backslash Si)\downarrow(Nt(s(m))\backslash Si))}, \rhd^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si)) \Rightarrow Si$$
$$\backslash R:\quad Np(n), 1, \rhd^{-1}((Nt(s(m))\backslash Si)\downarrow(Nt(s(m))\backslash Si)), \rhd^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si)) \Rightarrow Nt(s(m))\backslash Si$$
$$Nt(s(m)) \Rightarrow Nt(s(m)) \qquad \boxed{Sf} \Rightarrow Sf \qquad \backslash L:\quad Nt(s(m)), \boxed{Nt(s(m))\backslash Sf} \Rightarrow Sf$$
$$\downarrow L:\quad Nt(s(m)), Np(n), \boxed{(Nt(s(m))\backslash Si)\downarrow(Nt(s(m))\backslash Sf)}, \rhd^{-1}((Nt(s(m))\backslash Si)\downarrow(Nt(s(m))\backslash Si)), \rhd^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si)) \Rightarrow Sf$$

$((want\ ((isable\ ((read\ books)\ j))\ j))\ j)$

$Nt(s(m)) : j, Np(n) : books, Q/\hat{}(Sf\uparrow((NA\backslash Si)\downarrow(NA\backslash Sf))) : \lambda B(B\ \lambda C\lambda D((want\ (C\ D))\ D)), \rhd^{-1}((NE\backslash Si)\downarrow(NE\backslash Si)) : \lambda F\lambda G((isable\ (F\ G))\ G), \rhd^{-1}(NH\backslash(NI\backslash Si)) : read \Rightarrow Sf$

(d(4)) **jan+alles+las** : Sf

$Nt(s(m)) : j, (SA\uparrow Nt(s(n)))\downarrow SA : \lambda B\forall C[(thing\ C) \to (B\ C)], ND\backslash(Nt(s(E))\backslash Sf) : read \Rightarrow Sf$

$$Nt(s(m)) \Rightarrow Nt(s(m)) \qquad \boxed{Sf} \Rightarrow Sf$$
$$\backslash L:\quad Nt(s(m)), \boxed{Nt(s(m))\backslash Sf} \Rightarrow Sf \qquad Nt(s(n)) \Rightarrow Nt(s(n))$$
$$\backslash L:\quad Nt(s(m)), Nt(s(n)), \boxed{Nt(s(n))\backslash(Nt(s(m))\backslash Sf)} \Rightarrow Sf$$
$$\uparrow R:\quad Nt(s(m)), 1, Nt(s(n))\backslash(Nt(s(m))\backslash Sf) \Rightarrow Sf\uparrow Nt(s(n)) \qquad \boxed{Sf} \Rightarrow Sf$$
$$\downarrow L:\quad Nt(s(m)), \boxed{(Sf\uparrow Nt(s(n)))\downarrow Sf}, Nt(s(n))\backslash(Nt(s(m))\backslash Sf) \Rightarrow Sf$$

$\forall B[(thing\ B) \to ((read\ B)\ j)]$

$Nt(s(m)) : j, (SA\uparrow Nt(s(n)))\downarrow SA : \lambda B\forall C[(thing\ C) \to (B\ C)], Q/\hat{}(Sf\uparrow(ND\backslash(Nt(s(E))\backslash Sf))) : \lambda F(F\ read) \Rightarrow Sf$

(d(5)) **jan+alles+kan+lezen** : Sf

$Nt(s(m)) : j, (SA\uparrow Nt(s(n)))\downarrow SA : \lambda B\forall C[(thing\ C) \to (B\ C)], (ND\backslash Si)\downarrow(ND\backslash Sf) : \lambda E\lambda F((isable\ (E\ F))\ F), \rhd^{-1}(NG\backslash(NH\backslash Si)) : read \Rightarrow Sf$

