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NUMERICAL SIMULATIONS OF CORNERS IN RC FRAMES USING STRUT-AND-TIE METHOD AND CDP MODEL

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Abstract. Corners of reinforced concrete frames under opening bending moment should be always considered in complex stress conditions. A proper reinforcement of these corners demands some special methods instead of traditional algorithms. In this paper two chosen method are described, namely Strut-and-Tie (S&T) and FEM using Abaqus software. The second method uses Concrete Damaged Plasticity model, implemented in Abaqus code. The corners taken into consideration differ from each other in a reinforcement pattern. Moreover, two different cases of section heights are analyzed: same and different heights of elements joining in a corner. Calculations in S&T method allow to establish proper reinforcement and corner efficiency factor. Then the corners are modeled in Abaqus using CDP and results of crack patterns, efficiency factor, yielding, non-linear behavior and history of load are gained. An extra scientific problem described in this paper is a proper choice of CDP parameters.

1 INTRODUCTION

Corners under opening bending moment are very common case in structural design. They appear in single and multi storey frames, underground water tanks, retain walls and foundations. In the Figure 1 a corner under opening bending moment with sample crack pattern is presented.

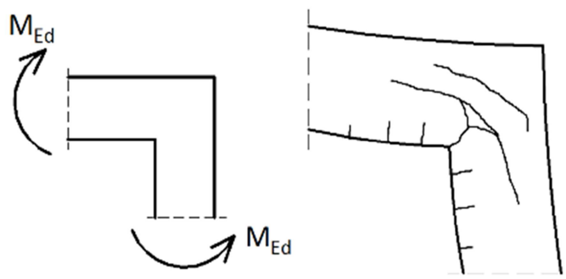


Figure 1: Corner under opening bending moment (crack pattern after [1])

Depending on the type of structure this corner can be considered in plane stress or plane strain state. The first case refers to single or multi storey frames and the second state is characteristic for tanks and retaining walls. Both these cases are useful for structural designers and are considered in this paper.

The choice of these corners for analysis is due to the fact that they are under complex stress conditions and a proper choice of their reinforcement requires some special methods. One of these methods is S&T, which is well described in Eurocode 2 [2] and FIB bulletins. However, S&T method for corners under opening bending moment is developed only for corner with the same section heights of beam and column. There is no guidance for corners with different section heights. Moreover, S&T allows only to calculate a required reinforcement and to check if maximal compression stress is not higher than admissible. The second method used by the authors is FEM, performed in Abaqus [3]. This method allows to recreate full history of load and yielding, including crack pattern and propagation. The reinforcement calculated in S&T is an input for FEM model and results gained in Abaqus allow to confirm which reinforcement pattern is worth recommendation.

2 INPUT DATA FOR FRAME CORNERS

The corners taken for analysis have some common properties, which are listed below. All corners are made of concrete C40/50 and reinforced with steel B500SP. Here are the material constants:

- concrete: f_{ck} =40 MPa, f_{cd} =34.30 MPa, E_{cm}=35 GPa, ν=0.167,
- steel: f_{yk} = 500 MPa, f_{yd} = 434.8 MPa, E_s=200 GPa, ν=0.3.

Each corner is loaded with an opening bending moment M=30 kNm modeled as a pair of forces of magnitude:

- 250 kN for the section height 200mm and the distance between the forces is 120mm,
- 71.43 kN for the section height 500mm and the distance between the forces is 420mm.

The geometry of corners for both cases is presented in the Figure 2. For the time being analysis is restricted to corners with the same section heights. In future authors plan to extend analysis for the case of different section heights. The reinforcement patterns taken into consideration are introduced in the Figure 3.

