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Fairness Adaptive Resource Allocation in OFDMA networks

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Abstract

This thesis work reviews contributions regarding dynamic resource allocation problems in Orthogonal Frequency Division Multiplexing (OFDM) systems, where various system metrics can be improved by periodically reassigning sub-carriers and transmit power to terminals depending on their current channel state. The following three classical problems have been reviewed: a) the sum rate maximization problem, b) the max min rate problem, and c) the sum rate maximization with rate proportionalities. System capacity is maximized in (a), by providing optimal spectral efficiency, but also poor system fairness index. In (b) and (c), fairness is very high but the capacity and spectral efficiency have been limited due to the fair policy; so the system capacity versus fairness trade off has been highlighted. The novel contribution of this thesis work is the formulation of a new problem which includes a system fairness target constraint enabling operators the ability to adjust fairness level. Operators, according to their needs, can get the most of spectral efficiency while providing a certain level of fairness among users. Several novel results regarding the new problem of system capacity maximization with a system fairness target constraint and various comparisons of different sub-optimal fairness-adaptive algorithm families are presented in this work. From the simulation results, including metrics such as system capacity, user fairness, user satisfaction and computational demand, it was possible to conclude about the most efficient fairness-adaptive approach from the perspective of both the user and the operator.

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1. Introduction and Motivation

Nowadays, the continuous growing need for higher data rates led to the development of *multiple transmition schemes* in order to overcome the major limiting factor in terms of performance; the frequency-selective nature of the wireless communication channels. With one single communication channel, frequency selective attenuation is decreasing performance and increasing the bit error rate (BER). With the well-known *Orthogonal Frequency Division Multiplexing* (OFDM), the frequency-selective problem has been solved. Instead of transmitting data in a serial way over the communication channel, by using parallelization, the channel is split into many sub-channels and the data transmition is done through these sub-channels in parallel. That way every sub-channel is a flat fading channel, means that experiences the same attenuation over its range, and performance increases greatly; therefore OFDM is an excellent system for high data rate wireless communications [1]. OFDM is adopted in some commercial systems as digital audio broadcasting (DAB), terrestrial digital video broadcasting (DVB-T), the IEEE 802.11a/g wireless local area network (WLAN), the IEEE 802.16, and also applied in digital subscriber line (DSL). A prominent example of this trend is represented by the OFDMA technology, which results from a combination of OFDM with a Frequency Division Multiple Access (FDMA) protocol [2]. About Beyond-3G (B3G) networks, OFDMA is also adopted in the Third Generation Partnership Project (3GPP) for Long Term Evolution (LTE), which supports up to 100Mbps peak rate for the downlink. [9]

Many unsolved problems are associated with the *radio resource allocation* (RRA), since many practical algorithms based on TDMA and OFDMA are static and that results spectral efficiency to be in significantly low levels. Since channels are varying over time, these schemes are not able to adapt in these changes and fail to efficiently use the resources. New generation algorithms are called *Dynamic Resource Allocation* (DRA) algorithms, since they dynamically redistribute the network resources according to the channel state and users' needs. In the following, the basic categories and problems associated with the resource allocation are presented, along with the motivation of this work.

1.1 Dynamic Resource Allocation (DRA) and Approaches

In the literature, most of the works follow either the *margin adaptive* (MA) approach, or the *rate adaptive* (RA) approach. The MA aims at the minimization of the total transmitted power, under rate requirement constraints of the users [4]. The rate adaptive aims at maximizing the total system's throughput, under a maximum transmitted power constraint [6-8]. The problems we are considering are rate adaptive sub-carrier and power allocation using optimization based on instantaneous user data rates, since rate tracking is performed in order to meet the users' rate requirements. The objective of this thesis is to study the trade-off between system *spectral efficiency* and *fairness* among the users when the presented RA algorithms are used. The problems presented in this thesis belong to *sum rate maximization* (SRM) problems, under a *system fairness target* (SFT) constraint.

The resources on a network problem are the sub-carriers/sub-channels and the power and the problem is the corresponding allocation. In most of the cases in the literature these two allocation procedures are split in order for the problem to be more simplified, but also joint approaches do exist. The sub-carrier allocation is associated with a matrix of correspondences between the sub-carriers and the users, or the *sub-carrier allocation indicator* $c_{k,n}$; note that each sub-carrier n should be assigned to at most one user k at a time. This indicator is zero if no connection is established for a given k-n pair, or one if it is. The power allocation defines how the power will be shared among the sub-carriers, and the problem lies in the specification of a power vector p that contain non-negative values that correspond to the allocated power for each sub-carrier. The objective function of most of the RA problems is rate maximization, and corresponds to the optimal selection of the resources in the sub-carrier and power allocation problems.

By using DRA algorithms, different approaches can be applied, based on the solutions provided for each maximization problem; either optimal ones, or suboptimal based on heuristics. Not all the problems have been solved, thus no optimal resource allocation has been found for every problem. In addition, when optimal solution has been found, it is not always feasible and usually is very complex, since an NP-hard non-linear problem has to be solved. Thus in terms of performance, the DRA algorithms strive to compete on their heuristics that are strictly-or-roughly approaching the mathematical optimum, when found.

1.2 Motivation

In the RRA problems there are many policies to choose from and also many parameters to adjust. In the following, the basic aspects are bulleted and commented afterwards.

- Operator: interested in both spectrum efficiency and a rough fairness level, usually not the maximum one.

- User: interested in receiving a particular data rate, no matter the circumstances, i.e. to have adequate fairness and high satisfaction index.

- *Spectral Efficiency versus Fairness Trade-off*

From the operator's point of view, high *spectral efficiency* and *capacity* are the main objectives, while for the users, *fairness* and *satisfaction* are more important. The operator's objectives are different from the users' ones, since capacity and fairness are reversely dependent metrics, thus, a capacity vs fairness trade-off appears. The main objective of this work is to analyze the aspects that vary this trade-off and conclude with a both efficient and balanced trade-off that satisfies both sides; operator and user.

Aspects of spectral efficiency, fairness and satisfaction of resource allocation have been well studied in economics, where utility functions are used to quantify the level of customers' satisfaction when they have been allocated certain resources by the system. In utility theory the optimization of a utility-pricing system is performed, which is established based on the mapping of some performance criteria (e.g. rate, delay) or resource usage (e.g. sub-carriers, power) into the corresponding pricing values [3], [4]. Thus the following questions come up:

- Which sub-carrier allocation algorithm should be chosen?

- Is equal power allocation sufficient or an adaptive one should be applied?

- Does it worth to use a more computationally complex algorithm and for what gain in performance?

There are many DRA algorithms with the same objective while differ in complexity; which algorithm should be chosen? In order to decrease complexity, usually the DRA problem is split into two procedures: a Sub-carrier and a Power Allocation. In sub-carrier allocation usually the complexity is linear, regarding the number of sub-carriers, and performance depends on the problem and the parameters chosen; thus simulations will answer which subcarrier allocation with which parameters fit better to our problem and assumptions. About Power Allocation, if an adaptive one is applied, usually is more complex than scheduling, therefore the following question arises: Does it worth to apply an adaptive power allocation? If the capacity gain is not important, then why should be applied such an algorithm? In contrast, what if a certain sub-carrier allocation along with the Equal Power Allocation, in which there is negligible complexity, performs similarly with greatly less computational burden?

In this work are examined some fairness-adaptive sub-carrier and power allocation algorithms using optimization based on instantaneous data rates. The objective of this work is first to study and second to balance the trade-off between system spectral efficiency and fairness among users when the previously mentioned RRA algorithms are used.

1.3 Chapters' Structure

In order to present in a concrete way the work in this thesis, the following structure has been chosen. The basic principles and characteristics of the OFDMA system are presented in Chapter 2, along with the system modelling, the propagation environment, and the link adaptation schemes. Also including the key terms of system fairness and capacity, which comprise the *Spectral Efficiency - Fairness* trade-off. In Chapter 3, the description of the classical Algorithms follows, in which the algorithms perform close to the extreme levels of fairness and capacity, along with the corresponding simulation results. Next in Chapter 4, in order to further investigate the spectral efficiency - fairness trade-off, *fairness-adaptive* algorithms are described and compared covering the whole range of this trade-off, and finally conclusions and perspectives are presented in Chapter 5.

