XIII International Conference on Computational Plasticity. Fundamentals and Applications
COMPLAS XIII
E. Oñate, D.R.J. Owen, D. Peric and M. Chiumenti (Eds)

SOURCES OF ERROR AND LIMITS OF APPLYING THE ENERGETIC-BASED-REGULARIZATION METHOD

N. AISSAOUI [*] & M. MATALLAH

[*] Risk Assessment & Management (RISAM)
Université de Tlemcen, Algeria
BP 230, Tlemcen, Algeria
e-mail: nassima.aissaoui@mail.univ-tlemcen.dz
e-mail: mohammed.matallah@gmail.com

Key words: Fracture energy, Localization, Structural analysis.

Summary. *In this work, the energetic regularization technique based on the Crack Band approach is studied. The main objective is to identify the sources of errors related to the application of such technique under 2D and 3D computational analysis. Mathematical developments and numerical simulations are proposed in order to assess the limits of application of this technique.*

1 INTRODUCTION

Numerical modeling is an essential tool in the field of analysis and study of the civil engineering structures. Numerical simulation of the nonlinear behavior of concrete suffers from a serious problem related to the localization mainly due to the softening parameter, which leads to a non-objective analysis.

The property of softening leads to a loss of ellipticity of the equilibrium equations under static and quasi-static analysis and a loss of hyperbolicity under dynamic analysis. In a conventional finite element modeling, within each element, the strain and the displacement fields are continuous and the strain discontinuities are "repulsed" to the boundaries of the finite elements. This leads to a strain localization in a band of a thickness controlled by the size of the finite element. If the FE mesh is very fine, the size of this zone decreases. Since the fracture energy is related to the volume of the dissipation area, if the size of the finite element approaches zero, a structure could collapse without dissipating energy.

An efficient technique widely used in practical engineering uses the fracture energy as a regularization parameter. Based on the "Crack Band approach" [1], where it is assumed that a crack (mode I) is spread over a bandwidth "h", the energy released to open a crack is directly related to the area under the stress-strain curve.

This energetic approach does not permit to solve the mathematical problem (loss of ellipticity or loss of hyperbolicity). The fracture energy is injected into a finite element calculation to preserve the energy dissipation and make it FE size-independent which could eliminate the pathological FE-mesh sensitivity.

From a practical point of view, the value of G_f controls the dissipation process during the degradation of the material through a parameter of the constitutive law used (damage or plastic-damage law). This energetic approach could be used only if the behavior-law formulation permits to establish a relationship between the parameter controlling softening and the parameter G_f. This relationship is generally obtained for a one-dimensional case, ie, for an element subjected to a uniaxial stress state. However, it has been shown [2][3] that, in some cases, even if the cracking state is under mode I (crack band approach basis), the stress state is not always uni-axial.

In [2], Jirasek and Bauer examined the numerical aspects related to the use of this energetic method. The influence of the type of the finite elements used, the integration scheme ... were studied to detect sources of errors related to the numerical aspects. In the present work, we propose a complementary study. The mathematical aspects of this approach are discussed including those related to its use in the case of a state of non-uniaxial stress (2D and 3D).. Mathematical developments are performed to understand the evolution of the fracture energy based on the relationships between the components of the stress tensor in 2D / 3D. These developments are validated by numerical examples.

2 THE ENERGETIC REGULARIZATION BASIS

Within a non-linear behavior formulation, the area under the stress-strain curve represents the dissipated energy density per unit volume

$$W = \int_0^{\infty} \sigma_{ij} d\varepsilon_{ij} \tag{1}$$

and the product of this quantity by the size of the localization area h gives the fracture energy G_f needed to create a cracking unit surface (energy dissipated per unit area)

$$G_f = \int_0^{\infty} \sigma du = hW \tag{2}$$

To represent the cracking process under mode I, Bazant and Oh [1] consider that the crack is spread over a band of width h, which allows calculating the displacement of the jump as the product of the fracture strain by the band width h

$$G_f = \int_0^{\infty} \sigma du = h\int_0^{\infty} \sigma d\varepsilon^f \tag{3}$$