$$Nt(s(m)) \Rightarrow Nt(s(m)) \qquad \boxed{Si\{1\}} \Rightarrow Si$$
$$\backslash L:\quad Nt(s(m)), \boxed{Nt(s(m))\backslash Si\{1\}} \Rightarrow Si \qquad Nt(s(n)) \Rightarrow Nt(s(n))$$
$$\backslash L:\quad Nt(s(m)), Nt(s(n)), \boxed{Nt(s(n))\backslash(Nt(s(m))\backslash Si)\{1\}} \Rightarrow Si$$
$$\rhd^{-1}L:\quad Nt(s(m)), Nt(s(n)), 1, \boxed{\rhd^{-1}(Nt(s(n))\backslash(Nt(s(m))\backslash Si))} \Rightarrow Si$$
$$\backslash R:\quad Nt(s(n)), 1, \rhd^{-1}(Nt(s(n))\backslash(Nt(s(m))\backslash Si)) \Rightarrow Nt(s(m))\backslash Si$$
$$Nt(s(m)) \Rightarrow Nt(s(m)) \qquad \boxed{Sf} \Rightarrow Sf \qquad \backslash L:\quad Nt(s(m)), \boxed{Nt(s(m))\backslash Sf} \Rightarrow Sf$$
$$\downarrow L:\quad Nt(s(m)), Nt(s(n)), \boxed{(Nt(s(m))\backslash Si)\downarrow(Nt(s(m))\backslash Sf)}, \rhd^{-1}(Nt(s(n))\backslash(Nt(s(m))\backslash Si)) \Rightarrow Sf$$
$$\uparrow R:\quad Nt(s(m)), 1, (Nt(s(m))\backslash Si)\downarrow(Nt(s(m))\backslash Sf), \rhd^{-1}(Nt(s(n))\backslash(Nt(s(m))\backslash Si)) \Rightarrow Sf\uparrow Nt(s(n)) \qquad \boxed{Sf} \Rightarrow Sf$$
$$\downarrow L:\quad Nt(s(m)), \boxed{(Sf\uparrow Nt(s(n)))\downarrow Sf}, (Nt(s(m))\backslash Si)\downarrow(Nt(s(m))\backslash Sf), \rhd^{-1}(Nt(s(n))\backslash(Nt(s(m))\backslash Si)) \Rightarrow Sf$$

$\forall B[(thing\ B) \to ((isable\ ((read\ B)\ j))\ j)]$

$Nt(s(m)) : j, (SA\uparrow Nt(s(n)))\downarrow SA : \lambda B\forall C[(thing\ C) \to (B\ C)], Q/\hat{}(Sf\uparrow((ND\backslash Si)\downarrow(ND\backslash Sf))) : \lambda E(E\ \lambda F\lambda G((isable\ (F\ G))\ G)), \rhd^{-1}(NH\backslash(NI\backslash Si)) : read \Rightarrow Sf$

(d(6)) **jan+cecilia+henk+de+nijlpaarden+zag+helpen+voeren** : Sf

$Nt(s(m)) : j, Nt(s(f)) : c, Nt(s(m)) : h, Nt(s(A))/CNA : the, CNp(n) : hippos, (Nt(s(B))\backslash Si)\downarrow(NC\backslash(Nt(s(B))\backslash Sf)) : \lambda D\lambda E((saw\ (D\ E))\ E), \rhd^{-1}((NF\backslash Si)\downarrow(NG\backslash(NF\backslash Si))) :$

$\lambda H \lambda I((help\ (H\ I))\ I), \rhd^{-1}(NJ\backslash(NK\backslash Si)) : feed \ \Rightarrow\ Sf$

$$\dfrac{CNp(n) \Rightarrow CNp(n) \qquad \boxed{Nt(s(p(n)))} \Rightarrow Nt(s(p(n)))}{\boxed{Nt(s(p(n)))/CNp(n)}, CNp(n) \Rightarrow Nt(s(p(n)))}\ /L$$

$$\dfrac{Nt(s(m)) \Rightarrow Nt(s(m)) \qquad \boxed{Si\{1\}} \Rightarrow Si}{Nt(s(m)), \boxed{Nt(s(m))\backslash Si\{1\}} \Rightarrow Si}\ \backslash L$$

