MASTER'S DEGREE THESIS

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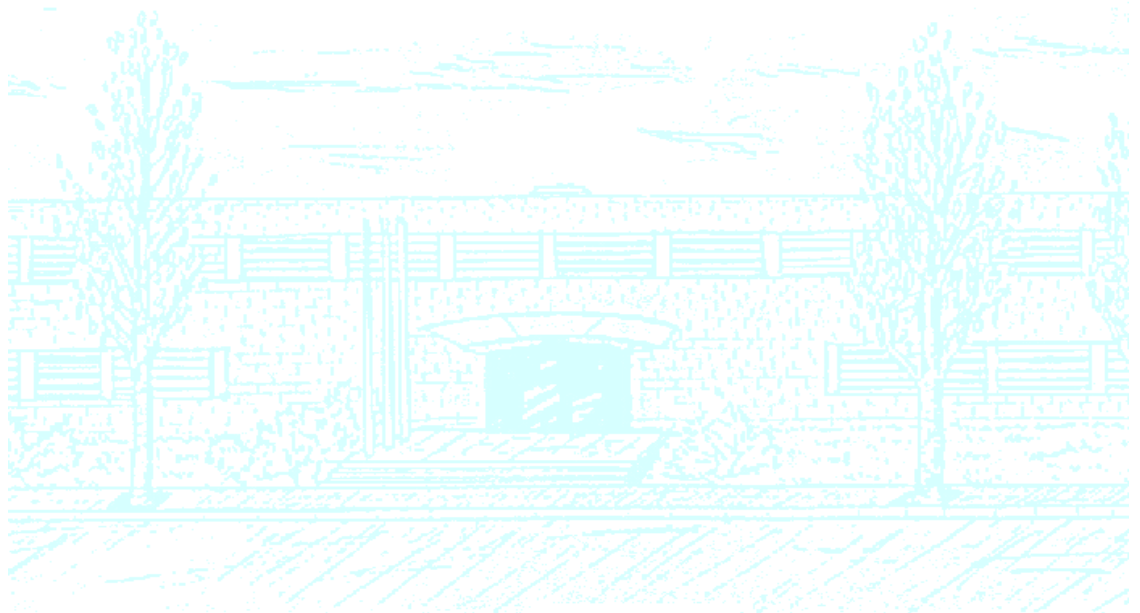
Title: Systematic Strategies for 3-dimensional Modular Robots

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Facultat de Matemàtiques i Estadística

Master Thesis
Master in Advanced Mathematics and
Mathematical Engineering

Systematic Strategies for
3-dimensional Modular Robots

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Abstract

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Modular robots have been studied an classified from different perspectives, generally focusing on the mechatronics. But the geometric attributes and constraints are the ones that determine the self-reconfiguration strategies. In two dimensions, robots can be geometrically classified by the grid in which their units are arranged and the free cells required to move a unit to an edge-adjacent or vertex-adjacent cell. Since a similar analysis does not exist in three dimensions, we present here a systematic study of the geometric aspects of three-dimensional modular robots.

We find relations among the different designs but there are no general models, except from the pivoting cube one, that lead to deterministic reconfiguration plans. In general the motion capabilities of a single module are very limited and its motion constraints are not simple. A widely used method for reducing the complexity and improving the speed of reconfiguration plans is the use of meta-modules. We present a robust and compact meta-module of M-TRAN and other similar robots that is able to perform the expand/contract operations of the Telecube units, for which efficient reconfiguration is possible. Our meta-modules also perform the scrunch/relax and transfer operations of Telecube meta-modules required by the known reconfiguration algorithms. These reduction proofs make it possible to apply efficient geometric reconfiguration algorithms to this type of robots.

Contents

Chapter 1
Introduction

Modular self-reconfigurable robots are connected sets of units of few different types that can change their connectivity, varying the shape of the robot. Thus, these systems can modify their morphology (reconfigure) to better suit different tasks, locomote, interact with the environment and self-repair. This makes them more versatile and robust than fixed-shape unique-purpose robots.

They can be classified according to different criteria: architecture and topology, connection mechanisms, degrees of freedom, propulsion method, etc. We are interested in reconfiguration strategies, that can be formulated as geometric algorithms given the properties and capabilities of each robotic unit. Neither of the above-mentioned criteria is precise enough for our purpose: the basic information we need comprises the geometric shape, the accessible cells and the conditions for each movement and connection (in terms of the orientation, neighbouring units and free space).

In two dimensions, only three regular polygons can tessellate the plane: squares, equilateral triangles, and regular hexagons. Universal reconfiguration is possible for all these grids and movements [**26**]. In contrast, in three dimensions only one Platonic solid (regular polyhedron) is space-filling, the cube, but we can consider other space-filling convex polyhedra, not necessarily cell-transitive, for the design of robotic units with surface-to-surface connections. Examples of these polyhedra are:

- Five polyhedra (Figure 1) that can tessellate space only with translations (each has forms of varied symmetry):
  (1) Parallelepiped.
  (2) Hexagonal prism.
  (3) Rhombic dodecahedron.
  (4) Elongated dodecahedron.
  (5) Truncated octahedron.
- Triangular prisms.
- Space-filling square bipyramid (irregular space-filling octahedron).
- Trapezo-rhombic dodecahedron.
- Triakis truncated tetrahedron.
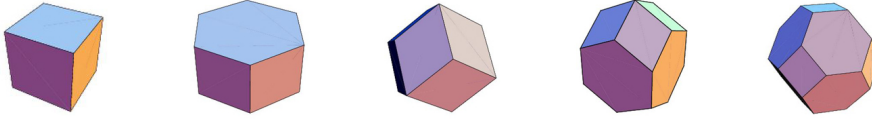- Gyrobifastigium, the 26th Johnson solid (J_{26}).

FIG. 1. Polyhedra tessellating space only with translations. From left to right: cube, hexagonal prism, rhombic dodecahedron, elongated dodecahedron, truncated octahedron.

- Hill tetrahedra, a family of space-filling tetrahedra.
- Isohedral simple tilings.

The most interesting polyhedra for the design of modular robots among the above-mentioned are those whose faces are all congruent, like the cube, the rhombic dodecahedron, the trigonal trapezohedron and the irregular space-filling octahedron, since less restrictions are needed for potential connections.

We can also consider polyhedra that are not space-filling (configurations would be less compact but the empty cell constraints for mobility can be less restrictive) and point-to-point connections. Some lattices in these cases are the same as the previous ones. For example, there are two different arrangements of equally sized spheres filling space with the highest possible average density ($\frac{\pi}{3\sqrt{2}} \simeq 0.74048$): the face-centered cubic (FCC) and the hexagonal close-packed (HCP). This is the Kepler conjecture, proven by Hales in 1998. The Voronoi polyhedra are produced augmenting each sphere with the points in space that are closer to it than to any other sphere. We obtain rhombic dodecahedra for FCC and trapezo-rhombic dodecahedra for HCP.

But not all three-dimensional robots have their units necessarily arranged in a lattice. Chain (or tree) architectures allow a continuous movement not restricted to a grid, what can contribute to locomotion, allowing different locomotion gaits. There are also hybrid robots combining features of the two previous groups.

Thus, there can be many abstract models for the design and reconfiguration of three-dimensional robots. The most simple one, the sliding cube model (SCM), considers the lattice robot units as identical cubes able to perform convex transition and horizontal advance with the free-space requirements shown in Figure 2. Several reconfiguration algorithms for this case are known [**1, 17, 61**], the last one restricted to configurations without holes. However, as far as we know, there is no physically implemented prototype for applying this model. It is an example of the difficulties and limitations of practical implementations.

There is also the pivoting cube model (PCM), in which the cubes are able to rotate around their edges, so the free-space requirements are different (Figure 3). A centralized algorithm is known for this model under the restriction
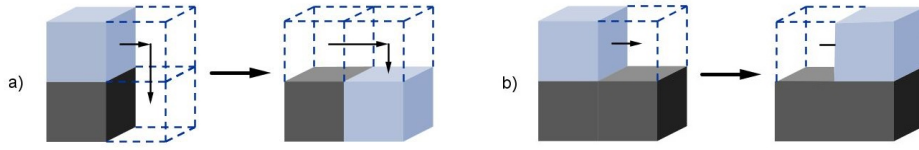
FIG. 2. Permissible movements in the sliding cube model: a) convex transition and b) horizontal advance.
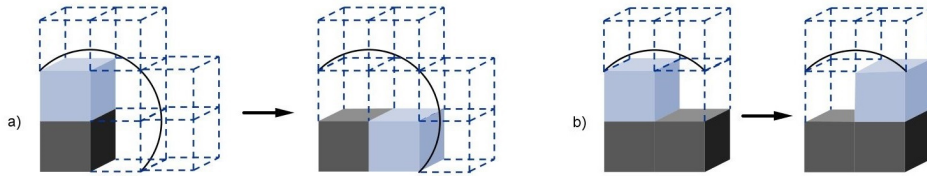

FIG. 3. Permissible movements in the pivoting cube model: a) convex transition and b) horizontal advance.
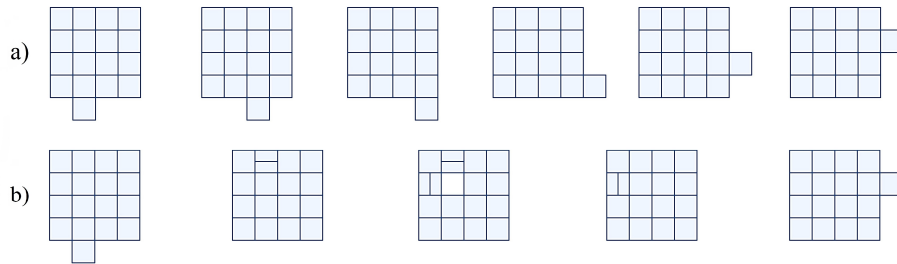

FIG. 4. Different reconfiguration strategies. In both cases, the leftmost configuration is the starting one, and the rightmost is the target one. a) Tunnelling algorithm for expandable and contractible modules. b) Surface algorithm.

that the initial and target configurations meet sufficient conditions for being reconfigurable [**78**]. In this case, the algorithm came after the robot, the M-Block developed at MIT.

