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## Parameter Uncertainty Modelling in Water Distribution Network Models

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### Abstract

The use of water distribution network (WDN) models is an extended practice [13]. Confidence on decisions taken upon such models depends highly on their accuracy [11]. The parameters uncertainty has to be defined in order to include it in the model. Some of the parameters in a network (e.g. pipes lengths and diameters) can be easily measured and their uncertainty can be calculated on a statistical basis [4]. Demands cannot be measured directly and they have to be estimated using other measurements [10][8]. The uncertainty in the measurements used for that estimation is propagated to the parameters [1]. Besides, demands have their own stochastic nature that induces uncertainty. This paper describes how the pressure measurements are used to infer the uncertainty model in demands for a real network. The real data are treated in order to avoid the effect of boundary conditions. An uncertainty model for demands is calculated to justify the observed behaviour of the measurements. Montecarlo simulations are used for the validation.

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### 1. Introduction

A model-based leak localisation method was successfully applied in a pilot test in a District Metered Area (DMA), called Nova Icaria, located within the Barcelona water distribution network (WDN). This study was the result of two different projects (PROFURED [6] and RTNM [7]) proposed and lead by CETAQUA, the technological Center of Barcelona Water Company managing the DMA (AGBAR), and mainly developed by the Advanced Control Systems (SAC) group of Technical University of Catalunya (UPC). This first approach motivated further steps on this work, related with the accuracy that could be achieved by the initial methodology when applied exhaustively to the whole WDN, if the only available information is coming from the measurements of the sensors already installed in the system, and how it improves as new sensors are introduced [6][5]. Furthermore, the accuracy of any model-based

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methodology is highly dependant on the model reliability [12][3]. The uncertainty of a model can have different origins. In this work the uncertainty is observed using measurements gathered from real scenarios. Measurements obtained at the same hour in different days show an uncertain behaviour of the network. Once the expected distribution of measurements is estimated, a hypothesis of the uncertainty source is assumed. This uncertainty source is modelled so that the simulated scenarios are a realistic representation of the system.

### 1.1. Problem statement

In general a DMA has its inputs monitored, both flows and pressures. This is the actual case in the Barcelona WDN where pressure measurements are used to set the model boundary conditions together with the demand distribution, based on registered water and the total demand provided by flow sensors at the network inputs [6]. The pressure values obtained by sensors installed within the DMA present a relevant dispersion. This dispersion includes uncertainties with different origins. The reproduction of these uncertainties in the simulation model allows the assessment of any methodology that will be applied in real networks beforehand. The questions that this work aims to answer are:

1. How can the uncertainty in pressure measurements be reduced by taking into account available information?
2. Which sources may be chosen to generate this uncertainty in the models?
3. How can this uncertainty be created in the simulation models?

### 1.2. Case Study

In this work, a DMA located in the Barcelona area is used as a case study. In order to simulate the DMA isolated from the water transport network, the boundary conditions (i.e. pressure and flow measurements from the network) are fixed. Generally, pressure is fixed using a reservoir and the overall demand is obtained as the sum of the inflow distributed through the DMA. The total inflow is distributed using a constant coefficient (base demand) in each consumption node. Hence, all the consumptions are assumed to share the same profile, whilst the billing information is used to determine the base demand of each particular consumption. A good estimation of the demand model is paramount for the real case application.

The DMA considered here (Fig. 1) is called *Canyars* and is located at the pressure level 80 within the Barcelona water transport network. This DMA has  $N_n = 694$  nodes and  $N_l = 719$  links, and delivers water to the end consumers by means of a single input point.

