Nonlinear Finite Element Method in Cable Marine Structures. Application on fishing cage

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Abstract. *There is no doubt that offshore technology has great relevancy. This leads to develop new techniques and methods to solve dynamics of devices which are placed on sea. Cable dynamics can be considered as special key into marine technology. Several structures are formed by cable. For instance, moorings allows to maintain floating offshore structures to be placed on a fix location into the ocean. Numerical methods are required to solve the nonlinear dynamic behaviour of cables. Mooring analysis can be considered as structural dynamic problem. Classical models which used quasi-static modelling based on catenary lines to solve dynamics. This work presents a strategy for solving the non-linear cable behaviour, based on Non-Linear Finite Element Method (NFEM) approach. Afterwards, formulation for first-order wave diffraction-radiation problem is described. A procedure for solving the coupled model between the wave loads and cable reaction forces is then described. Finally, an application on fishing cage simulation is performed several using fully coupled simulations. Some relevant conclusions will be obtained.*

Keywords - *Onshore power supply, ports, renewable energies.*

1 INTRODUCTION

The study of marine cables structures has reached great interest due to wide range of application in deep-ocean engineering. The range of cable application and line structures can be considered as wide from marine risers to net application. These applications include mooring line for marine structures, flexible risers, umbilical catenaries, towing hawser, subsea installations or fishing cages composed by suspended cables subjected to marine currents [1]. Cable structures are composed by a set of cables, chains or wire ropes, which are attached to offshore structures at different points with lower ends of these cables anchored at the seabed. The installation of subsea cables on seabed becomes significant importance due to communication technologies or floating offshore wind turbines require a precise study of mooring arrangements. Authors are shown that cables deployed in undersea conditions are susceptible of vibration since they interact with flow field, floating devices or seabed [2]. Most marine cable applications require to predict their dynamic behaviour and interaction with attached floater. Some authors are investigated mooring structures and their influence on behaviour of marine structures [2, 3, 4].

Other researchers have investigated marine slender structures as cables, umbilical and risers. For instance, Neto and Martins [5] have been studied the structural stability of flexible lines under torsion. Loops are very common in catenary risers in installations stages when tension is low and it is combined with torsion moments [5].

Flexible pipes and umbilical cables have importance to subsea production system [6]. Pipes can transport oil and gas from wells to marine facilities, and cable can provide electrical and hydraulic energy. The design of these flexible structures involves the operational loads, longitudinal strength, bending and torsional stiffness or collapse strength [6].

The coupled dynamic studies between cables, moorings and offshore structures become great relevance on cable and riser studies. For instance, Kim et al. [7] have compared two approach of cable treatments, linear spring method and Nonlinear Finite Element Method (NFEM). The first approach consists of adding spring constant of the mooring line's stiffness to motion equation of the ship. This method gives unreliable solution, because of the dynamics of cable motion is not considered. However, Nonlinear FEM allows to consider new effects such as bending and torsion, which can become critical in special situations, giving reliable solutions. Yang et al. [8] have developed coupled dynamic analysis of marine structures combining Higher-order Boundary Element Methods (HOBEM) to solve second-order wave equations and rod theory with FEM to estimate the dynamic behaviour of risers. In line with this, the authors presents in this work an application to couple NFEM cable model with second-order wave environment and ambient loads.

2 PROBLEM STATEMENT OF CABLE DYNAMICS

Nonlinear dynamic behaviour of cable can be modelled using Finite Element Approach. In this section the formulation of element kinematics is described. The implemented cable model uses updated Lagrangian formulation combined with corotational formulation. Joining both formulation can be achieved several advantages on computation of cable dynamics.

2.1 Governing equations of cable dynamics

In brief, the governing equations of mooring dynamic equilibrium was formulated by Tjavaras [9].

The set of equations for two dimensions are

$$m\left(\frac{\partial u}{\partial t}-v\frac{\partial \phi}{\partial t}\right)=\frac{\partial T}{\partial s}-\omega_o sin\phi-\frac{\partial \phi}{\partial s}\left(EI\frac{\partial^2 \phi}{\partial s^2}\right)-f_x,$$

$$m\left(\frac{\partial v}{\partial t}-u\frac{\partial \phi}{\partial t}\right)+m_a\frac{\partial v}{\partial t}=T\frac{\partial \phi}{\partial s}-\omega_o cos\phi-\frac{\partial}{\partial s}\left(EI\frac{\partial^2 \phi}{\partial s^2}\right)-f_y,$$

$$\frac{\partial T}{\partial t}=EA_o\left(\frac{\partial u}{\partial s}-v\frac{\partial \phi}{\partial s}\right),$$

$$\frac{\partial \phi}{\partial t}=\frac{\partial v}{\partial s}-u\frac{\partial \phi}{\partial s},$$

being m, and m_a, the mass and the added mass of the line respectively, EI, and EA_o, the flexure rigidity and the stiffness of the line, T the tension of the line, s the spatial coordinate along the unstretched length of the line, t the time variable, and ϕ the angle formed by line with coordinate axis.