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2. System Modelling

The system model, scenarios and the parameters of the simulator are presented here. This system modelling is conceptually based generally on OFDM and OFDMA systems, and also exposing details about the channel, the modulation schemes available and the traffic models. In the following some system assumptions and parameters are given:

- A single-cell scenario, with hexagonal cells is considered. Point-to-multi-point communication scheme while considering the downlink only. Interference due to adjacent cells is not considered.
- Transmission Time Interval (TTI) is the fundamental time unit of the simulator. Scheduling and power allocation is performed at each TTI.
- Mobility of the users is not considered. At the beginning of the simulations the users' positions are generated and they remain at this place until the end of the simulation. Though the sub-channels' gains are varying at each TTI because of the fast fading.
- Scheduling and power allocation algorithms are applied at each TTI, independently as separate modules.
- Full buffer traffic model is considered (users can transmit always when having a connection)
- The BS, at each TTI, has full information about channel conditions of all the users over all the sub-carriers. (i.e. perfect Channel State Information (CSI)).

2.1 The OFDMA System

OFDMA is based on parallelization; instead of transmitting data in a serial way over the communication channel, the channel is split into many sub-channels and the data transmition is done through these sub-channels in parallel. By decreasing the bandwidth of a sub-channel, the transmition time of a given amount of data is being increasing, and vice versa. The key feature of OFDM is that every sub-channel is a flat fading channel, means that it experiences the same attenuation over its frequency range. In contrast, with one single communication channel, frequency selective attenuation is decreasing performance and increasing the bit error rate (BER).

Since this thesis is not focusing on the physical layer aspects, an overview only of the key concepts about OFDM will be presented to emphasize the importance of some parameters which will be used through the entire chapter. Some of the information presented here is based on the thesis work of [5]. More details regarding OFDM system and its implementation can be found in [6].

Compared to conventional single-carrier systems, OFDM offers increased robustness against multi-path distortions as channel equalization can be easily performed in the frequency domain through a bank of one-tap multipliers. Furthermore, it provides larger flexibility by allowing independent selection of modulation and coding schemes over each sub-carrier. Due to these favourable characteristics, OFDM is already adopted in many commercial systems, such as DAB, DVB-T, in the IEEE 802.11a/g WLANs, in the IEEE 802.16, in DSL, and in the 3GPP-LTE. A prominent example of this trend is represented by the OFDMA technology, which results from a combination of OFDM with a Frequency Division Multiple Access (FDMA) protocol [7].

The main idea in OFDM is multicarrier modulation. The total available bandwidth B is divided into N sub-bands, each one with bandwidth $\Delta f = \frac{B}{N}$. In this way, instead of transmitting symbols in a serial way over the whole bandwidth at a given baud rate R, the data is converted into parallel streams with rates, over each sub-carrier, equal to $R_C = \frac{R}{N}$ and symbol duration equal to $T_C = \frac{1}{R_C}$. The resulting transmitted signal over the channel is given by:

$$x(t) = \sum_{m=-\infty}^{+\infty} \left(\sum_{n=0}^{N-1} (a_n \cdot p_n(t - m \cdot T_C)) \right) \tag{2.1}$$

where a_n is the data symbol modulating the n^{th} sub-carrier in the m^{th} signalling interval. Now, the choice of N is a fundamental task in designing the OFDM system. A reasonable range for N can be derived as:

$$\frac{B}{B_{coh}} \ll N \ll R \cdot T_{coh} \tag{2.2}$$

where B_{coh} and T_{coh} are the coherence bandwidth and the coherence time of the channel, respectively. This choice can be justified by the fact that the duration of an OFDM symbol should not be much smaller than the coherence time of the channel, yielding the detection of the symbol impossible. Finally, a set of orthogonal waveforms p_n should be considered in order to reduce the Inter-Channel Interference (ICI) due to the overlapping channels. A possible set is the following

$$p_n(t) = \begin{cases} \frac{1}{\sqrt{T_s}} e^{j\omega_n t} & , t \in [0, T_S] \\ 0 & , otherwise \end{cases}$$

where $\omega_n = \omega_0 + n \cdot \Delta f$, $n = 1,.., N$ is the carrier frequency of the n^{th} sub-carrier, and T_S is the duration of the modulated signal. Applying the definition of orthogonality for complex functions and taking into account condition (2.2), the following equation occurs, and guarantees orthogonality among different sub-carriers.

$$\int_0^{T_S} p_n(t) p_m^*(t) dt = \delta(n-m) \quad (2.3)$$

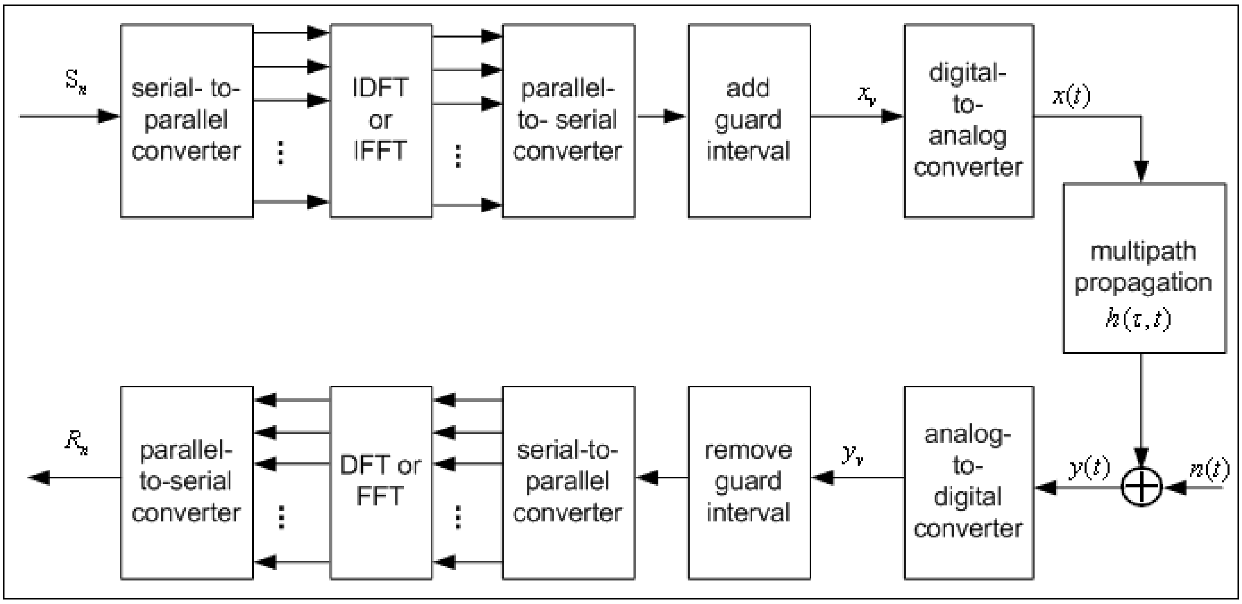

Figure 2.1: simplified OFDM scheme

In the OFDM system shown in the figure above, the incoming data stream is split by an S/P converter in N OFDM symbols. Then an IFFT operation is performed on each subcarrier. The resulting signals are then summed, and the baseband equivalent OFDM signal is transmitted over the channel, obtaining at the receiver the same signal affected by distortion and noise, both due to the channel. Finally, the receiver performs the inverse operations of the transmitter.

Main Advantages and Drawbacks of OFDM

In OFDM, high spectral efficiency can be achieved due to nearly rectangular frequency spectrum for high numbers of subcarriers. With a sufficient long guard interval, low complexity receivers can be used due to the avoidance of ISI and ICI. Different modulation and coding schemes can be used on individual sub-carriers which are adapted to the transmission conditions on each sub-carrier (link adaptation).

The main drawbacks of OFDM are the following. The use of the guard interval causes loss in spectral efficiency. Multi-carrier signals with high peak-to-average power ratio (PAPR) require high linear amplifiers and imperfections of the transmitter and receiver oscillators causes phase noise, so performance degradations occur and the out-of-band power will be enhanced. Its sensitivity to Doppler spreads is higher than single-carrier modulated systems, and more accurate synchronization in both time and frequency is required. More details can be found in [1].