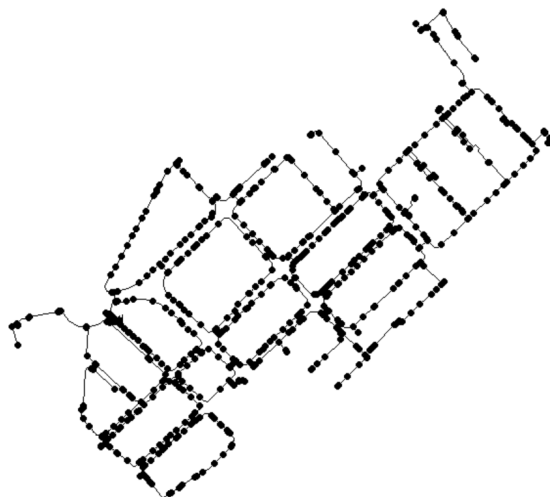


Fig. 1. Canyars DMA

The paper is organized as follows: Section 2 describes the uncertainty estimation methodology and the results obtained with the historical data available from the WDN. This uncertainty is generated in simulation using the methodology described in Section 3, which also presents the results obtained applying the methodology to the case study described in Section 1.2. Finally, the conclusions that show up from the results are discussed in Section 4.

## 2. Uncertainty estimation

The pressure measurements distribution in a node within the DMA is studied. At the beginning of this work, only pressure sensors were available in a DMA similar to our case study. Historical data, provided every 10 minutes are processed in order to obtain one filtered value every hour for every pressure sensor. The distributions studied correspond to the same hour of a week day so that demand conditions are similar. The range of pressures is rather wide and a first topological conditions identification (e.g. valve status) is carried on. The data taken at the same topological conditions are selected reducing the pressure variability. Boundary conditions induce part of the remaining variability in the pressure measurements. In order to not include this known information in the uncertainty model, a linear relation of the pressures with the measured inflows and the boundary pressures is estimated. Eq. 1 expresses the linear model for pressure in a node  $i$  considering a DMA with two inputs.

$$p_i = p_{i_0} + \sum_{j=1}^{N_j} \frac{\partial p_i}{\partial S P_j} |_{x_0} \Delta S P_j + \frac{\partial p_i}{\partial Q_t} |_{x_0} \Delta Q_t + \Delta p_i \tag{1}$$

where  $p_i$  is the pressure measured in node  $i$ ;  $p_{i_0}$  is the nominal pressure in node  $i$  under nominal boundary conditions  $x_0$ ;  $\Delta S P_j$  and  $\Delta Q_t$  are the pressure difference in input  $j$  and the total demand difference respect to the nominal boundary conditions respectively;  $N_j$  is the number of inputs.

$\Delta p_i$  is the remaining pressure uncertainty in node  $i$  after extracting the that uncertainty coming from the boundary conditions. The pressure measurements,  $p_i$ , are corrected in order to make them independent of the boundary conditions using this linear model. Eq. 2 shows the correction applied. The coefficients  $a_j$  and  $b$  are estimated by linear regressions.

$$p_i^* = p_i - \sum_{j=1}^{N_j} a_j \Delta S P_j - b \Delta Q_t = p_{i_0} + \Delta p_i \tag{2}$$

This remaining uncertainty,  $\Delta p_i$ , comes from different sources (e.g. demand behaviours, modelling and measurement uncertainties). It is expected to have a distribution normally distributed, so it can be characterised by its standard deviation. The target here is to replicate the uncertainty in the pressure measurements taken in identical conditions, so that the uncertain model lets us test the model-based methodologies.

The first row of Fig. 2 presents the pressure measurements for the five sensors considered at 0:00 weekdays. The corresponding distributions present high deviations and their shape are barely Gaussian.

In order to use all the available data to infer the uncertainty, the influence of the known boundary conditions has to be extracted. Fig. 3 and 4 show the relation of the boundary conditions, total inflow and input pressures, respectively, with the DMA inner pressures considered. A linear regression produces the coefficients  $a_j$ , and  $b$  in Eq. 2. Applying this linear correction the distributions obtained (second row in Fig. 2) can be assumed gaussian and also reduce their standard deviation from the former distributions.

