Reconstruction of Correlated Sources with Energy Harvesting Constraints

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***Abstract*—In this paper, we investigate how to minimize the distortion in the reconstruction of correlated sources. We consider a communication scenario where a sensor node is capable of harvesting energy from the environment and where the Fusion Center (FC), in order to exploit correlation, uses past observations as side information for decoding. We provide a convex formulation of the problem and derive the optimal transmission policies (i.e., power and rate allocation). We also propose an iterative procedure based on the subgradient method by means of which a solution can be iteratively found. Interestingly, each iteration entails the interaction (coupling) of a directional water-filling and a reverse water-filling schemes. Numerical results are provided in order to illustrate the impact of correlation in the resulting transmission policies.**

I. Introduction

Sensor nodes are usually powered by batteries which can be costly, difficult or even impossible to replace (e.g., when nodes are deployed in remote locations). In recent years, energy harvesting has emerged as a technology capable of overcoming (or, at least, alleviating) the limitations imposed by non-rechargeable batteries. Specifically, nodes equipped with an energy harvesting device are capable of scavenging e.g., solar, wind, thermal, kinetic energy from the environment [1] and, by doing so, extend their operational lifetime.

Energy harvesting has received considerable attention by the wireless communications and information theory communities. For point-to-point scenarios, and under the assumption of known energy and data arrivals (*offline* optimization), the main focus has been on the derivation of optimal transmission strategies at the sensor node. In [2], the authors study the problem of minimizing the time by which all data packets are transmitted to the destination. A number of authors go one step beyond and investigate the impact of *finite* energy storage capacity [3] or battery leakage [4]; generalize the analysis to fading channels [5]; or explicitly take into consideration the energy needed for data processing (in addition to data transmission) [6].

A number of works [7], [8] address the problem of source and channel coding with side information in Wireless Sensor Networks (WSNs) contexts. Other authors have also investigated source-channel coding aspects in an *energy harvesting* context. In [9], for instance, the case of a source generating *independent* samples of a Gaussian distribution is analyzed.

In this paper, we consider a point-to-point communication scenario where an energy harvesting sensor node collects and encodes observations from a series of (temporally) *correlated* underlying sources, and wirelessly transmits them to a remote Fusion Center (FC). For simplicity, correlation is modeled as a first order autoregressive process. In order to exploit correlation, the FC uses past observations as side information for decoding and adopts the well-known Wyner-Ziv approach [10]. We derive the optimal transmission policy (i.e., power and rate allocation) which minimizes the average distortion in the reconstructed observations at the FC. We also propose a procedure based on the subgradient method [11] to iteratively solve the problem. We show that such scheme encompasses the interaction of a directional [5] and reverse [12] water-filling schemes.

The paper is organized as follows. In Section II we introduce the system model and provide details on the encoding and decoding processes with side information. In Section III, we pose the problem in a convex optimization framework and derive the optimal power and rate allocation policies in terms of the solution to the dual problem. Next, we review the subgradient method and propose an iterative scheme based on it to solve the problem of interest (Section IV). Some numerical results are then presented in Section V. The paper closes by summarizing the main conclusions in Section VI.

II. System Model

Consider the point-to-point communication scenario of Fig. 1, where time is slotted with M time slots. An energy harvesting sensor measures a time-varying physical phenomenon modeled by multiple correlated and memoryless sources, where each source models the phenomenon in a given time slot[1]. In each time slot, the sensor node (i) collects a *large* number of independent and identically distributed (i.i.d.) samples from the corresponding source; (ii) subsequently encodes the sequence of measurements and (iii) wirelessly conveys

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[1]This model could apply to image observations, such as in a Wireless Video Sensor Network (WVSN) or spectrum sensing in a cognitive radio node.