Sources of Diversity

As briefly outlined in the Introduction, the adoption of an OFDMA-based system and the presence of multiple users in the cell, give to RRM the possibility to exploit different sources of diversity. In the following a more detailed explanation of these diversities is given.

Time. The time diversity is a consequence of the time-varying nature of the mobile radio channel. Since fast fading is calculated at each TTI, if we fix a sub-carrier it would experience different attenuations at different time instants.

Frequency. Also this diversity is a consequence of the fast fading process, since it varies also with frequency. Now, if we fix a time instant instead of a frequency, the sub-carriers, in the same TTI, would have different attenuations.

Multi-user. As several terminals are located in the cell (point-to-multipoint scenario), subcarriers are likely to have completely different attenuations for several users. In other words, the multi-user communication scenario is characterized by a spatial selectivity of the sub-carriers. The reason for the spatial selectivity is the fact that the fading process (as well as path loss and shadowing) is statistically independent for different terminals, as long as their receive antennas are separated by one wavelength [8].

2.2 Traffic Model

The simulator is supporting two types of services: Non Real Time (NRT) services and Real Time (RT) services. The first category includes several applications as World Wide Web (WWW), File Transfer Protocol (FTP) and e-mail, while the second is mainly characterized by Voice over IP (VoIP) [5]. In this work, only NRT services have been simulated, a traffic model based on WWW was considered, with a full buffer model.

Since we are considering a point-to-multi-point scenario in which the users receive data in downlink, the generation of traffic can be considered as done by an unknown transmitter at the application level. The data is then encapsulated by the IP protocol and transmitted to the BS. Once the packets arrive to the BS, then the downlink session starts.

The case we are considering is the full buffer model where the packet call size is set to infinity, which means that the user is downloading an ideal packet of infinite dimensions. At the end of the simulation, the size of the packet is not infinite of course, but equal to the amount of data that have been transmitted up to that moment when the session ends.

2.3 Propagation environment

The system model simulates path loss, shadowing and fast fading in order to approach realistic conditions for a mobile wireless system. A mobility model is absent, generating the users' initial positions, which do not vary for the whole simulation. A typical urban model is implemented as defined in [11].

The propagation environment is comprised of the following three factors:

1. *Distance dependent path loss attenuation* $L_k(d_k)$
2. *Slow fading* (or *shadowing*) G_k^{slow}
3. *Fast* (or *Rayleigh*) *fading* $G_{k,n}^{fast}$

The total path loss that the subcarrier n of user k is experiencing is the multiplication of the above factors in the linear, or the following summation in the logarithmic scale:

$$G_{k,n} = L_k(d_k) + G_k^{slow} + G_{k,n}^{fast} \quad \text{dB} \tag{2.4}$$

For calculating path loss attenuation $L_k \; d_k$, the single slope model [6] is adopted:

$$L_k(d_k) = 128.1 + 37.6 \log_{10}(d_k) \text{ dB} \tag{2.5}$$

where d_k (m) is the distance of user k from the BS.

Shadowing G_k^{slow} is a zero-mean, log-normal random variable with standard deviation σ .

Both factors $L_k(d_k)$ and G_k^{slow} are not changing during the simulation since the positions of the users (i.e. their distance from the BS) are generated at the beginning of the simulations and do not vary.

Fast fading $G_{k,n}^{fast}$ is implemented according to Jake's model [6], and power delay profile according to [11].

Fast fading varies with time and frequency; thus a user is experiencing different total channel gain in a particular subcarrier at each TTI.

2.4 Link adaptation

Most of the information presented in this section is based on the thesis work of [5], since it is continued by this work. The combination between the OFDM system in the physical layer and a medium access protocol in the MAC layer, yields to an OFDMA - based system. Assuming that K users share the same medium following a generic scheduling algorithm, and that N sub-carriers are available from the PHY layer, the set U containing the indexes of all the sub-carriers can be viewed as the union of the sets $U_1, \ldots, U_k, \ldots, U_K$, each set containing the indexes of the sub-carriers allocated to a specific user. For these sets the condition $U_i \cap U_j = \emptyset, \forall i \neq j$ holds. This equation constraint states that one sub-carrier or a set of sub-carriers can be assigned, by the MAC-layer protocol, to one user. If one sub-carrier is assigned to different users, transmission over that subcarrier would be impossible due to interference.

Now, given that the Base Station (BS), exploiting the multi-user diversity and based on a generic algorithm, has built the sets $U_1, \ldots, U_k, \ldots, U_K$, we want to determine suitable expressions for the achievable bit-rate by one user and the total bit-rate of the cell. Suppose that the n^{th} sub-carrier is assigned to the k^{th} user (i.e. $n \in U_k$), the achievable rate for this user in this sub-carrier would be a function of $SNR_{k,n}$, the signal-to-noise ratio of user k on sub-carrier n.

The received SNR of user k at subcarrier n is $SNR_{k,n} = \dfrac{P_n G_{k,n}^{tot}}{N_0 \dfrac{B}{N}}$ (2.6)

Where $SNR_{k,n}$ is in linear expression, N_0 is the power spectral density of the thermal noise and P_n is the transmitted power at subcarrier n.

Since the bandwidth of a sub-carrier is equal to $\Delta f = B/N$, the maximum bit-rate achievable by user k on sub-carrier n is given by the Shannon's formula:

$$r_{k,n} = \frac{B}{N}\log_2(1 + SNR_{k,n}) \tag{2.7}$$

which is also the theoretical upper bound for transmission capacity on a single sub-carrier.

The total rate that user k can achieve is given by the sum of the contributions of each sub-carrier that belongs to the set U_k (the subcarriers assigned to user k):

$$r_k = \sum_{n \in U_k} r_{k,n} \tag{2.8}$$

And the total rate of the system is the sum of r_k among all users:

$$R_{sys} = \sum_{k=1}^{K} r_k \tag{2.9}$$

Advantages and drawbacks of OFDMA are strictly connected to the ones of OFDM since it inherits from OFDM the robustness to ISI and channel distortions. Furthermore, orthogonality among the sub-carriers guarantees intrinsic protection against multiple access interference (MAI) [7].

In [10] a correction factor for SNR is introduced (known as *SNR gap*). The SNR gap is connected with a QoS requirement, the Bit Error Rate (BER), in the following way:

$$SNR^{gap} = -\frac{\ln(5 \cdot BER)}{1.5} \tag{2.10}$$

By applying the SNR gap, the new SNR is equal to $\frac{SNR_{k,n}}{SNR^{gap}}$, and equation (2.6) becomes

$$SNR_{k,n} = \frac{P_n G_{k,n}^{tot}}{N_0 \frac{B}{N} SNR^{gap}} \tag{2.11}$$

Where equations (2.7), (2.8), and (2.9) are updated and equation (2.12) gives the upper bound of the spectral efficiency in a realistic scenario:

$$S_{k,n} = \log_2(1 + \frac{SNR_{k,n}}{SNR^{gap}}) \quad (bits/s/Hz) \tag{2.12}$$

Thus, by setting the desired BER is possible first to determine SNR^{gap} through equation (2.10) and second to determine the maximum spectral efficiency that user k is able to achieve on sub-carrier n, by using equation (2.12).

Note that equation (2.12) gives us a continuous function of the *SNR,* but our objective is to reach a piece-wise constant curve that allows simple mapping between the *SNR* and a set of modulation schemes. A possible way to build such a curve is to fix a set of spectral efficiencies and associate a modulation scheme to each element of this set. For instance, if we fix the set of efficiencies $\{2,4,6\}$, we can associate, respectively, QPSK, 16-QAM and 64-QAM modulations. By manipulating Equation (2.12), we can determine the minimum required *SNR* necessary to reach the desired modulations. Furthermore, using Equation (2.7) we can point out the bit-rate associated to each modulation. In Table 2.1, the transmission rates corresponding to levels {2, 4, 6} are presented with the relatives values of *SNR* (in *dB), by considering* SNR^{gap}. The calculation is made using Δf = 15 *KHz.*

Table 2.1: Values of achievable rates and required SNRs

	QPSK	16-QAM	64-QAM
Rate(*Kbit/s)*	*30*	*60*	*90*
SNR	13.88	20.87	27.10

The piece-wise constant function that performs the mapping is called *link adaptation curve,* which is presented in Figure 2.2 in the case of efficiencies $\{2,4,6\}$. The theoretical curve of Equation (2.7) and the realistic curve with correction factor given by Equation (2.12) are also plotted to show a comparison.