In general, there are two different reconfiguration strategies. All the algorithms mentioned before correspond to surface strategies, in which the units move along the external boundary of the configuration, as we can see in the example in Figure 4a. There are robots whose square or cubic units are able to expand and contract by a factor of two. For this kind of units movements interior to the robot configuration are possible. Figure 4b shows how the same reconfiguration is achieved by means of a tunnelling strategy, in which modules travel through the volume of the robot.
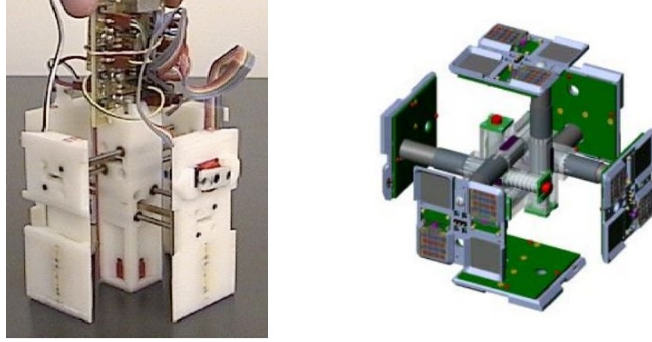
FIG. 5. Expandable and contractible units. Left: Crystalline robot. Font: [**64**]. Right: Telecube robot. Font: [**77**].

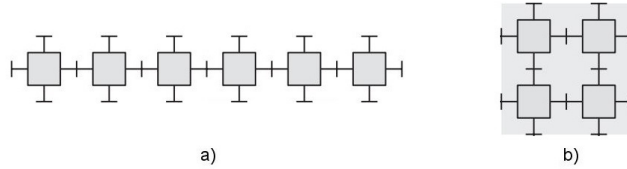a)   b)

FIG. 6.   The need of meta-modules for expandible and contractible units. a) Non-reconfigurable structure. b) Meta-module.

Physical prototypes of self-reconfiguring systems with square or cubic units that can can expand and contract by a factor of two in each of their dimensions are Crystalline robots [**64**] in two dimensions and Telecube [**77**] in three dimensions (Figure 5).

Several tunnelling algorithms for universal reconfiguration have been proposed for Crystalline and Telecube robots. The *melt-grow* [**63**] is a centralized algorithm which reconfigures any connected robot of n units in $O(n^2)$ moves and steps. The *Pac-Man* algorithm [**9**] and the algorithm in [**84**] use $O(n^2)$ parallel steps. Maintaining the assumptions of constant velocity and strength, *in-place* reconfiguration (space requirement is just the union of the the source and target configurations) is possible in linear time [**3**]. The total number of unit moves is $O(n^2)$, which is optimal in this setting. Requiring modules to have linear strength, i.e., to be able to pull or push a linear number of other modules, the total number of unit moves can be reduced to $O(n)$ [**4**]. With this force requirements and allowing velocities to build up over time, reconfiguration is possible in $O(\log n)$ parallel steps and $O(n \log n)$ total moves [**5**].

For all of these algorithms the units are grouped into meta-modules of at least $2 \times 2(\times 2)$ units and of precisely this size (Figure 6b) in the case of the linear and in-place algorithm [**3**]. The use of meta-modules is necessary since, for example, in the line configuration shown in Figure 6a no unit can be moved to a position outside the line without disconnecting the robot. In contrast, meta-modules can perform the srunch/relax and the transfer operations shown in Figure 7.
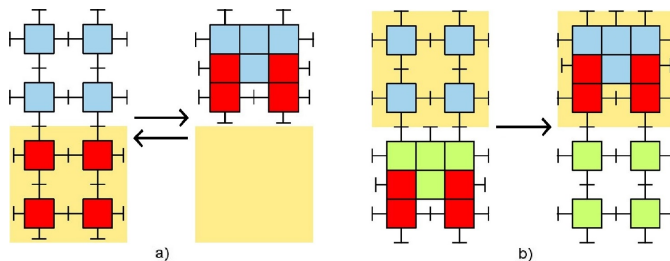
FIG. 7.   The two operations of the Crystalline and Telecube meta-modules. a) Scrunch/relax. b) Transfer.
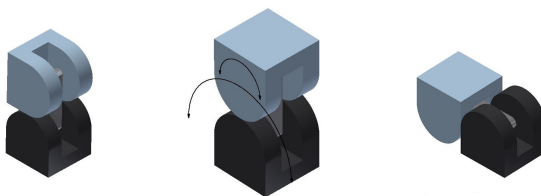

FIG. 8. A M-TRAN unit.

Many current modular robots prototypes have other very convenient properties but cannot expand and contract. We would like to apply the previously described algorithms also to some of these robots by constructing meta-modules with their units, which cannot expand and contract, such that the whole meta-module can.

One of the most interesting robotic systems developed so far is the M-TRAN series, from M-TRAN I to M-TRAN III [**38**]. It was the first hybrid robot, and similar shape and capabilities as those of their units (Figure 8) can be found in posterior robots.

For these reasons, there has been interest in designing meta-modules of M-TRAN units that can expand and contract. Murata and Kurokawa present in [**49**] a small and compact meta-module, but it can only expand and contract in 2 dimensions. For 3D, we are aware of two such meta-modules: one for Molecule units [**37**] and another for M-TRAN [**2**]. The latter meta-module is also valid for Molecube units [**100**]. However, the meta-module of [**2**] is formed by 58 units and the side length of its minimum axis-aligned bounding cube when expanded is 32 units. In addition, it is much less compact than the one by Murata and Kurokawa, making it less robust.

We would be interested in a more realistic meta-module, both in size and number of modules, and also capable of performing the Telecube meta-module operations of the reconfiguration algorithms, since then the use of meta-meta-modules of M-TRAN units would be avoided, reducing the grain of the configurations.

Given the variety of structures and possible designs of three-dimensional robots and the existing gap between abstract models and physical implementations, in contrast to the two-dimensional case, a systematic study of

3D prototypes from a geometric point of view is presented in Chapter 2. Other surveys have studied modular robots, but they have used different criteria [**93, 21**]. As far as we know, this is the first systematic one in the geometric context.

In Chapter 3, restricting ourselves to M-TRAN, we present a more robust and compact meta-module that is able to simulate the operations performed by Crystalline and Telecube units. From the results of Chapter 2 we derive that the meta-module we propose is also valid for other robotic systems. Moreover, since the meta-modules of expandable and contractible units required in the algorithms would lead to meta-meta-modules of M-TRAN units, in Chapter 3 we also show that the use of these meta-meta-modules can be avoided. A shorter version of the last Chapter was presented at the XVI Spanish Meeting on Computational Geometry [**58**].

Chapter 2

Study of self-reconfigurable modular robots from a geometric point of view

In two dimensions self-reconfigurable modular robots can be arranged in a square or in a hexagonal lattice. Examples of the former systems are: Vertical [**25**], Crystalline [**64**], Micro units [**95**], Pneumatic [**27**], CHOBIE [**28**], XBot [**89**], EM-Cube[**6**] and Smart Blocks [**46**]. Examples of the latter are: Metamorphic [**11, 57**], Fractum [**50**], Gear Type Units [**81**], Octabot [**73**] and HexBot [**66**]. All modular robots whose units can be arranged in a triangular lattice are stochastically assembled, like Stochastic [**88**] and Programmable Parts [**7**].

In square grids there are two basic movements: horizontal advance and convex transition. There are different free-space requirements depending on whether the units or the meta-modules pivot, slide or squeeze. In Figure 1 convex transition for these different models is shown. In the hexagonal case there is only one basic move with different free-space requirements depending on the unit. Centralized and distributed reconfiguration and locomotion algorithms have been presented for these lattices and models [**26, 16, 78, 54**].
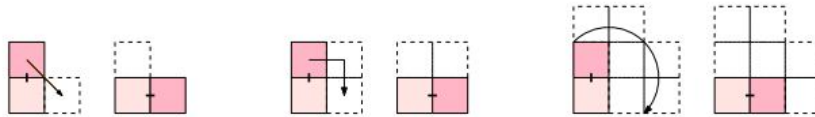
Fig. 1. 2D convex transition. Depending on the model a different set of cells is required to be empty. Left: squeezing. Center: sliding. Right: pivoting. Font: [**26**].

In this Chapter we present a study and an analysis of physically implemented three-dimensional self-reconfigurable modular robots. Since our goal is to provide useful information for designing models and deterministic reconfiguration strategies, we limit ourselves to robots able to scale up to large

enough ensembles. Chain robots are omitted since they are not designed for general reconfiguration and so are stochastic ones.

1. 3D Prototypes

3D Fractum, 1998. [**51**]
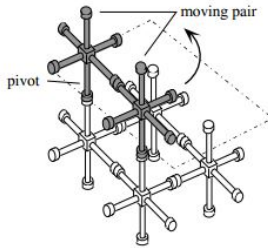
3D Fractum robot.
Source: [**51**].

Country and affiliation: Japan; AIST, MEL, MITI.

Type: Lattice: cubic.

Unit and connectors: Each unit consists of six arms attached to the six sides of a central immobile and small cube located in the center of the corresponding cubic cell. The end of each arm has a genderless connecting mechanism.

Moves: A unit can rotate each arm independently, an therefore has 6 rotational degrees of freedom[a]. However a singe unit cannot move to another cell without the help of another adjacent unit. This robot presents the 3D checker board limitation: units in black cells can never be moved to a white cell and vice versa. Also, as it occurs for the Crystalline and Telecube units, universal reconfiguration is not possible: for example, we cannot reconfigure a linear string of the unit into any other shape. The classification of the reachable configuration set is not known.

Algorithms: A stochastic reconfiguration algorithm was presented in [**51**].

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[a]We do not include the degrees of freedom of connectors in our counting.

Molecule, 1998. [**35**]
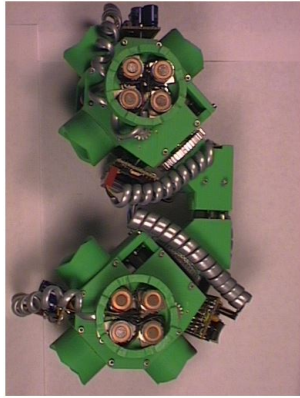
Molecule unit.
Source: [**35**].