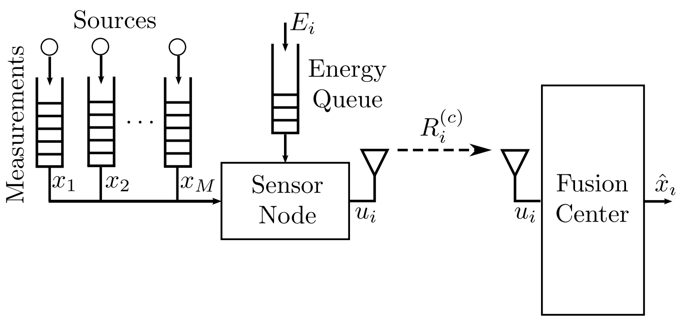

Fig. 1. System Model.

them to a remote fusion center; where we model the sensor-to-FC channel as a Gaussian channel. Therefore, the channel rate in the i-th time slot must satisfy[2]

$$R_i^{(c)} \leq \log\left(1 + |h_i|^2 p_i\right), \tag{1}$$

with $|h_i|^2$ and p_i standing for the corresponding channel gain and average transmit power, respectively. The underlying energy harvesting process can be modeled as a counting process with energy arrivals of E_i Joules at the beginning of time slot i. For simplicity, we assume that energy is stored in a rechargeable battery of infinite capacity. Hence, considering transmit power as the only energy cost, any transmission (power allocation) policy $\{p_i\}$ designed for the sensor node must satisfy the following energy causality constraint

$$T_s \sum_{j=1}^{i} p_j \leq \sum_{j=1}^{i} E_j, \qquad i = 1, \ldots, M, \tag{2}$$

where T_s denotes the duration of the time slot which, in the sequel, we will normalize (i.e., $T_s = 1$).

As discussed earlier, the physical phenomena is modeled by a series of temporally correlated sources. During the i-th time slot, the sensor node collects a sequence of n i.i.d. samples from the corresponding source, which we will denote by $\{x_i^k\}_{k=1}^n$. Furthermore, these samples are correlated over time slots, and modeled as a first-order autoregressive process. Therefore a given sample of the i-th source is modeled by

$$x_i^k = \sqrt{\rho} x_{i-1}^k + w_i^k, \qquad \begin{array}{c} k = 1, \ldots, n, \\ i = 1, \ldots, M, \end{array} \tag{3}$$

with $\rho = \mathbb{E}\left[x_i^k x_{i-1}^k\right]$ denoting the correlation coefficient; and w_i^k standing for an i.i.d. zero-mean Gaussian random variable with variance σ_w^2.

Our goal is to reconstruct at the FC the sequence of measurements $\{x_i^k\}_{k=1}^n$ in each time slot. Due to the continuous-valued nature of the sources and the capacity constraint (1) of the point-to-point channel, the reconstructed source measurements $\{\hat{x}_i^k\}_{k=1}^n$ will be unavoidably subject to some distortion. Such distortion will be characterized by the Mean Squared Error (MSE) metric, that is

$$D_i = \frac{1}{n} \sum_{k=1}^{n} \left(x_i^k - \hat{x}_i^k\right)^2. \tag{4}$$

A. *Source Coding and Distortion*

As for the encoding process at the sensor node, we assume separability of source-channel coding. The source sequence is then encoded into a length-n codeword (with a sufficiently large n) given by $\{u_i^k\}_{k=1}^n$, and which we model as [8]

$$u_i = x_i + z_i, \qquad i = 1, \ldots, M, \tag{5}$$

where for ease of notation we have omitted the sample index and where z_i denotes i.i.d. zero-mean Gaussian random noise of variance $\sigma_{z_i}^2$ which plays the role of encoding noise. Assuming that, in order to decode the received data, the FC exploits the available side information (i.e., all the preceding u_i), the average rate per sample R_i at the output of the encoder at the i-th time slot must satisfy [12]

$$R_i \geq I(x_i; u_i | u_1, \ldots, u_{i-1}), \tag{6}$$

where $I(\cdot;\cdot|\cdot)$ stands for the conditional mutual information. From (5), this last expression can be rewritten as follows

$$\begin{aligned} I(x_i; u_i | u_1, \ldots, u_{i-1}) =& H\left(u_i | u_1, \ldots, u_{i-1}\right) - \\ & H\left(u_i | u_1, \ldots, u_{i-1}, x_i\right) \\ =& \log\left(1 + \frac{\sigma^2_{x_i|u_1,\ldots,u_{i-1}}}{\sigma^2_{z_i}}\right), \end{aligned} \tag{7}$$