Table 1. Table with the standard deviations of data (in m)

Conditions	Sensor 1	Sensor 2	Sensor 3	Sensor 4	Sensor 5
Raw data	1.23	1.34	1.08	1.19	1.18
Extracting boundary conditions	0.21	0.19	0.24	0.19	0.52

Table 1 presents the standard deviations of the distributions in Fig. 2 before and after the regression model is applied. From second row in Table 1 a mean standard deviation is defined for all the sensors. Namely in our case

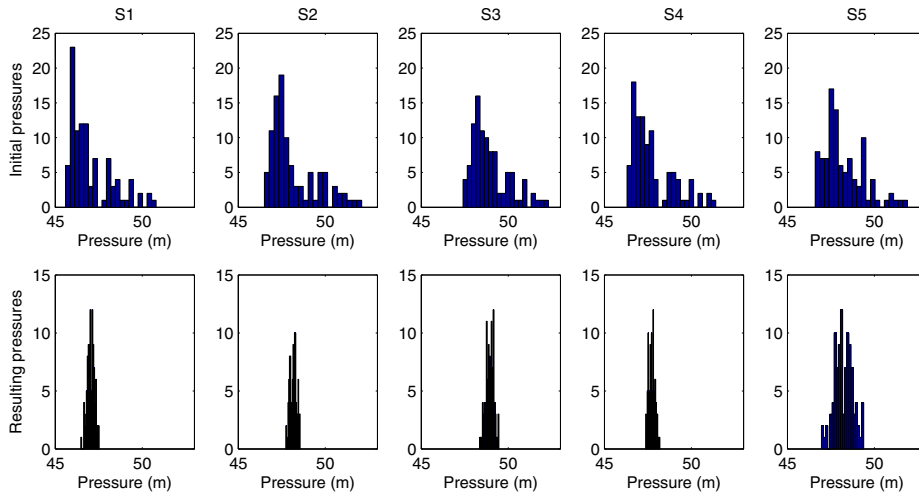


Fig. 2. Histogram of five pressure measurements (columns) at 0:00 a weekdays. Total data points: 133. First row: raw data, second row: row data without boundary conditions

study  $\sigma(p_i + \Delta p_i)$  is around  $20\text{cm}$ . In Section 3 this uncertainty in the pressure measurements is induced modelling the uncertainty of parameters.

### 3. Uncertainty modelling

Uncertainty is originated from parameter estimation, measurement errors, incorrect boundary conditions, inherent model structural errors or unknown status of valves [3][14]. In this work we aim to replicate the effect of the uncertainties in the measurements rather than model all these uncertainty sources. Demands have a variable behaviour compared with other parameters, thus they have inherent uncertainty added to the estimation uncertainty. We choose demands as our uncertainty source coinciding with [2] where demands are assessed as the principal source of uncertainty.

Our model assumes that the remaining uncertainty depends on the uncertainty in the demand, Eq. 3. The methodology for the demand uncertainty definition is to evaluate its effect on the uncertainty in pressures by means of Monte Carlo simulations. The uncertainty in demands is increased until the uncertainty in pressures equals the observed in Section 2.

$$p_i^* = p_{i_0} + \Delta p_i(\Delta \mathbf{d}) \quad (3)$$

#### 3.1. Basic model

Firstly, the simplest demand model of those applied in water networks is considered. It uses the inflow measurements in a DMA that are generally obtained on-line and the percentage of consumption of each demand that comes from billing (usually monthly or quarterly). Eq.4 expresses the demand  $d_i(t)$  at node  $i$  at each sample time  $t$ , hourly in this work.

$$d_i(t) = \frac{bd_i}{\sum(bd_i)} q_i(t) + \Delta d_i(t) \quad (4)$$

where  $bd_i$  is the so-called base demand that weighs the demand of node  $i$  within the global DMA and  $\Delta d_i$  is the uncertainty in the nodal demand. The uncertainty is simulated assuming a gaussian distribution with zero mean and standard deviation  $\sigma$  for node  $i$ , Eq.5.