Country and affiliation: USA; Dartmouth Robotics Laboratory.

Type: Lattice: cubic.

Unit and connectors: Each unit is composed of two cubical blocks linked by a rigid 90-degree link. Each block has five connection points. There are two kinds of units: the male Molecules have the active connectors and the female units have the passive ones. This does not cause a problem because of the 3D checker board property that this robot shows.

Moves: Each unit has four rotational degrees of freedom, two in each block. Two degrees of freedom allow each block to rotate 180 degrees relative to its bond connection, and the other two degrees of freedom correspond to two rotating connectors, one in each block, that allow them (and therefore the entire Molecule) to rotate 180 degrees relative to another connected unit. This rotating connector can never be placed in the face of the block that is opposite to the one connected with the link, but there is no need for that since the rotation of the link can lead to the same result. In [**35**] the class of three-dimensional objects that can be built out of Molecules is described. In the same paper it is also shown how Molecules can perform, on top of a lattice of identical units, linear translations in a plane and convex and concave transitions between two planar surfaces.

Algorithms and meta-modules: In [**37**] a meta-module allowing tunneling was proposed. It simplifies reconfiguration planning dividing it into two layers: the simpler planning problem for meta-modules and the trajectory planning for a single unit. A variation of Dijkstra's shortest path algorithm is used for solving the last problem.

Two types of locomotion algorithms have been presented in [**36**].

I-Cube, 1999. [**83**]
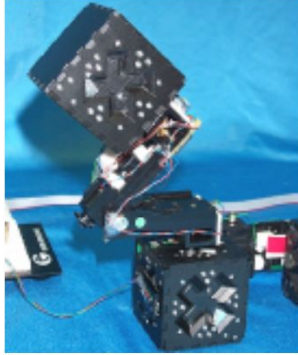
ICubes.
Source: [**82**].
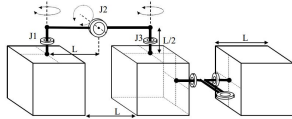
Degrees of freedom
of ICubes.
Source: [**83**].

Country and affiliation: USA; CMU, ICES.

Type: Lattice: cubic.

Unit and connectors: Two distinct elements compose the I-Cubes system: actuated links and cubes not capable of moving by themselves. The cubes, of side length d, can have up to six attachment points (one in each face) for the link connectors and can also be used as task-oriented units. Links are bent and have four sections of length $d/2$, d, d and $d/2$. They are capable of connecting to and disconnecting from the faces of the cubes.

Moves: Links have three degrees of freedom. They have two joints able to rotate 360° (between the sections of length $d/2$ and d, called J1 and J3 in the bottom figure on the left) and another (J2, between the two sections of length d) which can only rotate 270 degrees.

Algorithms and meta-modules: A reconfiguration algorithm using meta-modules of 8 cubes and 16 links was presented in [**82**]. By using these meta-modules the planning is simplified and separated into two different layers: meta-module motions and cube motions.

Telecube, 1998. [**77**]

Telecube unit.
Source: [**77**].

Country and affiliation: USA; Xerox PARC.
Type: Lattice: cubic extendible.
Unit and connectors: A unit has 6 moveable faces arranged in a cube. There are actuated genderless connectors in all of them.
Moves: The 6 prismatic degrees of freedom of each unit allow it to expand and contract each face independently.
Algorithms and meta-modules: We can apply tunnelling strategies for reconfiguration: [**63, 9, 84, 3, 4, 5**].

ATRON, 2004. [29]
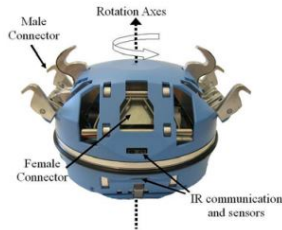
ATRON unit.
Source: [**13**].

An ATRON configuration.
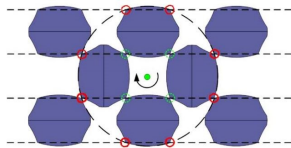
Collisions are avoided when the four modules surrounded by a dashed circle are moved by the rotation of a fifth module in the plane behind the represented one.
Source: [**56**].

Country and affiliation: Denmark; USD.

Type: Lattice: surface-centered cubic. The units are not space-filling, but considering their Voronoi regions we obtain irregular space-filling octahedra.

Unit and connectors: Each unit is nearly spherical, composed of two hemispheres. Each half has four connectors, two male and two female, placed so that opposite ones have the same gender. Connections between two neighbouring modules are possible in two different orientations.

Moves: The units have a single degree of freedom: the two halves of a unit can be rotated relative to each other. The module shape has been designed to prevent collisions during basic motions, as we can see in the bottom figure on the left.

In this case the checker board property, that can prevent connector-gender clashes, does not hold. Therefore, we need to be careful about this, placing all ATRONs in the lattice in a way that no conflicts occur. For example, we can place all x-ATRONs (those whose rotation axis is parallel to the x-axis) with their male connectors in the y-direction, y-ATRONs with their male connectors in the z-direction and z-ATRONs with their male connectors in the x-direction. But as soon as one ATRON-half makes a 90° rotation, the defined pattern is no longer adopted and we cannot guarantee the absence of conflicts. Thus, every rotation of 90° requires another rotation of 90° of the same half if we want to preserve the connections pattern.

Reconfiguration is not universal for ATRON units not even using meta-modules: consider one unit attached to another two in different halves, then we cannot obtain the configuration with one unit attached with the other two in the same half.

Algorithms and meta-modules: Mobility
can be increased by using meta-modules.
In [**13**] several meta-modules that are not
lattice-based (the meta-structure is not
permanent, it is only considered when
moving) are proposed and analyzed.
In [**12**] an algorithm and examples in 3D us-
ing one non-lattice-based meta-module with
three ATRON units are presented.
In [**8**] a 2D meta-module that consists of
four ATRON modules connected in a square
configuration was proposed. It allows us to
apply two-dimensional reconfiguration algo-
rithms [**26, 16**]. Also a centralized greedy
planer (that can get stuck in local min-
ima) and a greedy distributed control al-
gorithm for performing locomotion for the
meta-modules were proposed.

M-blocks, 2013. [**62**]

M-blocks.
Source: [**62**].

Country and affiliation: USA; MIT.

Type: lattice: cubic.

Unit and connectors: each unit is cube-shaped and each face can be connected to any other in all four possible relative orientations.

Moves: provided that no collision occurs, every unit can pivot around an edge shared with an adjacent (face-connected) unit. This robot implements the pivoting cube model (PCM) in three dimensions. The free-space requirements for basic movements are shown in Figure 3. Since the face diagonal of a cube is longer than the side, the volume swept is greater when pivoting than when sliding (under the sliding cube model, see Figure 2). Not all configurations can be reconfigured into any other. Sufficient conditions for the reconfigurable ones are known but they are far from being necessary.

M-Blocks are also independently mobile. A single unit can roll, spin in place, and jump over obstacles up to twice its height.

Relations with other prototypes: see Giant Helium Catoms.

Algorithms: In [**78**] a centralized algorithm for reconfiguration was presented under the restriction that the initial and target configurations have only convex holes and meet three sufficient conditions guaranteeing feasibility. The algorithm in these cases is able to reconfigure the robot into a line in $O(n^2)$ pivot moves.

Giant Helium Catoms (GHC), 2006. [**30**]
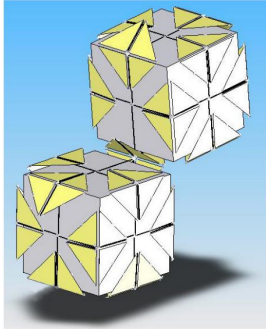
Design of GHC.
Source: [**30**].

Country and affiliation: USA; CMU.

Type: lattice: cubic.

Unit and connectors: Each unit is cube-shaped with four triangular actuated flaps on each face. Any two flaps can be adhered to one another.

Moves: the units can rotate around each other by attaching a flap to a neighbouring cube and then moving it back together with the cube as shown in the figure on the left. For this purpose the flaps can be actuated and controlled independently.

Relations with other prototypes: as M-blocks, Catoms are pivoting cubes.

Other: The Giant Helium Catoms are currently the largest units, with a cube side length of a approximately 1.9 m. As far as we know only two catoms have been constructed (`http://www.cs.cmu.edu/~claytronics/hardware/helium.html`).

Molecube, 2004. [100, 99]

Robot.
Source: [99].

Country and affiliation: USA; Cornell U.

Type: Hybrid: cubic.

Unit and connectors: Each unit is cube-shaped with rounded corners and all faces can be actively connected in any possible orientation.

Moves: Each cube is decomposed into two approximately triangular prisms and the two halves of a unit can rotate relative to each other. This is the only degree of freedom. The symmetry axis passes through the center of the cube and through two opposite vertices (at their furthest distance). Therefore, rotating one of the halves by 120° cycles through three faces of the cube. So, a single unit is not able to move alone. When connected to another one a unit can reach at most two neighbouring cells by rotating. This robots present the 3D checker board limitation, and even if this constraint is satisfied there are pairs of configurations between which we cannot move. For example, similarly to the ATRON counterexample for universal reconfigurability, if one unit is attached to another two in different halves, then we cannot reconfigure into a configuration with two units attached to the same half of a third unit.

Relations with other prototypes: See Roombot.

Algorithms and meta-modules: A meta-module simulating Crystalline and Telecube units was proposed in [2].

Roombot, 2008. [**74**]

Roombot unit.
Source: [**76**].
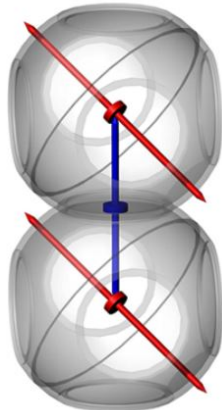
Degrees of freedom of
a Roombot unit.
Source: [**74**].

Country and affiliation: Switzerland; EPFL.
Type: Hybrid: cubic.
Unit and connectors: Each unit is cube-shaped with rounded corners. A single connection mechanism has both male and female connector features integrated so it can be attached to another connector or to a socket. In some cases only a few connectors per unit are needed (two are sufficient in [**76**]) so in each case we can decide how many sockets to equip active connection mechanisms with.
Moves: Each unit has two parallel axes of rotation, as the M-TRAN units, and an additional degree of freedom perpendicular to the two parallel ones, like in Superbot. All three degrees of freedom (bottom figure on the left) allow 360° rotations.