$$\dfrac{\boxed{Nt(s(p(n)))/CNp(n)}, CNp(n) \Rightarrow Nt(s(p(n))) \qquad Nt(s(m)), \boxed{Nt(s(m))\backslash Si\{1\}} \Rightarrow Si}{Nt(s(m)), Nt(s(p(n)))/CNp(n), CNp(n), \boxed{Nt(s(p(n)))\backslash(Nt(s(m))\backslash Si)\{1\}} \Rightarrow Si}\ \backslash L$$

$$\dfrac{Nt(s(m)), Nt(s(p(n)))/CNp(n), CNp(n), \boxed{Nt(s(p(n)))\backslash(Nt(s(m))\backslash Si)\{1\}} \Rightarrow Si}{Nt(s(m)), Nt(s(p(n)))/CNp(n), CNp(n), 1, \boxed{\rhd^{-1}(Nt(s(p(n)))\backslash(Nt(s(m))\backslash Si))} \Rightarrow Si}\ \rhd^{-1}L$$

$$\dfrac{Nt(s(m)), Nt(s(p(n)))/CNp(n), CNp(n), 1, \boxed{\rhd^{-1}(Nt(s(p(n)))\backslash(Nt(s(m))\backslash Si))} \Rightarrow Si}{Nt(s(p(n)))/CNp(n), CNp(n), 1, \rhd^{-1}(Nt(s(p(n)))\backslash(Nt(s(m))\backslash Si)) \Rightarrow Nt(s(m))\backslash Si}\ \backslash R$$

$$\dfrac{Nt(s(m)) \Rightarrow Nt(s(m)) \qquad \boxed{Si\{1\}} \Rightarrow Si}{Nt(s(m)), \boxed{Nt(s(m))\backslash Si\{1\}} \Rightarrow Si}\ \backslash L$$

$$\dfrac{Nt(s(m)) \Rightarrow Nt(s(m)) \qquad Nt(s(m)), \boxed{Nt(s(m))\backslash Si\{1\}} \Rightarrow Si}{Nt(s(m)), Nt(s(m)), \boxed{Nt(s(m))\backslash(Nt(s(m))\backslash Si)\{1\}} \Rightarrow Si}\ \backslash L$$

$$\dfrac{Nt(s(p(n)))/CNp(n), CNp(n), 1, \rhd^{-1}(Nt(s(p(n)))\backslash(Nt(s(m))\backslash Si)) \Rightarrow Nt(s(m))\backslash Si \qquad Nt(s(m)), Nt(s(m)), \boxed{Nt(s(m))\backslash(Nt(s(m))\backslash Si)\{1\}} \Rightarrow Si}{Nt(s(m)), Nt(s(m)), Nt(s(p(n)))/CNp(n), CNp(n), \boxed{(Nt(s(m))\backslash Si){\downarrow}(Nt(s(m))\backslash(Nt(s(m))\backslash Si))\{1\}}, \rhd^{-1}(Nt(s(p(n)))\backslash(Nt(s(m))\backslash Si)) \Rightarrow Si}\ {\downarrow}L$$

$$\dfrac{Nt(s(m)), Nt(s(m)), Nt(s(p(n)))/CNp(n), CNp(n), \boxed{(Nt(s(m))\backslash Si){\downarrow}(Nt(s(m))\backslash(Nt(s(m))\backslash Si))\{1\}}, \rhd^{-1}(Nt(s(p(n)))\backslash(Nt(s(m))\backslash Si)) \Rightarrow Si}{Nt(s(m)), Nt(s(m)), Nt(s(p(n)))/CNp(n), CNp(n), 1, \boxed{\rhd^{-1}((Nt(s(m))\backslash Si){\downarrow}(Nt(s(m))\backslash(Nt(s(m))\backslash Si)))}, \rhd^{-1}(Nt(s(p(n)))\backslash(Nt(s(m))\backslash Si)) \Rightarrow Si}\ \rhd^{-1}L$$