Using the energetic approach, the cracking process in concrete is thus governed by the fracture energy. The value of the fracture energy controls the dissipation process during the degradation of the material via a parameter of the nonlinear behavior law that controls softening. In the following analysis, a simple isotropic damage model is used [4][5]. The stress-strain relationships is written as

$$\sigma_{ij} = (1-d)\sigma_{ij} = (1-d)C^0_{ijkl}\varepsilon_{kl} \tag{4}$$

The damage evolution is described by an exponential evolution function of the equivalent strain. For the equivalent strain, we use the Mazars's definition [6].

$$d = 1 - \frac{\tilde{\varepsilon}_e}{\varepsilon_{d0}} \exp(B(\tilde{\varepsilon}_e - \varepsilon_{d0})) \quad \tilde{\varepsilon}_e = \sqrt{\langle \varepsilon_e^1 \rangle_+^2 + \langle \varepsilon_e^2 \rangle_+^2 + \langle \varepsilon_e^3 \rangle_+^2} \tag{5}$$

For a one-dimensional case with a mode I of crack propagation, we obtain [5]

$$G_f = h\int_0^\infty E(1-d)\varepsilon \, d\varepsilon = h\int_0^\infty E\left(\frac{\varepsilon_{d0}}{\varepsilon}\exp[B(\varepsilon_{d0}-\varepsilon)]\right)\varepsilon \, d\varepsilon = h\frac{E\varepsilon_{d0}^2}{2} + h\frac{E\varepsilon_{d0}}{B} \tag{6}$$

With h the size of the finite element and B the parameter controlling the softening.

Under a uniaxial stress stat, using the formulation given in equation (6) ensures that the model dissipates the same fracture energy injected.

2.1 For two-dimensional analysis

Using the same behavior law, Matallah et al [3] shows, for a beam under three point bending, that even if the cracking process is under mode I, the presence of a biaxial stress state modifies the dissipation inside the element.

If we consider the same damage behavior law presented above and if γ is the stress ratio σ_2/σ_1, the following formulas are obtained for the G_f evolution (0.2 is the value of the Poisson's ratio):

$$\gamma < 0.2 \Rightarrow G_f = h\eta_1 E\left(\frac{\varepsilon_{d0}}{B} + \frac{\varepsilon_{d0}^2}{2}\right) \tag{7}$$

$$\gamma \geq 0.2 \Rightarrow G_f = h\eta_2 E\left(\frac{\varepsilon_{d0}}{B} + \frac{\varepsilon_{d0}^2}{2}\right) \tag{8}$$

$$\eta_1 = \left(\frac{(1-2\gamma\nu+\gamma^2)}{(1-\gamma\nu)^2}\right), \quad \eta_2 = \left(\frac{(1-2\gamma\nu+\gamma^2)}{(1-\gamma\nu)^2+(\gamma-\nu)^2}\right) \tag{9}$$

These formulas are only valid for positive stress ratios. In the case of the presence of a negative stress (compression), the formulas are more complex.

Numerically speaking, let us consider an elementary volume subjected to a biaxial stress state with a ratio $\gamma = 1$. The simulations are performed with the same parameter set used for the 1D simulation. Two types of analysis are performed: the first one with a G_f issue form a 1D formulation (equation 6) and the second one by considering the 2D formula (equation 8). Table 1 shows the real dissipation in the element in the two case study. The error induced by considering a 1D formulation for the fracture energy is about 25%.

Table 1: Real energy dissipation in the case of 2D analysis (G_f injected = 150 N/m)

Dimensions L (m)	Simulation carried out with the formula 2D (equation 8)	Simulation carried out with the formula 1D (equation 6)
0.001	150.01	187.51
0.01	150.01	187.51
0.1	149.99	187.48

3 3D THEORETICAL ASPECTS OF THE CRACK BAND APPROACH

In this present paper, we propose an analytical expression for the evolution of the fracture energy in the case of a three-dimensional analysis. For an elementary volume submitted to a triaxial tension state, the fracture energy is evaluated by:

$$G_f = h \times \left(\sum_{I=1}^{3} \int_0^{\infty} \sigma_I d\varepsilon_I \right) \tag{10}$$

with $\sigma_I, \varepsilon_I, I = 1,2,3$ the principal stresses and the principal strains in the direction I.