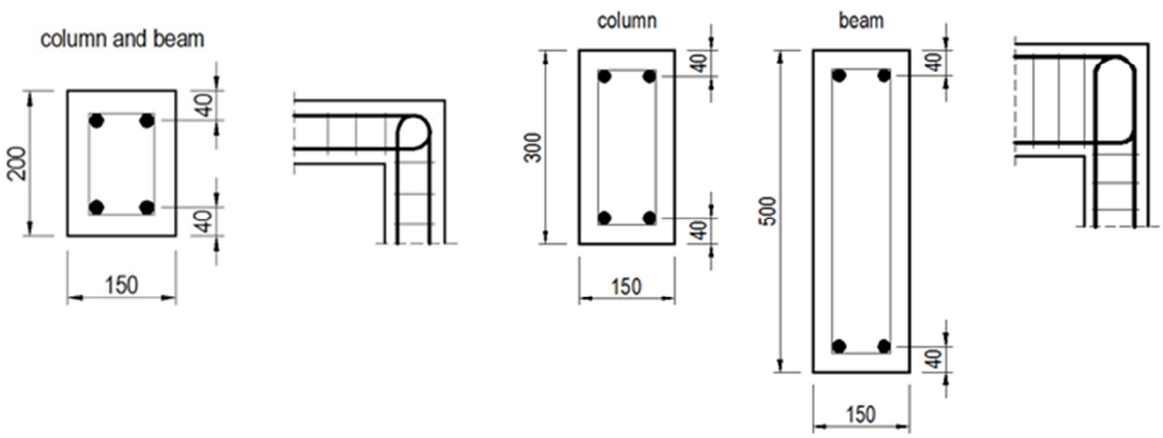


Figure 2: Dimensions of corners

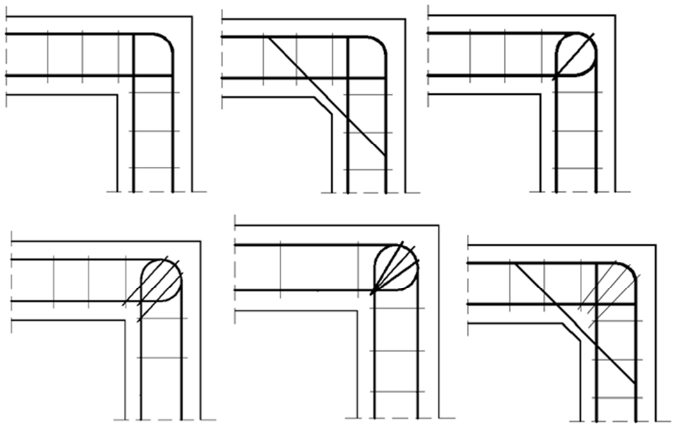


Figure 3: Reinforcement patterns for corners

The stress-strain curve for concrete in uniaxial compression is approximated by piecewise linearization of σ–ε curve given in EC2 [2]. The coordinates of assumed points are listed in the Table 1. For description of concrete in uniaxial tension a linear σ–ε relation is assumed in postcritical state for given fracture energy G_f.

Table 1: Compresive behavior of concrete

Yield stress [MPa]	Inelastic strain [$^o/_{oo}$]
13.70	0.00
29.30	0.95
34.30	1.75
33.70	2.25

The Abaqus user should also input some other important CDP model parameters. The authors of this paper analyze how the choice of these parameters affects the results of calculations. All these parameters are listed in the Table 2.

Table 2: CDP parameters range

Fracture energy G_f [Nm^{-1}]:	146.5, 500, 1000
Dilation angle [degrees]:	0, 5, 15, 30
Relaxation time [s]:	0.0001
Eccentricity:	0.1
fb0/fc0:	1.16
K:	0.667

For steel rebars elastic-idealplastic relation in uniaxial σ–ε state is assumed (Figure 4).

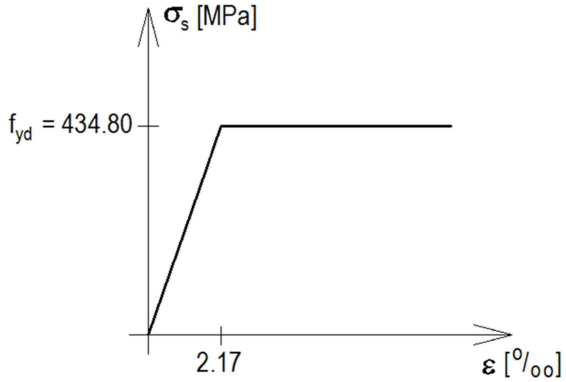


Figure 4: Reinforcing steel behavior

3 CONCRETE DAMAGED PLASTICITY MODEL

One of the concrete models available in Abaqus software is the so-called Concrete Damaged Plasticity. The theoretical framework of the model has been defined by Lubliner, Onate, Oller and Oliver [4]. A fundamental of this model is the assumption that there are two mechanisms of concrete damage: tensile cracking and compressive crushing. The amount of plastic deformation is controlled by the equivalent plastic strain in tension and compression. There are two damage parameters namely, d_t and d_c, where the subscript t denotes tension and c denotes compression of the concrete. These parameters are functions of plastic strain and take a value from 0 (in virgin state) to 1 (a total loss of capacity). Due to these parameters the reduction of the material stiffness is defined according to the formula [5]:

$$\boldsymbol{D} = \boldsymbol{D_o}(1 - d) \quad (1)$$

where $\mathbf{D}$ is a material stiffness operator, $\mathbf{D_0}$ is the stiffness operator in elastic range and d denotes one of these two parameters, d_t or d_c, depending on whether the material is in a compression or tension.

Abaqus allows piecewise linearization of σ–ε curve both in compression and tension according to values specified by user. Values used by authors are described in section 2. The program allows also to specify fracture energy $G_{f.}$

An important procedure that improves the convergence of the nonlinear problem is a regularization. One of the ways to carry it out is the use of visco-plastic model which takes into account the viscous properties of concrete. This possibility exists in Concrete Damaged Plasticity model in Abaqus and assumes Duvaut-Lions' viscoplastic model [6]. This regularization is expressed using formula:

$$\dot{\boldsymbol{\varepsilon}}_v^{pl} = \frac{1}{\mu}\left(\boldsymbol{\varepsilon}^{pl} - \boldsymbol{\varepsilon}_v^{pl}\right) \quad (2)$$

where $\boldsymbol{\varepsilon}_v^{pl}$ denotes the viscoplastic strain of concrete and $\boldsymbol{\varepsilon}^{pl}$ is the plastic strain of the concrete without taking into account the viscosity, μ is the relaxation time (in Abaqus called the viscosity parameter). In addition, in an analogous manner the degradation parameter of stiffness is regularized according to formula:

$$\dot{d}_v = \frac{1}{\mu}(d - d_v) \qquad (3)$$

where d is one of two degradation parameters d_c or d_t and d_v is the degradation parameter taking into account both viscoplasticity and viscous damage. The final relationship between stress and strain taking into account both viscoplasticity and viscous damage has form:

$$\boldsymbol{\sigma} = (1 - d_v)\boldsymbol{D_o}:(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_v^{pl}) \qquad (4)$$

where : denotes a scalar product of tensors.