Roombots present the 3D checker board limitation, and the same counterexample for Molecubes shows that there are pairs of configurations satisfying the checker board limitation between which we cannot move.
Relations with other prototypes:
Diametrical degrees of freedom were first presented in the Molecube system. Both robots are geometrically similar, but because of the extra degree of freedom mobility is significantly increased in the Roombot configurations.
Algorithms and meta-modules: In [**74**] two new heuristics were presented for self-reconfiguration planning. In [**76**] off-grid meta-modules of two units are used for increasing mobility during reconfiguration and locomotion.

M-TRAN, 1998 [I], 2002 [II], 2005 [III]. [52, 38]
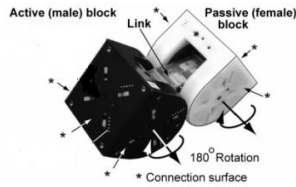
M-TRAN unit.
Source: [**38**].

Country and affiliation: Japan; AIST.

Type: hybrid: cubic.

Unit and connectors: M-TRAN units consist of two linked semi-cylindrical cubes. We refer to these semi-cylindrical cubes as *blocks*. Each block has a gender (male/female) and connectors (different for the two genders) on its three flat surfaces. Connectors of different gender can be attached in all four possible relative orientations. As long as the blocks stay in the lattice no connector-gender clashes can occur because of the 3D checker board property.

Moves: The units have two degrees of freedom: each semi-cylindrical block can rotate from $-90°$ to $90°$ with respect to the link joining both blocks. Reconfiguration is again not universal: two units attached at their square faces and with the links aligned cannot rotate any of their links.

Relations with other prototypes: See Superbot, iMobot, Ubot and SMORES.

Algorithms and meta-modules: Different meta-modules of M-TRAN units that can expand and contract have been designed. For 2 dimensions Murata and Kurokawa present in [**49**] a small and compact meta-module. For 3D there is the one of Aloupis et al. [**2**]. In Chapter 3 a three-dimensional compact meta-module is presented.
Small scale reconfiguration has been shown between different patterns including different locomotion modes. For large scale reconfigurations different meta-modules have been presented [**49**].

Superbot, 2005. [**67**]

Superbot unit in a mode
similar to CONRO.
Source: [**67**]

Superbot unit in a mode
similar to M-TRAN.
Source: [**67**]

Country and affiliation: USA; USC.
Type: hybrid: cubic.
Unit and connectors: SuperBot units, as M-TRAN ones, consist of two linked semi-cylindrical cubes. Connectors are placed on all six flat surfaces and in this case they are genderless. When first proposed no actuated connecting mechanism was included. In [**71**] the SINGO genderless connector was presented.
Moves: The units have three degrees of freedom: each semi-cylindrical block can rotate from $-90°$ to $90°$ and the middle joint can rotate continuously in both directions.
Relations with other prototypes: When the semicircular square faces both blocks are aligned, the Superbot unit can reproduce the M-TRAN moves. If the middle joint is rotated $90°$ then the Superbot has the same shape as CONRO.
Algorithms and meta-modules: SuperBot system supports different locomotion gaits for a wide range of terrains [**72**].

Ubot, 2009. [**79**]

UBot units.
Source: [**79**].

Country and affiliation: China; HIT.

Type: hybrid: cubic.

Unit and connectors: UBot units have a cubic shape and are composed of two L-type blocks connected by a right angle shaft. Each unit has four connecting surfaces, two in each block. There are female units with only passive connectors, and male units with only active ones. Connectors of different gender can be attached in all four possible relative orientations. As long as the blocks stay in the lattice no connector-gender clashes can occur because of the 3D checker board property.

Moves: Each unit has two rotational degrees of freedom. Both blocks of a unit can rotate $\pm 90°$ around the right angle shaft alone or simultaneously. However, the two blocks cannot rotate from the zero angle position towards the same direction since they would collide.

Relations with other prototypes: If we connect two units such that in the zero angle position two faces of the first unit are aligned with two faces of the second one, we obtain a meta-unit with similar (but not equal) shape and motion capabilities.

Sambot, 2010. [**85**]
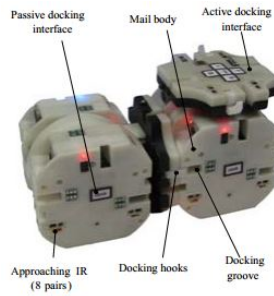
Sambot robots.
Source: [**86**].
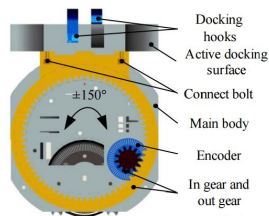
Source: [**85**].

Country and affiliation: China; Beihang University.

Type: hybrid: cubic and mobile.

Unit and connectors: Each unit is divided into a cube-shaped body with two wheels and an active docking surface. This active docking surface can be connected to the grooves (passive) in the four lateral faces of a cubic body.

Moves: There are three degrees of freedom, two of them corresponding to the wheels in the body. The other one corresponds to the $\pm150°$ rotation of the active docking surface around the central axle of the main body. Therefore, the reconfiguration capability within the lattice is very limited.

Other: each Sambot module can move autonomously as a swarm robot.

SMORES, 2012. [**15**]
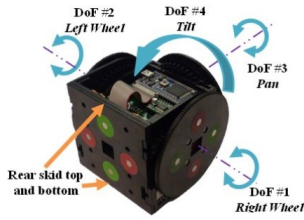
SMORES unit.
Source: [**15**].

Country and affiliation: Australia;   U.   of South Wales. USA; Penn.

Type: hybrid: cubic and mobile.

Unit and connectors: SMORES   units   are cube-shaped with genderless connectors in the four lateral faces of the cube. Three connectors are active (able to attach and detach) and are located in rotating wheels; the other is passive. Two passive ports can get attached but then they cannot disconnect. By connecting SMORES modules in a head-to-tail fashion we can prevent this from happening.

Moves: each unit has four rotational degrees of freedom. Three of them correspond to the continuous rotation of the wheels in which active connectors are placed. The other, parallel to two of the previous ones, allows the passive port to be rotated, this time not continuously but with a $\pm 90°$ limit. It produces a bending joint while the other three can twist connected units. Since the three active connectors can rotate to any angle, connected units can be placed in any relative orientation.

Relations with other prototypes: The SMORES system can emulate other robots, as we can see in the following Figure from [**15**].

The following table contains many modular 2D and 3D robots. The red ones are out of scope for the reason presented in the last column. Black robots have been already detailed in this chapter, blue ones are chain robots and green ones have only been simulated so far.

Robot	Country, Affiliation	Reference	Year	Dim.	Others
CEBOT	Japan, Science U. of Tokyo	[18]	1988	2D	Mobile
RMMS	USA, CMU	[70, 59]	1988	3D	Manually reconfigurable
Metamorphic	USA, Johns Hopkins U.	[11, 57]	1993	2D	
PolyPod	USA, Stanford, PARC	[91]	1993	3D	
Fractum	Japan, AIST, MEL, MITI	[50]	1994	2D	
Biomorphs	USA, LANL	[24]	1995	2D	Mobile
MARS	JP, NDCL, Nagoya U.	[45]	1995	2D	Mobile
TETROBOT	USA, RPI	[23]	1996	3D	Not self-reconfigurable
3D Fractum	Japan, AIST, MEL, MITI	[51]	1998	3D	
Molecule	USA, DRL	[35]	1998	3D	
Vertical	Japan, Riken, Meiji U.	[25]	1998	2D	
Telecube	USA, Xerox PARC	[77]	1998	3D	
CONRO	USA, USC, ISI	[90, 10]	1999	3D	
Crystalline	USA, Dartmouth College	[64]	1999	2D	
I-Cube (ICES Cubes)	USA, CMU, ICES	[83]	1999	3D	
Micro units	Japan, AIST	[95]	1999	2D	
M-TRAN	Japan, AIST	[52, 38]	1998	3D	
PolyBot	USA, Xerox PARC	[92]	2000	3D	
Proteo	USA, Xerox PARC	[94]	2000	3D	Only simulated so far
Pneumatic	Japan, TiTech	[27]	2002	2D	
SMC Rover	Japan, TiTech	[31]	2002	2D	Mobile
CHOBIE	Japan, TiTech	[28]	2003	2D	
Gear-Type Units	Japan, U. of the Ryukyus	[81]	2003	2D	
S-BOT	Switzerland, EPFL	[48]	2003	2D	Mobile
ATRON	Denmark, USD	[29]	2004	3D	
Molecube	USA, Cornell U.	[100, 99]	2004	3D	
Stochastic	USA, Cornell U.	[88]	2004	2D	Stochastic
Y1	Spain, UAM	[22]	2004	3D	Manually reconfigurable
Amoeba-I	Japan, Hokkaido U.	[41, 40]	2005	2D	Mobile
Catom	USA, CMU, Intel Research P.	[33, 32]	2005	2D	
HYDRON	CH, UZ; DK, USD; UK, U. Edin	[55]	2005	3D	Fluid scenario. No connections
Programmable Parts	USA, U. of Washington	[7]	2005	2D	Stochastic
Stochastic-3D	USA, Cornell U.	[87]	2005	3D	Stochastic
Superbot	USA, USC	[67, 72]	2005	3D	
YaMoR	Switzerland, EPFL	[47]	2005	2D	Manual assemble
CKBot	USA, Penn, FAMU, FSU	[68, 69]	2006	3D	
Giant Helium Catoms	USA, CMU	[30]	2006	3D	
GZ-I	DE, U. Hamb.; ES, UAM; CN, ZUT	[96]	2006	3D	Manual assemble
HitMSR	China, HIT	[97, 98]	2006	3D	
Miche	USA, MIT	[20]	2006	3D	Manual assemble