with $H(\cdot|\cdot)$ standing for the conditional entropy and $\sigma^2_{x_i|u_1,\ldots,u_{i-1}}$ for the conditional variance of the i-th observation given all the previous data available at the FC. Hence, from (6), the variance of the encoding noise is lower bounded by

$$\sigma^2_{z_i} = \frac{\sigma^2_{x_i|u_1,\ldots,u_{i-1}}}{\mathrm{e}^{R_i} - 1}. \tag{8}$$

For a reliable transmission to occur, the source coding rate of (6) must satisfy the channel capacity constraint of (1)[3], that is

$$R_i = R_i^{(c)} \leq \log\left(1 + |h_i|^2 p_i\right). \tag{9}$$

Finally, at each time slot, the FC produces the optimal Minimum Mean Squared Error (MMSE) estimate given the past encoded observations, namely

$$\hat{x}_i = \mathbb{E}\left[x_i | u_1, \ldots, u_i\right], \qquad i = 1, \ldots, M. \tag{10}$$

Bearing this in mind, the distortion in the reconstruction of x_i in the i-th time slot reads:

$$D_i = \sigma^2_{x_i|u_1,\ldots,u_i}, \tag{11}$$

which, after some algebra, can be expressed as

[2]Here, we assume that the duration of each time slot is such that Shannon's law holds.

[3]For simplicity, we have assumed that the number of samples per time slot equals the number of channel uses per time slot.

$$D_i = \sigma_x^2 \Bigg((1-\rho) \sum_{j=2}^{i} \rho^{i-j} \mathrm{e}^{-\sum_{k=j}^{i} R_k} + \rho^{i-1} \mathrm{e}^{-\sum_{k=1}^{i} R_k} \Bigg). \tag{12}$$

III. Minimization of the Average Distortion

In this section, we attempt to find the optimal power and rate allocation that minimizes the average distortion subject to the energy causality constraint of (2) and the capacity constraint of (1). Accordingly, the optimization problem can be posed as:

$$\min_{\{p_i\},\{R_i\}} \frac{\sigma_x^2}{M} \sum_{i=1}^{M} \Bigg((1-\rho) \sum_{j=2}^{i} \rho^{i-j} \mathrm{e}^{-\sum_{k=j}^{i} R_k} + \rho^{i-1} \mathrm{e}^{-\sum_{k=1}^{i} R_k} \Bigg) \tag{13a}$$

$$\text{s.t.} \quad R_i \le \log\left(1 + |h_i|^2 p_i\right), \quad i = 1, \ldots, M, \tag{13b}$$

$$\sum_{j=1}^{i} p_j \le \sum_{j=1}^{i} E_j, \quad i = 1, \ldots, M, \tag{13c}$$

$$-p_i \le 0, \quad i = 1, \ldots, M, \tag{13d}$$

$$-R_i \le 0, \quad i = 1, \ldots, M. \tag{13e}$$

Due to the coupling (over time slots) of the rates in the exponential terms, this optimization problem cannot be solved analytically. To circumvent this, we define the *cumulative* rates as $r_{ij} \triangleq \sum_{k=j}^{i} R_k$, for $i = 1, \ldots, M$ and $j = 1, \ldots, i$. By doing so, the optimization problem can be rewritten as follows:

$$\min_{\{p_i\},\{R_i\},\{r_{ij}\}} \frac{\sigma_x^2}{M} \sum_{i=1}^{M} \Bigg((1-\rho) \sum_{j=2}^{i} \rho^{i-j} \mathrm{e}^{-r_{ij}} + \rho^{i-1} \mathrm{e}^{-r_{i1}} \Bigg) \tag{14a}$$

$$\text{s.t.} \quad r_{ij} = \sum_{k=j}^{i} R_k \quad i = 1, \ldots, M, j = 1, \ldots, i \tag{14b}$$