If bending and torsional stiffness are negligible, as usually occurs, the set of equations can be formulated in vector form as,

$$(\rho_w C_m A_o+\rho_o)\frac{\partial r_l^2}{\partial t^2}=\frac{\partial}{\partial s}\left(EA\frac{e}{e+1}\frac{\partial r_l}{\partial s}\right)+f(t)(e+1),$$

where ρ_w is the water density, C_m is the added mass coefficient, ρ_o is the mass per unit length of the unstretched cable, r_l is the position vector, e is the strain, and $f(t)$ are the external loads applying over catenary mooring cable.

2.2 Spatial discretization using FEM formulation

The weak form of the equation of the motion of the element given by Borst et al. on [10] in current configuration as

$$\int_V(\rho\delta u^T u+(L\delta u)^T\sigma)dV=\int_V(\rho\delta u^T g)dV+\int_S\delta u^T t\, dS,$$

being V the volume, S the surface, u the vector displacement, L an appropriate operator, t the stress vector on surface, g the gravity acceleration vector, ρ the cable density and σ the stress tensor.

Introducing the vector r_k in which the components (i,j,k) of the displacement vector at node k are gathered, the continuous displacement field u of each element can be approximated as,

$$u=\sum_{k=1}^{n}h_k r_k=Hr,$$

where h_k are the interpolation functions, H is a $3\ x\ 3n$ matrix and r_k is the vector which stores all degree of freedom of the nodes.

The element vector r_k can be related to the global displacements contained in a global displacement vector a via the matrix Z, when the system consists of N global degrees of freedom Z is a $3n\times N$ matrix. So, the last Equation becomes as,

$$\sum_{e=1}^{n}Z_e^T\int_{V_e}\rho H^T H Z_e dV\ \ddot{r}=$$

$$=\sum_{e=1}^{N}Z_e^T\int_{V_e}\rho H^T g\, dV+\sum_{e=1}^{N}Z_e^T\int_{S_e}H^T t\ dS-\sum_{e=1}^{N}Z_e^T\int_{Ve}B^T\sigma dV.$$

With B = LH, the last equations arrives as balance of momentum. The associated matrix structure of the equations of dynamic equilibrium of forces at time t can be written as

$$f=M\ddot{r}+C\dot{r}+P^0+R,$$

where f is the external loads vector, M is the mass matrix of the line, considering inertia and added mass, P^0 is the pretension vector in the initial configuration, and R is the internal forces vector of the cable. Damping effects of cable are introduced through a Rayleigh proportional damping matrix of C.

2.3 Direct time integration

To solve the dynamic equilibrium of cable an implicit time integration scheme based on the called Bossak-Newmark method [11] is applied. This provides a set of algebraic equations that can be solved in an iterative manner.

$$\left[(1-\alpha)M^{t+\Delta t,i}+\Delta t\gamma C^{\Delta t+t,i}+\Delta t^2\beta\left(K_L^{t+\Delta t,i}+K_{NL}^{t+\Delta t,i}\right)\right]\dddot{r}^{t+\Delta t,i+1}=$$
$$\Delta t^2\beta\left(K_L^{t+\Delta t,i}+K_{NL}^{t+\Delta t,i}\right)\dddot{r}^{t+\Delta t,i}+f^{t+\Delta t,i}+P_o-R^{t+\Delta t,i}$$
$$-C^{t+\Delta t}[\dot{r}^t-\Delta t(1-\gamma)\ddot{r}^t]-\alpha M^{t+\Delta t.i}\dddot{r}^t,$$

where Δt is the time step, i denotes iteration, α is a parameter related with the Bossak-Newmark implicit method, and γ and β are parameters related to the Newmark time integration scheme.