Robot	Country, Affiliation	Reference	Year	Dim.	Others
XBot	USA, Penn	[**89**]	2007	2D	
EM-Cube	Republic of Korea, DCS lab	[**6**]	2008	2D	
Octabot	Taiwan, NTU, HUST, TMUST	[**73**]	2008	2D	
Odin	Denmark, USD	[**42**]	2008	3D	Manual assemble
Roombots	Switzerland, EPFL	[**75, 76**]	2008	3D	
HexBot	American University of Sharjah	[**66**]	2009	2D	
Ubot	China, HIT	[**79**]	2009	3D	
CrossCube	USA, SIT; UK, U. Surrey	[**43**]	2010	3D	Only simulated so far
iMobot	USA, U. of California	[**65**]	2010	3D	Manual assemble
JHU	USA, JHU	[**39**]	2010	3D	No space filling 3D lattice
Pebbles	USA, MIT	[**19**]	2010	3D	Manual assemble
Rob. Stoch. Fluidic Assembly	USA, Cornell U.	[**53**]	2010	3D	Stochastic
Sambot	China, Beihang U.	[**85**]	2010	3D	
Cross-Ball	USA, SIT; UK, U. Surrey	[**44**]	2011	3D	Only simulated so far
ModRED	USA, UNL, UNO	[**14**]	2011	3D	
Milli-Motein	USA, MIT	[**34**]	2012	3D	Manual assemble
Smart Blocks	FR, UFC; DE, TU Ilmenau	[**46**]	2012	2D	
SMORES	AU, U. of South Wales; USA, Penn	[**15**]	2012	3D	
M-blocks	USA, MIT	[**62**]	2013	3D	
ReBiS	India, VNIT	[**80**]	2014	3D	Manual assemble

2. Analysis and conclusions

Compared to the two dimensional case, in three dimensions the mechatronic design and the motion restrictions are more complex. A unit like ATRON, that has only one degree on freedom, needs the help of another neighbouring unit to be moved. In general, mechanical design can be simpler at the expense of the reconfiguration plan and the other way around, so the challenge is to simplify both aspects at the same time.

In most of the cases, for a large number of units it is too hard to generate a deterministic motion plan for the units since we would have to consider an exponential number of configurations. Thus, plans can consider just local information.

Meta-modules are a widely used method that can eliminate the need for online trajectory planning. They can increase the mobility, reduce the dependence upon the help of neighbouring units and simplify the motion constraints. Thus, reconfiguration is simplified but the grain of the configurations is increased.

An exception is the pivoting cube model that can be applied to the M-blocks without needing meta-modules. It shows that a balance between mechanical and algorithmic constraints is possible. We consider that one of the major issues of self-reconfigurable modular robots is the convergence of theoretical models designed from the algorithmic perspective and mechanical designs.

Chapter 3
A new meta-module for efficient robot reconfiguration

The M-TRAN system was the first hybrid robot and it is one of the most interesting robotic systems developed so far. As we have seen in the previous chapter, other robots including the SuperBot, and the SMORES are able to simulate the mobility and connectivity of the M-TRAN units. It is therefore interesting to study the reconfiguration problem for the M-TRAN robot.

Expandable and contractible units allow tunnelling strategies. This capability can be exploited to reconfigure in linear time and only using the cells of the source and target configurations [**3, 60**]. Moreover, as the volume of a configuration grows there is proportionally less surface area per module. This is an impediment to parallelism in surface strategies, causing the reconfiguration speed to decrease as the number of modules increases.

The way to apply tunnelling algorithms to other kinds of units is constructing an expandable and contractible meta-module. This reduction was proven for Molecules by Kotay and Rus [**37**] and for the M-TRAN units (and also for the Molecube ones) by Aloupis et al [**2**].

In the remaining of this chapter we describe a more compact and robust meta-module that is able to expand and contract. It will also be proven that there is no need to use meta-meta-modules to be able to apply tunnelling algorithms. This was presented at the XVI Spanish Meeting on Computational Geometry [**58**].

1. Design and correctness of the meta-module

Our meta-module, illustrated in Figure 1, consists of 6 *arms*, aligned in three directions that are parallel to the x, y and z axes.

Each arm is implemented using a *2-unit chain*: two units attached at square flat faces and with the direction of their links aligned, as shown in Figure 2. The key property of 2-unit chains is that the rotation of the blocks within the units allows them to contract an expand, while preserving potential connections.
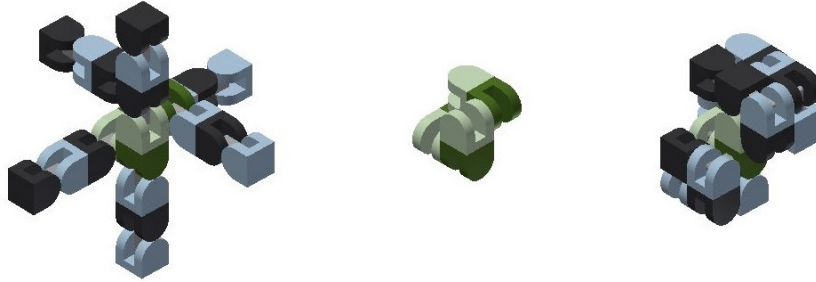
FIG. 1. Our M-TRAN meta-module. Left: all arms expanded. Center: central blocks. Right: all arms contracted.
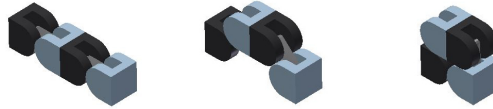
FIG. 2. The 2-unit chain is able to contract while maintaining the alignment and the orientation of its potential connections at both ends.

LEMMA 1. *The 2-unit chain can be contracted to half its length. During this operation its two extremal blocks stay aligned and keep their orientation.*

PROOF. The contraction operation is shown in Figure 2. Its realization is allowed by the two rotational degrees of freedom and the semi-cylindrical shape of the blocks. It is easy to see that this operation does not change neither alignment nor the orientation of the extremal blocks of the chain. □

The pairs of arms of the meta-module that are oriented in the same direction are connected to each other, resulting in a 4-unit chain whose blocks are all aligned. However, the linkages of the two connected arms differ in their orientation (see Figure 3 left). We call the blocks connecting the two arms *central*. The end blocks of a 4-unit chain are called *tips*.

Since the linkages of the two arms forming a 4-unit chain have different directions, their contraction and expansion movement takes place on two orthogonal planes, as illustrated in Figure 3 right.

The six arms of the meta-module form three 4-unit chains, one for each of the x, y and z directions, attached through their central blocks at their semicircular faces.

The meta-module can contract or expand each arm independently while keeping the six central blocks still. An important property of the meta-module is that the expansion or contraction of an arm can never interfere with another arm of the same meta-module.
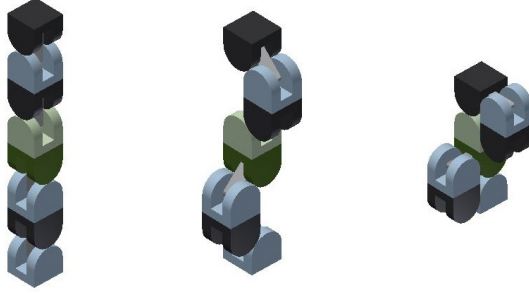
FIG. 3. Left: connecting two arms into a 4-unit chain. The central blocks are highlighted in green. Right: the compression movement.

LEMMA 2. *No self-intersection is produced when expanding or contracting any of the six arms of the meta-module.*

PROOF. Consider the minimum axis-aligned cube containing the expanded meta-module and decompose it into eight octants. It is easy to see that each expanded arm is contained in a different octant. The plane on which the contraction of an arm occurs always has a region in the corresponding octant, in such a way that when contracting an arm, the module can always use the octant that is exclusive to that arm. This guarantees that collisions cannot occur. □

LEMMA 3. *During the expansion and contraction of any subset of arms of a meta-module the structure remains connected.*

PROOF. While expanding and contracting any arm, the central blocks remain immobile. These six blocks maintain the meta-module connected at all times. Moreover, connectivity with neighbouring meta-modules is preserved: if the tip of an arm is attached to the tip of another meta-module arm, Lemma 1 guarantees that this attachment can be maintained during expansion and contraction. □

THEOREM 4. *The meta-module can perform the Crystalline and Telecube unit operations: expand, contract, attach and detach.*

PROOF. From the previous lemmas we conclude that, in any direction, the length of the meta-module can be reduced by half (when expanded arms are contracted) or doubled (when contracted arms are expanded) in any of the x, y and z directions. This can be done while preserving connectivity (Lemma 3) and avoiding collisions (Lemma 2). □

The meta-module we have presented uses 12 M-TRAN units. When expanded, its length is 8 units. Thus, the number of units is significantly reduced with respect to the 58-units meta-module presented in [2] and its size is scaled down to half. It is also very compact if compared with the 54-Molecule metamodule [37].

Furthermore, robustness is also improved over the previous meta-module: when contracted, our meta-module has only two corner joints per arm, as opposed to the four used in previous work, and leaves no gaps, making it much more compact.

2. Avoiding meta-meta-modules

By Theorem 4, we can apply the algorithms in [**63, 9, 84, 3**] for Crystalline and Telecube units to our meta-module. These algorithms, in turn, use meta-modules of Crystalline or Telecube units that are able to perform the scrunch/relax (Figure 4a) and the transfer (Figure 4b) operations.

a)   b)
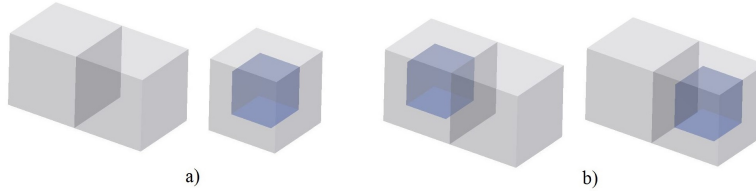
Fig. 4.   Crystalline and Telecube meta-module operations. a) Scrunch and Relax. b) Transfer.

In the previous section we showed that our meta-module is able to perform the Crystalline and Telecube unit operations. In this section we show that meta-meta-modules of M-TRAN units are not required since our meta-module is also able to simulate the scrunch/relax and transfer operations. This decreases the resolution of the configurations that we can handle, both in size and number of units.

Figure 5 illustrates two adjacent meta-modules before and after a scrunch/ relax operation. In a scrunch operation one of the meta-modules stays still, guaranteeing the connectivity of the overall structure. The other meta-module adopts a position that we call *canonical*, and has the following properties:

- The 4-unit chains of the moving meta-module are parallel to those of the still meta-module, and they are all connected at their central blocks.
- The symmetry of the resulting configuration allows to perform a relax operation on the moving meta-module to place it in any of the six adjacent lattice cells.