$$R_i \le \log\left(1 + |h_i|^2 p_i\right), \quad i = 1, \ldots, M, \tag{14c}$$

$$\sum_{j=1}^{i} p_j \le \sum_{j=1}^{i} E_j, \quad i = 1, \ldots, M, \tag{14d}$$

$$-p_i \le 0, \quad i = 1, \ldots, M, \tag{14e}$$

$$-R_i \le 0, \quad i = 1, \ldots, M, \tag{14f}$$

where now the optimization is with respect to variables $\{p_i\}$, $\{R_i\}$ and $\{r_{ij}\}$. Since the objective function (14a) is convex and the constraints (14b)-(14f) define a convex feasible set, the optimization problem (14) is convex and, thus, has a global solution [13]. By satisfying the Karush-Kuhn-Tucker (KKT) conditions, we identify the necessary and sufficient conditions for optimality. The Lagrangian of (14) reads

$$\begin{aligned} \mathcal{L} = &\frac{\sigma_x^2}{M} \sum_{i=1}^{M} \Bigg((1-\rho) \sum_{j=2}^{i} \rho^{i-j} \mathrm{e}^{-r_{ij}} + \rho^{i-1} \mathrm{e}^{-r_{i1}} \Bigg) \\ &+ \sum_{i=1}^{M} \sum_{j=1}^{i} \lambda_{ij} \left(r_{ij} - \sum_{k=j}^{i} R_k \right) \\ &+ \sum_{i=1}^{M} \mu_i \left(R_i - \log\left(1 + |h_i|^2 p_i\right)\right) \\ &+ \sum_{i=1}^{M} \beta_i \left(\sum_{j=1}^{i} p_j - \sum_{j=1}^{i} E_j \right) \\ &- \sum_{i=1}^{M} \eta_i p_i - \sum_{i=1}^{M} \delta_i R_i, \end{aligned} \tag{15}$$

where $\{\mu_i\} \ge 0$, $\{\beta_i\} \ge 0$, $\{\eta_i\} \ge 0$, $\{\delta_i\} \ge 0$ and $\{\lambda_{ij}\}$ stand for the corresponding Lagrange multipliers. The additional complementary slackness conditions are given by

$$\mu_i \left(R_i - \log\left(1 + |h_i|^2 p_i\right)\right) = 0, \quad \forall i, \tag{16}$$

$$\beta_i \left(\sum_{j=1}^{i} p_j - \sum_{j=1}^{i} E_j \right) = 0, \quad \forall i, \tag{17}$$

$$\eta_i p_i = 0, \quad \forall i, \tag{18}$$

$$\delta_i R_i = 0, \quad \forall i. \tag{19}$$

Finally, by taking the derivative of the Lagrangian with respect to p_i, R_i, r_{ij} and letting them be equal to zero we obtain the set of optimality conditions, namely,

$$\frac{\partial \mathcal{L}}{\partial p_i} = -\frac{\mu_i |h_i|^2}{1 + |h_i|^2 p_i} + \sum_{j=i}^{M} \beta_j - \eta_i = 0, \tag{20}$$

$$\frac{\partial \mathcal{L}}{\partial R_i} = -\sum_{k=i}^{M} \sum_{j=1}^{i} \lambda_{kj} + \mu_i - \delta_i = 0, \tag{21}$$

$$\frac{\partial \mathcal{L}}{\partial r_{ij}} = \begin{cases} -\frac{1}{M} \sigma_x^2 \rho^{i-j} \mathrm{e}^{-r_{ij}} + \lambda_{ij} = 0, & \text{if } j = 1, \\ -\frac{1}{M} \sigma_x^2 (1-\rho) \rho^{i-j} \mathrm{e}^{-r_{ij}} + \lambda_{ij} = 0, & \text{if } j \ne 1. \end{cases} \tag{22}$$