$$\Delta d_i(t) = \frac{bd_i}{\sum(bd_i)} q_i(t) \mathcal{N}(0, \sigma). \quad (5)$$

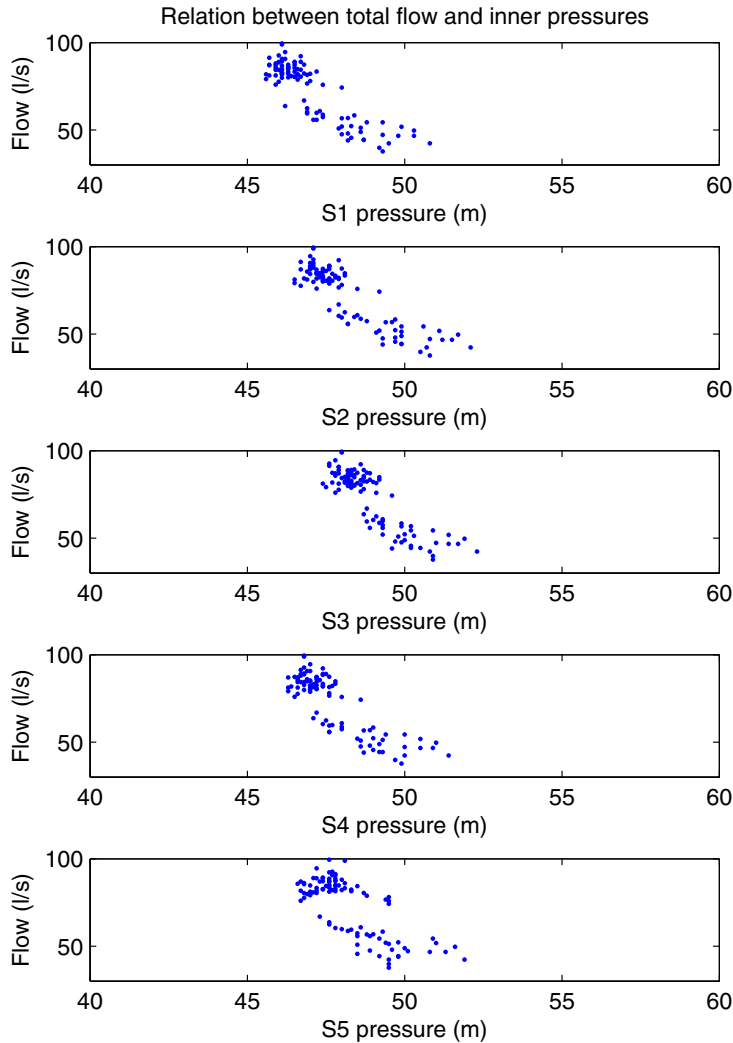


Fig. 3. Total inflow in the DMA versus pressure in five sensors within this DMA.

This uncertainty is conditioned by the knowledge of the total demand:

$$\sum_j \Delta d_j = 0 \tag{6}$$

We try to estimate the standard deviation ( $\sigma$ ) value that induces a similar uncertainty in the predicted pressure measurements as the observed in Section 2.

Using the model presented in Section 3.1 it is not possible to generate the uncertainty in the pressures even when considering rather high uncertainties in the demands. Table 2 shows the uncertainties obtained which are far from the 20 cm, observed in the real measurements, even with an uncertainty standard deviation of 10 times the demand in each node.

Computing the pressure uncertainty as a function of the demand uncertainty gives insight into how the demand model may be changed to accommodate the observed uncertainty.