$$\dfrac{Nt(s(m)), Nt(s(m)), Nt(s(p(n)))/CNp(n), CNp(n), 1, \boxed{\rhd^{-1}((Nt(s(m))\backslash Si){\downarrow}(Nt(s(m))\backslash(Nt(s(m))\backslash Si)))}, \rhd^{-1}(Nt(s(p(n)))\backslash(Nt(s(m))\backslash Si)) \Rightarrow Si}{Nt(s(m)), Nt(s(p(n)))/CNp(n), CNp(n), 1, \rhd^{-1}((Nt(s(m))\backslash Si){\downarrow}(Nt(s(m))\backslash(Nt(s(m))\backslash Si))), \rhd^{-1}(Nt(s(p(n)))\backslash(Nt(s(m))\backslash Si)) \Rightarrow Nt(s(m))\backslash Si}\ \backslash R$$

$$\dfrac{Nt(s(m)) \Rightarrow Nt(s(m)) \qquad \boxed{Sf} \Rightarrow Sf}{Nt(s(m)), \boxed{Nt(s(m))\backslash Sf} \Rightarrow Sf}\ \backslash L$$

$$\dfrac{Nt(s(f)) \Rightarrow Nt(s(f)) \qquad Nt(s(m)), \boxed{Nt(s(m))\backslash Sf} \Rightarrow Sf}{Nt(s(m)), Nt(s(f)), \boxed{Nt(s(f))\backslash(Nt(s(m))\backslash Sf)} \Rightarrow Sf}\ \backslash L$$

$$\dfrac{Nt(s(m)), Nt(s(p(n)))/CNp(n), CNp(n), 1, \rhd^{-1}((Nt(s(m))\backslash Si){\downarrow}(Nt(s(m))\backslash(Nt(s(m))\backslash Si))), \rhd^{-1}(Nt(s(p(n)))\backslash(Nt(s(m))\backslash Si)) \Rightarrow Nt(s(m))\backslash Si \qquad Nt(s(m)), Nt(s(f)), \boxed{Nt(s(f))\backslash(Nt(s(m))\backslash Sf)} \Rightarrow Sf}{Nt(s(m)), Nt(s(f)), Nt(s(m)), Nt(s(p(n)))/CNp(n), CNp(n), \boxed{(Nt(s(m))\backslash Si){\downarrow}(Nt(s(f))\backslash(Nt(s(m))\backslash Sf))}, \rhd^{-1}((Nt(s(m))\backslash Si){\downarrow}(Nt(s(m))\backslash(Nt(s(m))\backslash Si))), \rhd^{-1}(Nt(s(p(n)))\backslash(Nt(s(m))\backslash Si)) \Rightarrow Sf}\ {\downarrow}L$$

$(((saw\ (((help\ ((feed\ (the\ hippos))\ h))\ h)\ c))\ c)\ j)$

(d(7)) **wil+jan+boeken+lezen** : Q

$(NA\backslash Si){\downarrow}(NA\backslash Sf) : \lambda B\lambda C((want\ (B\ C))\ C), Nt(s(m)) : j, Np(n) : books, \rhd^{-1}(ND\backslash(NE\backslash Si)) : read\ \Rightarrow\ Q$

$Q/\hat{}(Sf{\uparrow}((NA\backslash Si){\downarrow}(NA\backslash Sf))) : \lambda B(B\ \lambda C\lambda D((want\ (C\ D))\ D)), Nt(s(m)) : j, Np(n) : books, \rhd^{-1}(NE\backslash(NF\backslash Si)) : read\ \Rightarrow\ Q$

$$\dfrac{Nt(s(m))\ \Rightarrow\ Nt(s(m)) \qquad \boxed{Si\{1\}}\ \Rightarrow\ Si}{Nt(s(m)), \boxed{Nt(s(m))\backslash Si\{1\}}\ \Rightarrow\ Si}\ \backslash L$$

$$\dfrac{Np(n)\ \Rightarrow\ Np(n) \qquad Nt(s(m)), \boxed{Nt(s(m))\backslash Si\{1\}}\ \Rightarrow\ Si}{Nt(s(m)), Np(n), \boxed{Np(n)\backslash(Nt(s(m))\backslash Si\{1\})}\ \Rightarrow\ Si}\ \backslash L$$