The flow potential is defined by the hyperbolic function being asymptotic to the Drucker-Prager cone [5]. The yield function implemented in CDP model is specified according to Lubliner et al. [4] with later modifications by Lee and Fenves [7]. The plastic flow is nonassociated and therefore an Abaqus user should define dilation angle separately. For a full definition of CDP model in Abaqus one should input the following parameters:

- compressive behavior of concrete including compression damage condition,
- tension behavior of concrete (optionally fracture energy) including tension damage condition,
- dilation angle ψ in the p-q plane,
- flow potential eccentricity ε,
- the ratio fb0/fc0 of biaxial compressive yield stress to uniaxial compressive yield stress,
- the ratio K of the second stress invariant on the tensile meridian to that on the compressive meridian for the yield function,
- the viscosity parameter (if viscous regularization is used).

4 CALIBRATION OF CDP MODEL PARAMETERS

The calibration of CDP model parameters is performed in Abaqus software as a uniaxial and biaxial compression of a concrete specimen. Results of the calculations are compared with the well-known laboratory tests of Kupfer [8]. The specimen has the same properties and dimensions as Kupfer's ones (20x20x5 cm). The FEM analysis is performed in three-dimensional strain state. Finite elements are modeled as 3D cubic elements (Figure 5).

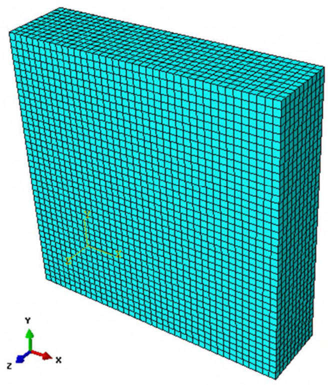


Figure 5: 3D view of a concrete specimen

One of the boundary condition of this model is a linear displacement of a top surface of the specimen (for uniaxial compression) or linear displacement of both top and lateral surface (for biaxial compression). The biaxial compression tests are divided into two separate cases: when compression for both directions is equal (1:1) and when compression for one direction is two times higher (1:0.5). Results of calculations are presented in the form of plots of linear strains and volumetric strain versus stress. A tested parameter is dilation angle, which varies from 0 through 5, 15 till 30 degrees.

In the Figures 6 and 7 linear and volumetric strains versus leading stress σ_{11} for uniaxial compression are shown. The Figures 8 and 9 present the same relationship for biaxial (σ_{11}:σ_{22}=1:1) compression and Figures 10 and 11 for σ_{11}:σ_{22}=1:0.5. Stress values are given in non-dimensional form scaled relative to uniaxial compressive strength f_c. Each graph contains also curves gained in tests by Kupfer [8]. For uniaxial compression the compressive behavior of concrete is defined according to Kupfer's results. The point is to find for which dilation angle the σ–ε curve for biaxial compression is the most similar to laboratory tests.

