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Supporting Information for

Wind-retrieval from multi-angle backscatter lidar profiles through anisotropic aerosol structures

Sergio Tomás[1] and Francesc Rocadenbosch[2,3]

[1]Institut de Ciències de l'Espai (ICE-CSIC/IEEC), Campus UAB, E-08193 Barcelona, Spain.

[2]Dept. of Signal Theory and Communications, Remote Sensing Lab. (RSLAB), Universitat Politècnica de Catalunya (UPC), E-08034 Barcelona, Spain.

[3]Institut d'Estudis Espacials de Catalunya (IEEC-CRAE), E-08034 Barcelona, Spain.

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Introduction

- Text S1 (Appendix A) is a summary review of aerosol-pattern correlation models in the literature.
- Text S2 (Appendix B) reviews and discusses the limitations of the classic wind-retrieval approach based on time/space maximization of the correlation function.
- Text S3 lists the additional references mentioned in the texts S1 and S2.

Text S1. Aerosol-pattern correlation models

The spatial-correlation model of Eq.(3) consists of a quadratic function, *q,* describing the spatial anisotropy (Eq.(5)) composed with a monotonically-decaying even (i.e., symmetric) function, $f(q)$. Two common models for f are the Booker-Gordon exponential model, $f(q)=\exp\left(-\sqrt{q}\right)$, and the Gaussian one, $f(q)=\exp(-q)$ (Ishimaru, 1978). The Gaussian model is widely used in the statistical description of aerosol structures in the ABL, where they can be spatially modeled as three-dimensional Gaussian fields or "puffs". For an *advected* aerosol concentration field, the space-time correlation function (Eq.(8)) can be expressed as

$$R(\boldsymbol{\rho},\tau)=\exp\left[-q(\boldsymbol{\rho}-\mathbf{U}\tau)\right]. \tag{A.1}$$

In Eq.(A.1), the locus of constant correlation is a four-dimension ellipsoid.

A more complete model of the aerosol concentration field includes the *diffusion* caused by turbulent wind eddies that decorrelate the inhomogeneous structures during transport. First applied by Little and Ekers (1971), and later by Kunkel et al. (1980) especially for an aerosol case, this model can explicitly be formulated for an isotropic, Gaussian medium as

$$R(\boldsymbol{\rho},\tau)=\left[K(\tau^2)\right]^{\frac{3}{2}}\exp\left[-K(\tau^2)q(\boldsymbol{\rho}-\mathbf{U}\tau)\right], \tag{A.2}$$

where

$$K(\tau^2)=\frac{1}{1+\frac{\sigma^2}{\rho_c^2}\tau^2} \tag{A.3}$$

is a Lorentzian function modelling eddy diffusion, σ^2 is the turbulence variance (assumed isotropic), and ρ_c is the characteristic correlation length (Sect. 2.2.1.1).

Other more accurate models based on power-law spectral modelling have been considered by Doviak et al. (1996) (study of the refractive index from ground-based antennas) and by Astafurov et al. (1992) (lidar aerosol-concentration sensing).

Text S2. Relationship with classical methods: Correlation-function maximization

In order to compare the method presented in Sect. 3 with previously published results, we discuss two well-known "classical methods" where the cross-correlation function, $R_z(\mathbf{r},\tau)$, is maximized by finding the optimum time delay, $\tau = \tau_{opt}$ (observations continuous in time, and made at two points) or, alternatively, the optimum spatial lag $\mathbf{r} = \mathbf{r}_{opt}$ (observations continuous in space, and made at two times). For historical reference, the reader can follow Briggs (1968).

(i) *Time optimization*

Statement 1. - "The approach in which the wind vector is inverted by finding the time delay where the correlation function has a maximum

$$\left.\frac{dR_z}{d\tau}\right|_{\tau=\tau_{opt}} = 0, \tag{B.1}$$

given lidar observations made at two points separated by $\mathbf{r}_0$ is not valid for the turbulent atmosphere case. The non-turbulent case requires isotropic media or, alternatively, a velocity- inversion volume densely sampled in all directions."

1. For the turbulent case of Eq.(A.2), is it obvious that the derivative of the correlation function (Eq.(B.1)) involves the "turbulent" diffusion function $K\left(\tau^2\right)$ and its derivative,

which cannot be determined unless $K\left(\tau^2\right)$ is known “a priori”. As a result, the statement fails.