$$\dfrac{Nt(s(m)), Np(n), \boxed{Np(n)\backslash(Nt(s(m))\backslash Si\{1\})}\ \Rightarrow\ Si}{Nt(s(m)), Np(n), 1, \boxed{\rhd^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si))}\ \Rightarrow\ Si}\ \rhd^{-1}L$$

$$\dfrac{Nt(s(m)), Np(n), 1, \boxed{\rhd^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si))}\ \Rightarrow\ Si}{Np(n), 1, \rhd^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si))\ \Rightarrow\ Nt(s(m))\backslash Si}\ \backslash R$$

$$\dfrac{Nt(s(m))\ \Rightarrow\ Nt(s(m)) \qquad \boxed{Sf}\ \Rightarrow\ Sf}{Nt(s(m)), \boxed{Nt(s(m))\backslash Sf}\ \Rightarrow\ Sf}\ \backslash L$$

$$\dfrac{Np(n), 1, \rhd^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si))\ \Rightarrow\ Nt(s(m))\backslash Si \qquad Nt(s(m)), \boxed{Nt(s(m))\backslash Sf}\ \Rightarrow\ Sf}{Nt(s(m)), Np(n), \boxed{(Nt(s(m))\backslash Si){\downarrow}(Nt(s(m))\backslash Sf)}, \rhd^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si))\ \Rightarrow\ Sf}\ {\downarrow}L$$

$$\dfrac{Nt(s(m)), Np(n), \boxed{(Nt(s(m))\backslash Si){\downarrow}(Nt(s(m))\backslash Sf)}, \rhd^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si))\ \Rightarrow\ Sf}{Nt(s(m)), Np(n), 1, \rhd^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si))\ \Rightarrow\ Sf{\uparrow}((Nt(s(m))\backslash Si){\downarrow}(Nt(s(m))\backslash Sf))}\ {\uparrow}R$$

$$\dfrac{Nt(s(m)), Np(n), 1, \rhd^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si))\ \Rightarrow\ Sf{\uparrow}((Nt(s(m))\backslash Si){\downarrow}(Nt(s(m))\backslash Sf))}{Nt(s(m)), Np(n), \rhd^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si))\ \Rightarrow\ \boxed{\hat{}(Sf{\uparrow}((Nt(s(m))\backslash Si){\downarrow}(Nt(s(m))\backslash Sf)))}}\ \hat{}R$$

$$\dfrac{Nt(s(m)), Np(n), \rhd^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si))\ \Rightarrow\ \boxed{\hat{}(Sf{\uparrow}((Nt(s(m))\backslash Si){\downarrow}(Nt(s(m))\backslash Sf)))} \qquad \boxed{Q}\ \Rightarrow\ Q}{\boxed{Q/\hat{}(Sf{\uparrow}((Nt(s(m))\backslash Si){\downarrow}(Nt(s(m))\backslash Sf)))}, Nt(s(m)), Np(n), \rhd^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si))\ \Rightarrow\ Q}\ /L$$

$((want\ ((read\ books)\ j))\ j)$

(d(8)) **jan+wil+boeken+lezen** : $Nt(s(m)){\bullet}\hat{}(Q{\uparrow}Nt(s(m)))$

$Nt(s(m)) : j, (NA\backslash Si){\downarrow}(NA\backslash Sf) : \lambda B\lambda C((want\ (B\ C))\ C), Np(n) : books, \rhd^{-1}(ND\backslash(NE\backslash Si)) : read\ \Rightarrow\ Nt(s(m)){\bullet}\hat{}(Q{\uparrow}Nt(s(m)))$