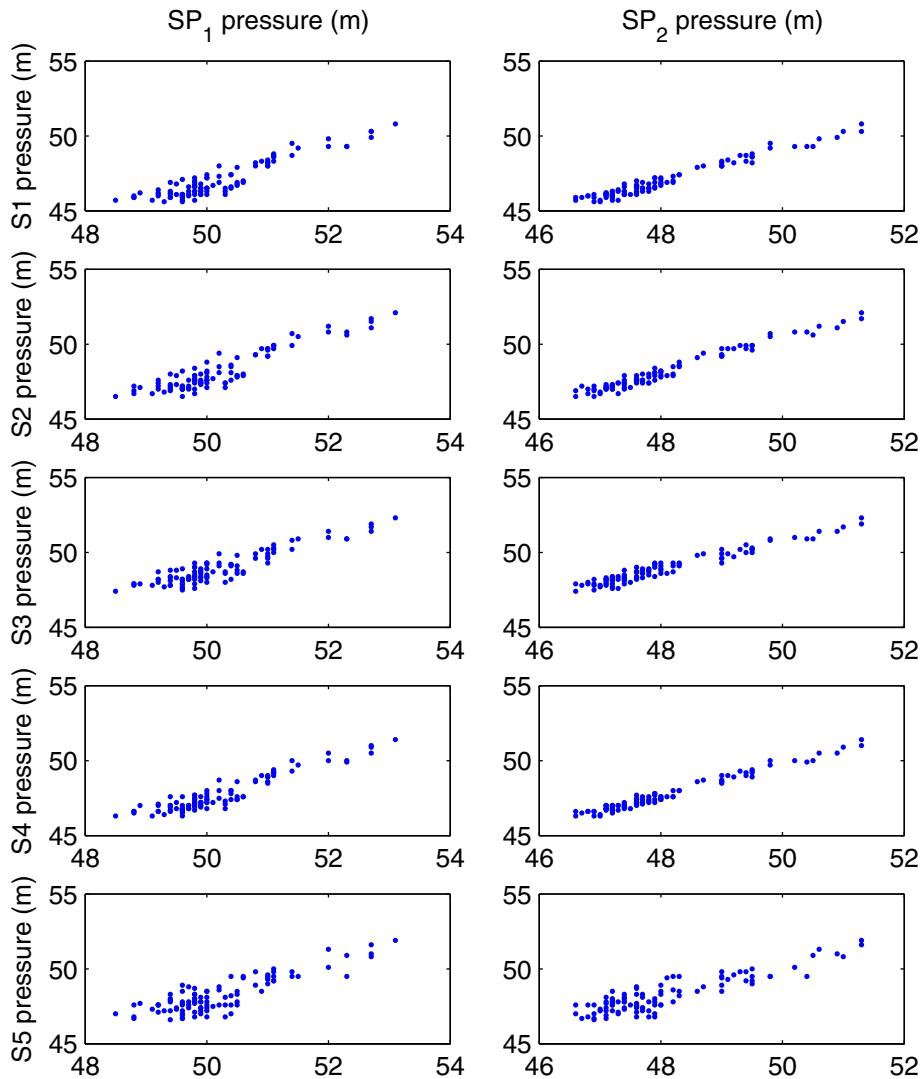


Fig. 4. Pressure at the two inputs of the DMA (columns) versus pressure in five sensors within this DMA (rows).

Consider first that pressure uncertainty is a linear function of the uncertainty in the demands

$$\Delta p_i = \sum_j \beta_j \Delta d_j. \quad (7)$$

The pressure variance is obtained squaring the previous equation and taking the expectation, denoted by  $E$

$$E\Delta^2 p_i = \sum_j \sum_k \beta_j \beta_k E\Delta d_j \Delta d_k. \quad (8)$$

As a simple application of Eq. 8 consider the uncorrelated uncertainty in the demands case, i.e.  $E\Delta d_j \Delta d_k = 0$  for  $j \neq k$ , then pressure variance is

$$E\Delta^2 p_i = \sum_j \beta_j^2 E\Delta^2 d_j. \quad (9)$$

Table 2. Table with the standard deviations of data (in m)

