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MASTER THESIS WORK

TRANSFORMATION OPTICS IN COUPLED WAVEGUIDES

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Transformation optics in coupled waveguides

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Abstract. The technique of transformation optics is applied to a system of three coupled waveguides undergoing adiabatic passage of a light beam. We show that it is possible, by applying different transformations to different parts of the system, to modify at wish the geometry of the waveguides, keeping the efficiency of the process. The performance of this technique is demonstrated through 2D numerical simulations. All the analysis is performed for non-magnetic optical materials and for electromagnetic waves in the microwave region.

Keywords: Transformation optics, Adiabatic passage of light in waveguides

1. Introduction: What is transformation optics?

The idea that light propagation in an artificial medium can be controlled, at will, by controlling spatially the material properties has led to a large growth in the field of metamaterials and transformation optics [1]. Transformation optics (TO) is a mathematical technique which is used in designing such artificial media [2, 3]. The basic idea behind this technique is that any spatial coordinate transformation can be interpreted as appropriate changes in the material's optical parameters (i.e. the relative permittivity and permeability tensors). The underlying principle behind this technique is the invariant nature of Maxwell's equations under coordinate transformations [4, 5]. The relation between light propagation and effective space-time geometries was considered, for instance, in earlier papers by Tamm [6, 7]. While the concept of TO was known since long [8, 9], it was recently, and especially in the context of metamaterials, that this field has been re-established.
Metamaterials are arrangement of artificially structured elements designed to achieve unusual electromagnetic properties. Typical distances between constituent elements are much smaller than the wavelength of the incident light and then they can be described appropriately by Maxwell's equations. To date a large number of metamaterials have been investigated; one of the most famous one being the so-called invisibility cloak [1, 3], which has the ability of camouflaging any object placed inside it from electromagnetic (EM) waves at a certain range of frequencies [10, 11, 12]. Using TO, this can be achieved if a region of free space is transformed such that a point maps to a circle. Then the area inside the circle is completely inaccessible from outside, in the transformed space (see figure 1).
Ideally speaking, it is possible to guide EM waves in any arbitrary manner through cleverly shaping the space in the material through appropriate transformations. However, this unprecedented control comes at a cost; the electric permittivity and magnetic permeability tensors obtained for the transformed material are in general highly anisotropic and cover a wide

range (even negative) of magnitude [13]. Apart from the invisibility cloak, a number of devices have been demonstrated such as perfect lenses [14], beam collimators [15] or right-angle bends [15] to only cite a few.

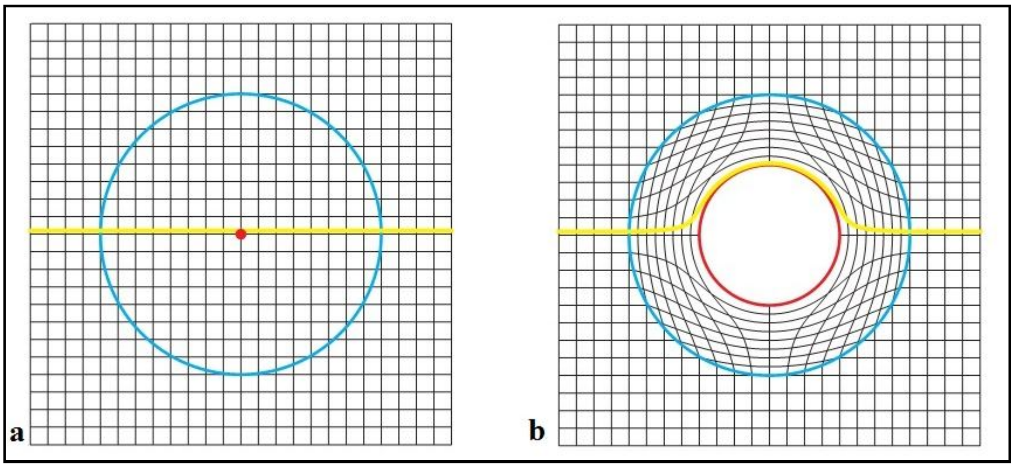

Figure 1: Illustration of the working principle behind the invisibility cloak. The central point (red) in (a) is mapped to a circle (red) in (b). A ray of light (yellow) propagating in a straight line in free space (a) will be guided smoothly around the 'hole' in the transformed space (b). Outside the blue circle the fields will remain the same. (From Ref. [16])