A. Optimal Power Allocation

From (20), (21), (18) and (19), the optimal power allocation follows,

$$p_i^\star = \left[\frac{\sum_{k=i}^{M} \sum_{j=1}^{i} \lambda_{kj}}{\sum_{j=i}^{M} \beta_j} - \frac{1}{|h_i|^2} \right]^+, \qquad i = 1, \ldots, M, \tag{23}$$

where $[\cdot]^+ = \max[\cdot, 0]$. It is worth nothing that, unlike in *classical* waterfilling, the solution here exhibits multiple water levels (i.e., for the i-th time slot, the waterlevel is given by $\nu_i = \frac{\sum_{k=i}^{M} \sum_{j=1}^{i} \lambda_{kj}}{\sum_{j=i}^{M} \beta_j}$). This is due to the fact that energy becomes available only when it is harvested. Moreover, as in [5], it turns out to be a *directional* waterfilling solution in that water (energy) can only flow forward (since energy cannot be consumed before it has been harvested). This results into the inclusion of the β_j multipliers, the ones associated with the causality constraints, in the denominator of the first term of equation (23).

B. Optimal Rate Allocation

Next, by solving (22) for r_{ij}, and noting that r_{ij} is necessarily positive due to constraints (14b) and (14f), the optimal *cumulative* rate allocation can be written as

$$r_{ij}^{\star} = \begin{cases} \left[\log\left(\frac{\frac{1}{M}\sigma_x^2\rho^{i-j}}{\lambda_{ij}}\right)\right]^+, & \text{if } j=1, \\ \left[\log\left(\frac{\frac{1}{M}\sigma_x^2\left(1-\rho\right)\rho^{i-j}}{\lambda_{ij}}\right)\right]^+, & \text{if } j\neq 1. \end{cases} \quad (24)$$

We can observe that $\{\lambda_{ij}\} > 0$, and therefore from (21), $\{\mu_i\} > 0$. This implies that the constraint (14c) is satisfied with equality. Moreover, expression (24) can be readily interpreted in terms of the classical *reverse* water-filling solution for the reconstruction of parallel Gaussian sources [12, Chapter 10]. To see that, we define

$$\gamma_{ij} = \begin{cases} \frac{1}{M}\sigma_x^2\rho^{i-j}, & \text{if } j=1, \\ \frac{1}{M}\sigma_x^2\left(1-\rho\right)\rho^{i-j}, & \text{if } j\neq 1, \end{cases} \quad (25)$$

and

$$D_{ij} = \begin{cases} \lambda_{ij}, & \text{if } \lambda_{ij} < \gamma_{ij}, \\ \gamma_{ij}, & \text{if } \lambda_{ij} \geq \gamma_{ij}. \end{cases} \quad (26)$$

Bearing all the above in mind, equation (24) can be rewritten as

$$r_{ij}^{\star} = \log\left(\frac{\gamma_{ij}}{D_{ij}}\right). \quad (27)$$

Clearly, this last expression mimics that of the classical reverse waterfilling of [12, Theorem 10.3.3]. However, the allocated rates $r_{ij}^{\star}$ here are *cumulative* rather than *individual*; and the reverse water level given by λ_{ij} is not constant (since, again, it depends on energy harvested up to time slot i). Besides, the numerator in the argument of (27) does not only depend on the variance of the sources σ_x^2 but also on the correlation coefficient ρ, as (25) illustrates.

Finally, by replacing (27) in (12), the optimal distortion in the i-th time slot reads

$$D_i^{\star} = \sum_{j=1}^{i} D_{ij}, \quad (28)$$

that is, it can be computed as the sum of the distortions associated to the corresponding cumulative rates.

IV. Optimization Algorithm

The expressions derived in the previous section reveal that the optimal power allocation (23) and the optimal rate allocation (24) can be expressed in terms of the Lagrange multipliers λ_{ij}. Moreover, one can easily prove that problem (14) satisfies Slater's condition and, hence, strong duality holds [13]. Since in these conditions the duality gap is zero, we propose to solve the corresponding *dual* problem in order to determine the *exact* solution (power and rates) to the original one in which we are interested. Specifically, we will resort to the *subgradient method* [11], an iterative scheme which we will adapt to the problem at hand. This is further explained in the following subsections.