	$\sigma = 0.5$	$\sigma = 1$	$\sigma = 3$	$\sigma = 5$	$\sigma = 10$
$\sigma_{S1}$	0.0009	0.0016	0.0041	0.0080	0.0359
$\sigma_{S2}$	0.0028	0.0046	0.0098	0.0165	0.0500
$\sigma_{S3}$	0.0015	0.0027	0.0074	0.0138	0.0561
$\sigma_{S4}$	0.0020	0.0037	0.0105	0.0195	0.0700
$\sigma_{S5}$	0.0046	0.0069	0.0127	0.0197	0.0478

In the present case demand uncertainty is conditioned by the knowledge of the total demand (Eq. 6) implying that demands are not uncorrelated

$$E\Delta^2 d_j = - \sum_{k \neq j} E\Delta d_j \Delta d_k. \tag{10}$$

The previous equation combined with Eq. 8 gives an expression for the variance of the pressure uncertainty that allows the interpretation of the present case

$$E\Delta^2 p_i = \sum_j \sum_{k \neq j} (\beta_j \beta_k - \beta_j^2) E\Delta d_j \Delta d_k. \tag{11}$$

To accommodate the observed pressure variance using only demand uncertainty, Eq. 11 suggests the use of a demand model that increases the correlation of the demand uncertainties considering the geographical information in the coefficients  $\beta_j \beta_k - \beta_j^2$ .

### 3.2. Demand component model

The second model considered here includes demand components that have a daily periodicity so that the variation in pressure gradients may be justified. These components have a geographical distribution. More information about this model can be found in [9]. Eq. 12 expresses the demand using this model.

$$d_i(t) = b d_i \sum_{j=1}^{n_c} (m_{ij}(c_j(t) + \Delta c_j(t))) q_i(t) \tag{12}$$

where  $c_j(t)$  is the value of demand component  $j$  at time instant  $t$ ;  $n_c$  is the number of components defined in the DMA;  $m_{ij}$  is the membership of demand  $i$  to component  $j$ ;  $\Delta c_j(t)$  is the uncertainty of the component  $j$ . The uncertainty is simulated assuming that the elements of a component are not fixed and well-known values but they have a gaussian distribution with standard deviation  $\sigma_j$  (Eq. 13). We try to estimate the standard deviation of these distributions  $\sigma_j$  that induces a similar uncertainty in the predicted pressure measurements as the observed in section 2.

$$\Delta c_j(t) = c_j(t) \mathcal{N}(0, \sigma_j). \tag{13}$$

These demand components are defined using the sensitivity matrix that relates the demands in the nodes with the pressures. Therefore these demand components have a geographical meaning. For the case study considered here the membership of each node to each of the three demand components is presented in Fig. 5. The number of components depends on the information available when the model is calibrated. Three components are considered as a reasonable demand modelling of the DMA for this particular case.

Fixing the boundary conditions, a Monte Carlo simulation with 1000 realisations gives the mean standard deviation in the pressures of the nodes depending on  $\sigma_j$  (Fig. 6). The standard deviations in the three demand components that produce de 20 cm of mean standard deviation in pressures are  $\sigma_1 = 2.9$ ;  $\sigma_2 = 3.7$ ;  $\sigma_3 = 3.1$ , respectively.

The standard deviation of the demands and pressures is not homogeneous in this model. Fig. 7 shows the demand and pressure standard deviation obtained by the Monte Carlo simulation. Fig. 8 presents the pressure distribution for 25 different nodes, geographically representative of all the network. The Uncertainty distributions obtained are similar to the ones observed in the measurements (Fig. 2) but their shape is not Gaussian. This deformation on the distributions shape can be produced by the constraints in the negative demands, which are not allowed.