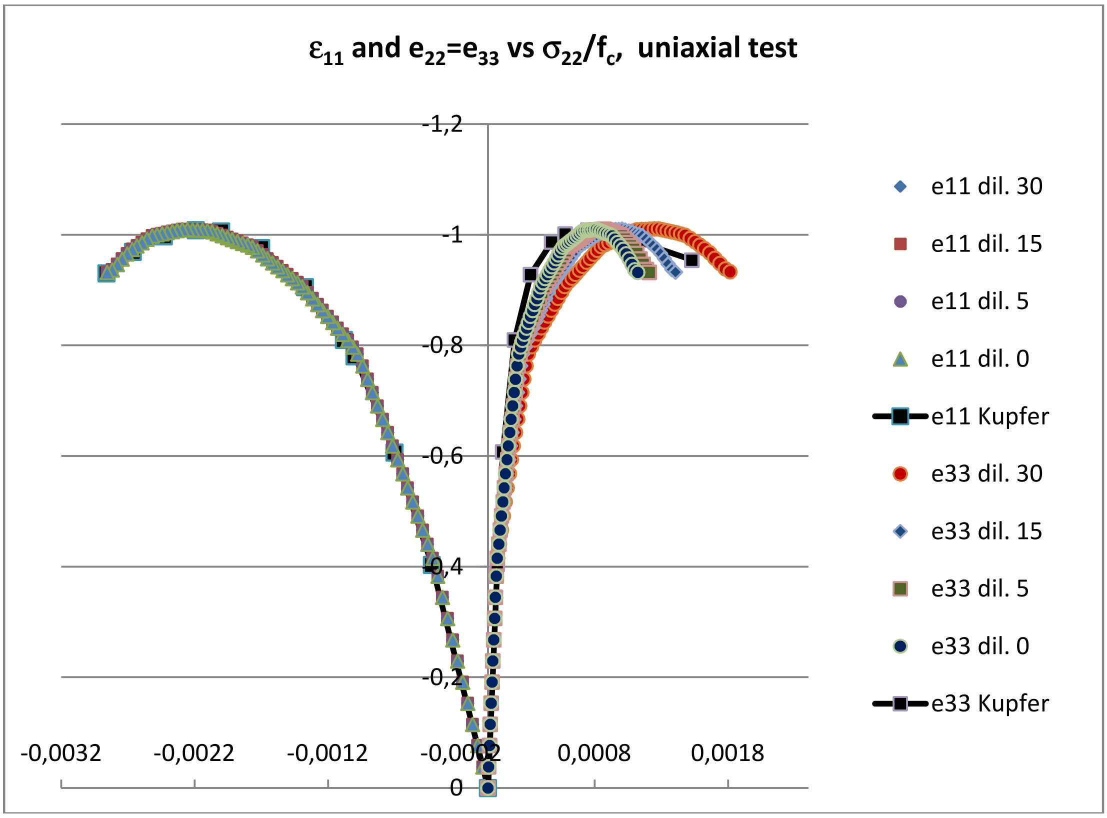


Figure 6: Linear strains versus σ_{11} for uniaxial compression

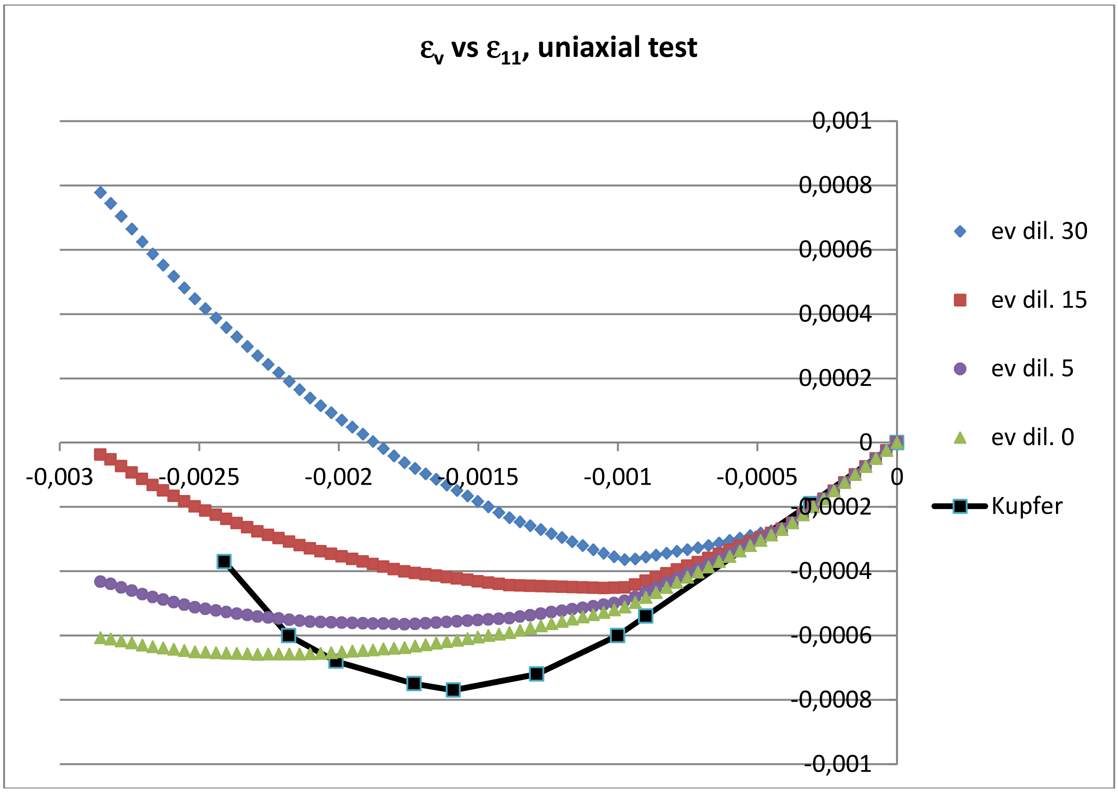


Figure 7: Volumetric strain versus σ_{11} for uniaxial compression

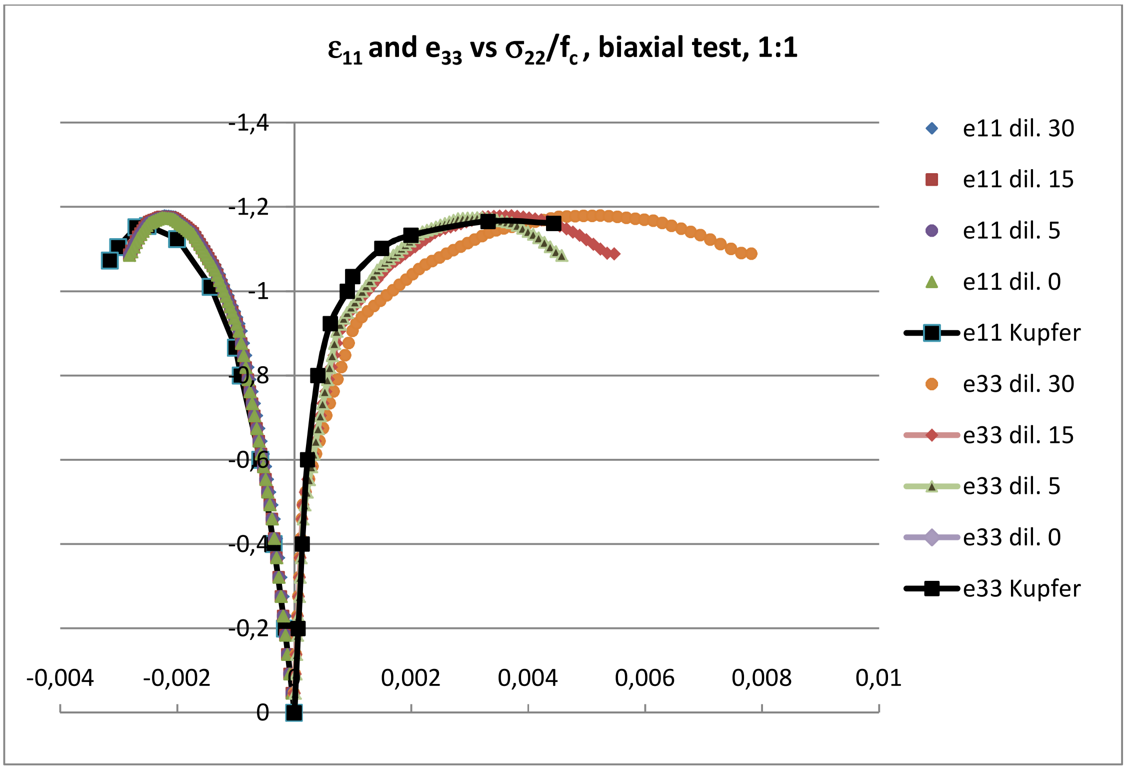


Figure 8: Linear strains versus σ_{11} for 1:1 biaxial compression

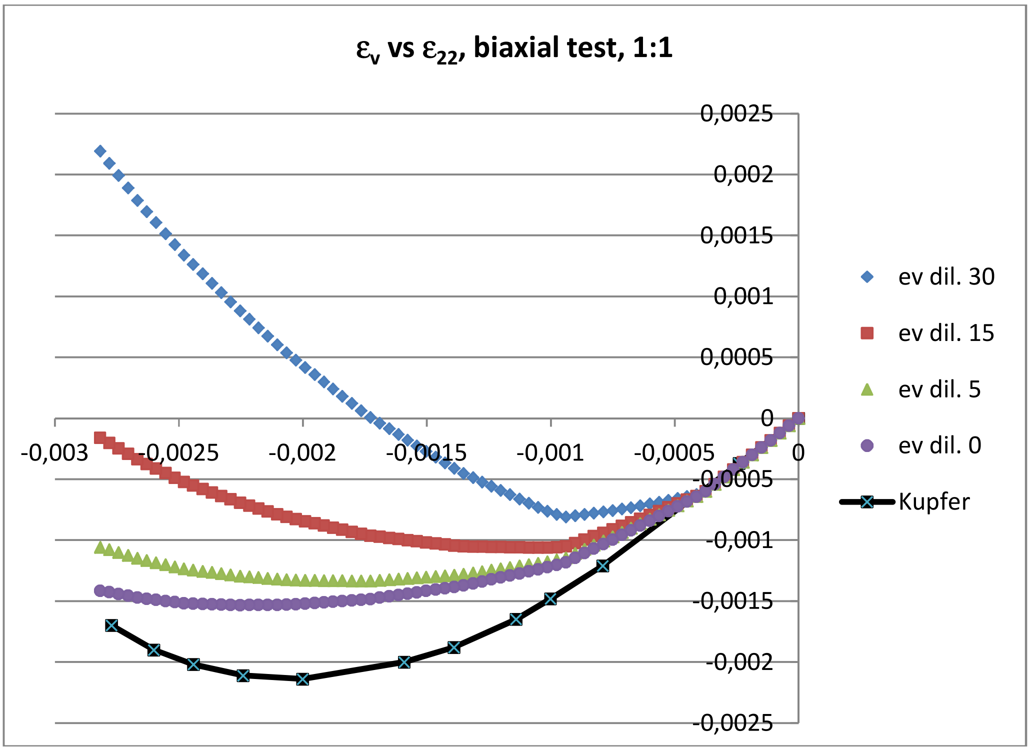


Figure 9: Volumetric strain versus ε_{11} for 1:1 biaxial compression