The main aim of this project is to apply the technique of TO to a system of three coupled waveguides undergoing adiabatic passage, to transform the original system of curved waveguides to a set of straight waveguides. The project is organised as follows. In section 2, we will explain the phenomenon of adiabatic passage in a system of coupled waveguides. Then, in section 3, we will introduce the main ideas of TO and we will apply them to transform the system of three curved waveguides in real space to a new geometry: a set of three straight waveguides. We will also characterize the phenomenon of adiabatic passage for both real and transformed space by means of 2D numerical simulations using a commercial finite-element method based software (COMSOL). Finally, in section 4, we will conclude.

2. Adiabatic passage in optical waveguides

We consider the system of three identical coupled waveguides depicted in figure 2. The geometry of this system is such that the coupling between each of the outer waveguides and the central waveguide changes along the propagation direction. The outer curved waveguides follow a parabolic profile, while the central waveguide is straight, with a little slope. The closest distance between the central waveguide and the outer waveguides is denoted by δ and occurs at points $P(x_1, y_1)$ and $P'(-x_1, -y_1)$. The distance between the points P and P' is X_0. The upper and lower parabolas are displaced in the y-direction by y_0 and $-y_0$ respectively.
The electric field in this system of single-mode weakly interacting coupled optical waveguides can be expressed as [17, 18]:

$$E(x, y, z) = \sum_{j=U,M,L} a_j(x)\, \vec{e}_j(y, z) exp(-i\beta x) \quad (1)$$

where, a_U, a_M, and a_L are the amplitude coefficients of the upper, middle and lower waveguides respectively, $\vec{e}_j(y,z)$ is the fundamental mode field of each waveguide and β is the propagation constant in the x- direction.

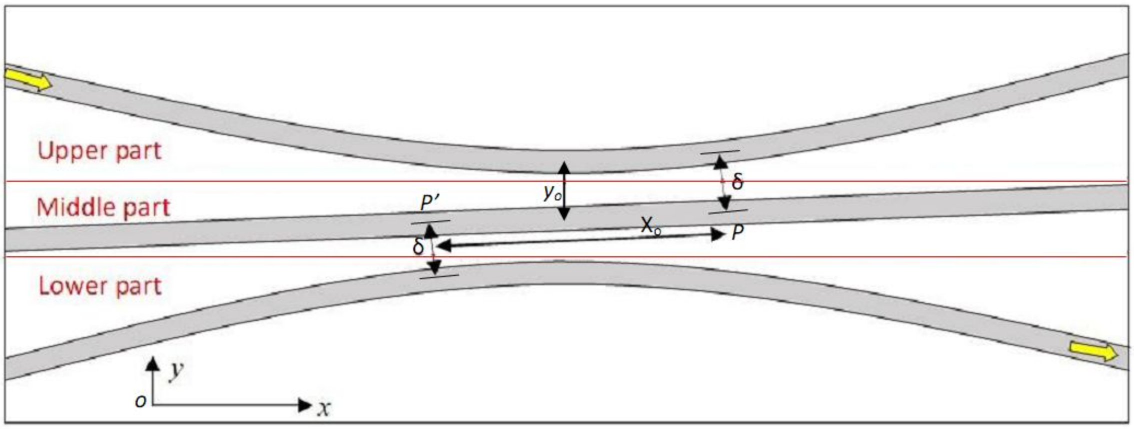

Figure 2: Schematic top view of a system of three coupled waveguides in which light can undergo adiabatic passage. The waveguides are depicted in gray, while the two red lines divide the structure into three parts. The closest distance between the central waveguide and the outer waveguides is denoted by δ, and occurs at $P(x_1, y_1)$ and $P'(-x_1, -y_1)$. The distance between P and P′ is X_0. The upper and lower parabolas are displaced in the *y*-direction by y_0 and $-y_0$ respectively.