A. Solving the Dual Problem with the Subgradient Method

Our constrained optimization problem (14), which in the sequel we will refer to as the *primal problem*, can be cast into the following general form[4]:

$$\begin{array}{ll} \text{minimize} & f_0\left(\mathbf{x}\right) \\ \text{subject to} & f_i\left(\mathbf{x}\right) \leq 0, \quad i=1,\ldots,m \\ & \mathbf{x}\in\mathcal{X} \end{array} \quad (29)$$

where $f_0 : \mathbb{R}^n \mapsto \mathbb{R}$ and $f_i : \mathbb{R}^n \mapsto \mathbb{R}$ are convex functions, and $\mathcal{X}$ is a convex closed subset of $\mathbb{R}^n$. The Lagrangian of this problem can be expressed as

$$\mathcal{L}\left(\mathbf{x},\boldsymbol{\lambda}\right) = f_0\left(\mathbf{x}\right) + \sum_{i=1}^{m}\lambda_i f_i\left(\mathbf{x}\right). \quad (30)$$

From here we can derive the *dual function* as

$$g\left(\boldsymbol{\lambda}\right) = \inf_{\mathbf{x}\in\mathcal{X}} \mathcal{L}\left(\mathbf{x},\boldsymbol{\lambda}\right) = f_0\left(\mathbf{x}^{\star}\left(\boldsymbol{\lambda}\right)\right) + \sum_{i=1}^{m}\lambda_i f_i\left(\mathbf{x}^{\star}\left(\boldsymbol{\lambda}\right)\right), \quad (31)$$

where $\boldsymbol{\lambda} \in \mathbb{R}^m$ corresponds to the vector collecting all the Lagrangian multipliers. Then, the corresponding *dual problem* can be expressed as

$$\begin{array}{ll} \text{maximize} & g\left(\boldsymbol{\lambda}\right) \\ \text{subject to} & \boldsymbol{\lambda} \succeq 0 \\ & \boldsymbol{\lambda}\in\mathbb{R}^m. \end{array} \quad (32)$$

Where $\succeq$ corresponds to the element-wise inequality. In order to solve the dual problem, we use the subgradient method which allows to update the current solution as follows

$$\lambda_i^{(k+1)} = \left[\lambda_i^{(k)} + \alpha f_i(\mathbf{x}^{(k)})\right]^+, \quad i=1,\ldots,m, \quad (33)$$

where α is the step size, and $f_i\left(\mathbf{x}^{(k)}\right)$ turns out to be the subgradient of g at $\lambda_i^{(k)}$, with

$$\mathbf{x}^{(k)} = \arg\min_{\mathbf{x}\in\mathcal{X}}\left(f_0\left(\mathbf{x}\right) + \sum_{i=1}^{m}\lambda_i^{(k)} f_i\left(\mathbf{x}\right)\right). \quad (34)$$

The convergence of the subgradient method is guaranteed under some mild conditions (e.g., for the case of a constant step size, the algorithm is guaranteed to converge to the optimal value, given a sufficiently small step size). As stated earlier, the resulting solution to the dual problem $\boldsymbol{\lambda}^{\star}$ allows to obtain the optimal solution to the primal problem $\mathbf{x}^{\star}$ (i.e., power and rate allocation, in our case).

[4]Here, we consider the general case with only inequality constraints.

Algorithm 1 Procedure for the computation of the optimal power and rate allocations