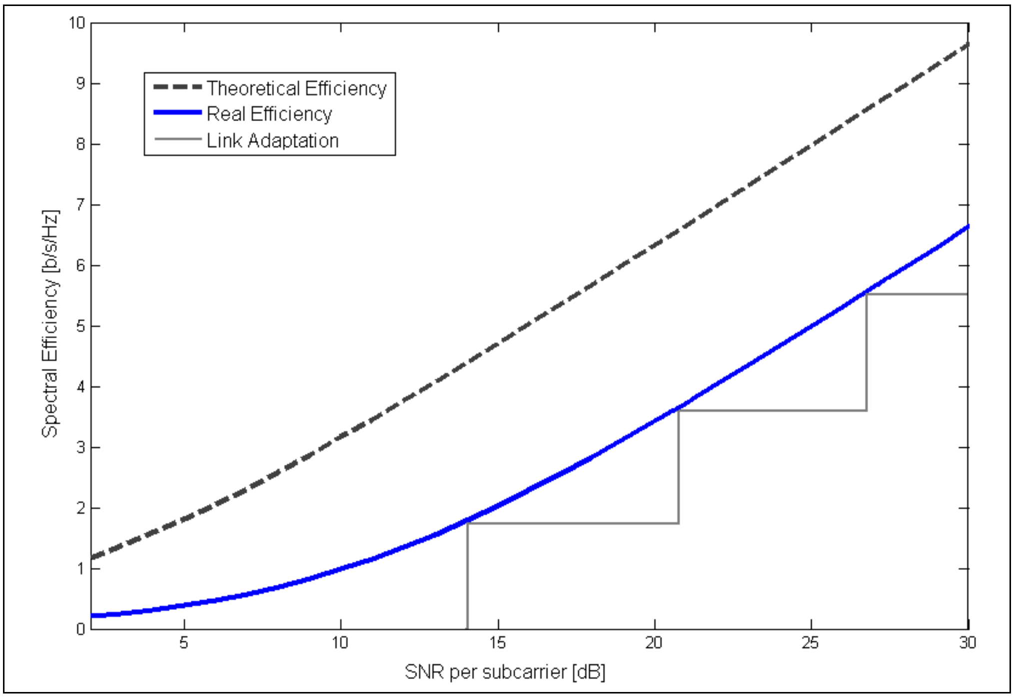

Figure 2.2: Link adaptation curve in the case of 3 efficiencies {2,4,6}.

In the simulations we use the real efficiency curve as depicted in the figure above, using the *continuous rate* mode. In the case of adapting the link adaptation curve as depicted in the figure above, in order to get discrete rate levels, then we should use the *discrete rate* mode.

2.5 Fairness and Satisfaction Metrics

In order to define *user fairness index* (UFI) based on *rate proportionalities*:

$$\varphi_k^{\gamma} = \frac{\tilde{R}_k}{\gamma_k} \tag{2.13}$$

Where $\tilde{R}_k$ is the normalized rate of user *k*, as defined in (2.14), γ_k is a constant value that indicates the desired proportion of the total system throughput that the user *k* receives. The rate proportion $\gamma_k \in [0,1]$, and φ_k^{γ} is the fairness index that shows how close the user from their required target is. The numerical value of UFI is always non negative, usually its maximum value is not significantly higher than 1. In order to better understand the proportion γ_k, consider the normalized rate of user *k* as:

$$\tilde{R}_k = R_k / \sum_{k=1}^{K} R_k \tag{2.14}$$

$\tilde{R}_k$ is also a proportion of the overall system rate assigned to user *k*, and $\sum_{k=1}^{K} \tilde{R}_k = 1$ holds as a result. Then the gamma proportion γ_k should be equal or very close to the respective normalized rate $\tilde{R}_k$ in order to achieve UFI $\varphi_k^{\gamma} \approx 1$. This way user *k* meets their rate proportional requirements and is considered as satisfied for that specific time, since we are considering instantaneous rates here.

The benefit of using this approach lies in the capability of applying different rate proportionalities among users, so will result them to get different rate proportions respectively.

Now, the definition of the *System Fairness Index (SFI)* (Jain based [16]) follows:

$$\Phi = \frac{\left(\sum_{k=1}^{K} \varphi_k\right)^2}{K \cdot \sum_{k=1}^{K} \varphi_k^2} \tag{2.15}$$

where φ_k is the UFI calculated according to equation (2.13). The system fairness index (SFI) is also called *instantaneous system fairness index*, since UFI φ_k's are calculated based on instantaneous user rates.

While the proportionalities have been achieved (i.e. $R_1 : R_2 : ... : R_K = \gamma_1 : \gamma_2 : ... : \gamma_K$), then the resulting status is the most fair for the users, given their proportional requirements, since the users' rates are proportional to their γ_k's. Because of the achieved proportionality, the UFI's are equal and this results the SFI $\Phi = 1$.

In order for the reader to become more familiar with the SFI and the way that it varies according to the user rates, the following Figure 2.3 depicts the possible values of SFI for a given range of rates that users can experience. For sake of simplicity, consider a simplified scenario where the system capacity is fixed to 128 kbps and all users are receiving the same SNR all the time. Assume that the system's capacity can be distributed to the users with not any possible combination for all users, but for the two out of three; means that the third users receives the same rate as the first for the sake of simplicity.

In the following Figure 2.3, x-axis is triple, where the rates correspond to user 1, user 2, and user 3 respectively. Note that every triplet of rates always sums up to 128 kbps, the total system capacity.

The lower extreme value of SFI is 1 / 3 = 0.333, since there are 3 users, and occurs only when one user takes all the capacity of 128 kbps and this is the unfairest combination. In contrast, the higher extreme (SFI = 1) occurs when all users are experiencing equal rates; this happens when all the 3 users receiving 128 / 3 = 42.66 kbps, and this is the fairest combination.

As it can be seen, for a given desired system fairness target (SFT), for example 0.8, it is obvious that 2 combinations of users rates appear. As the number of users and the possible discrete user rates increases, the possible combinations that results the desired SFT is increasing in a polynomial manner. For further information check Appendix B. Thus, to reverse the procedure and find the best rate combination given a desired SFT is a hard problem and might be computationally inefficient in a RRA procedure.

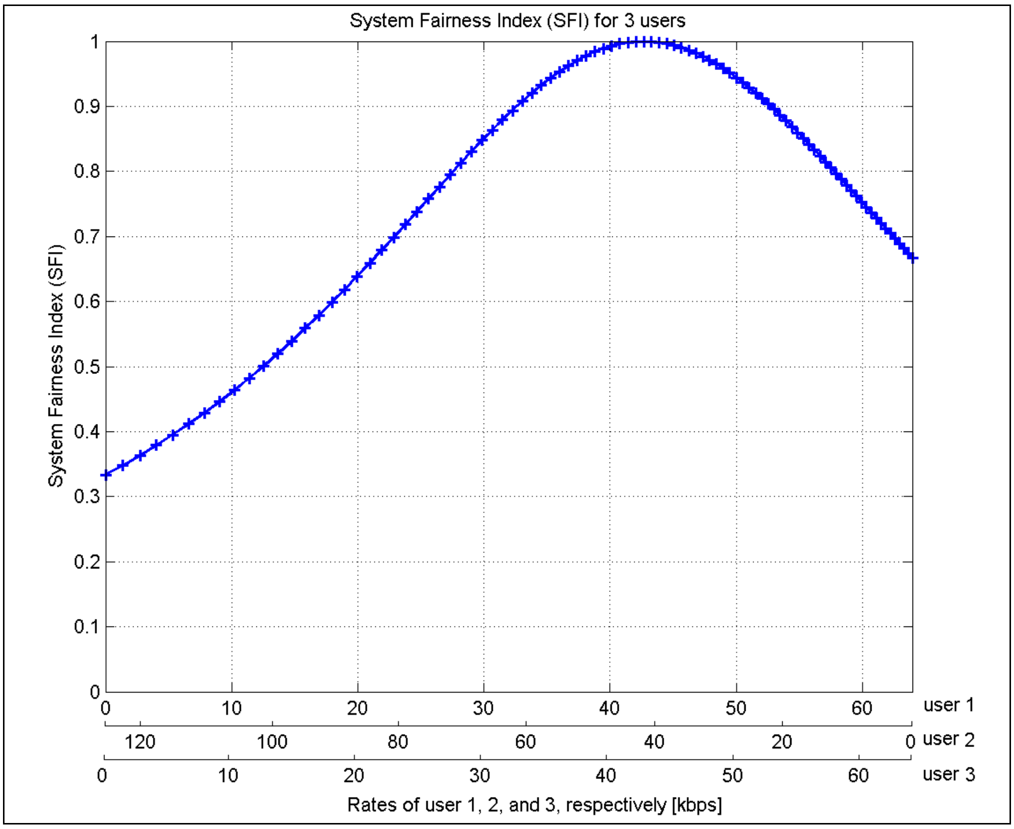

Figure 2.3: Simple example of System Fairness Index for 3 users receiving 128 kbps of system capacity