It is assumed that since the couplings are through evanescent fields, and therefore weak, the two outer waveguides are not directly coupled to each other. Then, the coupled mode equations that give the evolution of the amplitude coefficients along the propagation direction, read [17, 18]:

$$i\frac{d}{dx}\begin{pmatrix} a_U \\ a_M \\ a_L \end{pmatrix} = \begin{pmatrix} 0 & \Omega_U & 0 \\ \Omega_U & 0 & \Omega_L \\ 0 & \Omega_L & 0 \end{pmatrix}\begin{pmatrix} a_U \\ a_M \\ a_L \end{pmatrix}. \tag{2}$$

Here, Ω_U and Ω_L are the coupling coefficients between upper-middle and lower-middle waveguides, respectively. The coupling coefficient is defined as $\Omega_i = \pi/(2L_c)$, where L_c is defined as the minimum distance required to have a maximum transfer of power between two parallel waveguides. From equations (1) and (2) we obtain the eigenmodes of the system. One of them is the so called ‘dark mode’, which involves only the outer waveguides and not the central one, in analogy with the dark-state of the STIRAP technique [19]:

$$D(\theta) = (a_U, a_M, a_L)^T = (cos\theta, 0, -sin\theta)^T \tag{3}$$

with,

$$tan\theta = \Omega_U / \Omega_L . \tag{4}$$

Thus, if light is launched into the upper waveguide and the couplings are set initially such that $\theta = 0$, then the initial state of the system is $D(\theta) = (1,0,0)^T$. Due to the geometry of the system of waveguides, the coupling coefficient between them changes along the propagation direction. If the couplings are varied such that θ goes from $\theta = 0$ to $\theta = 90^o$ adiabatically, then the state of the system follows the dark state from $D(\theta) = (1,0,0)^T$, corresponding to $\theta = 0$, to $D(\theta) = (0,0,1)^T$ corresponding to $\theta = 90^o$. In other words, the light injected in the upper waveguide is transferred completely to the lower waveguide with almost no intensity in the middle waveguide. The couplings between the waveguides follow a counterintuitive sequence along the propagation direction: initially the lower and central waveguides are approached; later on and with a certain overlap, upper and central waveguides are approached whereas lower and central ones are separated (see figure 2). The approach and separation of the waveguides must occur adiabatically in order to follow the eigenmode (3). Note that this is a very robust process, as shown in [17, 18], in the sense that it is quite insensitive to the fluctuations of the parameters as long as the adiabaticity condition $\sqrt{(\Omega_U{}^2 + \Omega_L{}^2)}\, X_0 > 10$ is fulfilled [18].

This process is analogous to the Stimulated Raman Adiabatic Passage technique [20], a well-known quantum mechanical phenomenon occurring in 3-level atomic systems in interaction with two laser pulses.

3. Transformation optics in coupled waveguides

From the theory of TO [4] we know that for a transformation from a real coordinate system $O(x, y, z)$ to a transformed coordinate system $O'(x', y', z')$ given by the mapping:

$$x' \rightarrow f(x, y, z) \tag{5a}$$
$$y' \rightarrow g(x, y, z) \tag{5b}$$
$$z' \rightarrow h(x, y, z) \tag{5c}$$

where *f(x, y, z)*, *g(x, y, z)* and *h(x, y, z)* are arbitrary functions, the values of the permittivity and permeability tensors in the transformed space are related to the original ones by [4]:

$$\epsilon'_{i',j'} = \left|\det(A_{i,j})\right|^{-1} A_{i,j} A^T{}_{i,j} \epsilon_{i,j} \quad and \quad \mu'_{i',j'} = \left|\det(A_{i,j})\right|^{-1} A_{i,j} A^T{}_{i,j} \mu_{i,j} \tag{6}$$

where, $i, j = x, y, z$ and $i', j' = x', y', z'$. Also, $\epsilon_{i,j}$ and $\mu_{i,j}$ ($\epsilon'_{i',j'}$ and $\mu'_{i',j'}$) are the permittivity and permeability tensors in the real (transformed) space, respectively, and the elements of the matrix *A* are:

$$A_{i,j} = di'/dj \quad . \tag{7}$$

In this work we will apply this general theory of TO to the specific case of three coupled waveguides in order to show how to engineer at wish the geometry of these waveguides without modifying the propagation of light throughout the system. In particular, we will transform the initial system of, in general, three curved waveguides into three straight waveguides. In the following subsection, 3.1, we will describe the real space and the adiabatic transfer in the waveguide system in real space. In the next subsection, 3.2, we will show how to transform the real space, and in subsection 3.3, the performance of the system in the transformed space.