2.4 First Order Wave problem

First Order solution of wave-radiation problem can be obtained assuming incompressible flow and irrotational flow. After using the perturbed solution based on the Stokes perturbation and applying Taylor series expansions, and retaining the first-order terms, the governing equation for First Order wave radiation problem become as [12],

$$\Delta\varphi=0 \qquad \text{in } \Omega,$$

$$\frac{\partial \xi}{\partial t}-\frac{\partial \varphi}{\partial z}=0 \qquad \text{in } Z=0,$$

$$\frac{\partial \varphi}{\partial t}-\frac{P_{fs}}{\rho}-\xi g=0 \qquad \text{in } Z=0,$$

$$v_p\cdot n_p+v_\varphi n_p=0 \qquad \text{in } S_b,$$

being φ the velocity potential, ξ the wave elevation, z the vertical coordinate, P_{fs} the pressure on free surface, Ω the fluid domain, v_p the first order body velocity over point P, v_φ is the first order potential velocity over the point P, and n_p the body surface normal vector at point P.

3 APPLICATION EXAMPLES

In this section, they are shown an examples of application of cable problem on marine structures. First, a semisubmersible platform is analysed. Then, an application on fishing cage is performed.

3.1 Fully coupled analysis of semisubmersible platform

An application example based on fully analysis of semisubmersible platform GVA 4000 [13] subject to first-order wave environment combined with NFEM mooring model is performed (see Fig. 1). The main particulars of the platform and key parameters of simulations are shown in Table 1.

Table 1. "Main particulars of semisubmersible and key parameter of simulations performed".

Ítem		Value
Characteristic Length	m	70.0
Depth	m	20.0
Mass	Kg	2.591×10^7
Centre of gravity	m	0.0; 0.0; 0.85
I_{xx}/mass	m^2	30.40
I_{yy}/mass	m^2	31.06
I_{zz}/mass	m^2	37.54
Wave spectrum		JONSWAP
Peak period	s	7.7
Significant wave height	m	3.0
Number of mooring lines		8
Stiffness	N	3.84×10^7
Length	m	520.0
Weight per unit length	N/m	698.1
Number of elements per mooring line		100

Table 2 shows a comparison between the mean, the amplitude and the *Root Mean Square* (RMS) values for first -order wave loads of semisubmersible platform.

Table 2. "Mean, amplitude and RMS values of semisubmersible motions for first-order wave environment".

	Surge (m)	Sway (m)	Heave (m)	Roll (deg)	Pitch (deg)	Yaw (deg)
Mean	-0.01	0.00	0.00	0.00	0.00	0.00
Amplitude	1.20	0.12	1.44	0.39	1.37	0.17
RMS	0.17	0.02	0.21	0.07	0.22	0.03

The mean values have similar trends in all cases combined with NFEM mooring model, as it can be observed. General values shows that mooring arrangements achieve good performance in operational conditions.

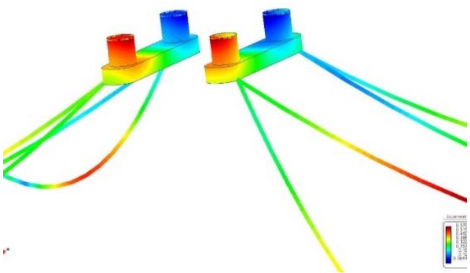

Fig 1. "General view of semisubmersible platform".

Table 3. "Comparison between maximum, minimum, mean, and RMS values of fairlead tension of semisubmersible platform".

Line	Max. (N)	Min. (N)	Mean (N)	RMS (N)
Line 1	4.309×10^5	3.930×10^5	4.118×10^5	4.119×10^5
Line 2	4.308×10^5	3.908×10^5	4.117×10^5	4.118×10^5
Line 3	4.305×10^5	3.922×10^5	4.118×10^5	4.118×10^5
Line 4	4.319×10^5	3.934×10^5	4.118×10^5	4.119×10^5
Line 5	4.309×10^5	3.930×10^5	4.118×10^5	4.119×10^5
Line 6	4.308×10^5	3.908×10^5	4.117×10^5	4.118×10^5
Line 7	4.305×10^5	3.922×10^5	4.118×10^5	4.118×10^5
Line 8	4.319×10^5	3.934×10^5	4.118×10^5	4.119×10^5

The key values of fairlead tension on semisubmersible platform are shown in Table 3. Similar values are recorded for all lines.

3.2 Application on fishing cage

Finally, an analysis of fishing cage subject to first-order wave environment combined with NFEM mooring model is performed.

The fishing cage analysed has circular shape (see Fig. 2). The main particulars of the fishing cage are shown on Table 3.

Figure 2. "Example of circular fishing cage. (www.akavgroup.com)".

The main particulars of the fishing cage and key parameters of simulations are shown in Table 4.

Table 4. "Main particulars of fishing cage and key parameter of simulations performed".

