L-Fuzzy Sets for Group Linguistic Preference Modeling: An application to assess a firm's performance

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***Abstract*— Given a finite totally ordered set of linguistic descriptions, the extended set of qualitative labels with different levels of precision *L* is constructed. In this framework, qualitative descriptions of a given set are *L*-fuzzy sets. A distance between *L*-fuzzy sets is introduced based on the properties of the lattice *L*. An illustrative example in the retail sector applied to assess a firm's overall performance using perceptions of managers in the firm's different departments is presented.**

***Index Terms*— Knowledge Management, Decision support, Uncertainty and fuzzy reasoning.**

I. Introduction

Several approaches in the fuzzy set framework have been developed to model linguistic preferences [7], [6], [19]. These approaches allow the imprecise and uncertain knowledge that characterizes human preference reasoning to be handled. Some of these approaches involve different levels of precision or multi-granularity in the fuzzy linguistic modeling and are therefore based on a non-totally ordered set of linguistic labels [3], [13], [15]. Linguistic preference modeling has been used in consensus approaches in decision making, an overview of soft consensus models in fuzzy environments can be found in [8].

L-fuzzy sets were defined by Goguen [5] as a generalization of the classic fuzzy sets by considering membership functions with range values in a lattice L. In this way, the classic fuzzy sets are a special case of the L-fuzzy sets when $L = [0,1]$. Topological and metric properties of L-fuzzy sets have been analyzed in [9], [18]. In [11], [12] representation theorems for L-fuzzy sets can be found. Using L-fuzzy set representations, ordered structures have been characterized in [17]. In addition, several studies have addressed the relation between L-fuzzy sets and other extensions of fuzzy sets, such as intuitionistic fuzzy sets and interval-valued fuzzy sets [2], [22].

On the other hand, qualitative reasoning was introduced in the 80's [4] to model real-world problems in which only incomplete qualitative knowledge is available [20]. Qualitative order-of-magnitude models are basic theoretical tools for qualitative reasoning involving different levels of precision or multi-granularity [21]. The adaptation of L-fuzzy sets theory to represent linguistic preferences via qualitative order-of-magnitude models was introduced in [14].

In this paper, we present a generalization of the existing qualitative order-of-magnitude models, which allows us to define qualitative descriptions of a set in terms of L-fuzzy sets, and formalize the concept of distance between qualitative descriptions by means of a multi-granular set of order-of-magnitude labels.

A formal mathematical model is developed to support experts in group decision-making under uncertainty. The proposed model focuses the situation where the group of decision makers has some subgroups, each subgroup composed by members with similar profiles. The perspectives of the different subgroups are analyzed and modeled via an aggregation of linguistic preferences. Distances between qualitative descriptions are used to measure differences between the subgroups. This contributes to measure the risk and assure the validity of the actions derived from a decision outcome.

The rest of this paper is organized as follows: First Section II introduces the theoretical framework for the new approach. Section III presents the new approach for group preference modeling based on an aggregation of qualitative descriptions and the distances among them. An illustrative example in the retail sector is presented in Section IV. Finally, Section V contains the main conclusions and lines of future research.

II. Preliminaries: Qualitative Descriptions as L-Fuzzy Sets

In this section, we briefly review some basic concepts that will be used in the next sections [1], [14], [16].

A. The lattice $(\mathbb{S}_n, \sqcup, \sqcap)$

From here on, let $\mathcal{S} = \{a_1, \ldots, a_n\}$ be a finite totally ordered set, with $a_1 < \ldots < a_n$. The following definitions were introduced in [14]:

Definition 1: The *basic qualitative labels (or basic labels) over* $\mathcal{S}$ are the singletons $\{a_i\}$, with $i \in \{1, \ldots, n\}$.

Definition 2: The *qualitative labels* (*or labels) over* $\mathcal{S}$ are the intervals $[a_i, a_j] = \{x \in \mathcal{S} \,|\, a_i \leq x \leq a_j\}$, for all $i, j \in \{1, \ldots, n\}$ with $i \leq j$.