System fairness index properties

- $\Phi \in [1/K, 1]$, the lower extreme value $\Phi = 1/K$ occurs when only one user gets all the resources from the system, so all other users are silent. In the case that α users are active, out of *K* (*α≤K*), the maximum SFI value is $\Phi_{\alpha \le K} = \alpha/K$. The highest extreme value $\Phi = 1$ occurs when all users are transmitting and were given exactly the rate proportion that their requirements indicate. This results all UFIs $\varphi_k = \tilde{R}_k / \gamma_k = 1, \forall k \in K$, which indicates equal fairness for all users and results the maximum SFI.
- SFI indicates how much the *rate proportionality fairness* is achieved among users. While the rate proportionalities are satisfactory met, the SFI reaches its maximum value, which is 1.
- The value of Φ increases while increasing the uniformity of users' fairness indexes, and vice versa. Note that no matter the actual values of UFIs, while they are close enough, results high levels of SFI. And while UFIs are far enough, results low levels of SFI. A simple rule to increase the SFI is to adjust the UFIs such as to become more equal, while to decrease it, they should become more unequal.

Satisfaction

Satisfaction is a more hedonic [14] based metric, which ranks the service quality from the user's perspective; is what the user apprehends from the connection experience and services.

The i*nstantaneous User Satisfaction Index (USI)* is defined as:

$$s_k = \begin{cases} 1 & ,\text{if } R_k \ge R_k^{req} \\ 0 & , \ otherwise \end{cases} \tag{2.16}$$

where R_k^{req} is a given rate requirement for user *k*. This metric indicates whether the user received the required bits at each fundamental time unit, which is one TTI. A time window can be set in order to introduce the S*hort-Term User Satisfaction Index* (ST-USI), where the time window can be one or more TTIs.

Now the *User Satisfaction Ratio* (USR) is defined as the number of times that the users are satisfied ($s_k = 1$) in a time widow (one TTI or more), divided by the total number of these time windows that comprise the whole session. This metric is closer to the user perspective and indicates the percentage of the time that the service provided to the user is adequate enough, according to their requirements.

A complementary index to USR is the *User Dissatisfaction Ratio* (UDR) [15], which is basically the percentage of time that the users are receiving service inadequate with respect to their requirements ($s_k = 0$).

Last, the *Long Term User Satisfaction Index* (LT-USI) is based on the cumulative sum of the user's transmitted bits divided by the total time of the session (session throughput). The resulting rate should exceed the requirement R_k^{req} in order for the LT-USI to be 1, otherwise is 0.

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3. Classical Algorithms

In this chapter, only the classical algorithms are described, which were used as a basis in this thesis work. In the literature, most of the works follow either the margin adaptive (MA) approach, or the rate adaptive (RA) approach. The MA aims at the minimization of the total transmitted power, under rate requirement constraints of the users. The RA aims at maximizing the total system throughput, under a maximum transmitted power constraint. The problems we are considering are RA sub-carrier and power allocation using optimization based on instantaneous user data rates, since rate tracking is performed in order to meet the users' rate requirements. The objective of this thesis is to study the trade-off between system spectral efficiency and fairness among the users when the aforementioned RRM algorithms are used.

The different policies applied by each algorithm have a specific objective, such as maximizing the capacity or the fairness of the system. Simulation results from the classical algorithms are presented at the end of this chapter.

The two algorithms presented in sections 3.1 [1] and 3.3 [4], are the basic ones that constitute the two basic performance extremes in terms of system fairness and capacity. These two metrics are reverse-dependent, which means that maximizing one results the other to be minimized. The problem of radio resource allocation (RRA) is split in two procedures; first the *sub-carrier allocation algorithm* assigns sub-carriers to the users, and afterwards the *power allocation algorithm* is allocating which amount of power each sub-carrier is assigned.

Notation

In order to describe mathematically the problems that the algorithms are solving, the following notation will be used:

Table 3.1: Notations used

	Symbols
k	$:user\ index,\ k \in [1,2,..,K]$
K	$:total\ number\ of\ users$
n	$:subcarrier\ index,\ n \in [1,2,..,N]$
N	$:total\ number\ of\ subcarriers$
$p_{k,n}$	$:subcarriers'\ power$
$P_{\max}$	$:maximum\ BS\ transmitted\ power$
B	$:total\ bandwidth\ available$
$c_{k,n}$	$:subcarrier\ allocation\ indicator, c_{k,n} \in \{0,1\}$
$H_{k,n}$	$:effective\ subcarrier\ SNR$
$r_{k,n}$	$:subcarriers'\ rate\ w.r.t. user\ k$
R_k	$:users'\ rate\ vector$
γ_k	$:user\ rate\ proportionality\ constrains,\ \gamma_k \in [0,1]$
R_k^{req}	$:user\ rate\ requirements$
φ_k	$:user\ fairness\ index\ (UFI)$
Φ	$:system\ fairness\ index\ (SFI)$
	Bullets
$+$	$:advantages$
$-$	$:dissadvantages$
$\pm$	$:positive\ and\ /\ or\ negative$
o	$:neutral$

Notation notes:

1. $p_{k,n}, c_{k,n}$, and $r_{k,n}$ in the general case can be matrices with size KxN. Constraints are applied so that sub-carrier *n* is assigned to a maximum of one user *k*. The connection matrix $c_{k,n}$ shows whether a sub-carrier is assigned to user *k* or not, by having values 1 and 0 respectively. Thus each of $c_{k,n}$'s columns will sum up to 1.
2. $p_{k,n}$ in many cases inside the algorithms is used as vector p_n. Note that *k* user index is absent and the size of these vectors is 1xN. By using another vector, the *channel allocation* vector, which is the output of the *channel allocation algorithm,* we have the correspondences about which sub-carrier is assigned to which user, so the sub-carrier power vector p_n, along with the channel allocation vector contain all the information we need.
3. $H_{k,n}$ and $r_{k,n}$ can be found as H_n and r_n, respectively. What described in 2 applies in the same way. $H_{k,n}$ is the effective subcarrier SNR as defined in [4], including channel gains, noise power, and SNR gap.

3.1 Sum Rate Maximization (SRM)

This is the most common case in maximizing the sum rate capacity of the system. Jang et al in [1] solved the problem of maximizing the total aggregate system rate and proved that the maximum capacity can be achieved. The maximum capacity objective of the Sum Rate Maximization (SRM) algorithm is succeeded; however (as mentioned in the capacity vs fairness section) this algorithm provides the maximum capacity, at the cost of very low SFI, and also there is no guarantee that all users will be assigned with at least one sub-carrier. This problem is not considering the users' requirements; its objective is to maximize the total system capacity. The problem formulation is the following:

SRM problem formulation

$$\begin{aligned} &\max_{p,c} && \sum_k \sum_n c_{k,n} \cdot r_{k,n} && (o) \\ &\text{subject to} && p_{k,n} \geq 0 \quad \forall k,n && (c1) \\ & && \sum_k \sum_n p_{k,n} \leq P_{\max} && (c2) \\ & && c_{k,n} \in \{0,1\} \quad \forall k,n && (c3) \\ & && \sum_k c_{k,n} \leq 1 \quad \forall k,n && (c4) \end{aligned} \tag{3.1}$$

The objective of this policy is to maximize the total rate of the system; this is shown by the objective function (o) of this problem.