1: t ▷ Iteration index
2: Set α ▷ Step size
3: Set stopping criteria
4: Initialize $\lambda_{ij}^{(t)} \quad \forall i, j$
5: **repeat**
6: **for all** i **do** ▷ Directional Water-Filling (23)
7: $$p_i^{(t+1)} \leftarrow \left[\frac{\sum_{k=i}^{M} \sum_{j=1}^{i} \lambda_{kj}^{(t)}}{\sum_{j=i}^{M} \beta_j} - \frac{1}{|h_i|^2} \right]^+$$
8: **end for**
9: **for all** i, j **do**
10: **if** $j = 1$ **then** ▷ Cumulative rate allocation (24)
11: $$r_{ij}^{(t+1)} \leftarrow \left[\log \left(\frac{\frac{1}{M} \sigma_x^2 \rho^{i-j}}{\lambda_{ij}^{(t)}} \right) \right]^+$$
12: **else**
13: $$r_{ij}^{(t+1)} \leftarrow \left[\log \left(\frac{\frac{1}{M} \sigma_x^2 \left(1 - \rho\right) \rho^{i-j}}{\lambda_{ij}^{(t)}} \right) \right]^+$$
14: **end if**
▷ Update Lagrangian multiplier (33)
15: $$\lambda_{ij}^{(t+1)} \leftarrow \Bigg[\lambda_{ij}^{(t)} +$$
16: $$\alpha \left(r_{ij}^{(t+1)} - \sum_{k=j}^{i} \log \left(1 + |h_k|^2 p_k^{(t+1)} \right) \right) \Bigg]^+$$
17: **end for**
18: **until** stopping criteria is met

B. Optimization Procedure

Algorithm 1 is a particularization of the subgradient method to problem (14). We start by initializing the Lagrange multipliers of the cumulative rates (which act as our dual variables) to an arbitrary yet positive value. Then, the iterative process begins. At iteration t, for the given value of the multipliers λ_{ij}, we compute in Step 7 the corresponding power allocation by following the directional water-filling algorithm of [5]. Then, the cumulative rate allocation is computed in Steps 10-14 and the value of the multipliers is updated through the subgradient method in Step 15. The procedure is repeated until the selected stopping criteria is met.

V. Numerical Results

In this section, we assess the performance of the proposed optimal power and rate allocation scheme. We are particularly interested in analyzing the impact of the correlation coefficient ρ in the resulting transmission policy. For this reason, in all numerical results we have set the channel gains to a (constant) unit value. The simulation setup considers a system with $M = 10$ time slots and, without loss of generality, an energy harvesting profile with energy arrivals given by $E_1 = 0.2$, $E_3 = 0.6$, $E_6 = 0.8$ and $E_7 = 1.4$.

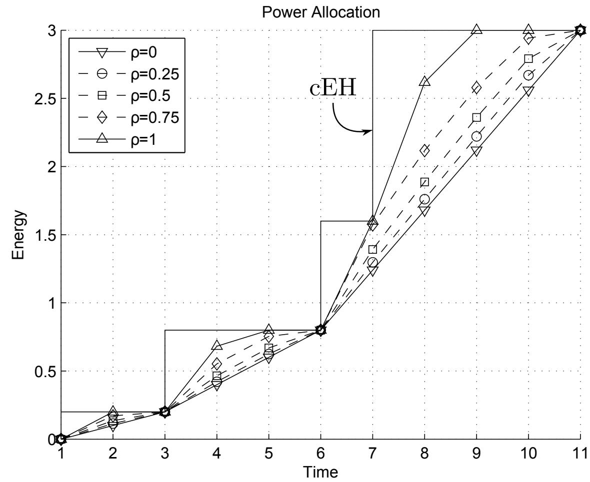

Fig. 2. Optimal power allocation for a varying correlation coefficient ρ.

Figure 2 reveals that, for uncorrelated sources (i.e., $\rho = 0$), the solution corresponds to the well-known geometrical solution of [2] and [14]. This corresponds with the tightest string below the cumulative energy harvesting (cEH) curve connecting the origin and the total harvested energy by the end of time slot M. However, as correlation increases, the harvested energy tends to be spent (i.e., allocated as transmit power) sooner. As a result, in Fig. 2 the slope of the energy consumption (EC) curves right after new energy arrivals (e.g., in the beginning of time slot 3) increases with ρ. This indicates that, in order to minimize the average distortion, one should encode the observations as accurately as possible when some new energy is made available. This stems from the fact that past observations are used here as side information at the receiver (see (10)). Intuitively, the earlier an observation is accurately encoded, the more estimates (in subsequent time slots) can benefit from such an increased accuracy. This holds true even at the expense of a reduced or zero (as in time slot 3, for $\rho = 1$) transmit power being allocated to some subsequent time slots in which case transmission is suspended. All the above is in stark contrast with the uncorrelated case of [2] where the transmit power is (i) strictly positive for all time slots and (ii) a monotonically increasing function.