Note that the basic labels are labels and the entire set $\mathcal{S} = [a_1, a_n]$ is a label. The label $\mathcal{S}$, which is the union of all basic labels, is frequently denoted by the symbol ? and referred to as the *"unknown"* label: ? $= \mathcal{S}$.

Definition 3: Let $\mathcal{P}(\mathcal{S})$ be the power set of $\mathcal{S}$. *The set* $\mathbb{S}_n^* \subseteq \mathcal{P}(\mathcal{S})$ *of all of the qualitative labels over* $\mathcal{S}$ is called

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the order-of-magnitude qualitative space with granularity n *over* $\mathcal{S}$:

$$\mathbb{S}_n^* = \{[a_i, a_j] \mid i, j \in \{1, \ldots, n\}, i \leq j\}.$$

The set $\mathbb{S}_n^*$ is extended to include the empty set $\emptyset$ to obtain a lattice structure.

Definition 4: The *extended set* $\mathbb{S}_n \subseteq \mathcal{P}(\mathcal{S})$ *of qualitative labels over* $\mathcal{S}$ is:

$$\mathbb{S}_n = \mathbb{S}_n^* \cup \{\emptyset\}.$$

The order relation *to be more precise or equal than* between qualitative labels, induced by the inclusion $\subseteq$, is defined as follows:

Definition 5: For any qualitative labels $[a_i, a_j]$ and $[a_{i'}, a_{j'}]$, we say that $[a_i, a_j]$ is *more precise or equal than* $[a_{i'}, a_{j'}]$ iff $[a_i, a_j] \subseteq [a_{i'}, a_{j'}]$, i.e., $i' \leq i$ and $j \leq j'$.

Example 1: Let us consider a simple example to illustrate the above definitions. Suppose that a firm's performance regarding its *market positioning* is qualitatively described by means of the set of qualitative labels over $\mathcal{S}$, with

$$\mathcal{S} = \{a_1, a_2, a_3, a_4\}$$

where associate linguistic labels are: *not very good, moderately good, very good, extremely good*.
Then an example of a *basic label* is $\{a_2\}$ = *moderately good* and two examples of *non-basic labels* are $[a_1, a_3]$ = *not extremely good* and ? $= [a_1, a_4]$ =*unknown*. The relation to be *more precise or equal than* among these three labels gives: $\{a_2\} \subseteq [a_1, a_3] \subseteq$?.

The binary operations on the extended set $\mathbb{S}_n$ of qualitative labels: the *connected union*, $\sqcup$ and the *intersection*, $\cap$ provide a lattice structure to $\mathbb{S}_n$ [14] (the *connected union* of two qualitative labels is the least element of $\mathbb{S}_n$, based on the subset inclusion relation $\subseteq$, that contains both qualitative labels). In Figure 1 the diagram of this lattice is depicted.

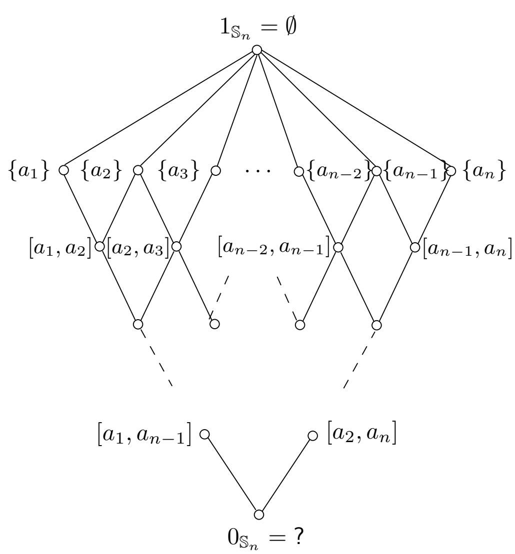

Fig. 1. Diagram of the lattice $(\mathbb{S}_n, \sqcup, \cap)$ [14]

