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# MODELLING OF PROPAGATION WITH SPH OF 1966 ABERFAN FLOWSLIDE: SPECIAL ATTENTION TO THE ROLE OF RHEOLOGY AND PORE WATER PRESSURE

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**Abstract.** Landslides can cause major economic damage and a large number of casualties as it is possible to see from past events occurred all over the world. Being able to predict these kind of hazards would then suppose the achievement of great benefits. Here a model that combines a depth integrated description of the soil-pore fluid mixture together with a set of 1D models dealing with pore pressure evolution within the soil mass is presented. The mathematical model is based on the Biot-Zienkiewicz equations, from where a depth averaged model is derived. Concerning the material behaviour, the approach used is the one suggested by the Perzyna viscoplasticity, which has been extensively used in the past to model solid behaviour prior to failure. In this framework, a simple shear rheological model is derived, providing the basal friction needed in depth integrated models. The Smoothed Particle Hydrodynamics (SPH) has been the numerical technique chosen to spatially discretised the depth integrated equations of the mathematical model. The purpose of this work is to apply the SPH depth integrated numerical model, together with the sub-model that predicts the evolution of the pore water pressure inside the landslide, to simulate the propagation phase of the Aberfan flowslide occurred in 1966.

## 1 INTRODUCTION

Flowslides are rapid flows, either saturated or unsaturated, where the material has a high compaction tendency, a low density and is characterised by a metastable structure. Since flow failures experience a sudden loss of strength followed by a very rapid development of deformations, their effects are often much more dramatic and devastating then other types of landslides. Thus, the prediction of flowlides propagation distances, velocity and pore pressure through mathematical modelling will suppose the achievement of great human and economic benefits and an effective way of identifying and designing appropriate preventive flowslide measures.

Here we propose to apply the model described by Pastor and collaborators [18] to predict their paths, velocities, depths, pore pressures and propagation distances.

The mathematical approach used in this framework is a depth integrated model resulting in an excelent compromise between accurancy and computational cost. The rheological approach is based on Perzyna viscoplasticity [19], while the numerical technique chosen is the Smoothed Particle Hydrodynamics (SPH) [14, 9, 15, 2, 16, 13], a Lagrangian meshless numerical technique, which has been enriched by adding a 1D finite differences grid associated at each SPH node in order to improve the description of pore water profiles in the avalanching soil.

The goal of this study is to apply the SPH depth integrated numerical model, which includes a sub-model able to predict the evolution of the pore water pressure, to simulate the propagation phase of the Aberfan flowslide occurred in 1966.

## 2 MODELING APPROACH

In what follows, a brief description of the methodology used is given. A more detailed explanation can be found in the papers of Pastor and collaborators [18, 17].

### 2.1 Mathematical model

The mathematical framework considered here is the one proposed by Biot [3, 4] where the coupling between the solid phase and the fluid phase is developed. Biot's theory has been further developped by researchers at Swansea University where it has been extended for non linear materials and for large deformation problems [24, 23, 22, 25, 21]. Among all the coupled models developed at Swansea University, the one followed here is the one cast in terms of velociy of the soil skeleton and pore water pressure $v - p_w$, developed by Biot- Ziekiewicz [23], which, depth integrated on the vertical axes (reference system showed in figure 1), is given by the following equations:

$$\frac{\overline{D}h}{Dt} + \frac{\partial \overline{v}_i}{\partial x_i} h = e_R \tag{1}$$