In the present study, we adopt $h = \sqrt[3]{V_{elem}}$ with V_{elem} the volume of the finite element. Note γ_1 , γ_2 the two stress ratios σ_2 / σ_1 and σ_3 / σ_1.

$$\frac{\sigma_2}{\sigma_1} = \frac{\widetilde{\sigma}_2}{\widetilde{\sigma}_1} = \gamma_1 \Rightarrow \frac{\varepsilon_2}{\varepsilon_1} = \frac{\gamma_1(1-2\nu) + (\gamma_2+1)(2\nu^2-\nu)}{1-2\nu+(2\nu^2-\nu)(\gamma_1+\gamma_2)} \tag{11}$$

$$\frac{\sigma_3}{\sigma_1} = \frac{\widetilde{\sigma}_3}{\widetilde{\sigma}_1} = \gamma_2 \Rightarrow \frac{\varepsilon_3}{\varepsilon_1} = \frac{\gamma_2(1-2\nu) + (\gamma_1+1)(2\nu^2-\nu)}{1-2\nu+(2\nu^2-\nu)(\gamma_1+\gamma_2)} \tag{12}$$

According to the strains ratios (Equation 11 et 12) we distinguish four domains. Table 2 illustrates the different zones and the corresponding values of G_f.

Table 2: Formulas of G_f in the case of 3D analysis.

Zone I	$\widetilde{\varepsilon} = \varepsilon_I$	$G_f = h\eta_1 E\left(\frac{\varepsilon_{d0}^2}{2} + \frac{\varepsilon_{d0}}{B}\right)$
Zone 2	$\widetilde{\varepsilon} = \varepsilon_1 \frac{\sqrt{(1-2\nu+(2\nu^2-\nu)(\gamma_1+\gamma_2))^2 + (\gamma_1(1-2\nu)}}{1-2\nu+(2\nu^2-\nu)(\gamma_1+\gamma_2)}$ $+\varepsilon_1 \frac{\sqrt{(2\nu^2-\nu)(\gamma_2+1))^2 + (\gamma_2(1-2\nu)+(2\nu^2-\nu)(\gamma_1+1))^2}}{1-2\nu+(2\nu^2-\nu)(\gamma_1+\gamma_2)}$	$G_f = h\eta_2 E\left(\frac{\varepsilon_{d0}^2}{2} + \frac{\varepsilon_{d0}}{B}\right)$
Zone 3	$\widetilde{\varepsilon} = \varepsilon_1 \frac{\sqrt{(1-2\nu+(2\nu^2-\nu)(\gamma_1+\gamma_2))^2}}{1-2\nu+(2\nu^2-\nu)(\gamma_1+\gamma_2)}$ $+\varepsilon_1 \frac{\sqrt{(\gamma_1(1-2\nu)+(2\nu^2-\nu)(\gamma_2+1))^2}}{1-2\nu+(2\nu^2-\nu)(\gamma_1+\gamma_2)}$	$G_f = h\eta_3 E\left(\frac{\varepsilon_{d0}^2}{2} + \frac{\varepsilon_{d0}}{B}\right)$
Zone 4	$\widetilde{\varepsilon} = \varepsilon_1 \frac{\sqrt{(1-2\nu+(2\nu^2-\nu)(\gamma_1+\gamma_2))^2}}{1-2\nu+(2\nu^2-\nu)(\gamma_1+\gamma_2)}$ $+\varepsilon_1 \frac{\sqrt{(\gamma_2(1-2\nu)+(2\nu^2-\nu)(\gamma_1+1))^2}}{1-2\nu+(2\nu^2-\nu)(\gamma_1+\gamma_2)}$	$G_f = h\eta_4 E\left(\frac{\varepsilon_{d0}^2}{2} + \frac{\varepsilon_{d0}}{B}\right)$