A simple calculation proves that the cardinality of $\mathbb{S}_n$ is

$$|\mathbb{S}_n| = 1 + \frac{n(n+1)}{2}.$$

The lattice $(\mathbb{S}_n, \sqcup, \cap)$ is not distributive. A counterexample in which the property $X \cap (Y \sqcup Z) = (X \cap Y) \sqcup (X \cap Z)$ does not hold is given in the case where $\mathcal{S}$ has at least three elements, considering $a_1, a_2, a_3 \in \mathcal{S}$ such that $a_1 < a_2 < a_3$ and $Y = \{a_1\}, X = \{a_2\}, Z = \{a_3\}$. In addition, $(\mathbb{S}_n, \sqcup, \cap)$ does not satisfy the modular condition if $n \geq 3$. A sublattice of $\mathbb{S}_n$ that is isomorphic to the pentagon lattice is given by the following five labels: $[a_1, a_3], [a_1, a_2], \{a_1\}, 1_{\mathbb{S}_n}, \{a_3\}$.

B. Qualitative descriptions as L-fuzzy sets

The concept of an *L-fuzzy set on a non-empty set* Λ was introduced by Goguen in [5] as a function $f : \Lambda \to L$, where L is a lattice. This concept is applied to the case of the lattice $(\mathbb{S}_n, \sqcup, \cap)$ of qualitative labels over a finite set $\mathcal{S}$ in the following definitions and theorem.

Definition 6: An *$\mathbb{S}_n$-fuzzy set on* Λ is a function $Q : \Lambda \to \mathbb{S}_n$.

Note that any $f : \Lambda \to \{0, 1\}$ defines an ordinary set on Λ, that is, a subset of Λ, whose characteristic function is f. If $f : \Lambda \to [0, 1]$, then f defines a fuzzy set on Λ, where for each $\lambda \in \Lambda$, $f(\lambda)$ is the degree of membership of λ. We can therefore consider an $\mathbb{S}_n$-fuzzy set $Q : \Lambda \to \mathbb{S}_n$ on Λ as a set whose elements are assigned qualitative labels from the extended set $\mathbb{S}_n$ over $\mathcal{S}$ rather than degrees of membership.

Definition 7: The *set* $\mathcal{Q}$ *of* $\mathbb{S}_n$*-fuzzy sets on* Λ is:

$$\mathcal{Q} = {\mathbb{S}_n}^{\Lambda} = \{Q \mid Q : \Lambda \to \mathbb{S}_n\}.$$

Definition 8: A *qualitative description of the set* Λ *by* $\mathbb{S}_n$ *(or using the labels of* $\mathbb{S}_n$*)* is an $\mathbb{S}_n$-fuzzy set on Λ such that for all $\lambda \in \Lambda$, $Q(\lambda)$ is a qualitative label, i.e., $Q(\lambda) \in \mathbb{S}_n^* = \mathbb{S}_n - \{\emptyset\}$.

C. A distance between qualitative descriptions

As proved in [14], formula:

$$D_{\mathbb{S}_n^*}(E_1, E_2) = \text{card}(E_1 \sqcup E_2) - \text{card}(E_1 \cap E_2)$$

for $E_1 = [a_i, a_j]$ and $E_2 = [a_{i'}, a_{j'}]$ in $\mathbb{S}_n^*$, provides a distance on $\mathbb{S}_n^*$. This distance between qualitative labels induces a distance between qualitative descriptions.

Let us consider a finite set $\mathsf{Q} = \{Q_1, \ldots, Q_k\} \subset \mathbb{S}_n^{\Lambda}$ of qualitative descriptions of a set Λ by $\mathbb{S}_n$. For every $Q_i \in \mathsf{Q}$, let $Q_i(\Lambda) = \{E_1^i, \ldots, E_{r_i}^i\} \subseteq \mathbb{S}_n^*$.

Let $\mathcal{P} = \{B_1, \ldots, B_m\}$ be the partition of Λ such that all functions $Q_1, \ldots, Q_k$ are constant on each part $B_t \in \mathcal{P}, t = 1, \ldots, m$ constructed in [14].