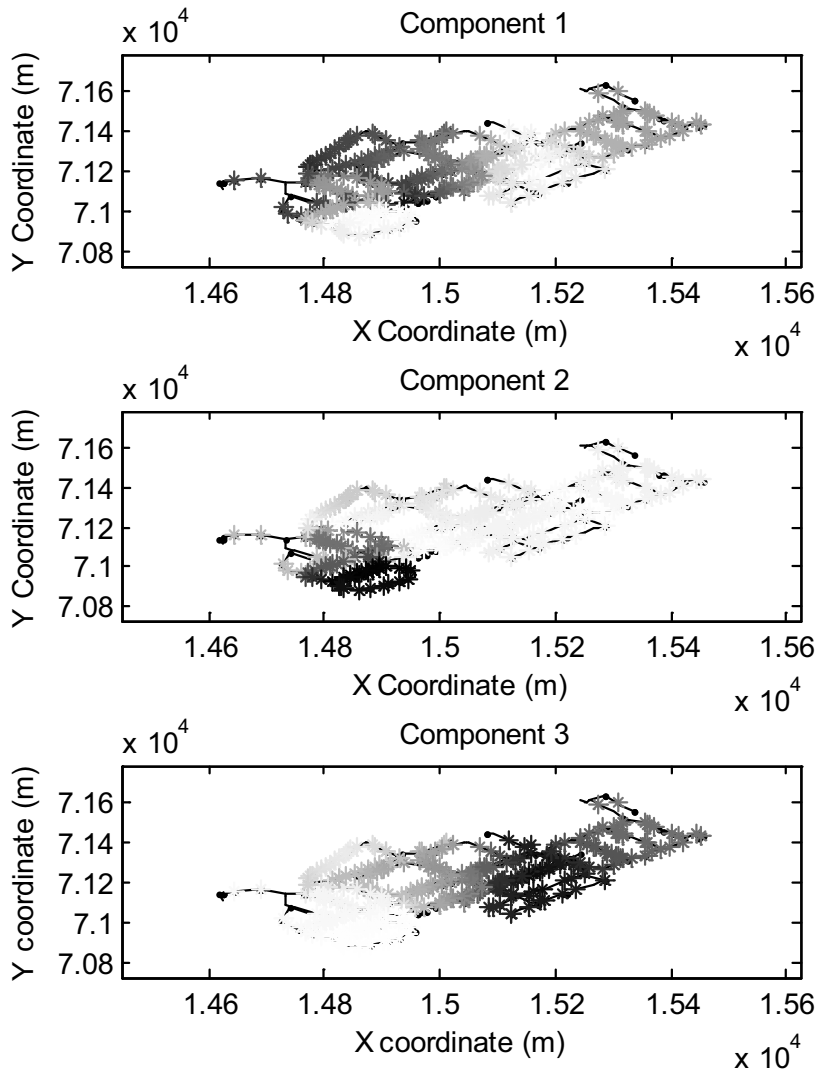


Fig. 5. Membership of each node (darker: higher membership) to the three demand components

#### 4. Conclusions

This paper describes a methodology for modelling the uncertainty observed in field data in a WDN. Firstly this uncertainty has been reduced by means of extracting the effect of varying boundary conditions. The application of a linear model estimated by regressions on real data has produced a Gaussian distribution with a reasonable standard deviation.

The uncertainty source in the model proposed is the demand. A first attempt using a basic model of demands can not justify the uncertainty observed in pressures. The analysis of the demand uncertainty effect on the pressure uncertainty has suggested to change the demand model. A one with more correlated demand uncertainty between nodes in a similar geographical location has been considered.