Figure 3 depicts the reconstruction distortion in *each* time slot resulting from the optimal policy. Unsurprisingly, the higher the correlation, the more predictable the sources are and, hence, the lower the distortion (curves are shifted downwards). For correlated sources, however, distortion does not monotonically decrease with the time slot index. As discussed in the previous paragraph, this follows from the *anticipated* consumption of the harvested energy for the encoding of previous observations (yet, in the end, the average distortion will be lower).

Finally, in Figure 4, we illustrate the convergence of the proposed algorithm. Specifically, we depict the residual error

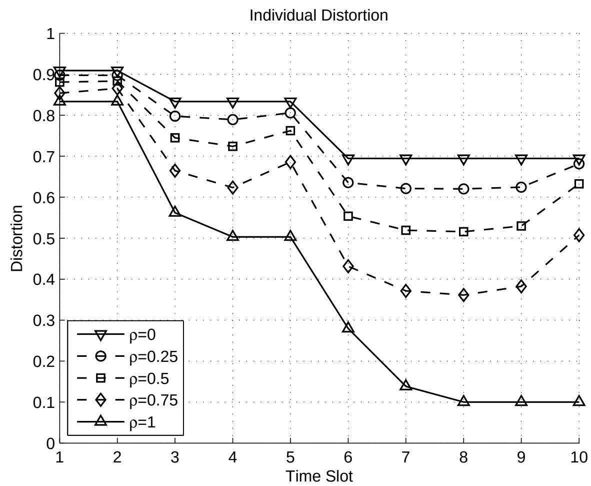

Fig. 3. Distortion per time slot for a varying correlation coefficient ρ.

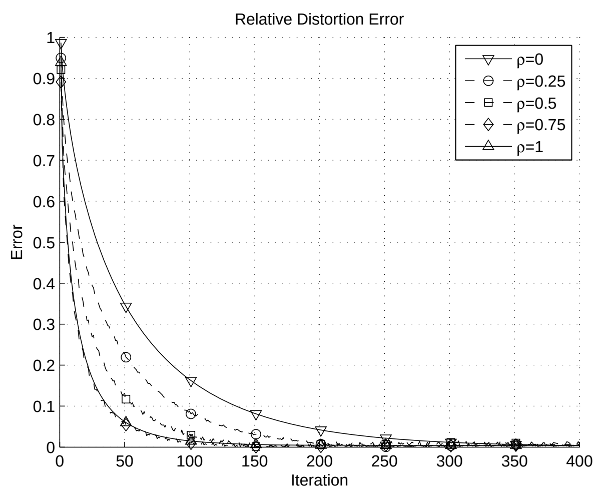

Fig. 4. Convergence error of the proposed algorithm ($\alpha = 0.01$).

in the attained distortion, namely

$$\varepsilon = \frac{\left|D_{opt} - D^{(t)}\right|}{D_{opt}}. \quad (35)$$

As expected, the algorithm converges in all cases. However, convergence (in relative terms) is faster when correlation is high. This is due to the fact that, for high ρ, the update of the cumulative rate allocation (Step 10 in Algorithm 1) is slower, and so is convergence.

VI. Conclusions

In this paper, we have investigated the impact of correlated sources in the design of optimal transmission policies. The goal was to minimize the average distortion in the decoded (reconstructed) observations when side information is used at the FC for decoding data. We have also proposed an iterative procedure based on the subgradient method to solve the problem which entails the interaction of a directional and reverse water-filling schemes in each iterations. Numerical results revealed that, differently from the uncorrelated case (without side information), minimizing the average distortion implies encoding observations as accurately as possible when some new energy is made available. This holds true even if the transmit power allocated to some subsequent time slots is lower or, eventually, zero (and, thus, distortion in such time slots in particular is potentially higher). The iterative scheme converges to the optimal solution with convergence time depending on the degree of correlation.

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