Let $Q_i(B_t) = \{F_t^i\}$, with $F_t^i \in \{E_1^i, \ldots, E_{r_i}^i\}$, for each $i = 1, \ldots, k$ and $t = 1, \ldots, m$. Then Formula 1:

$$D_{\mathsf{Q}}(Q_i, Q_j) = \sum_{t=1}^{m} \text{card}(F_t^i \sqcup F_t^j) - \sum_{t=1}^{m} \text{card}(F_t^i \cap F_t^j) \quad (1)$$

provides a distance in Q, i.e., a distance between qualitative descriptions (see [14]).

Example 2: Let us consider a simple example to illustrate the computation of this distance between qualitative descriptions. Suppose $\Lambda = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5\}$ and $\mathtt{Q} = \{Q_1, Q_2\}$, where Q_1 and Q_2 are the qualitative descriptions of the set Λ by $\mathbb{S}_4$ given in Table I.

TABLE I

QUALITATIVE DESCRIPTIONS Q_1 AND Q_2

	Q_1	Q_2
λ_1	$[a_2, a_3]$	$[a_3, a_4]$
λ_2	$\{a_4\}$	$\{a_4\}$
λ_3	$[a_2, a_3]$	$[a_3, a_4]$
λ_4	$\{a_3\}$	$[a_3, a_4]$
λ_5	$\{a_4\}$	$[a_2, a_4]$

Then, the partition of Λ such that Q_1 and Q_2 are constant on each part is $\mathcal{P} = \{B_1, \ldots, B_4\}$, where $B_1 = \{\lambda_1, \lambda_3\}, B_2 = \{\lambda_2\}, B_3 = \{\lambda_4\}$ and $B_4 = \{\lambda_5\}$. Their respective values are:

$F_1^1 = [a_2, a_3], F_2^1 = \{a_4\}, F_3^1 = \{a_3\}, F_4^1 = \{a_4\}$,

$F_1^2 = [a_3, a_4], F_2^2 = \{a_4\}, F_3^2 = [a_3, a_4], F_4^2 = [a_2, a_4]$.

Therefore:

$$D_{\mathtt{Q}}(Q_1, Q_2) = \sum_{t=1}^{4} \text{card}(F_t^1 \sqcup F_t^2) - \sum_{t=1}^{4} \text{card}(F_t^1 \cap F_t^2) =$$

$$(3 + 1 + 2 + 3) - (1 + 1 + 1 + 1) = 5.$$

Having in mind that, in this case, the distance between two qualitative descriptions ranges from 0 to 16, this value indicates that both qualitative descriptions are quite close.

III. MODELING GROUP PREFERENCES AND DISTANCES BETWEEN GROUPS

The proposed model focusses the situation where the group of decision makers can be split into subgroups which are composed by members with similar profiles. In other words, there is a previous segmentation of the group of experts that is considered relevant for the stated decision-making problem. For instance, we can consider decision-makers with different professional profiles, e.g. nurses, doctors or familiars in a health-care decision-making problem, or managers from different departments of a firm in a management decision-making problem.

The lattice structure of $\mathbb{S}_n$-fuzzy sets and the distance in Subsection II-C will allow us to deal with the subgroups' evaluations of features or alternatives in a group decision-making process. To this end, we consider the connected union in $\mathbb{S}_n$-fuzzy sets to model subgroups' evaluations.

Let Λ be a set of features and G a group of decision makers. Let $G = G_1 \cup \cdots \cup G_k$, where each G_i is a group of decision makers with a similar profile. Let $Q_1^i, \ldots, Q_{r_i}^i$ be the qualitative descriptions of Λ provided by the decision makers in G_i.

Definition 9: For each $\lambda \in \Lambda$, the qualitative description of λ corresponding to each G_i is:

$$Q_{G_i}(\lambda) = Q_1^i(\lambda) \sqcup \ldots \sqcup Q_{r_i}^i(\lambda)$$

From this definition, the *qualitative description of* Λ *corresponding to the subgroup* G_i *of decision makers,* $i = i, \ldots, k$ is:

$$\begin{aligned} Q_{G_i} : \Lambda &\longrightarrow \mathbb{S}_n \\ \lambda &\mapsto Q_{G_i}(\lambda) = Q_1^i(\lambda) \sqcup \ldots \sqcup Q_{r_i}^i(\lambda) \end{aligned}$$