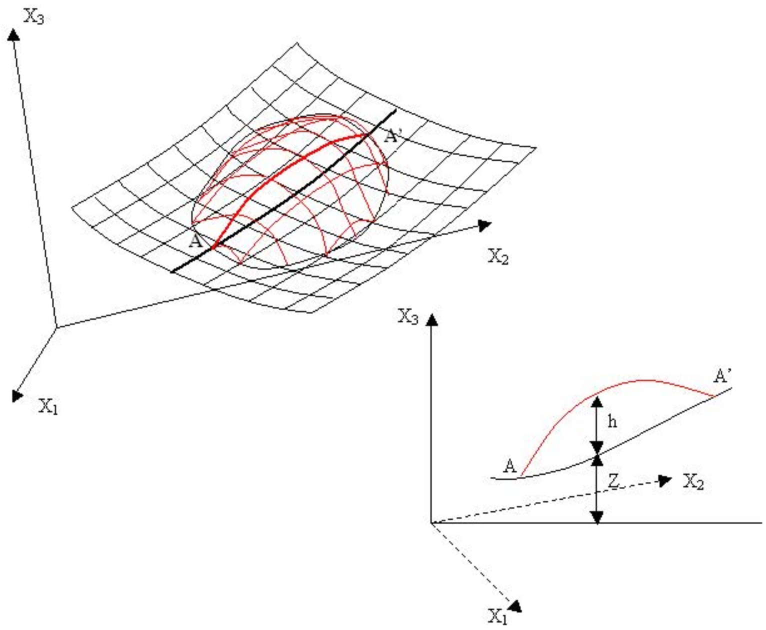


**Figure 1**: Reference system and notation used in the analisys

$$h\frac{\overline{D}\overline{v}_i}{Dt} - \frac{\partial}{\partial x_j} b_3 \frac{h^2}{2}\delta_{ij} = -\overline{v}_i\ e_R + b_i h + \frac{1}{\rho}\frac{\partial}{\partial x_j} h\overline{\sigma}^*_{ij} - \alpha\frac{\partial}{\partial x_j}\left(h\overline{v}_i\overline{v}_j\right) + \frac{1}{\rho}\left|N^B\right| t^B_i \qquad (2)$$

$$\frac{dp_w}{dt} = \rho g\frac{dh}{dt}\left(1 - \frac{x_3}{h}\right) + \frac{K_v}{\alpha}\dot{\epsilon}_v - \frac{K_v}{\alpha}k_w\frac{\partial^2 p_w}{\partial x_3^2} \qquad (3)$$

where $e_R$ is the erosion rate, h is the depth of the flow, $\overline{v}_i$ is the i-th component of the velocity of the soil skeleton, $\overline{\sigma}^*_{ij}$ is the effective stress tensor, $t^B_i$ is the i-th component of the surface stress acting on the basal surface, $\left|N^B\right|$ is the normal vector to the basal surface, $p_w$ is the pore pressure, $\dot{\epsilon}_v$ is the rate of deformation tensor, $k_w$ is the permeability, $K_v$ is a suitable stiffness module and $\alpha$ is a general coefficient.

Equations 1 and 2 represent the balance of mass and momentum respectively and are discretised spatially through the SPH tecnique. Equation 3 represents a full approximation of pore pressures inside the landslides which is discretised with a 1D Finite Differences Scheme used at each SPH node.

### 2.2 Constitutive model

In this framework, a new simple rheological law based on Perzyna viscoplasticity [18] for frictional materials has been used to model the Aberfan flowslide. When obtaining the depth-integrated equations, the flow structure along the vertical, which is needed to obtain both the basal friction and the depth-integrated stress tensor, is lost. A possible solution that is widely used consists in assuming that the flow at a given point and time with known depth and depth-averaged velocities has the same vertical structure as a uniform steady-state flow. In the case of flow-like landslides, this model is often referred to as the infinite landslide, as it is assumed to have a constant depth and move at a constant velocity along a constant slope.

The expression found in order to get the basal shear friction is the following:

$$\tau_b = s_b \left[ \left( \frac{\overline{v} 2\mu}{h} \right)^{\frac{1}{N}} + 1 \right] \tag{4}$$

where $s_b = \sigma_{3b} \tan \varphi$ is the basal shear strength ($z = 0$).

## 3 THE ABERFAN FLOWSLIDE

### 3.1 Introduction

Aberfan is today a former coalmining village in South Wales (UK). In 1966 a liquefaction-induced flowslide of coal waste occured, propagating onto the village itself and provoking 144 fatalities. Information about the failure mechanism and material properties have been provided by Bishop [5, 6] and Hutchinson [11]. Other raw material is also available at the UK National Archive.