In a transfer operation two adjacent meta-modules stay still, while the other moves from the canonical position attached to one of the still meta-modules to the canonical position attached to the other.

The low density [**54**] of the configuration with two meta-modules in the same bounding box, as shown in Figure 5, allows performing the scrunch/relax and transfer operations. Their actual implementation is rather involved.
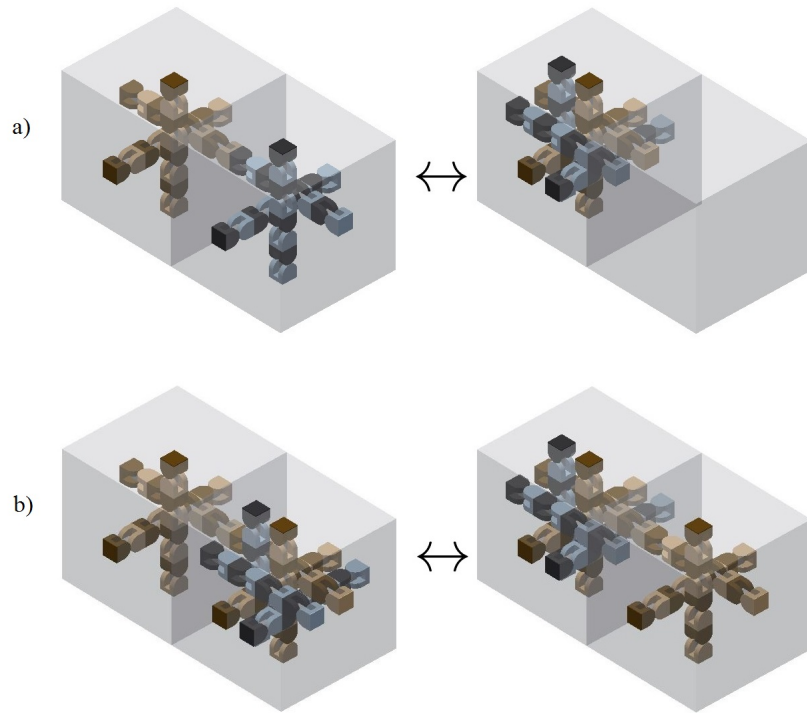
FIG. 5. a) The scrunch/relax operation. b) The transfer operation. Notice the canonical position of the blue meta-module in the bottom figure.

It comprises 56 independent moves of the six 2-unit chains of the moving meta-module for the scrunch operation and 68 for the transfer operation. This leads to the following result.

THEOREM 5. *The meta-module can perform the Crystalline and Telecube meta-module operations* scrunch/relax *and* transfer.

The details of the implementation are provided in the two following subsections. Geometric and visual proofs are presented for some steps whose feasibility might not be immediate. The view from the opposite perspective is also provided when needed.

2.1. Scrunch/Relax operation.

(0)

(1)

(2)

(3)

(4)

(5)

(6)

(7)

(8)

(9)　　　　　　　(10)　　　　　　　(11)

(12)　　　　　　　(13)　　　　　　　(14)

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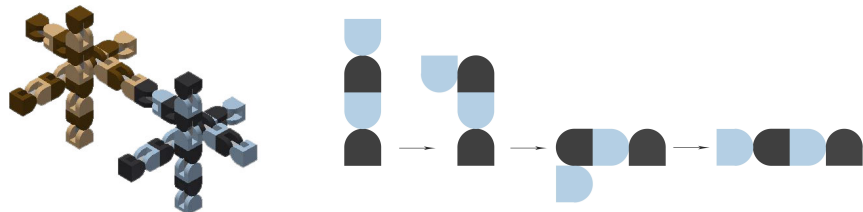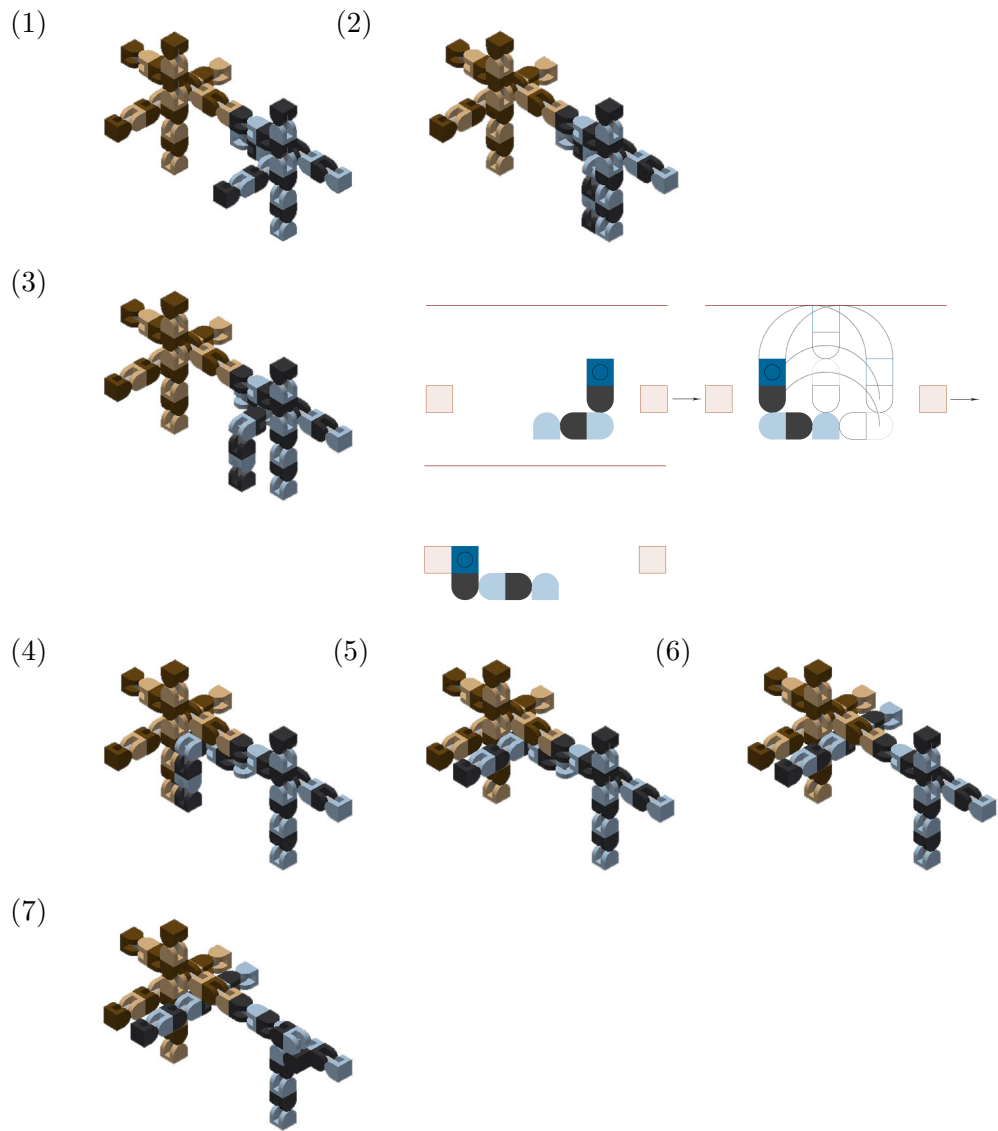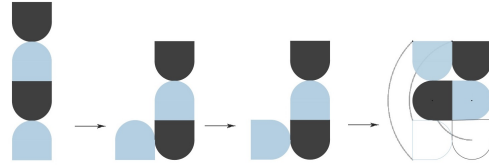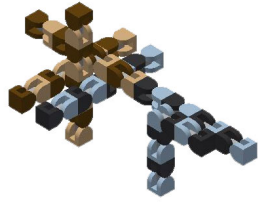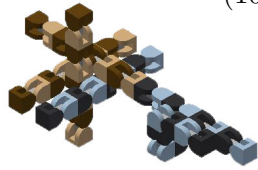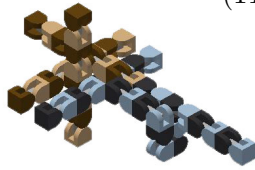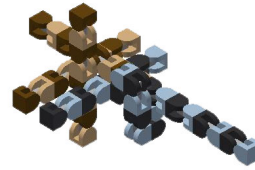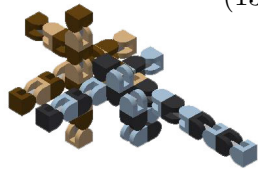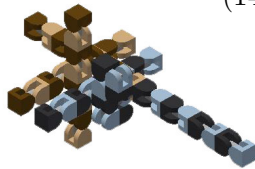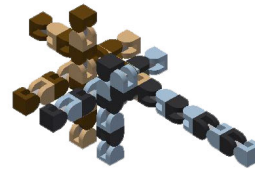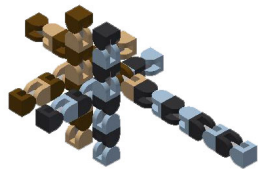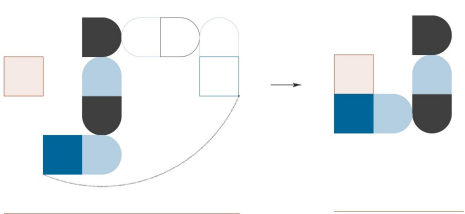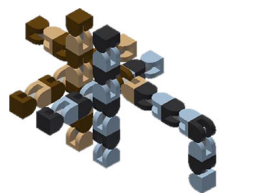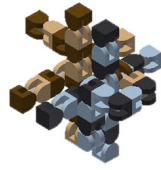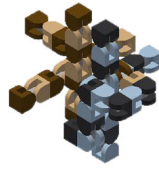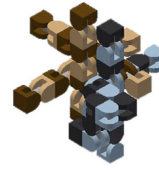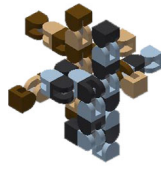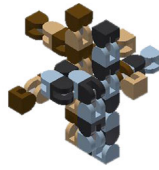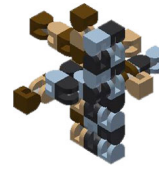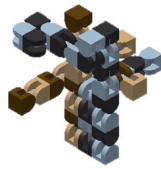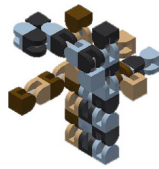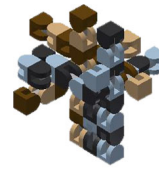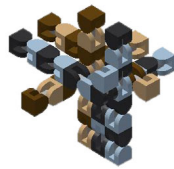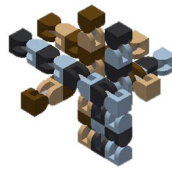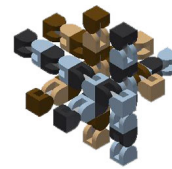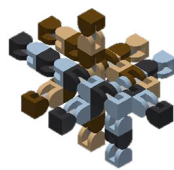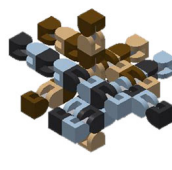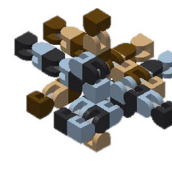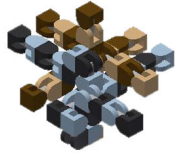
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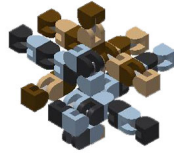
(34)
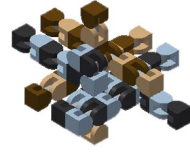
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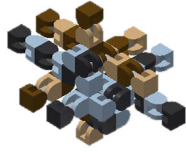
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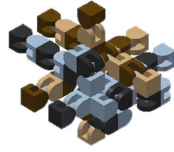
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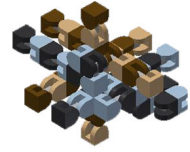
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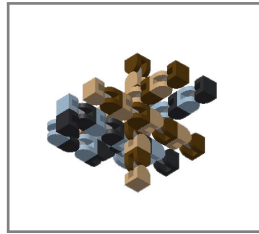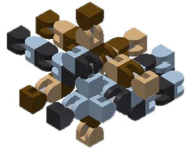
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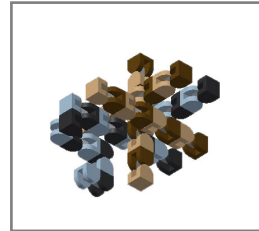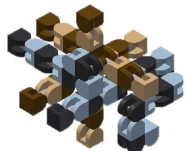
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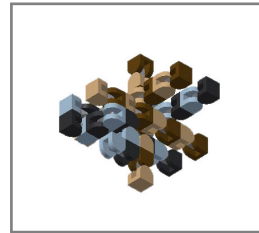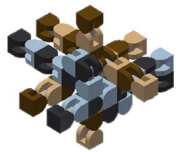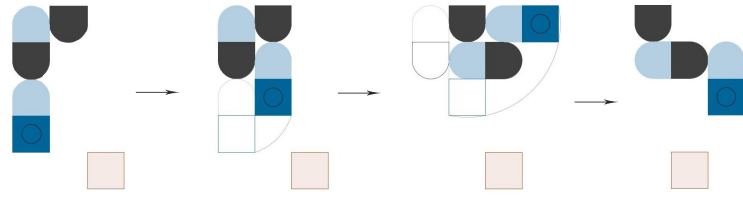
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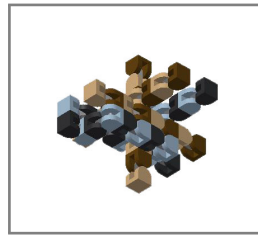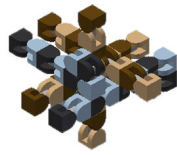
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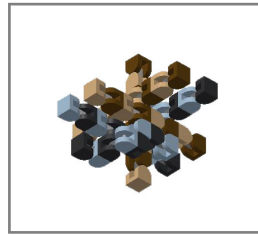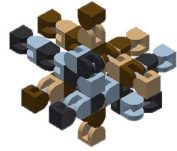
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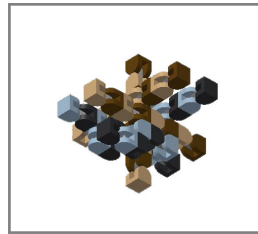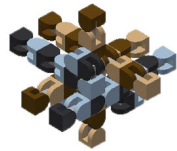
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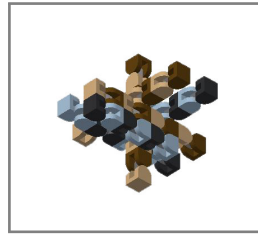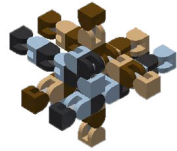
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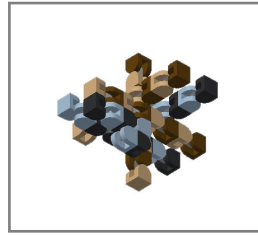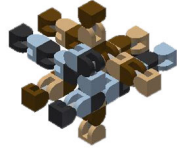
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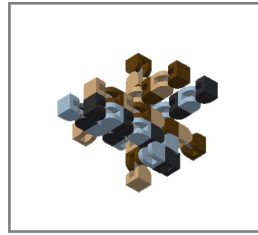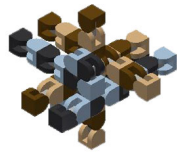
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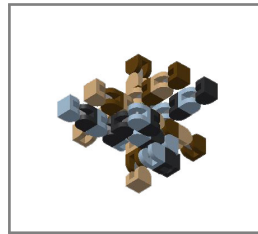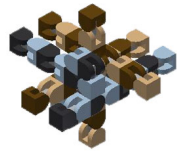
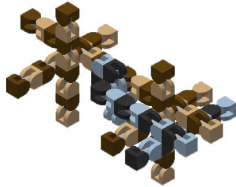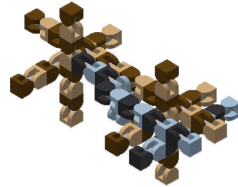(48)
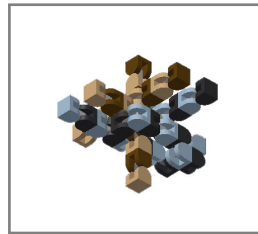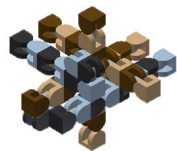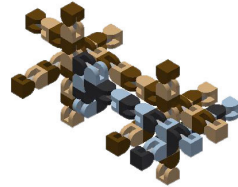