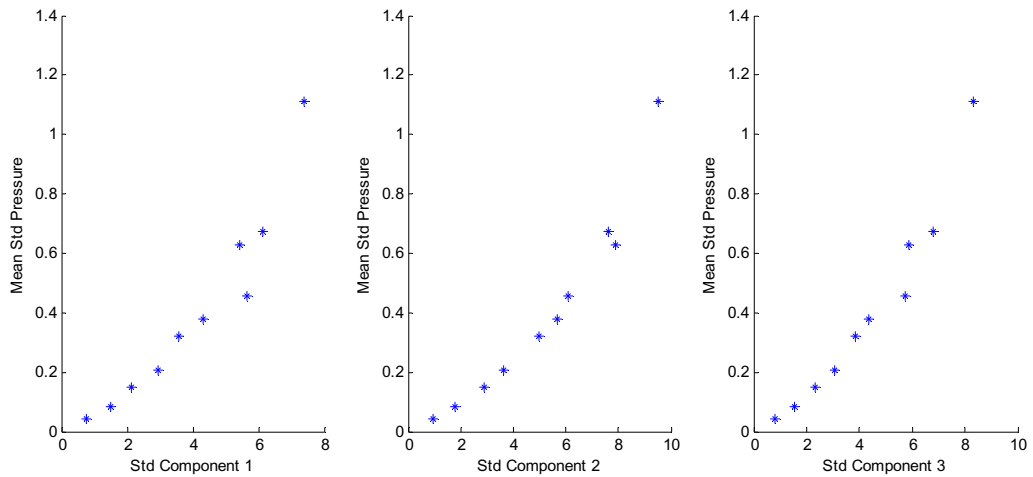


Fig. 6. Mean standard deviation in pressures depending on standard deviation in demand components. Obtained by Monte Carlo simulation.

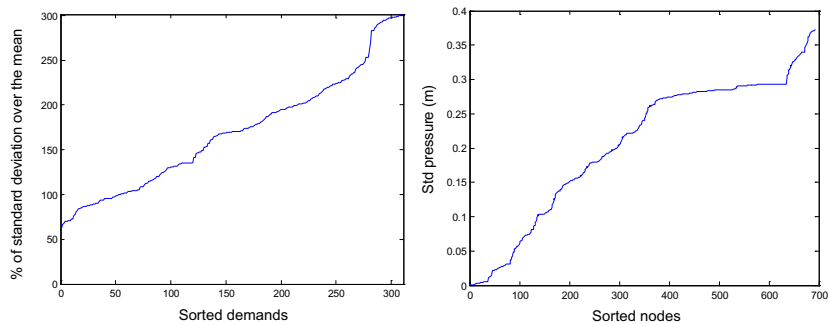


Fig. 7. Standard deviation of demands and pressures in each node.

The demand components model has a geographical meaning that produces distribution in predicted pressures similar to the measurements available. The standard deviation defined for each component has not implied high deviation in individual demand or pressures (Fig. 6).

The model including the uncertainty allows the simulation of realistic scenarios for developing and validating model-based methodologies like leak localisation.

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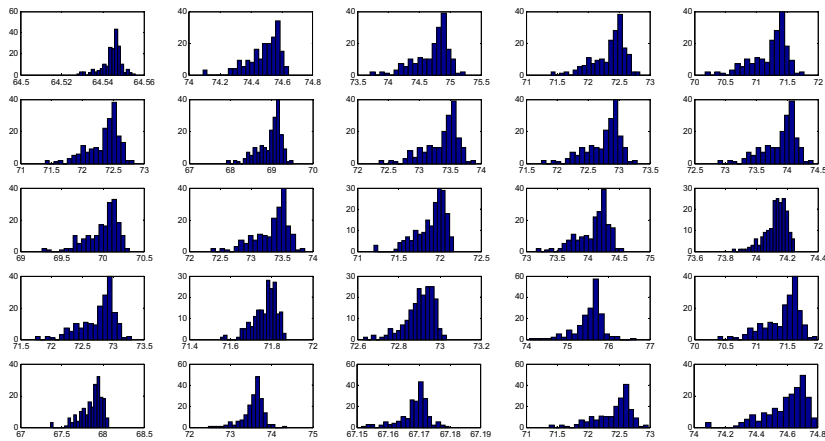


Fig. 8. Pressure distribution in 25 nodes of the network with the modelled uncertainty, obtained by Monte Carlo simulation.

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