$Nt(s(m)) : j, Q/\hat{}(Sf{\uparrow}((NA\backslash Si){\downarrow}(NA\backslash Sf))) : \lambda B(B\ \lambda C\lambda D((want\ (C\ D))\ D)), Np(n) : books, \rhd^{-1}(NE\backslash(NF\backslash Si)) : read\ \Rightarrow\ Nt(s(m)){\bullet}\hat{}(Q{\uparrow}Nt(s(m)))$

$$\dfrac{Np(n)\ \Rightarrow\ Np(n) \qquad \boxed{Si\{1\}}\ \Rightarrow\ Si}{Np(n), \boxed{Np(n)\backslash Si\{1\}}\ \Rightarrow\ Si}\ \backslash L$$

$$\dfrac{Nt(s(m))\ \Rightarrow\ Nt(s(m)) \qquad Np(n), \boxed{Np(n)\backslash Si\{1\}}\ \Rightarrow\ Si}{Np(n), Nt(s(m)), \boxed{Nt(s(m))\backslash(Np(n)\backslash Si\{1\})}\ \Rightarrow\ Si}\ \backslash L$$

$$\dfrac{Np(n), Nt(s(m)), \boxed{Nt(s(m))\backslash(Np(n)\backslash Si\{1\})}\ \Rightarrow\ Si}{Np(n), Nt(s(m)), 1, \boxed{\rhd^{-1}(Nt(s(m))\backslash(Np(n)\backslash Si))}\ \Rightarrow\ Si}\ \rhd^{-1}L$$

$$\dfrac{Np(n), Nt(s(m)), 1, \boxed{\rhd^{-1}(Nt(s(m))\backslash(Np(n)\backslash Si))}\ \Rightarrow\ Si}{Nt(s(m)), 1, \rhd^{-1}(Nt(s(m))\backslash(Np(n)\backslash Si))\ \Rightarrow\ Np(n)\backslash Si}\ \backslash R$$

$$\dfrac{Np(n)\ \Rightarrow\ Np(n) \qquad \boxed{Sf}\ \Rightarrow\ Sf}{Np(n), \boxed{Np(n)\backslash Sf}\ \Rightarrow\ Sf}\ \backslash L$$

$$\dfrac{Nt(s(m)), 1, \rhd^{-1}(Nt(s(m))\backslash(Np(n)\backslash Si))\ \Rightarrow\ Np(n)\backslash Si \qquad Np(n), \boxed{Np(n)\backslash Sf}\ \Rightarrow\ Sf}{Np(n), Nt(s(m)), \boxed{(Np(n)\backslash Si){\downarrow}(Np(n)\backslash Sf)}, \rhd^{-1}(Nt(s(m))\backslash(Np(n)\backslash Si))\ \Rightarrow\ Sf}\ {\downarrow}L$$

$$\dfrac{Np(n), Nt(s(m)), \boxed{(Np(n)\backslash Si){\downarrow}(Np(n)\backslash Sf)}, \rhd^{-1}(Nt(s(m))\backslash(Np(n)\backslash Si))\ \Rightarrow\ Sf}{Np(n), Nt(s(m)), 1, \rhd^{-1}(Nt(s(m))\backslash(Np(n)\backslash Si))\ \Rightarrow\ Sf{\uparrow}((Np(n)\backslash Si){\downarrow}(Np(n)\backslash Sf))}\ {\uparrow}R$$

$$\dfrac{Np(n), Nt(s(m)), 1, \rhd^{-1}(Nt(s(m))\backslash(Np(n)\backslash Si))\ \Rightarrow\ Sf{\uparrow}((Np(n)\backslash Si){\downarrow}(Np(n)\backslash Sf))}{Np(n), Nt(s(m)), \rhd^{-1}(Nt(s(m))\backslash(Np(n)\backslash Si))\ \Rightarrow\ \boxed{\hat{}(Sf{\uparrow}((Np(n)\backslash Si){\downarrow}(Np(n)\backslash Sf)))}}\ \hat{}R$$

$$\dfrac{Np(n), Nt(s(m)), \rhd^{-1}(Nt(s(m))\backslash(Np(n)\backslash Si))\ \Rightarrow\ \boxed{\hat{}(Sf{\uparrow}((Np(n)\backslash Si){\downarrow}(Np(n)\backslash Sf)))} \qquad \boxed{Q}\ \Rightarrow\ Q}{\boxed{Q/\hat{}(Sf{\uparrow}((Np(n)\backslash Si){\downarrow}(Np(n)\backslash Sf)))}, Np(n), Nt(s(m)), \rhd^{-1}(Nt(s(m))\backslash(Np(n)\backslash Si))\ \Rightarrow\ Q}\ /L$$