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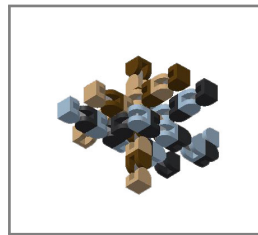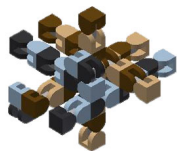
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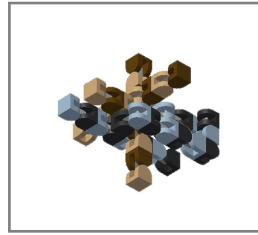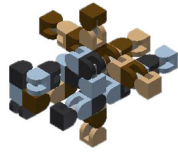
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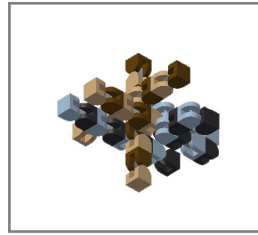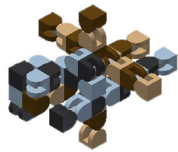
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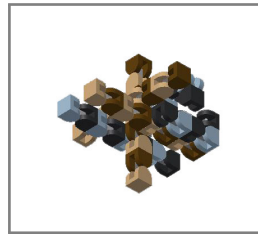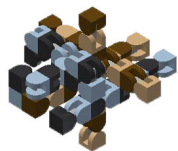
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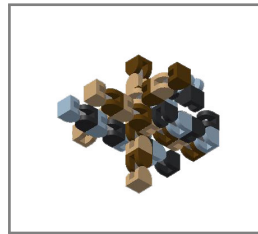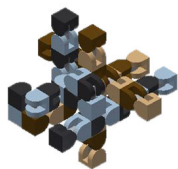
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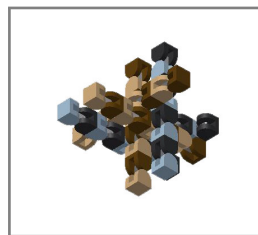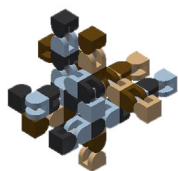
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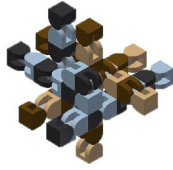
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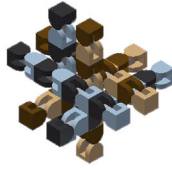
2.2. Transfer operation.