The Aberfan colliery waste was tipped on the side of a hill (Tip 7) facing the village. The triggering mechanisms of the flowslide lay in the hydrogeology of the site. Due to heavy rain, in fact, artesian pore pressure rose up in the sandstone beneath the less permeable glacial deposit at the toe of the slope causing the liquefaction of the loose waste material dumped.

Tip 7 was about 67 meters high from toe at the moment the slide occured on October 21st and the underlaying terrain had a slope of 12 degrees. The slide moved for 275 meters before dividing itself into two lobes. The larger south lobe travelled for a distance of 500 meters before impacting Aberfan buildings and stopped 100 meters after, for a total propagation length of 600 meters with estimated velocities in the range of $4,5-9\ \frac{m}{s}$. Table 1 resumes the characteristics of the flowslide.

### 3.2 Numerical Simulation

In order to simulate the Aberfan flowslide, the topographic mesh and the SPH nodes representing the initial mass are needed. Authors have built both of them by using the topographic information and maps available at the UK National Archive and they are shown in figure 2.

In table 2 the parameters used to model the Aberfan flowslide which gives the best

**Table 1**: Aberfan flowslide characteristics

| Concept | Value | Meaning |
|---|---|---|
| Height | 67 m | Height measured from the toe |
| Slope Terrain | 12 degrees | Slope of the underlying terrain |
| Distance | 275 m | Distance before division in to two lobes |
| Total distance | 600 m | Longest distance travelled |
| Velocity | $4,5 - 9\frac{m}{s}$ | Estimated velocity of the flowslide |

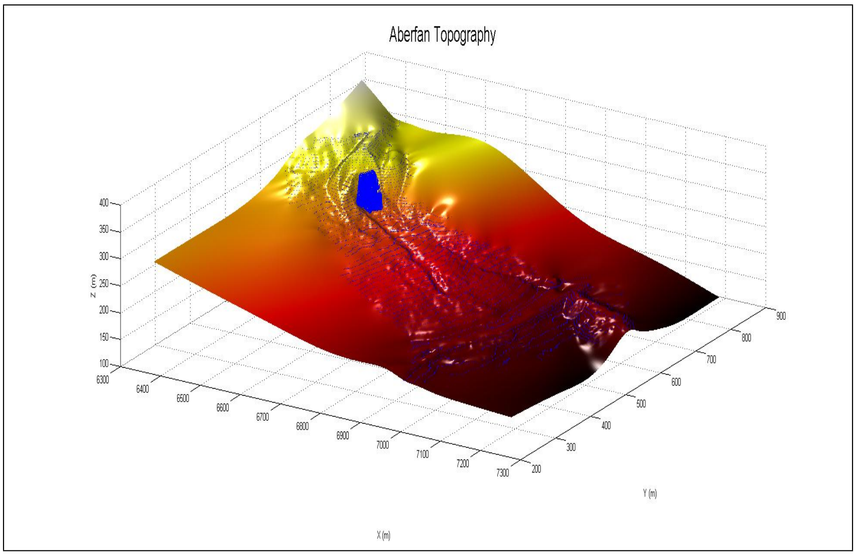


**Figure 2**: The Aberfan sliding portion topography used in the simulation

agreement with field observation are presented. Erosion has been taken into account through the erosion coefficient of the Hungr erosion law [10]. In fact, with a careful reading of the report written right after the disaster and available at the UK National Archives, it is possible to see that erosion is widely mentioned by the author. Moreover, as for the maximum excess pore water pressure at the basal surface ($p_w^{rel}$), it has been assumed to vary between 0 and 1; this last value corresponds to liquefaction. Finally, the relative width of the basal saturated layer to the total depth ($h_w^{rel}$) was assumed to range between 0.4 and 1.