$$\dfrac{\boxed{Q/\hat{}(Sf{\uparrow}((Np(n)\backslash Si){\downarrow}(Np(n)\backslash Sf)))}, Np(n), Nt(s(m)), \rhd^{-1}(Nt(s(m))\backslash(Np(n)\backslash Si))\ \Rightarrow\ Q}{Q/\hat{}(Sf{\uparrow}((Np(n)\backslash Si){\downarrow}(Np(n)\backslash Sf))), Np(n), 1, \rhd^{-1}(Nt(s(m))\backslash(Np(n)\backslash Si))\ \Rightarrow\ Q{\uparrow}Nt(s(m))}\ {\uparrow}R$$

$$\dfrac{Q/\hat{}(Sf{\uparrow}((Np(n)\backslash Si){\downarrow}(Np(n)\backslash Sf))), Np(n), 1, \rhd^{-1}(Nt(s(m))\backslash(Np(n)\backslash Si))\ \Rightarrow\ Q{\uparrow}Nt(s(m))}{Q/\hat{}(Sf{\uparrow}((Np(n)\backslash Si){\downarrow}(Np(n)\backslash Sf))), Np(n), \rhd^{-1}(Nt(s(m))\backslash(Np(n)\backslash Si))\ \Rightarrow\ \boxed{\hat{}(Q{\uparrow}Nt(s(m)))}}\ \hat{}R$$

$$\dfrac{Nt(s(m))\ \Rightarrow\ Nt(s(m)) \qquad Q/\hat{}(Sf{\uparrow}((Np(n)\backslash Si){\downarrow}(Np(n)\backslash Sf))), Np(n), \rhd^{-1}(Nt(s(m))\backslash(Np(n)\backslash Si))\ \Rightarrow\ \boxed{\hat{}(Q{\uparrow}Nt(s(m)))}}{Nt(s(m)), Q/\hat{}(Sf{\uparrow}((Np(n)\backslash Si){\downarrow}(Np(n)\backslash Sf))), Np(n), \rhd^{-1}(Nt(s(m))\backslash(Np(n)\backslash Si))\ \Rightarrow\ \boxed{Nt(s(m)){\bullet}\hat{}(Q{\uparrow}Nt(s(m)))}}\ {\bullet}R$$

$(j, \lambda A((want\ ((read\ A)\ books))\ books))$

$$\frac{\overline{Nt(s(m)) \Rightarrow Nt(s(m))} \qquad \overline{\boxed{Si\{1\}} \Rightarrow Si}}{Nt(s(m)), \boxed{Nt(s(m))\backslash Si\{1\}} \Rightarrow Si}\ \backslash L$$

$$\frac{\overline{Np(n) \Rightarrow Np(n)} \qquad Nt(s(m)), \boxed{Nt(s(m))\backslash Si\{1\}} \Rightarrow Si}{Nt(s(m)), Np(n), \boxed{Np(n)\backslash(Nt(s(m))\backslash Si\{1\})} \Rightarrow Si}\ \backslash L$$

$$\frac{Nt(s(m)), Np(n), \boxed{Np(n)\backslash(Nt(s(m))\backslash Si\{1\})} \Rightarrow Si}{Nt(s(m)), Np(n), 1, \boxed{\triangleright^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si))} \Rightarrow Si}\ \triangleright^{-1}L$$

$$\frac{Nt(s(m)), Np(n), 1, \boxed{\triangleright^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si))} \Rightarrow Si}{Np(n), 1, \triangleright^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si)) \Rightarrow Nt(s(m))\backslash Si}\ \backslash R$$

$$\frac{\overline{Nt(s(m)) \Rightarrow Nt(s(m))} \qquad \overline{\boxed{Sf} \Rightarrow Sf}}{Nt(s(m)), \boxed{Nt(s(m))\backslash Sf} \Rightarrow Sf}\ \backslash L$$