Items		Value
Diameter	m	30.0
Depth	m	16.0
Mass	Kg	12.85×10^5
Centre of gravity	m	0.0; 0.0; -1.0
I_{xx}/mass	m^2	112.
I_{yy}/mass	m^2	112
I_{zz}/mass	m^2	224
Wave spectrum		Monochromatic
Period	s	7.5
Wave height	m	0.75
Stiffness of mooring lines	N	6.04×10^7
Weight per unit length of mooring lines	N/m	1.0

Ambient conditions are chosen according to Galanis et al. [15]. A general view of cable arrangement used of fishing cage analysed can be observed on Fig 3.

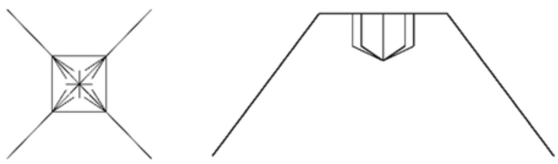

Figure 3. "General view of cable arrangement of fishing cage".

Table 5 shows a comparison between the mean, the amplitude and the RMS values of motions for first-order wave loads on fishing cage. As can be shown, the fishing cage has great amplitudes of yaw motion compared with roll and pitch. Heave amplitude is according to wave spectrum, and as can be observed the fishing cage gets good performance in operational conditions.

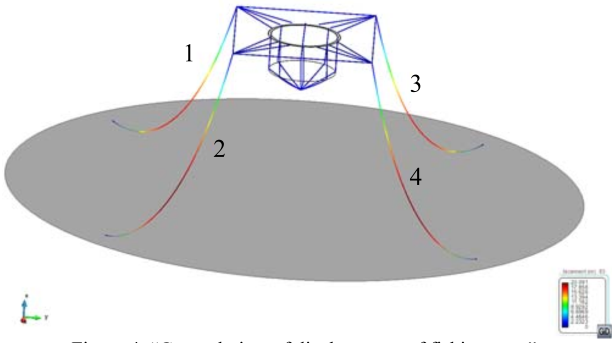

Figure 4. "General view of displacement of fishing cage".

Table 5. "Mean, amplitude and RMS values of fishing cage motions for first-order wave environment".

	Surge (m)	Sway (m)	Heave (m)	Roll (deg)	Pitch (deg)	Yaw (deg)
Mean	0.19	0.05	0.00	0.00	0.00	0.00
Amplitude	1.52	0.10	1.10	1.20	5.25	7.14
RMS	0.39	0.06	0.32	0.01	1.54	0.76

The values of fairlead tension on each mooring line have similar behaviour according to heave motion of fishing cage. The mean values are close to itself weight. This fact indicates that valued obtained are realistic.

Table 6. "Comparison between maximum, minimum, mean, and RMS values of fairlead tension of fishing cage".

Line	Max. (N)	Min. (N)	Mean (N)	RMS (N)
Line 1	$1{,}400 \times 10^4$	$5{,}012 \times 10^2$	$1{,}382 \times 10^4$	$1{,}389 \times 10^4$
Line 2	$1{,}996 \times 10^4$	$2{,}570 \times 10^2$	$1{,}389 \times 10^4$	$1{,}393 \times 10^4$
Line 3	$1{,}490 \times 10^4$	$8{,}922 \times 10^1$	$1{,}386 \times 10^4$	$1{,}389 \times 10^4$
Line 4	$1{,}400 \times 10^4$	$8{,}922 \times 10^1$	$1{,}386 \times 10^4$	$1{,}389 \times 10^4$

Finally, time evolution of fairlead tension of each mooring line is shown on Fig. 5. The period obtained for each line are similar in all case. These values are close to wave period, as it is expected.

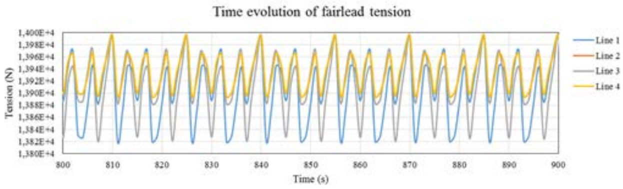

Figure 5. "Time evolution of each mooring line analysed".

4 CONCLUSION

A FEM coupled seakeeping and mooring model for the analysis of offshore structures has been presented. From the obtained results, the following concluding remarks can be remarked. Seakeeping model for solving the governing equations for first-order wave diffraction-radiation problem in the time domain based on [14] has been shown. An introduction to cable dynamics has been described. Formulation based on Nonlinear FEM dynamic cable model to solve realistic problems has been presented. Fully coupled simulations to determine the dynamic behaviour of two representative examples of offshore structures has been performed: semisubmersible platform and fishing cage. The NFEM mooring combined with first-order waves produces a successful simulation of the performance of the floating structures.

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