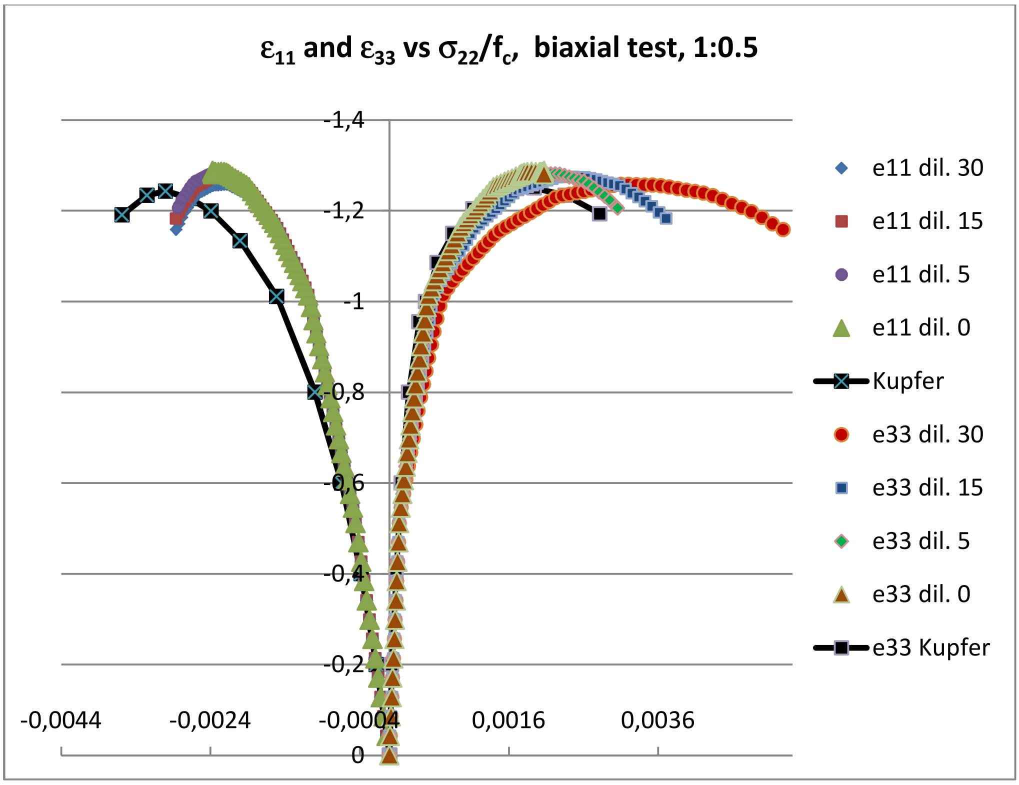


Figure 10: Linear strains versus σ_{11} for 1:0.5 biaxial compression

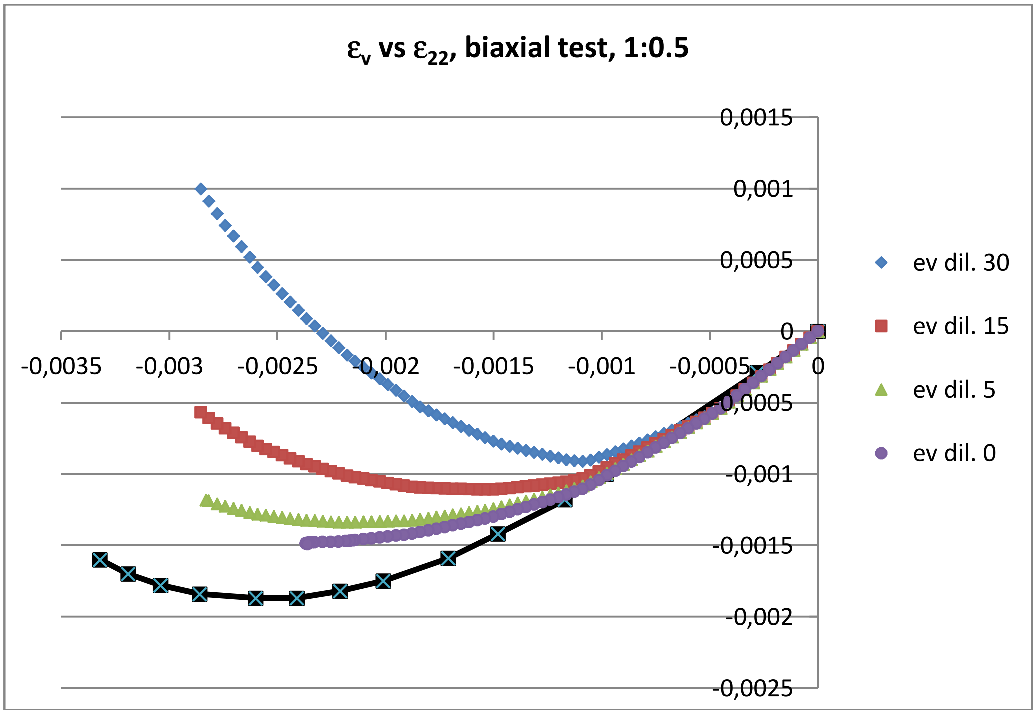


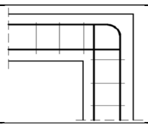
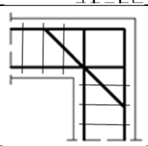
Figure 11: Volumetric strain versus σ_{11} for 1:0.5 biaxial compression

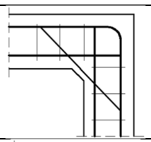
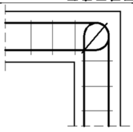
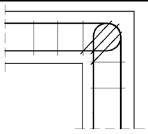
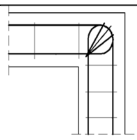
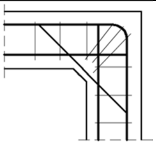
The results of this calibration show that for biaxial compression (1:1) the most compatible with Kupfer's tests is the curve for dilation angle 15 degrees. For the other cases the dilation of 5 degrees seems to be rational. Using dilation angle of 30 degrees, Abaqus user should be aware that positive volumetric strains in concrete can appear.