$$\frac{Np(n), 1, \triangleright^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si)) \Rightarrow Nt(s(m))\backslash Si \qquad Nt(s(m)), \boxed{Nt(s(m))\backslash Sf} \Rightarrow Sf}{Nt(s(m)), Np(n), \boxed{(Nt(s(m))\backslash Si){\downarrow}(Nt(s(m))\backslash Sf)}, \triangleright^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si)) \Rightarrow Sf}\ {\downarrow}L$$

$$\frac{Nt(s(m)), Np(n), \boxed{(Nt(s(m))\backslash Si){\downarrow}(Nt(s(m))\backslash Sf)}, \triangleright^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si)) \Rightarrow Sf}{Nt(s(m)), Np(n), 1, \triangleright^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si)) \Rightarrow Sf{\uparrow}((Nt(s(m))\backslash Si){\downarrow}(Nt(s(m))\backslash Sf))}\ {\uparrow}R$$

$$\frac{Nt(s(m)), Np(n), 1, \triangleright^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si)) \Rightarrow Sf{\uparrow}((Nt(s(m))\backslash Si){\downarrow}(Nt(s(m))\backslash Sf))}{Nt(s(m)), Np(n), \triangleright^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si)) \Rightarrow \boxed{{}^\wedge(Sf{\uparrow}((Nt(s(m))\backslash Si){\downarrow}(Nt(s(m))\backslash Sf)))}}\ {}^\wedge R$$

$$\frac{Nt(s(m)), Np(n), \triangleright^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si)) \Rightarrow \boxed{{}^\wedge(Sf{\uparrow}((Nt(s(m))\backslash Si){\downarrow}(Nt(s(m))\backslash Sf)))} \qquad \overline{\boxed{Q} \Rightarrow Q}}{\boxed{Q/{}^\wedge(Sf{\uparrow}((Nt(s(m))\backslash Si){\downarrow}(Nt(s(m))\backslash Sf)))}, Nt(s(m)), Np(n), \triangleright^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si)) \Rightarrow Q}\ /L$$

$$\frac{\boxed{Q/{}^\wedge(Sf{\uparrow}((Nt(s(m))\backslash Si){\downarrow}(Nt(s(m))\backslash Sf)))}, Nt(s(m)), Np(n), \triangleright^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si)) \Rightarrow Q}{Q/{}^\wedge(Sf{\uparrow}((Nt(s(m))\backslash Si){\downarrow}(Nt(s(m))\backslash Sf))), 1, Np(n), \triangleright^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si)) \Rightarrow Q{\uparrow}Nt(s(m))}\ {\uparrow}R$$

$$\frac{Q/{}^\wedge(Sf{\uparrow}((Nt(s(m))\backslash Si){\downarrow}(Nt(s(m))\backslash Sf))), 1, Np(n), \triangleright^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si)) \Rightarrow Q{\uparrow}Nt(s(m))}{Q/{}^\wedge(Sf{\uparrow}((Nt(s(m))\backslash Si){\downarrow}(Nt(s(m))\backslash Sf))), Np(n), \triangleright^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si)) \Rightarrow \boxed{{}^\wedge(Q{\uparrow}Nt(s(m)))}}\ {}^\wedge R$$

$$\frac{\overline{Nt(s(m)) \Rightarrow Nt(s(m))} \qquad Q/{}^\wedge(Sf{\uparrow}((Nt(s(m))\backslash Si){\downarrow}(Nt(s(m))\backslash Sf))), Np(n), \triangleright^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si)) \Rightarrow \boxed{{}^\wedge(Q{\uparrow}Nt(s(m)))}}{Nt(s(m)), Q/{}^\wedge(Sf{\uparrow}((Nt(s(m))\backslash Si){\downarrow}(Nt(s(m))\backslash Sf))), Np(n), \triangleright^{-1}(Np(n)\backslash(Nt(s(m))\backslash Si)) \Rightarrow \boxed{Nt(s(m)){\bullet}{}^\wedge(Q{\uparrow}Nt(s(m)))}}\ {\bullet}R$$

$(j, \lambda A((want\ ((read\ books)\ A))\ A))$