3.1 Real Space

We begin with the system of three coupled waveguides with a curved geometry which allows adiabatic passage as described in section 2. The waveguide geometry is shown in figure 2. The system described is for EM radiation in the range of the microwaves, with free space wavelength λ_o=3 cm. The length of the waveguides in the x-direction is 90 cm. The three waveguides have the same width, equal to 2 cm. The centre of each waveguide, y_c, follows the dependences:

Upper waveguide: $y_c = ax^2 + y_0$ (8a)
Central waveguide: $y_c = bx$ (8b)
Lower waveguide: $y_c = -(ax^2 + y_0)$ (8c)

Therefore, the two extreme curved waveguides have a parabolic geometry and are symmetric to each other with respect to the x-axis. The central waveguide is straight, with a small positive slope. Each of the waveguides has a step refractive index profile with $\epsilon = 1.4$ and $\mu = 1$. The values for the substrate are $\epsilon = 1.1$, $\mu = 1$.
For optimized values of the parameters, a= 0.05, b= 0.27, y_0= 3 cm, light launched into the upper waveguide undergoes adiabatic transfer to the lower waveguide. The 2D numerical simulation for the power flow was performed using COMSOL. Figure 3 shows the top view of the power flow in the waveguide system (left) and the power transfer as calculated from boundary integration at the input and output (right). The efficiency of the system was 90%.

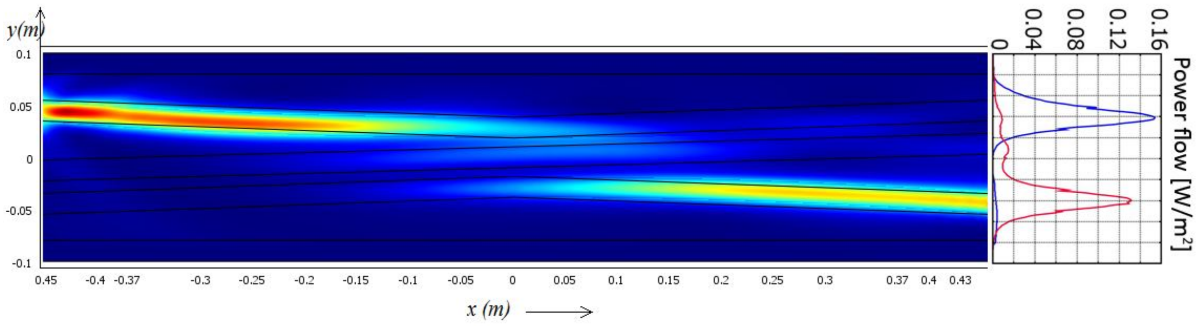

Figure 3: The power flow in the curved waveguides system in the real space. The EM beam launched in the upper waveguide undergoes adiabatic passage and is transferred to the lower waveguide with almost no intensity in the central one. The graph on the right compares the input power (blue) in the upper waveguide with the output power (red), mainly located at the position of the lower waveguide. The efficiency in this case was 90%.

3.2 Transformation

To transform the curved waveguides described above to a set of straight waveguides, we must modify the space in such a way that the extreme parabolic waveguides map to a straight waveguide while the central waveguide maps either to itself or to another straight waveguide with a different slope. We divide the coordinate space of the geometry into three parts – upper, middle and lower as shown in figure 2, and apply the following transformations by parts:

Upper part:

$$y' \rightarrow \left({a'}/{\sqrt{a}}\right)\sqrt{y-y_0} + y_0' \quad ; \; x \geq 0 \tag{9a}$$

$$y' \rightarrow -\left({a'}/{\sqrt{a}}\right)\sqrt{y-y_0} + y_0' \quad ; \; x < 0 \tag{9b}$$

$$x' \rightarrow x \tag{9c}$$

$$z' \rightarrow z \tag{9d}$$

Middle part:

$$y' \rightarrow \left({b'}/{b}\right) y \tag{10a}$$

$$x' \rightarrow x \tag{10b}$$

$$z' \rightarrow z \tag{10c}$$

Lower part:

$$y' \rightarrow \left({a'}/{\sqrt{a}}\right)\sqrt{-(y-y_0)} - y_0' \quad ; \; x \geq 0 \tag{11a}$$

$$y' \rightarrow -\left({a'}/{\sqrt{a}}\right)\sqrt{-(y-y_0)} - y_0' \quad ; \; x < 0 \tag{11b}$$

$$x' \rightarrow x \tag{11c}$$

$$z' \rightarrow z \tag{11d}$$

where, $a'(b')$ is the slope of the outer (central) waveguides in the transformed space, and y_o' is the distance between the outer waveguides and the central waveguide at $x' = 0$.
This results in a new geometry in the 'transformed space'. The three waveguides have been mapped to the following three straight waveguides:

Upper waveguide: $$y_c' = a'x + y_o' \tag{12a}$$
Middle waveguide: $$y_c' = b'x \tag{12b}$$
Lower waveguide: $$y_c' = a'x - y_o' \tag{12c}$$

It should be kept in mind that the real space is divided into three parts, with different transformations for each part. Therefore, for the method to work properly, the material parameters at the two boundaries (between upper and middle and between lower and middle parts) should be matched. Note that, the choice of the slopes for the upper/lower and the central waveguides affects the material parameters of the transformed space.
We have used the particular set of transformations given in equations (9), (10) and (11), because it gives zero values for the non-diagonal terms of the permittivity and permeability matrix (14). Thus, leading to more easily realizable materials and simplifying the matching of the material parameters at the boundaries.

3.3 Transformed space

The transformed space, obtained by equations (9), (10) and (11) is shown in figure 4. The centre of each of the waveguides, y_c' , in the transformed space follows the dependence given by equations (12 (a), (b) and (c)) with $a' = b'$ =0.06. Thus, the waveguides are parallel to each other with the separation between them equal to y_0' =3 cm. The width of the upper and lower waveguide is 1.4 cm and of the central waveguide is 0.5 cm. These compressions of the widths with respect to the original ones are a consequence of the transformations.

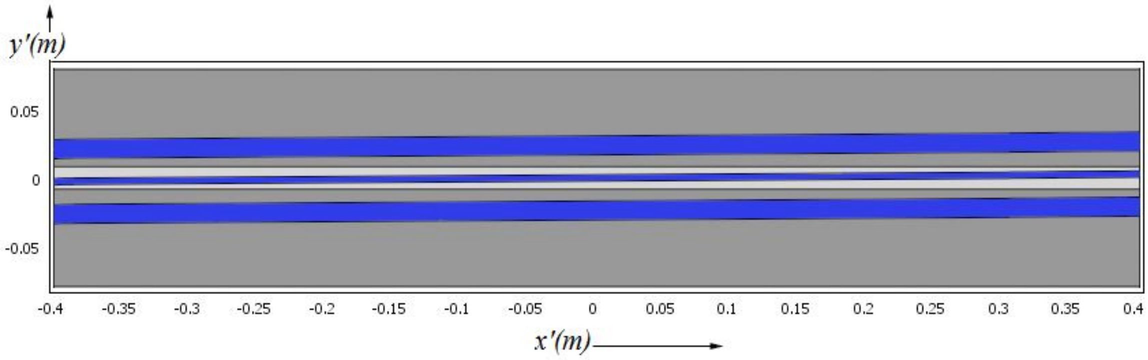

Figure 4: The geometry in the transformed space comprises a set of straight waveguides. The upper and lower parts are shown in dark-gray while the middle part is in light-gray. The waveguides (blue) have different thickness as a consequence of the transformation. The two boundaries between the different parts are horizontal, which ensures that the material parameters can be matched.

Using the transformations given in equations (9), (10) and (11) we can calculate, from equation (6), the material parameters for the three parts. The electric permittivity tensors thus obtained are:

$$\epsilon'_{upper} = \epsilon_r \begin{pmatrix} \frac{y'}{\rho} & 0 & 0 \\ 0 & \frac{\rho}{y'} & 0 \\ 0 & 0 & \frac{y'}{\rho} \end{pmatrix}, \quad \epsilon'_{middle} = \epsilon_r \begin{pmatrix} \frac{1}{\sigma} & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & \frac{1}{\sigma} \end{pmatrix}, \quad \epsilon'_{lower} = \epsilon_r \begin{pmatrix} \frac{-y'}{\rho} & 0 & 0 \\ 0 & \frac{-\rho}{y'} & 0 \\ 0 & 0 & \frac{-y'}{\rho} \end{pmatrix} \tag{14}$$

where, $\rho = (a'^2/2a)$ and $\sigma = (b'/b)$.