2. For the non-turbulent (frozen-atmosphere) case, we return to Eq.(8)) and recall that the model function f (Eq.(3)) is by definition monotonically decreasing. As a result, $df/dq < 0$ and maximizing the correlation function ($dR_z(\boldsymbol{\rho},\tau)/d\tau = 0$) is equivalent to

$$\frac{dR_z}{d\tau} = \frac{df}{dq}\frac{dq}{d\tau} = 0 \Rightarrow \frac{dq}{d\tau} = 0 . \tag{B.2}$$

That is, maximising the quadratic form q for $\tau = \tau_{opt}$. Departing from Eq.(12), the condition (B.2) under the most general case of anisotropic media is equivalent to

$$\left(\mathbf{r}^T - \tau_{opt}\mathbf{V}^T\right)\mathbf{A}\mathbf{V} = 0 \Rightarrow \begin{cases} \left(\mathbf{r} - \tau_{opt}\mathbf{V}\right) \perp \mathbf{A}\mathbf{V} \\ \mathbf{r}_p = \tau_{p,opt}\mathbf{V} \end{cases} . \tag{B.3}$$

By interpreting this equation as the dot product, $\mathbf{V}_C^T\mathbf{V}_D = 0$, between vector $\mathbf{V}_C = \mathbf{r} - \tau_{opt}\mathbf{V}$ and vector $\mathbf{V}_D = \mathbf{A}\mathbf{V}$, Eq.(B.3) has two possible solutions:

2.1) The general solution, $\mathbf{V}_C \perp \mathbf{V}_D$, implies that $\mathbf{V}_C = \mathbf{r} - \tau_{opt}\mathbf{V}$ is orthogonal to the anisotropy-rotated wind vector $\mathbf{V}_D = \mathbf{A}\mathbf{V}$.

If (as is always the case) the anisotropy matrix, $\mathbf{A}$, is not known “a priori”, assuming isotropic media when in fact it is not, Eq.(B.4), next, yields a “false” *apparent direction* of drift that combines the contributions of true wind direction and the anisotropy-dominant direction of the aerosol pattern.

For the isotropic case ($\mathbf{A} = \mathbf{I}$, $\mathbf{V}_D = \mathbf{V}$), $\mathbf{r} - \tau_{opt}\mathbf{V}$ is orthogonal to the true wind vector, $\mathbf{V}$, and Eq.(B.3) can also be written as

$$\frac{r}{\tau_{opt}} = \frac{V}{\cos(\mathbf{r},\mathbf{V})}, \quad \text{(B.4)}$$

where $\mathbf{r}$ is a user-given baseline, $r = |\mathbf{r}|$, $V = |\mathbf{V}|$, and the relative direction of wind and baseline appear explicitly ($\cos(\mathbf{r},\mathbf{V})$).

If **A** is known (be it via the wind-retrieval method outlined in Sect. 3.2, or any other means, including, obviously, the isotropic case **A** = **I**), direct solution of Eq.(B.3) is cumbersome, for it involves quadratic and cross-product terms of the wind components (V_x, V_y, V_z). This is a main argument in favour of the linear approach presented in Sect. 3.3.

2.2) The particular solution of Eq.(B.3), $\mathbf{V}_C = \mathbf{0}$ (equivalently, Eq.(B.4) for the isotropic case), occurs when the chosen baseline, $\mathbf{r}_p$, and the wind vector, $\mathbf{V}$, are parallel. In that case, the particular solution $\mathbf{r}_p = \tau_{p,opt}\mathbf{V}$ (yielding the wind vector solution, $\mathbf{V} = \mathbf{r}_p / \tau_{p,opt}$) indicates that only for the wind-parallel baseline, $\mathbf{r}_p$, there is a time shift, $\tau_{p,opt}$, giving the absolute maximum correlation. However, this particular solution cannot be found when the set of baselines is very limited, as in the case of a few scanning LOS (e.g., the three-angle azimuth scan). That is, unless the high resolution and the radial extension of the velocity-inversion volume permit very dense sampling and hence the possibility of finding in the volume data lattice such a wind-parallel baseline.