The constraints are showing the limitations of the problem:

(c1) limits the power assigned to every sub-carrier to be non negative

(c2) limits the total power assigned to all sub-carriers not to exceed the total transmitted power of the BS

(c3) indicates the connection between channels and users.

$c_{k,n} = 0$, indicates that sub-carrier *n,* is not assigned to user *k,* and

$c_{k,n} = 1$, indicates that sub-carrier *n,* is assigned to user *k.*

This constraint is responsible for the non-convexity of the problem, since its domain is integer. The reader may check Annex I for more details about convexity.

(c4) limits each given sub-carrier to be assigned to one user only. Each channel cannot be shared by many users at each given time.

The SRM sub-carrier allocation algorithm is initially considering *Equal Power Allocation* (EPA) across all channels. Its policy is to assign each sub-carrier to the user that experiences the best channel gain on that sub-carrier. Therefore, every channel is allocated to the best user for it, and results the total system rate to be absolutely the maximum, compared with any other sub-carrier allocation policy. The sub-carrier allocation algorithm is shown in the following Algorithm 3.1 (Jang).

All notations used are explained in the Table 3.1 at the beginning of this chapter.

The sub-carrier allocation algorithm pseudo code description is the following:

Initialization
    set $R_k = 0 \ \forall k$, $c_{k,n} = 0$ zero rates and connections
    $\forall k \in \{1, 2, .., K\}$, *and* $\forall n \in \{1, 2, .., N\}$
Sub-carrier assignment
    for $n = 1 : N$ find best user
      find *k* satisfying $H_{k,n} \geq H_{i,n}$, $\forall i \in [1, 2, ..K]$
      $c_{k,n} = 1$ set the connection
      $R_k = R_k + r_{k,n}$ update rate of user *k*
  end
end

Algorithm 3.1: SRM Sub-carrier Allocation (Jang) [1]

At the end of the sub-carrier allocation, however, some users may have been allocated no channels, so they remain without connection, perhaps for the whole session, especially when they are located quite far from the BS and are experience low channel gains. On the other hand, best users, are usually the ones that are located very close to the BS and receiving the biggest part of the resources, and their total rate is very high. The most usual case is that very few users are given sub-carriers, because they are experiencing the best channel gains, and all other users are remaining silent even for the whole transmitting session. As expected, the system fairness index (SFI) levels are very low, maybe close to the minimum limit of 1/K.

The power allocation is performing the well known waterfilling [1],[5],[6] among best channels of all users, which is pouring most of the power to the sub-carriers that are experiencing higher channel gain. So best sub-carriers are getting more power, resulting more rate to the users that have been assigned to. Therefore the policy applied here is getting more radical and best users are getting even more rate after the power allocation procedure, which is shown below:

> **Power Allocation**
> Sub-carrier allocation is known, thus
> assigned channel gains are known,
> perform **waterfilling** over all channels,
> according to [1] and [6]
> **end**

Algorithm 3.2: SRM Power Allocation - Waterfilling (Jang) [1]

Summarizing the features of the SRM algorithm

+ Maximum system throughput is achieved
+ Some users are assigned extremely high rates
– Very low system fairness index (SFI) levels
– Many users lacking connection

3.2 Maximize the Minimum Rate (MMR)

Max Min Rate (MMR) adaptive algorithm based on Rhee et al in [2] is maximizing the minimum rate of the users; tries to equalize their rates and provides high levels of SFI. The problem formulation follows:

Rhee's problem formulation

$$\begin{aligned}
&\max_{p,c}\min_{k} && \sum_{n} c_{k,n}\cdot r_{k,n} && (o) \\
&\text{subject to} && p_{k,n}\geq 0 \quad \forall k,n && (c1) \\
& && \sum_{k}\sum_{n} p_{k,n}\leq P_{\max} && (c2) \\
& && c_{k,n}\in\{0,1\} \quad \forall k,n && (c3) \\
& && \sum_{k} c_{k,n}\leq 1 \quad \forall k,n && (c4)
\end{aligned} \qquad (3.2)$$

The objective of this policy is to maximize the minimum user rate; this is indicated by the objective function (o) of the problem.

The constraints of the problem are the same as the previous problem (3.1). This problem is non-convex, since the domain of the 3rd constraint is integer. More information about convexity analysis can be found in Annex I. Authors in [2] found the optimal solution for a reformulated problem in which constraint c3 is relaxed. The sub-optimal solution solves the problem exactly as it is shown in (3.2). The suboptimal algorithm offers significant computational advantage while slightly degrades performance.

Note that this MMR problem (3.2) is a special case of the following problems (3.3) and (3.4), when all rate proportionalities are equal. In order to handle also unequal proportional rate requirements, we can slightly modify the problem and the algorithm described above as:

MMR problem formulation

$$\begin{aligned}
&\max_{p,c}\min_{k} && \sum_{n} (c_{k,n}\cdot r_{k,n})/\gamma_k && (o) \\
&\text{subject to} && p_{k,n}\geq 0 \quad \forall k,n && (c1) \\
& && \sum_{k}\sum_{n} p_{k,n}\leq P_{\max} && (c2) \\
& && c_{k,n}\in\{0,1\} \quad \forall k,n && (c3) \\
& && \sum_{k} c_{k,n}\leq 1 \quad \forall k,n && (c4) \\
& && R_i : R_j = \gamma_i : \gamma_j \ \forall i,j\in\{1,..,K\}, i\neq j && (c5)
\end{aligned} \qquad (3.3)$$

Note that the constraint (c5) is added, since the proportionalities are now taken into account, and the objective function (o) has been changed by dividing the rate of user *k* with their corresponding rate proportion γ_k.

The following suboptimal sub-carrier allocation algorithm (Algorithm 3.3) is divided in two steps. In the first step, the best available sub-carrier is assigned to each user, one for each user. In the second step, the maximization of the proportional minimum rate is performed by assigning the best available channel (from the remaining ones) to the user whose current proportional rate is the lowest of all. The second step is repeated until all channels are finally assigned to all users, resulting proportional rates to be quite equal, and the system fairness index (SFI) level to be extremely high (close to one).

All notations used are explained in the Table 3.1 at the beginning of this chapter.

The sub-carrier allocation algorithm pseudo code description is the following:

Initialization
    set $R_k = 0 \ \forall k$, $c_{k,n} = 0 \ \forall k, n$, $\Omega = \{1, 2, .., N\}$
        $k \in \{1, 2, .., K\}, and \ n \in \Omega$
Sub-carrier assignment
1. for $k = 1 : K$
      find n satisfying $H_{k,n} \geq H_{k,i}$, $\forall i \in \Omega$
      set $c_{k,n} = 1$ **,and update** $R_k = R_k + r_{k,n}$, $\Omega = \Omega - \{n\}$
    end
2. while $\Omega \neq \emptyset$
      find k satisfying $\frac{R_k}{\gamma_k} \leq \frac{R_i}{\gamma_i}$, $\forall i \in [1, 2, .., K]$
      for the found k, find n satisfying $H_{k,n} \geq H_{k,j}$, $\forall j \in \Omega$
      set $c_{k,n} = 1$ **,and update** $R_k = R_k + r_{k,n}$, $\Omega = \Omega - \{n\}$
    end
end

Algorithm 3.3: MMR Sub-carrier Allocation (Modified Rhee) [2]

Apart from the extremely high SFI that this algorithm provides, all users are given channels so that they have roughly the same proportional rates, according to their rate requirements. Therefore there are no users that do not have any connection, differently from the previous SRM algorithm.

About power allocation, authors claim that if any waterfilling solution is used, it is known that the total data throughput of a zero-margin system is close to the maximum capacity. This applies, even if flat transmit power spectral density is used, as long as the energy is poured only into sub-channels with good channel gains [7]. Therefore an Equal Power Allocation (EPA) among all channels would hardly reduce the data throughput of a multiuser OFDM system. The complexity of a power allocation algorithm is avoided by using an EPA. Thus the power distribution among sub-carriers is flat, which means equal amount of power assigned for each sub-carrier.