With the following formulation of the parameters η_1 , η_2 , η_3 and η_4 :

$$\eta_1 = \frac{(1-3\nu+4\nu^3)[(1-2\nu)(1+\gamma_1^2+\gamma_2^2)+2(2\nu^2-\nu)(\gamma_1+\gamma_2+\gamma_1\gamma_2)]}{(1+\nu)(1-2\nu)[1-2\nu+(2\nu^2-\nu)(\gamma_1+\gamma_2)]^2} \tag{13}$$

$$\eta_2 = \frac{1}{(1+\nu)(1-2\nu)} \times \frac{(1-3\nu+4\nu^3)[(1-2\nu)(1+\gamma_1^2+\gamma_2^2)+2(2\nu^2-\nu)(\gamma_1+\gamma_2+\gamma_1\gamma_2)]}{[(1-2\nu+(2\nu^2-\nu)(\gamma_1+\gamma_2))^2+(\gamma_1(1-2\nu)+(2\nu^2-\nu)(\gamma_2+1))^2+(\gamma_2(1-2\nu)+(2\nu^2-\nu)(\gamma_1+1))^2} \tag{14}$$

$$\eta_3 = \frac{(1-3\nu+4\nu^3)[(1-2\nu)(1+\gamma_1^2+\gamma_2^2)+2(2\nu^2-\nu)(\gamma_1+\gamma_2+\gamma_1\gamma_2)]}{(1+\nu)(1-2\nu)[(1-2\nu+(2\nu^2-\nu)(\gamma_1+\gamma_2))^2+(\gamma_2(1-2\nu)+(2\nu^2-\nu)(\gamma_1+1))^2]} \tag{15}$$

$$\eta_4 = \frac{(1-3\nu+4\nu^3)[(1-2\nu)(1+\gamma_1^2+\gamma_2^2)+2(2\nu^2-\nu)(\gamma_1+\gamma_2+\gamma_1\gamma_2)]}{(1+\nu)(1-2\nu)[(1-2\nu+(2\nu^2-\nu)(\gamma_1+\gamma_2))^2+(\gamma_1(1-2\nu)+(2\nu^2-\nu)(\gamma_2+1))^2]} \tag{16}$$

The results of simulations are presented in Table 3. The simulation concerns an element subjected to a three-dimensional stress state (triaxial tension) with $\gamma_1 = 1$ and $\gamma_2 = 1$ which corresponds to a formula G_f of zone 2.

Table 3: Real dissipation in the case of 3D analysis (G_f injected = 150 N/m)

Dimensions L (m)	Simulation carried out with the formula 3D (Zone 2)	Simulation carried out with the formula 1D (equation 6)
0.001	150.00	250
0.01	149.99	250
0.1	149.82	249.85

Table 3 shows the real dissipation within the elementary volume in case of a 3D analysis. By using a formulation of G_f issued from a one-dimensional analysis the committed error is considerable. The fracture energy really dissipated in the finite element is very different from that injected in the computation. Using a correct 3D formulation allows a conservation of the dissipated energy.

4 CONCLUSIONS

The energetic regularization technique is a practical method which allows to eliminate (completely or partially) the pathological FE-mesh sensitivity. However, it should be used with precaution. In a structural analysis that generates a state of cracking under mode I, the finite elements are subjected to different stress states. Mathematically speaking, The parameter of the constitutive law used to adjust the softening in order to control the dissipation is calculated by

assuming a uniaxial stress state. This aspect could induce a large error (overestimation or underestimation) regarding the real dissipated energy.

REFERENCES

[1] Bazant, Z.B. and Oh, B.H. Crack band theory for fracture of concrete. *Mater. Structu* (1983) **16**:155-177.

[2] Jirasek, M. and Bauer, M. Numerical Aspects of the crack band approach. *Computer and Structures* (2012) **110**:60-78.

[3] Matallah, M., Farah, M., Grondin, F. and Loukili, A. Size Independent Fracture Energy of Concrete at Very Early Ages by Inverse Analysis. *Engineering FracturebMechanics* (2013) **109**:1-16.

[4] Fichant, S. Endommagement et anisotropie induite du béton de structures. Modélisations approchées. Phd Thesis, ENS Cachant France. (1996).

[5] Matallah, M., La Borderie, C., and Maurel, O. A practical method to estimate crack openings in concrete structures. *Engineering Fracture Mechanics* (2010) **34**:1615-1633.

[6] Mazars, J. *Application de la mécanique de l'endommagement au comportement non linéaire et à la rupture du béton de structure*. Thèse de Doctorat, ParisVI, France (1984).