## 4 RESULTS AND DISCUSSION

The results of the propagation and height of the soil obtained by the simulation are shown in figure 4 at time 0, 10, 20, 30, 45 and 50 s. Results are satisfactorily reproducing

**Table 2**: Parameters used for the numerical simulation

| Parameter | Value | Meaning |
|---|---|---|
| $tan\phi'$ | 0.726 | Tangent of the friction angle |
| N | 1 | Perzyna model parameter |
| $\gamma$ | 0.001 s | Viscosity |
| $\rho$ | 1740 $\frac{kg}{m^3}$ | material density |
| $e_R$ | 65.e-4 | erosion coefficient |
| $C_v$ | 65.e-5 | Consolidation coefficient |
| $p_w^{rel}$ | 0.8 | Initial pore pressure |
| $h_h^{rel}$ | 0.4 | Initial height of basal saturated layer |

the flowslide. It is possible to note that the final height of the soil of the left lobe at 50 seconds is almost 10 meters which match with the real height that hit the school. Furthermore, the SPH program reproduce well the division of the flowslide in to two lobes. Results of the soil height also well match with the one dimensional results obtained by Pastor et al [17, 18].

In figure 3 the pore pressure contours evolution is presented at 0, 2, 5 and 10 s. Please note that in order to improve readability, the saturated layer has been expanded and now it occupies the whole mass. This is possible because we are considering the depth of the basal saturated layer proportional to the one of the landslide.

## 5 CONCLUSIONS

It has been presented here a framework which has been applied to model the case of the Aberfan flowslide occured in 1966.

The mathematical model is based on the $v - p_w$ Biot- Ziekiewicz model [23], for propagation and consolidation of avalanches, debris flow and fast landslide problems.

Then, the numerical model combines a SPH model for landslide propagation and a set of Finite Difference meshes associated to each of the SPH nodes where the pore pressure evolution is solved.

Concerning the constitutive model, viscoplastic models have been applied both to cohesive [1, 7] and frictional materials. They have been found to reproduce well slow landslide movements [20, 12, 8]. This is the reason why, Perzyna viscoplasticity has been taken as a base model, presenting the advantage of being close to some rheological models of interest. Consequently, the new depth integrated rheological model based on Perzyna viscoplasticity [18], under the hypothesis of simple shear infinite landslide models, has been used to model the Aberfan flowslide behaviour.