Summarizing the features of the MMR algorithm (Rhee) for the simple case where rate proportions γ_k are not considered

+ Approximately equal user fairness indexes (UFI) and extremely high SFI levels provided
+ No users lacking connection

± All users are assigned roughly equal rate

– Spectral efficiency has been limited due to the equal rates assigned

Summarizing the features of the MMR algorithm (Modified Rhee) for the general case where rate proportions γ_k are considered

+ Approximately equal user fairness indexes (UFI) and extremely high SFI levels provided
+ All users are assigned rates proportional to their rate requirements / constraints, so that proportionalities are met

– Spectral efficiency has been limited due to the proportionality constraints

3.3 Sum Rate Maximization with Proportional Rate Constraints (SRM-P)

This is the case that the proportional rate constraints $R_1 : R_2 : ... : R_K = \gamma_1 : \gamma_2 : ... : \gamma_K$ are taken into consideration by the original classical algorithm that is presented in the following. In practical systems, the proportionalities are used to differentiate various services, where the provider can give different priority depending on several service and billing policies. The work presented in this section is based on Shen et al [3], where the non-linear problem was first formulated and a suboptimal solution found. However the non-linear solution cannot always be found and requires the use of intelligent numerical methods in order to find a solution. Shen also presented the solution of two special cases: the linear case, when the proportionalities are integer quantities and high SNR case.

Initial problem formulation

$$\begin{aligned}
\max_{c_{k,n},\, p_{k,n}} \quad & \frac{B}{N}\sum_k \sum_n c_{k,n} \cdot \log_2(1 + p_{k,n} \cdot H_{k,n}) && (o) \\
\text{subject to} \quad & c_{k,n} \in \{0,1\} \quad \forall k,n && (c1) \\
& p_{k,n} \geq 0 \qquad \forall k,n && (c2) \\
& \sum_k c_{k,n} = 1 \quad \forall n && (c3) \\
& \sum_k \sum_n p_{k,n} \leq P_{\max} && (c4) \\
& R_i : R_j = \gamma_i : \gamma_j \ \forall i, j \in \{1,..,K\},\, i \neq j && (c5)
\end{aligned} \tag{3.4i}$$

According to [4], this is an NP-hard combinatorial optimization problem with non-linear constraints, and the computational complexity is such that it is highly improbable that polynomial time algorithms will be used to solve it optimally. The authors based on the assumptions of [9], made a simplification for the last constraint (c5) and introduced the predefined N_k, which is the number of channels that users will be allocated. This way they satisfy constraint (c5) as the transformed one $N_1 : N_2 : ... : N_K = \gamma_1 : \gamma_2 : ... : \gamma_K$, implying that the amount of rate that a user may require will be proportional to the number of sub-carriers they should be assigned. This way, after sub-carrier allocation, the objective (o) of the problem (3.4i) is simplified into a maximization over continuous power variables:

SRM-P problem formulation

$$\begin{aligned}
\max_{p_{k,n}} \quad & \frac{B}{N}\sum_k \sum_n \log_2(1 + p_{k,n} \cdot H_{k,n}) && (o) \\
\text{subject to} \quad & p_{k,n} \geq 0 \qquad \forall k,n && (c2) \\
& \sum_k \sum_n p_{k,n} \leq P_{\max} && (c4) \\
& R_i : R_j = \gamma_i : \gamma_j \ \forall i, j \in \{1,..,K\},\, i \neq j && (c5)
\end{aligned} \tag{3.4}$$

Note the absence of the sub-carrier allocation indicator $c_{k,n}$.

All notations used are explained in the Table 3.1 at the beginning of this chapter. The sub-carrier allocation algorithm pseudo code description is the following:

Number of sub-carriers per user and Initialization

a. $N_k = \lfloor \gamma_k \cdot N_{tot} \rfloor$, $N_{res} = N_{tot} - \sum_k N_k$

$R_k = 0 \ \forall k$, $c_{k,n} = 0 \ \forall k,n$, $p = P_{max}/N$,

$\Omega = \{1,2,..,N\}$, $\Omega^* = \{1,2,..,N\}$, $k \in \{1,2,..,K\}$, *and* $n \in \Omega$

sort $H_{k,n}$ **in ascending order**

Sub-carrier assignment

b. for $k = 1 : K$ — for each user k

- $n = \arg\max_{n \in \Omega^*} |H_{k,n}|$ — find best available channel
- $c_{k,n} = 1$ — set the connection
- $R_k = R_k + \frac{B}{N}\log_2(1 + p \cdot H_{k,n})$ — update user rates
- $N_k = N_k - 1,\ \Omega^* = \Omega^* - \{n\}$ — exclude given channel n

end

$Z = \{1,2,..,K\}$ — enable all users

c. while $\|\Omega^*\| > N_{res}$

- $k = \arg\min_{k \in Z}(\frac{R_k}{\gamma_k})$ — find min prop rate user
- $n = \arg\max_{n \in \Omega^*} |H_{k,n}|$ — find best available channel
- **If** $N_k > 0$ — if they should be given any channel
  - $c_{k,n} = 1$ — set the connection
  - $R_k = R_k + \frac{B}{N}\log_2(1 + p \cdot H_{k,n})$ — update user rates
  - $N_k = N_k - 1,\ \Omega^* = \Omega^* - \{n\}$ — exclude given channel
- **else**
  - $Z = Z - \{k\}$ — exclude that user
- **end**

end

$Z = \{1,2,..,K\}$ — enable all users

d. for $n = 1\, to\, N_{res}$ — for each remaining channel

- $k = \arg\max_{k \in Z}(H_{k,n})$ — find the user with max channel gain
- $c_{k,n} = 1$ — set the connection
- $R_k = R_k + \frac{B}{N}\log_2(1 + p \cdot H_{k,n})$ — update user rates
- $\Omega^* = \Omega^* - \{n\}$ — exclude channel n

end

end

Algorithm 3.4: SRM-P Sub-carrier Allocation (Wong) [4]

In the sub-carrier algorithm R_k keeps track of each user's capacity. Steps a-d are very interesting as a policy for handling performance in terms of maximizing throughput (steps b, d) and in terms of increasing fairness (step a, c).

step a: Determine the number of sub-carriers that each user will be assigned. The sum of these channels may be less than the total number of channels available; the remaining channels are assigned at step d

step b: Assign to each user one best channel. An inherent advantage is gained, since the users can choose their best channel in this step

step c: Track users with minimum proportional rate and assign them the best channel available. The greedy policy of assigning resources to the user that needs a sub-carrier the most is performed here; they also are able to choose the best channel available.

step d: For the remaining channels, assign each of them to the user that gains the most. This step is performed in order to allow users with the best channel gains to get the rest of the channels for maximizing the system throughput.

As a consequence of this sub-carrier allocation scheme $N_1 : N_2 : \ldots : N_K \approx \gamma_1 : \gamma_2 : \ldots : \gamma_K$, this policy achieves approximated rate proportionality fairness while increasing overall capacity. While $N \to \infty$ and $N >> K$, the approximation above is getting tighter, and this assumption appears reasonable, since contemporary OFDMA wireless systems satisfy these conditions.

The power allocation algorithm formulas used are the following:

V,W parameters calculation

$$V_k = \sum_{n=2}^{N_k} \frac{H_{k,n} - H_{k,1}}{H_{k,n} H_{k,1}}$$

$$W_k = \left(\prod_{n=2}^{N_k} \frac{H_{k,n}}{H_{k,1}} \right)^{\frac{1}{N_k}}$$

a,b parameters calculation

$$a_k = -\frac{N_1}{N_k} \frac{H_{k,1} W_k}{H_{1,1} W_1}$$

$$b_k = \frac{N_1}{H_{1,1} W_1} \left(W_k - W_1 + \frac{H_{1,1} V_1 W_1}{N_1} - \frac{H_{k,1} V_k W_k}{N_k} \right)$$

Individual powers (per user) calculation

$$P_1 = \left(P_{\max} - \sum_{k=2}^{K} \frac{b_k}{a_k} \right) \Big/ \left(1 - \sum_{k=2}^{K} \frac{1}{a_k} \right)$$

$$P_k = b_k - P_1 \; / a_k \, , \; for \; k = [2,..,K]$$

Waterfilling across sub-carriers per user

$$p_{k,1} = \frac{P_k - V_k}{N_k}$$

$$p_{k,n} = p_{k,1} + \frac{H_{k,n} - H_{k,1}}{H_{k,n} H_{k,1}}$$

Algorithm 3.5: SRM-P Power Allocation (Wong) [4]

About power allocation, the authors in [4], used Lagrangian multiplier techniques in order to define the calculated channel gain related parameters V and W as defined in the Algorithm 3.5. Note that channel gains $H_{k,n}$ should be sorted in ascending order per sub-carrier, and V parameter sum calculation is up to N_k: the number of sub-carriers assigned to a generic user *k*. These parameters were also met in Shen [3], as the *linear case*. The authors in [4], linearized the problem and then followed Shen's solution. The system of the simultaneous linear equations that the authors formulated can be easily solved due to its symmetric and sparse structure. The only parameters that the sparse system contains are the a_k and b_k parameters; and the variables are the individual powers P_k. Then, by using LU decomposition and forwards-backwards substitution led to the individual powers P_k calculation. The formulas used are illustrated in the Algorithm 3.5, and further information can be found in [4]. A very strong advantage of this algorithm is that it is not iterative (such as Han's algorithm [10], which deals with the same problem) and its computational complexity is very low compared with all other techniques studied in the reference list.