5 STRUT-AND-TIE AND ABAQUS ANALYSIS

All the chosen reinforcement details are at first analyzed with Strut-and-Tie method. Thanks to these calculations a required reinforcement and a corner efficiency factor are established. The corner efficiency factor is a ratio of permissible stress in struts to compressive stress calculated for each strut for the external moment equal to 30 kNm. All the results are listed in the Table 3.

Table 3: Results gained with Strut-and-Tie method

Nº	Reinforcement detail	Efficiency factor	Provided reinforcement
1.		0.83	Main reinforcement: 4 bars each ϕ20mm
2.		0.76	Main reinforcement: 4 bars each ϕ20mm Diagonal bar: 2 bars each ϕ12mm

3.		0.79	Main reinforcement: 4 bars each ϕ20mm Diagonal bar: 2 bars each ϕ12mm
4.		1.13	Main reinforcement: 4 bars each ϕ16mm Diagonal stirrup: 2 bars each ϕ20mm
5.		1.44	Main reinforcement: 4 bars each ϕ16mm Diagonal stirrup: central - 2 bars each ϕ20mm, rest: ϕ12mm
6.		1.49	Main reinforcement: 4 bars each ϕ16mm Diagonal stirrup: 6 bars each ϕ12mm
7.		1.56	Main reinforcement: 4 bars each ϕ16mm Diagonal stirrup: central - 2 bars each ϕ20mm, rest: ϕ12mm Diagonal bar: 2 bars each ϕ12mm

The provided reinforcement is used for defining the same corner details in Abaqus software. The corners are calculated both in plane stress and plane strain states. The applied load is defined using load parameter λ which takes value 1 for external moment equal to 30 kNm (the load level used in Strut-and-Tie method). In computations value of load parameter λ has varied from 0 (no load) to 2.

One of the calculated values is the relationship between a displacement of a chosen node and the load ratio λ. The chosen node and its displacement is shown in the Figure 12. The expected relationship between these two quantities is non-linear with a clear horizontal end fragment (plastic plateau).

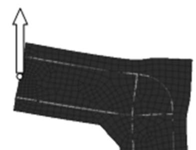


Figure 12: Location of a node and its displacement

In the Figures 13 to 15 results of FEM calculations for reinforcement detail Nº 1 in plane stress state are presented. The results show a variation of the nodal displacement versus the load ratio λ. Different values of fracture energy and dilation angle values are taken into consideration and also different sizes of finite elements. With a dashed horizontal line the efficiency factor calculated with Strut-and-Tie method is marked.