And these functions model the subgroups' linguistic preferences by means of $\mathbb{S}_n$-fuzzy sets. From these functions, the total group's linguistic preferences can be obtained as the *qualitative description of* Λ *corresponding to the total group* G *of decision makers*:

$$\begin{aligned} Q_G : \Lambda &\longrightarrow \mathbb{S}_n \\ \lambda &\mapsto Q_G(\lambda) = Q_{G_1}(\lambda) \sqcup \ldots \sqcup Q_{G_k}(\lambda). \end{aligned}$$

The distance in $\mathbb{S}_n^\Lambda$ between qualitative descriptions, as defined in Subsection II-C, can be computed either for each pair of subgroups G_i, G_j or for each subgroup G_i and the total group G. These distances allow the analysis of the topology of the group of decision makers.

On the one hand, similarities and differences among subgroups' preferences are evidenced. To this end a matrix of distances between pairs G_i, G_j can be computed.

On the other hand, similarities and differences between each subgroup and the total group of decision makers can also be revealed.

This knowledge is crucial in decision-making consensual processes that require several rounds of assessments to converge to a final solution. The topological analysis of the group of decision makers, allows to focus in the dissident subgroups. As a result, this analysis can significantly reduce the necessary number of rounds and the moderator's task.

IV. AN ILLUSTRATIVE EXAMPLE IN THE RETAIL SECTOR

This section focuses on the empirical study that was conducted on a working session in which 70 senior managers from a major chain store organization participated. The objective of the study was to identify the most relevant features with regards to the performance of the firm. President Chain Corporation is a multinational retailing company operating in the regular chain convenience stores sector based in Taiwan. Managers were divided into four main subgroups depending on broad functional area: marketing (15); operations and store operations (17); accounting, finance and audit (24); R&D and information systems (14). Previous to the working session, a state-of-the-art study and a set of in-depth qualitative interviews were conducted to identify 170 performance-related variables in their sector. From this list, 44 features or variables related to resources used in retailing were selected as the main performance variables (see Table II).

A. Data description

An one-dimensional absolute order-of-magnitude model with 4 basic labels corresponding to the 4 ordered responses of the Likert scale used by the managers: (1) $\{a_1\}$ = extremely good; (2) $\{a_2\}$ = very good; (3) $\{a_3\}$ = moderately good; (4) $\{a_4\}$ = not very good. As a result we consider Λ as the set of the 44 selected features. Data considered for this

TABLE II

THE SELECTED RESOURCE FEATURES

Resource area	Feature
Physical resource	λ_1: Number of customer visits λ_2: Store location
Legal resource	λ_3: Sales of private brand products λ_4: Social responsibility
Human resource	λ_5: Employee turnover rate λ_6: Staff training
Organizational resources	λ_7: Franchise system λ_8: Store opening strategy λ_9: Sales per store λ_{10}: Spending-per-visit rate λ_{11}: Internal procedures λ_{12}:Achievement of year-end goals λ_{13}: Investments in technology development λ_{14}: Quality of data collection and process sys. λ_{15}: Empowerment of staff λ_{16}: Response to staff issues λ_{17}: Inventory loss control λ_{18}: Inventory service level λ_{19}: Market positioning λ_{20}: Store renovation/redecoration λ_{21}: Expense control ability λ_{22}: Percentage of part-time staff λ_{23}: Shelf-life of new products λ_{24}: Speed of new products development λ_{25}: Past credit history λ_{26}: Financial support from stockholders λ_{27}: Internet channel development λ_{28}: Maintaining target customers in market diversification
Informational resources	λ_{29}: Following fashion trends λ_{30}: Facing seasonal demands λ_{31}: Openness to criticism λ_{32}: Willingness to innovate
Relational resources	λ_{33}: Customer complaints management λ_{34}: Cost sharing with suppliers on promotions λ_{35}: Joint venture opportunity with competitors
External factors	λ_{36}: Changes in customer preferences λ_{37}: Changes in supplier contract content λ_{38}: Innovation and imitation from competitors λ_{39}: Change in government laws λ_{40}: Stability of government λ_{41}: Innovation of new technology equipment λ_{42}: New management system software devel. λ_{43}: Change of population structure λ_{44}: Change of lifestyle

study are the qualitative descriptions of Λ provided by the 70 managers, considering the group of managers split into the four above-mentioned main subgroups.