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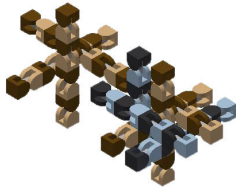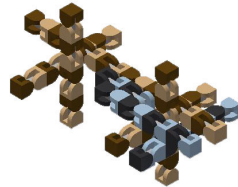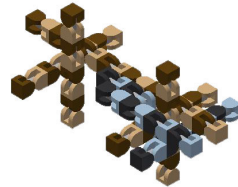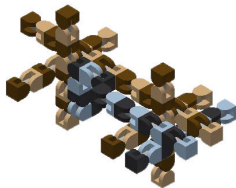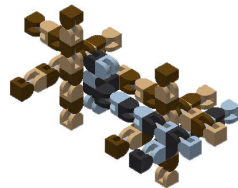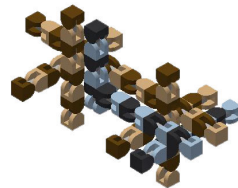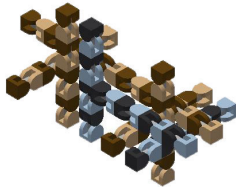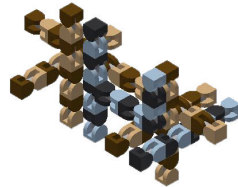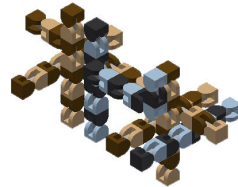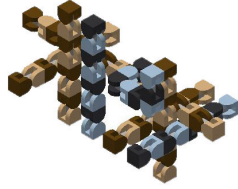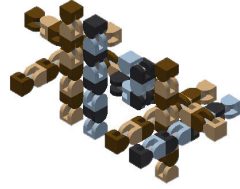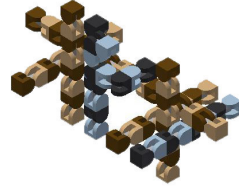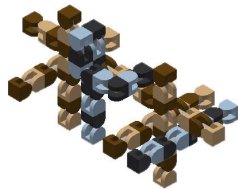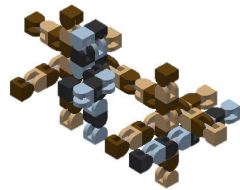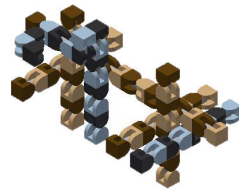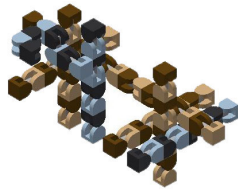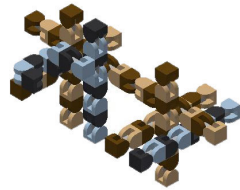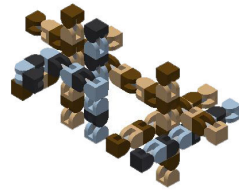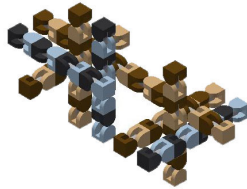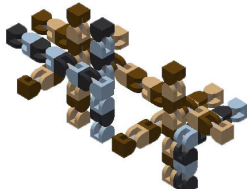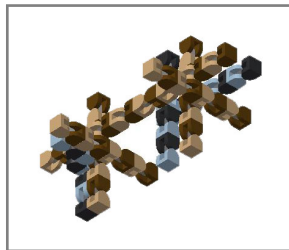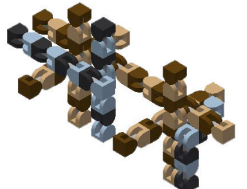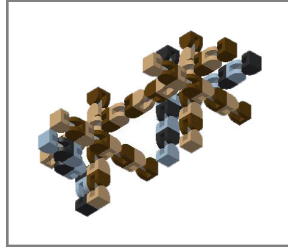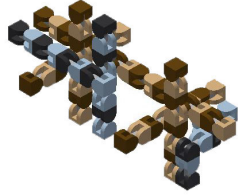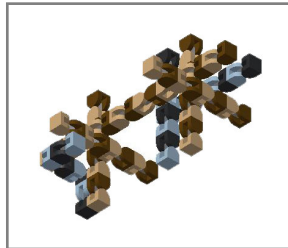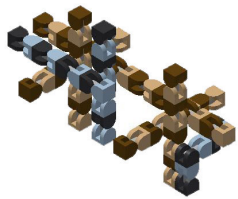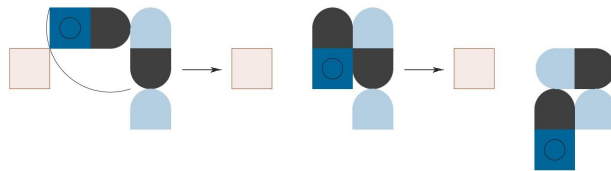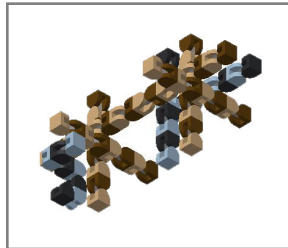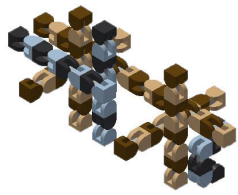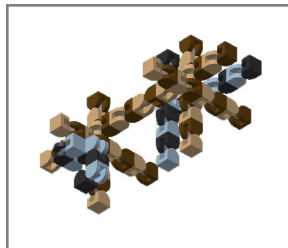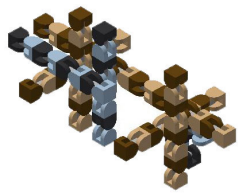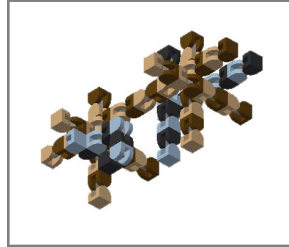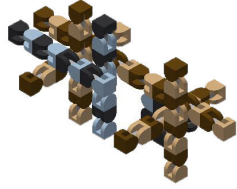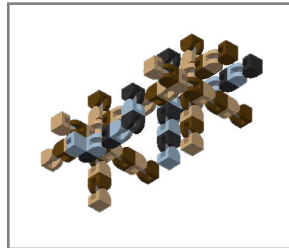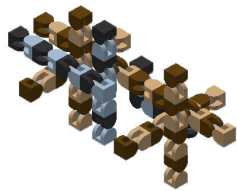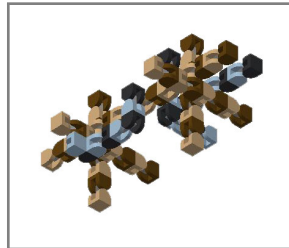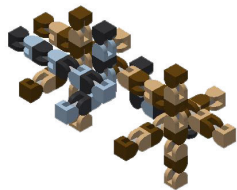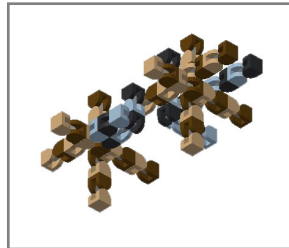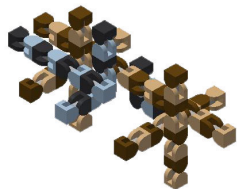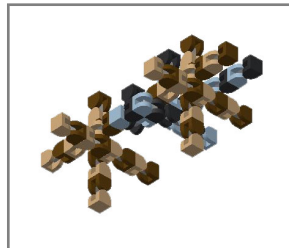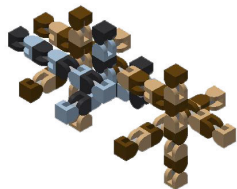
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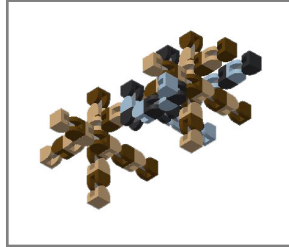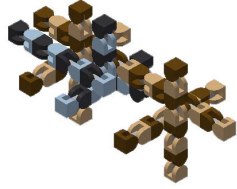
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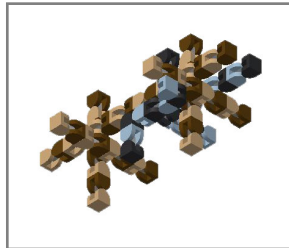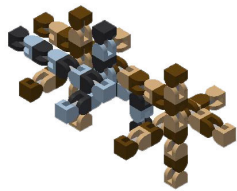
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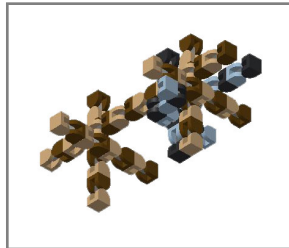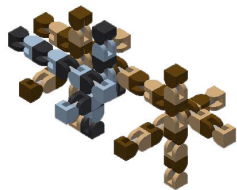
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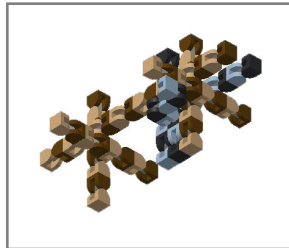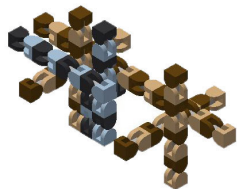
(37)
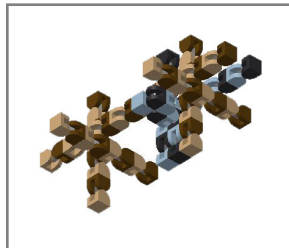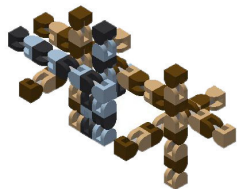
(38)
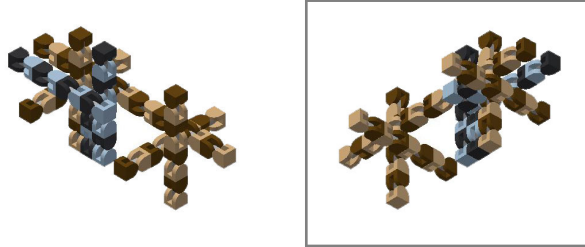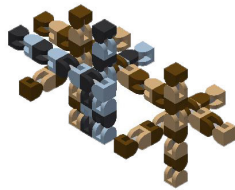
(39)   (40)
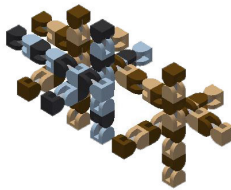
The configuration after step 40 is almost the same as after step 28 of the scrunch/relax operation of the meta-module. The only difference is the presence of an immobile meta-module in the adjacent cell of the moving (blue) one. Along steps 29-56 of the scrunch/relax operation the space occupied by the adjacent immobile meta-module is not used for reconfiguration, so steps 41-68 of the transfer operation are exactly the same.

3. Conclusions

In this section we have presented a new compact and robust meta-module that can simulate Telecubes. Moreover, it is the first time that it has been proved that the tunnelling algorithms, that have been shown to satisfy the most interesting properties, can be applied avoiding the use of meta-meta modules. We hope that similar reductions can be proved to other modular robotic systems other than M-TRAN, SuperBot and SMORES.

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