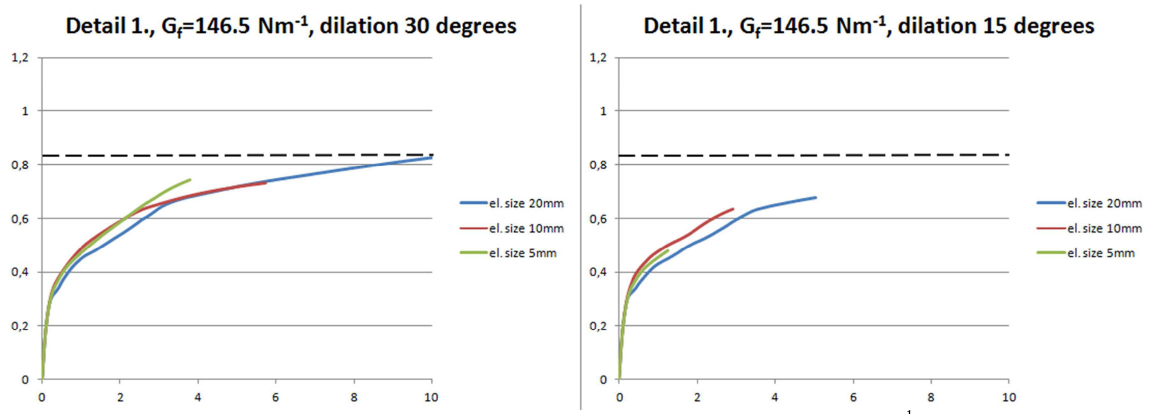


Figure 13: Displacement [mm] vs load ratio for G_f=146.5 Nm^{-1}

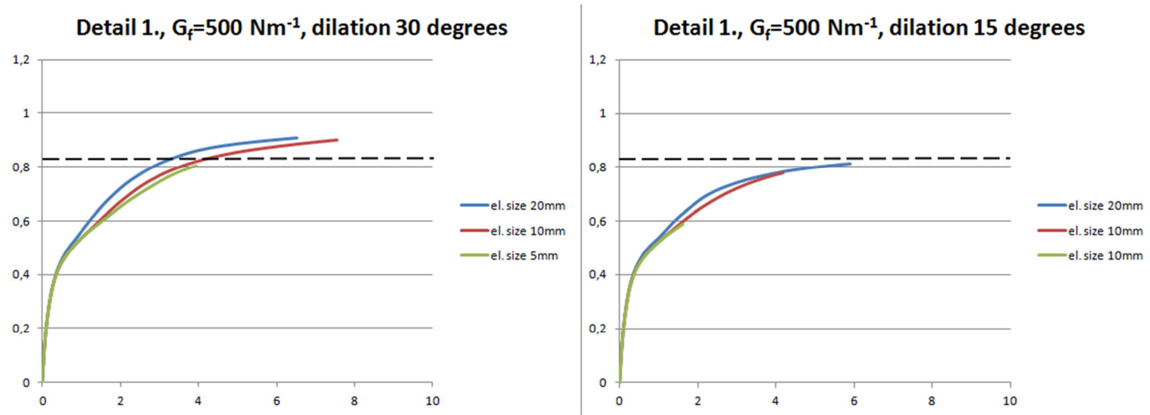


Figure 14: Displacement [mm] vs load ratio for G_f=500 Nm^{-1}

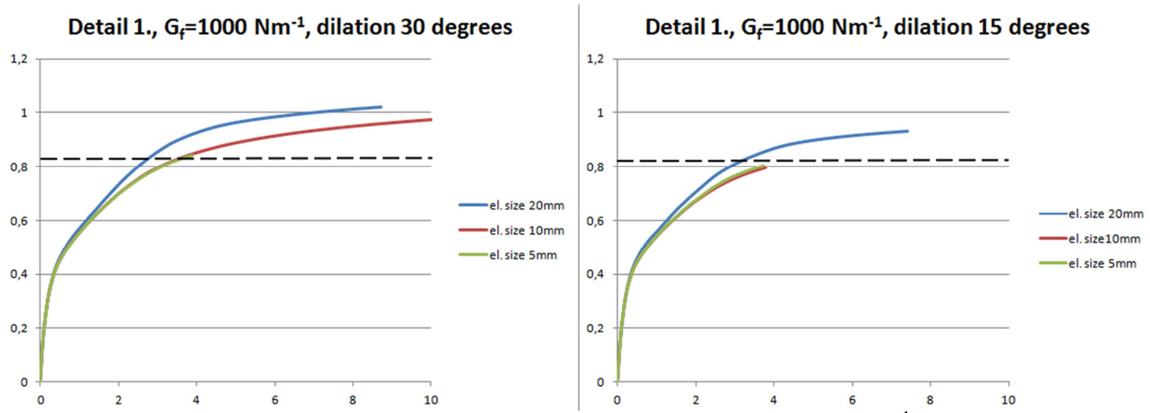


Figure 15: Displacement [mm] vs load ratio for G_f=1000 Nm^{-1}

We can clearly see that the value of fracture energy for plane stress state should be carefully considered. This value calculated according to FIB model code [9] (146.5 Nm^{-1}) concerns concrete without reinforcement. For FEM modeling of RC structures it is seems to be reasonable to choose a higher value of G_f.

In the Figure 16 nodal displacements for plane strain state and dilation angle 15 degrees are presented. This time all these curves have a clear plastic plateau and they present similar efficiency factors to those gained in Strut-and-Tie method. In this case dilation angle 15 degrees and fracture energy according to FIB recommendation seem to be reasonable.

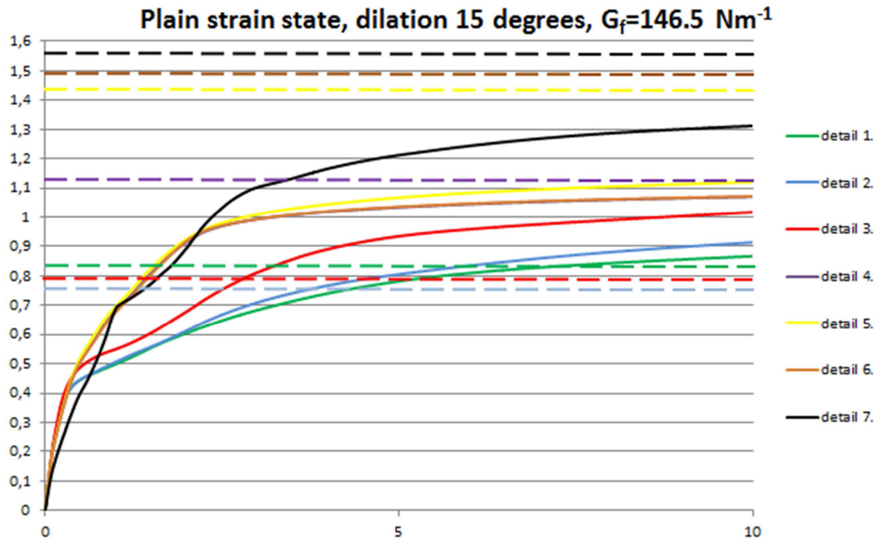


Figure 16: Displacement [mm] vs load ratio for plane strain state

6 CONCLUSIONS

A proper choice of CDP model parameters for numerical simulations is still difficult and demands many tests such as calibration and case studies. The authors of this paper suggest that the values of dilation angle and fracture energy should be established in compliance with results gained in Strut-and-Tie method and laboratory tests. Moreover, it is also very important whether simulations are executed in plane stress or plane strain state. As the results of calculation show, it is easier to gain a proper displacement-load relation for concrete if plane strain state is assumed.

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