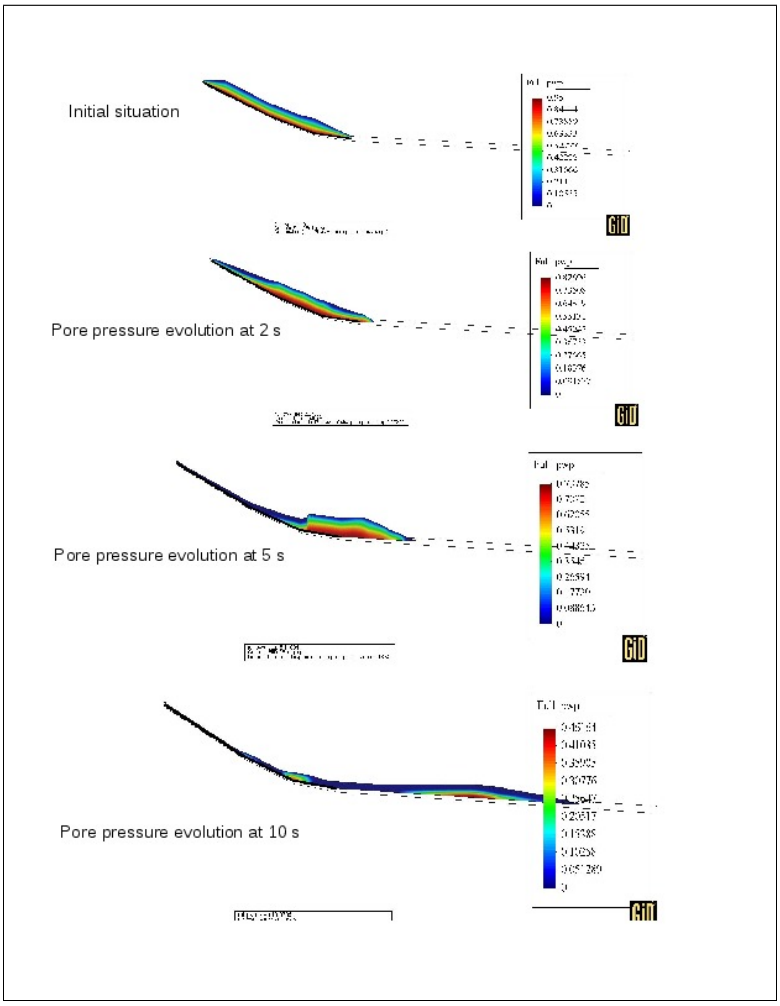


**Figure 3**: Pore pressure contours evolution at 0, 2, 5 and 10 seconds

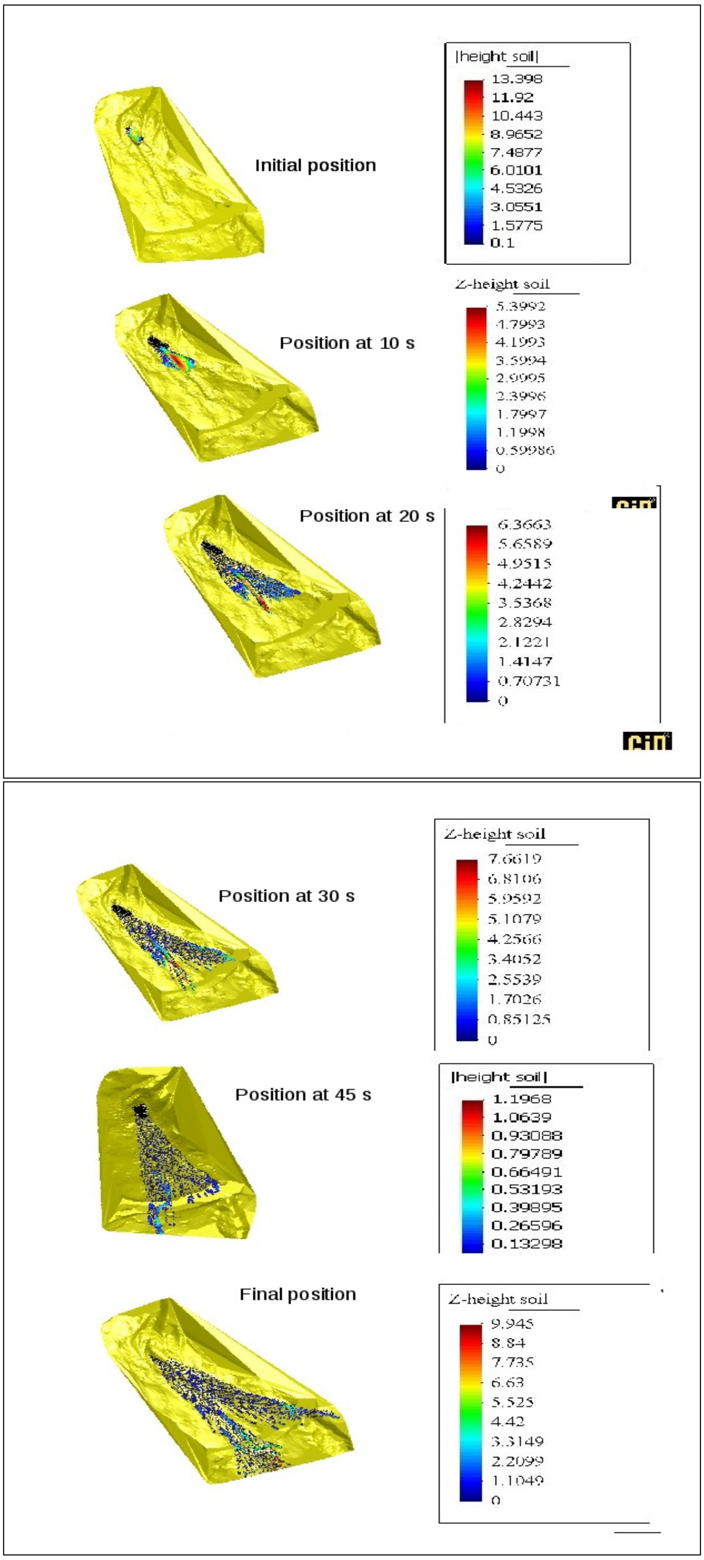


**Figure 4**: Results sequence of Aberfan flowslide simulation at 0, 10, 20, 30, 45 and 50 seconds

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