The magnetic permeability tensor remains, $\mu' = 1$ since we assume non-magnetic materials. As seen from equation (14), the electrical permittivity values for the upper and lower parts depend only along the y'-direction while in the middle part they are constants. It is therefore possible to match their values along a perfectly horizontal boundary (x-direction).

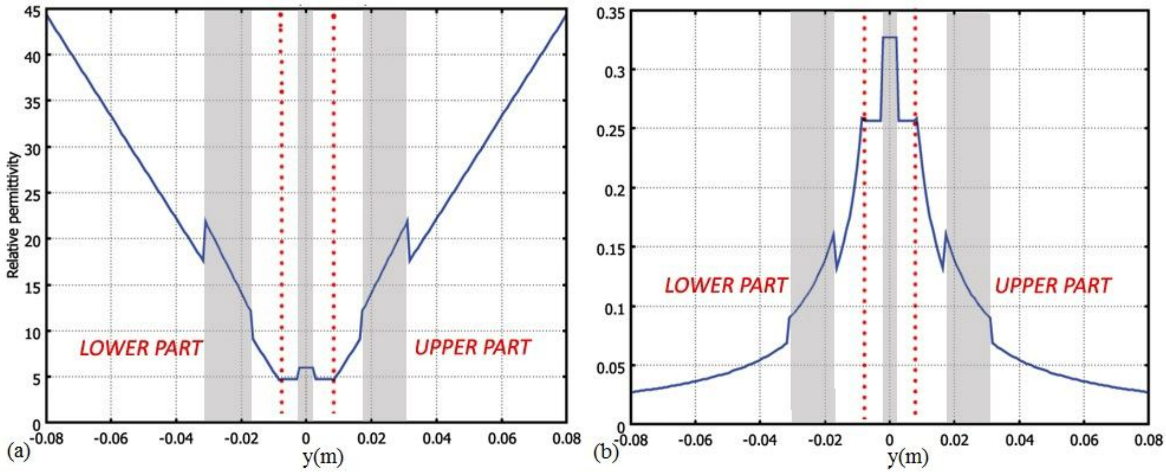

Figure 5: Variation of the electric permittivity in the y-direction (it is constant in the x-direction) in the transformed space. The variation of ϵ_{xx} is shown in (a) and of ϵ_{yy} in (b). The dotted red lines mark the boundary between the different parts. It can be seen that the boundaries are well matched. The limits of the waveguides are marked by the gray shaded regions.