(ii) *Spatial optimization*

Statement 2.- "The space shift $\mathbf{r}_{opt}$ giving the maximum correlation value,

$$\left.\frac{dR_z}{d\mathbf{r}}\right|_{\mathbf{r}=\mathbf{r}_{opt}}=0, \tag{B.5}$$

for lidar observations separated by a time interval τ_0 gives the mean wind velocity (drift velocity) as $\mathbf{V}=\mathbf{r}_{opt}/\tau_0$ provided that the velocity-inversion volume is densely sampled in virtually all directions."

Departing from any of the correlation models discussed so far (the non-turbulent "advective" of Eq.(8), the Little-Ekers' turbulent one of Eq.(A.2)), it is straightforward to show that Eq.(B.5) (i.e., the directional derivative of the correlation function in the $\mathbf{r}_{opt}$ direction) is equivalent to that of the quadratic form, q (compare with Eq.(B.2) above). Formally,

$$\left.\frac{dR_z}{d\mathbf{r}}\right|_{\mathbf{r}=\mathbf{r}_{opt}}=0\Rightarrow\left.\frac{dq}{d\mathbf{r}}\right|_{\mathbf{r}=\mathbf{r}_{opt}}=0,\ \frac{dq}{d\mathbf{r}}=\lim_{h\to 0}\frac{q(\mathbf{r}+\Delta\mathbf{r},\tau)-q(\mathbf{r},\tau)}{h}, \tag{B.6}$$

where $\Delta\mathbf{r}=h\hat{\mathbf{u}}$ and $\hat{\mathbf{u}}$ is a unit vector in the direction of interest ($\hat{\mathbf{u}}=\mathbf{r}_{opt}/r_{opt}$). Since this direction is not known "a priori" the user must search for it in all baseline directions of the volume.

Substitution of Eq.(12) into Eq.(B.6) above and considering lidar observations separated by a time, τ_0, yields

$$2\mathbf{A}\mathbf{r}_{opt}-2\tau_0\mathbf{A}\mathbf{V}=0. \tag{B.7}$$

Eq.(B.7) means that there is one baseline, $\mathbf{r}=\mathbf{r}_{opt}$, that maximizes the correlation function (among any other direction) for the atmospheric wind vector, **V**. Therefore, the wind vector is obtained as $\mathbf{V}=\mathbf{r}_{opt}/\tau_0$, which is a vector parallel to the baseline vector $\mathbf{r}_{opt}$. In the multiple-angle azimuth technique, because all baselines and LOS are nearly parallel, if the gradient of a large-scale aerosol feature is not parallel to the LOS (or more generally, if $\mathbf{A} \neq \mathbf{I}$) then a "false apparent motion" is retrieved.

Again, while this result is feasible for area scanning schemes and horizontal wind retrievals, it is not for scanning schemes based on a few scanning LOS (e.g., the three-angle-azimuth scan). This is because of the limited number of baselines available from the lattice of measurements in the velocity-inversion volume.

Text S3. References

Astafurov, V. G., E. Y. Ignatova, and G. G. Matvienko, 1992: "Efficiency of lidar measurements of wind velocity by a correlation lidar," *Atmos. Oceanic. Opt.,* **5,** 318.

Briggs, B. H., 1968: "On the analysis of moving patterns in geophysics – I. Correlation analysis", *J. Atmos and Terrestrial Physics,* **30,** 1777-1788.

Doviak, R. J., R.J. Lataitis and C. L. Holloway, 1996: "Cross correlation and cross spectra for spaced antenna wind profilers," *Radio Science*, **31**(1), 157-178.

Ishimaru, A., 1978: Scattering of waves from random continuous and turbulent media. *Wave propagation and scattering in random media.* Vol. 2. Academic Press Inc, San Diego, CA, pp. 334-336.

Kunkel, K. E., E. W. Eloranta, and J. A. Weinman, 1980: "Remote Determination of Winds, Turbulence Spectra and Energy Dissipation Rates in the Boundary Layer from Lidar Measurements," *J. Atmos. Sci.*, **37,** 978-985.

Little, L. T., and R. D. Ekers, 1971: "A Method for Analysing Drifting Random Patterns in Astronomy and Geophysics," *Astron. Astrophys.,* **10,** 306.