The qualitative descriptions of the set Λ given by managers are aggregated to obtain the qualitative descriptions corresponding to the different subgroups. For each group G_i, $i = 1, \dots, 4$, the qualitative description Q_{G_i} can be represented by means of a 44-dimensional vector of qualitative labels. Component j of the vector Q_{G_i}, for each $j = 1, \dots, 44$, is the connected union of the responses of the managers with respect to the feature importance of the feature λ_j.

Let us consider, for instance, G_1 = "Department of Marketing", in this case Q_{G_1} is represented by 44-dimensional vector of qualitative labels, containing four different labels:

$Q_{G_1}(\Lambda) = \{[a_2, a_3], [a_2, a_4], [a_1, a_3], [a_1, a_4]\}$ and the partition of Λ associated to Q_{G_1}, i.e., the subsets of features described by these 4 qualitative labels are respectively:

$P_1 = \{\{\lambda_1, \lambda_2, \lambda_9, \lambda_{10}, \lambda_{11}, \lambda_{12}, \lambda_{17}, \lambda_{18}, \lambda_{19}, \lambda_{20}, \lambda_{21}, \lambda_{22}, \lambda_{23}, \lambda_{33}, \lambda_{44}, \}\{\lambda_3, \lambda_5, \lambda_6, \lambda_7, \lambda_{13}, \lambda_{14}, \lambda_{15}, \lambda_{16}, \lambda_{27}, \lambda_{28}, \lambda_{31}, \lambda_{34}, \lambda_{35}, \lambda_{37}, \lambda_{39}, \lambda_{40}, \lambda_{41}, \lambda_{42}, \lambda_{43}\}, \{\lambda_4, \lambda_8, \lambda_{24}, \lambda_{25}, \lambda_{26}, \lambda_{29}, \lambda_{30}, \lambda_{32}, \lambda_{36}\}, \{\lambda_{38}\}\}$.

TABLE III

DISTANCES AMONG DEPARTMENTS AND BETWEEN EACH DEPARTMENT AND THE TOTAL GROUP

D	G_1	G_2	G_3	G_4	G
G_1	0	0.87	0.57	0.67	0.63
G_2	0.87	0	0.9	1	0.7
G_3	0.57	0.9	0	0.83	0.57
G_4	0.67	1	0.83	0	0.9

Note that, when for a specific feature, managers in G_i extremely disagree, the connected union of the managers' opinions in G_i is the qualitative label $? = [a_1, a_4]$. In this case, only the feature λ_{38} is described by ?. In addition, let us remark that few outlier managers in the group have been removed for some features (there was one outlier for features $\lambda_4, \lambda_{11}, \lambda_{18}, \lambda_{19}, \lambda_{20}, \lambda_{21}, \lambda_{39}, \lambda_{41}, \lambda_{42}, \lambda_{44}$ and two outliers for λ_{29}). Outliers have been determined using the median absolute deviation method [10], which in most cases corresponds to take the central 85% of data. Alternative approaches could be considered by taking the central 50% o 75% of data, which would result in tighter intervals of data.

B. Experimental results

Considering the partitions of Λ associated to Q_{G_i}, $i = 1, \dots, 4$, corresponding to each one of the four departments, the partition $P = \{B_1, \dots, B_{34}\}$ of Λ such that all functions $Q_{G_1}, \dots, Q_{G_4}$ are constant on each part $B_i \in \mathcal{P}$, $i = 1 \dots, 34$, as constructed in [14] is:

$P = \{B_1 = \{\lambda_1, \lambda_2, \lambda_9, \lambda_{12}, \lambda_{20}, \lambda_{23}\}, B_2 = \{\lambda_3, \}, B_3 = \{\lambda_4\}, B_4 = \{\lambda_5\}, B_5 = \{\lambda_6, \lambda_{37}, \lambda_{43}\}, B_6 = \{\lambda_7\}, B_7 = \{\lambda_8\}, B_8 = \{\lambda_{10}\}, B_9 = \{\lambda_{11}\}, B_{10} = \{\lambda_{13}\}, B_{11} = \{\lambda_{14}\}, B_{12} = \{\lambda_{15}\}, B_{13} = \{\lambda_{16}\}, B_{14} = \{\lambda_{17}, \lambda_{19}\}, B_{15} = \{\lambda_{18}\}, B_{16} = \{\lambda_{21}\}, B_{17} = \{\lambda_{22}\}, B_{18} = \{\lambda_{24}\}, B_{19} = \{\lambda_{25}\}, B_{20} = \{\lambda_{26}, \lambda_{32}\}, B_{21} = \{\lambda_{27}\}, B_{22} = \{\lambda_{28}, \lambda_{34}\}, B_{23} = \{\lambda_{29}\}, B_{24} = \{\lambda_{30}\}, B_{25} = \{\lambda_{31}\}, B_{26} = \{\lambda_{33}\}, B_{27} = \{\lambda_{35}\}, B_{28} = \{\lambda_{36}\}, B_{29} = \{\lambda_{38}\}, B_{30} = \{\lambda_{39}\}, B_{31} = \{\lambda_{40}\}, B_{32} = \{\lambda_{41}\}, B_{33} = \{\lambda_{42}\}, B_{34} = \{\lambda_{44}\}\}$.

And, considering $F_i^j = Q_{G_i}(B_j)$, the distances among departments are computed using $D_{\mathbb{Q}}(Q_{G_i}, Q_{G_j}) = \sum_{t=1}^{34} \operatorname{card}(F_t^i \sqcup F_t^j) - \sum_{t=1}^{34} \operatorname{card}(F_t^i \cap F_t^j)$. Their normalized values are summarized in Table III.

Results in Table III show that G_3, i.e. the department of accounting, finance and audit, is the most representative department of the total group of managers, being the one closer to G. Departments G_1, marketing, and G_3, accounting, finance and audit, are those that have expressed more similar opinions with respect to the 44 features related to the firm's performance. Finally, G_2, operations and store operations, and G_4, R&D and information systems, are the department with more different opinions compared to the rest.

V. CONCLUSIONS

This paper puts forward a method, based on a distance defined among groups of experts, for analyzing the topology of the group. Experts' evaluations are expressed using a set of linguistic labels describing order-of-magnitude. The method enables the handling of imprecise information given by evaluators. The approach has three main advantages. First, it takes into account the different degrees of strictness of the evaluators' opinions. Second, it removes the need to calculate an average value of ordinal data. Third, the method accommodates "unknown values" by using the label "?" defined in the absolute order-of-magnitude qualitative model.

From a well-ordered set $\mathcal{S}$ of basic labels, the extended set of qualitative labels $\mathbb{S}_n$ over $\mathcal{S}$ has been considered. The qualitative descriptions of a set Λ are defined as $\mathbb{S}_n$-fuzzy sets. When there is a previous segmentation of the group of decision makers that is considered relevant for the stated decision-making problem, a $\mathbb{S}_n$-fuzzy set is defined for each subgroup of decision makers. A distance between $\mathbb{S}_n$-fuzzy sets allows us to analyze similarities and differences among subgroups and between each subgroup and the total group of decision makers.

A real-case application in the retail sector has been used to capture the differences between a firm's departments when assessing variables related to the performance of the firm. The real-case application gives us an example of how the model presented could benefit managerial decision-making processes.

Three main lines of future research are currently under consideration. First, using the concepts presented in this paper, to develop a web-based software tool capable of gathering and summarizing opinions and working simultaneously with different levels of precision for group decision-making processes. Second, the definition of a feedback process based on recommendations will be studied. These recommendations will be generated from rules induced by the distances between each subgroup and the total group. Finally, regarding the real case study, from the presented analysis that separately considers the functional area of managers, a study to improve consensus reaching will be addressed.

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