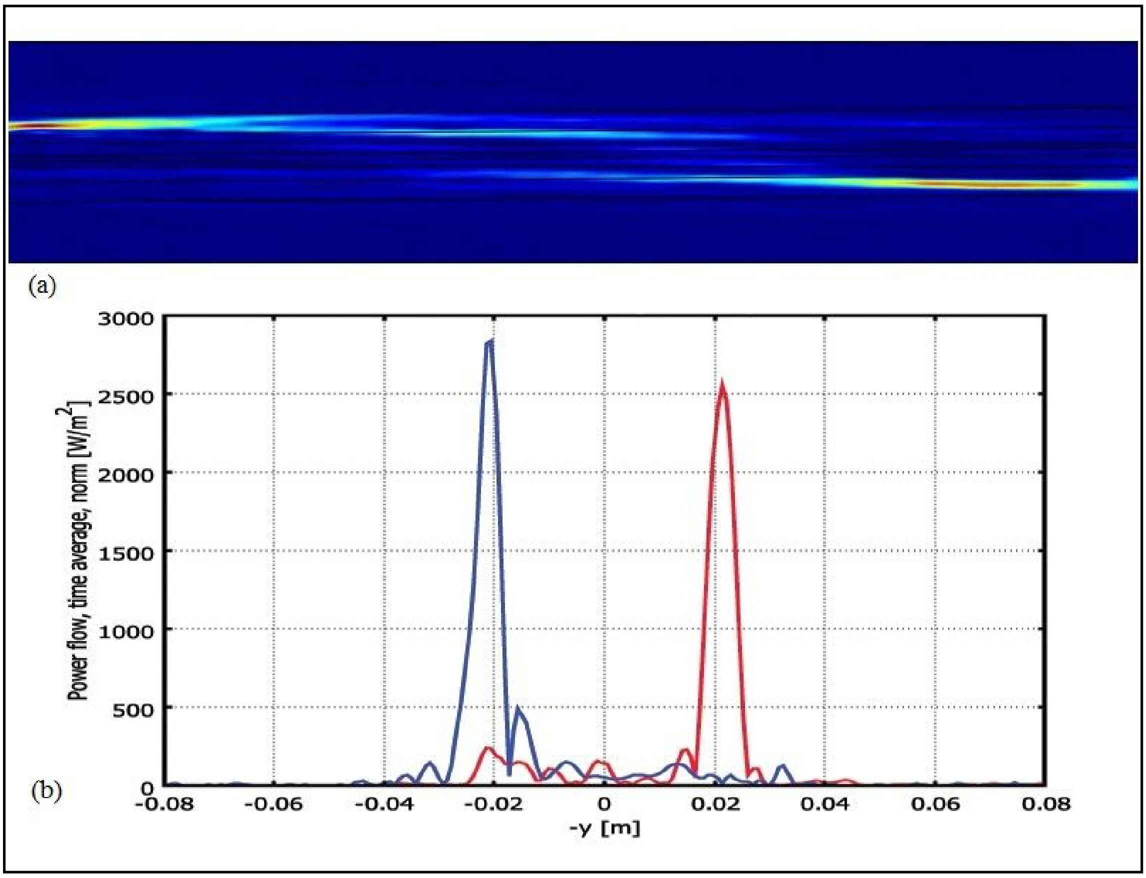

Figure 6: (a) Power flow in the transformed-waveguides. Light launched in the upper waveguide undergoes adiabatic passage and is almost totally transferred to the lower waveguide. (b) A comparison of the light input in the upper waveguide (blue) and the light output centered at the position of the lower waveguide (red). The efficiency of the passage in this case is 91%. Note that the horizontal axis denote the –y-direction, so the peak on the left (blue) is the power in the upper waveguide and the peak on the right is the power in the lower waveguide.

The plots for ϵ_{xx} and ϵ_{yy} are given in the figure 5 (a) and (b), respectively. In order to check the working of the transformed waveguide system, we performed 2D numerical simulations using COMSOL. Figure 6(a) shows the top view of the power flow in the transformed waveguide system, where the power input in the upper waveguide is transferred to the lower waveguide, with nearly no power in the central waveguide. As seen in the figure, the power transfer occurs near the central region (near $x' = 0$), which corresponds to the overlap region in the real-space, as should be expected. We calculated the power transfer through boundary integration at the input and the output boundaries, and the graph is shown in figure 6(b). The vertical axis in figure 6(b) gives the input power (blue curve) and the output power (red curve), while the horizontal axis denotes the y'-axis of the transformed space. Note that the horizontal axis is plotted in the negative; so that the left side denotes the upper part and the right side denotes the lower part. The calculated ratio for the power transfer is 91%. The power transfer here is in fact slightly better than the real-space, probably due to better coupling of the light input source to the upper waveguide.

4. Conclusion

In this study we have applied the technique of transformation optics to a coupled waveguide system undergoing adiabatic passage of an EM wave. We have transformed the system of curved waveguides in the real space, to a system of straight waveguides in the transformed space. We have used different transformations for the different parts of the overall structure and matched the material parameters at the boundaries. We have checked the performance of adiabatic passage in the transformed waveguide system through 2D numerical simulations using COMSOL. In the case of both, the real space and the transformed space, we found that the phenomenon of adiabatic passage was almost equally efficient. Transformation optics can be a very useful tool in creating novel structures or in situations where the control of the structure of the materials or the trajectory of light inside a medium is required. By using part-wise transformations, we may have much more flexibility in designing such structures.

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