At the end of the power allocation procedure, individual waterfilling is performed. This means that after deciding the total power P_k that each user will be assigned throughout all their channels, a waterfilling across these channels is performed in order to maximize capacity.

Summarizing the features of Wong's SRM-P algorithm

- \+ Approximately equal user fairness indexes (UFI) and very high SFI levels provided
- \+ All users are assigned rate roughly equal to their proportional rate requirements / constraints, so that proportionalities are satisfactory met
- \+ Extremely low computational complexity (as linear algorithm)
- ± Spectral efficiency has been limited due to the required proportionalities, but noticeable maximization has been made over MMR algorithm

3.4 Simulation Results

In the following results, some performance metrics are presented, such as System Fairness Index (SFI), Cell Throughput, User Satisfaction, and a quantitative bar plot and CDF of the rates allocated to users, in order to present the behavior of the algorithms under certain circumstances, and different sets of user loads. The simulations performed in a Windows XP server 2003 x64 Intel Xeon machine with 4 cores fully occupied, each by one simulation in parallel. 1000 TTIs (the fundamental time unit) are considered with 100 different user placements, all result 10.000 different channel realizations per algorithm per user load.

The main simulation parameters are shown in the following Table 3.2:

Table 3.2: Simulation Parameters

Parameter	Value
Number of cells	1 hexagonal
Maximum BS transmission power ($P_{\max}$)	1 W
Cell radius (R)	500 m
Mobile terminal speed	static
Carrier frequency	2 GHz
Number of sub-carriers (N)	192
Sub-carrier bandwidth (B/N)	15 KHz
Path loss attenuation (L_k)	Using equation (2.5)
Log-normal shadowing std deviation (σ)	8 dB
Fast/Rayleigh fading	Typical Urban (TU)
AWGN power per sub-carrier ($N_0 \cdot B/N$)	-123.24 dBm
BER requirement	10^{-6}
Link adaptation	Continuous using equation (2.12)
Transmission Time Interval (TTI)	0.5 ms
Traffic model	Full buffer
User Rate Proportionality Constraints	follow the probability mass function $\zeta_k = \begin{cases} 1 \;\; \textit{with probability} \; 0.5 & [320 kbps] \\ 2 \;\; \textit{with probability} \; 0.3 & [640 kbps] \\ 4 \;\; \textit{with probability} \; 0.2 & [1280 kbps] \end{cases}$

Simulation parameter values are chosen such as most of the literature standard values for simulating the environment conditions. Other parameters, such as user rate requirements were multiplied by 5, and their values are as displayed on Table 3.2. These values are sufficient to see a difference in performance among the algorithms (i.e. to make the system satisfaction sensitive).

The first metric presented is the System Fairness Index (SFI) plot, shown in the following Figure 3.1. As presented in section 3.1, SFI in **SRM** algorithm [1] is very low, since its objective is to maximize rate, even if many users are not given any sub-carrier. This is obvious in Figure 3.1 where SFI is very close to its lower extreme 1/K, where K is the number of users, and this happens when only one user is given all the available resources. As seen in Figure 3.4, on average, only three users are being given approximately all sub-carriers, since all remaining users have weak channel gains and are not assigned with any channel. The algorithm is choosing to assign each sub-carrier to the user that experiences stronger connection; the one that has the best channel gain. The algorithm appears to decrease SFI as the number of users increases, since strongest users are taking all resources.

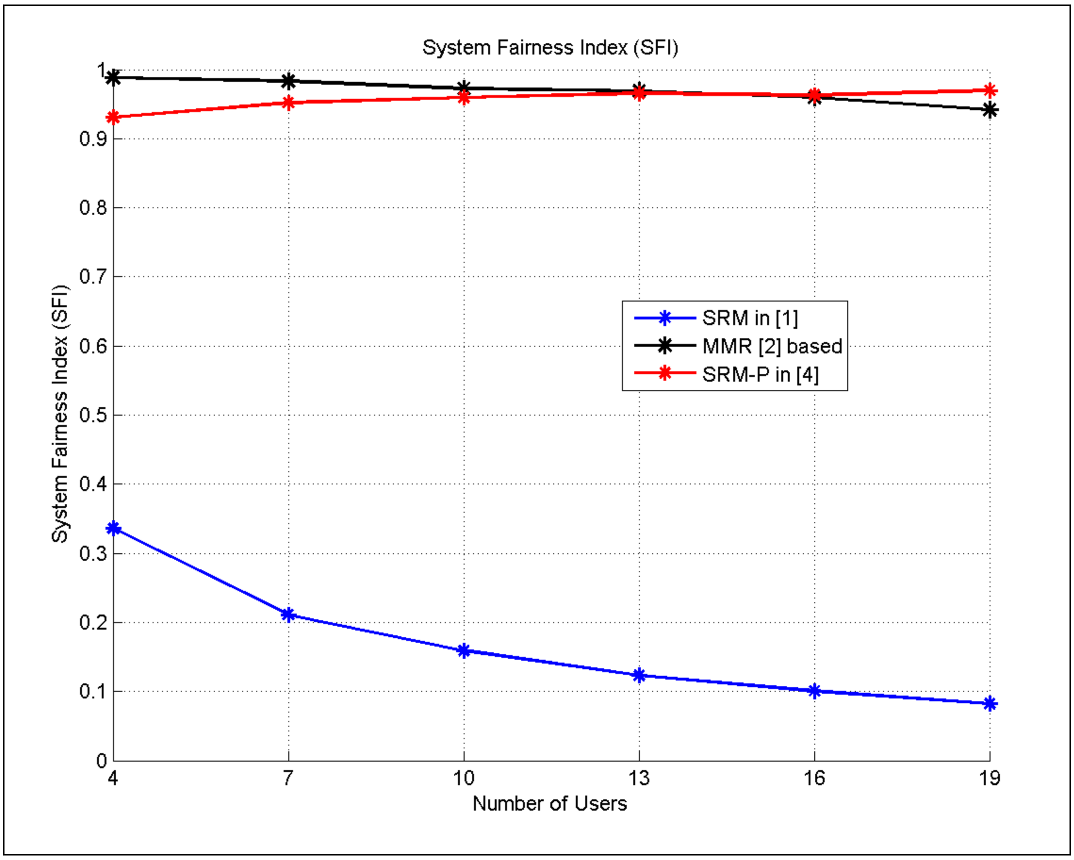

Figure 3.1: System Fairness Index (SFI)

In contrast, the other two algorithms, **MMR** (acc. to [2]) and **SRM-P** [4], are quite fair algorithms, since their policy is maximizing sum rate while satisfying user requirements. These algorithms are distributing resources quite fairly, according to the proportional rate requirements of the users, and SFI, as a result, fluctuates in high levels, close to its maximum, which is one.

MMR sub-carrier allocation (Algorithm 3.3), at each iteration is allocating sub-carriers to the user with minimum proportional rate, so in the end all the user proportional rates are quite equal. After that, Equal Power Allocation (EPA) is applied. **SRM-P**, instead, as presented in section 3.3, after their sub-carrier allocation (Algorithm 3.4), is applying a power allocation (Algorithm 3.5), that improves both SFI and cell throughput, as shown in the following Figure 3.2.

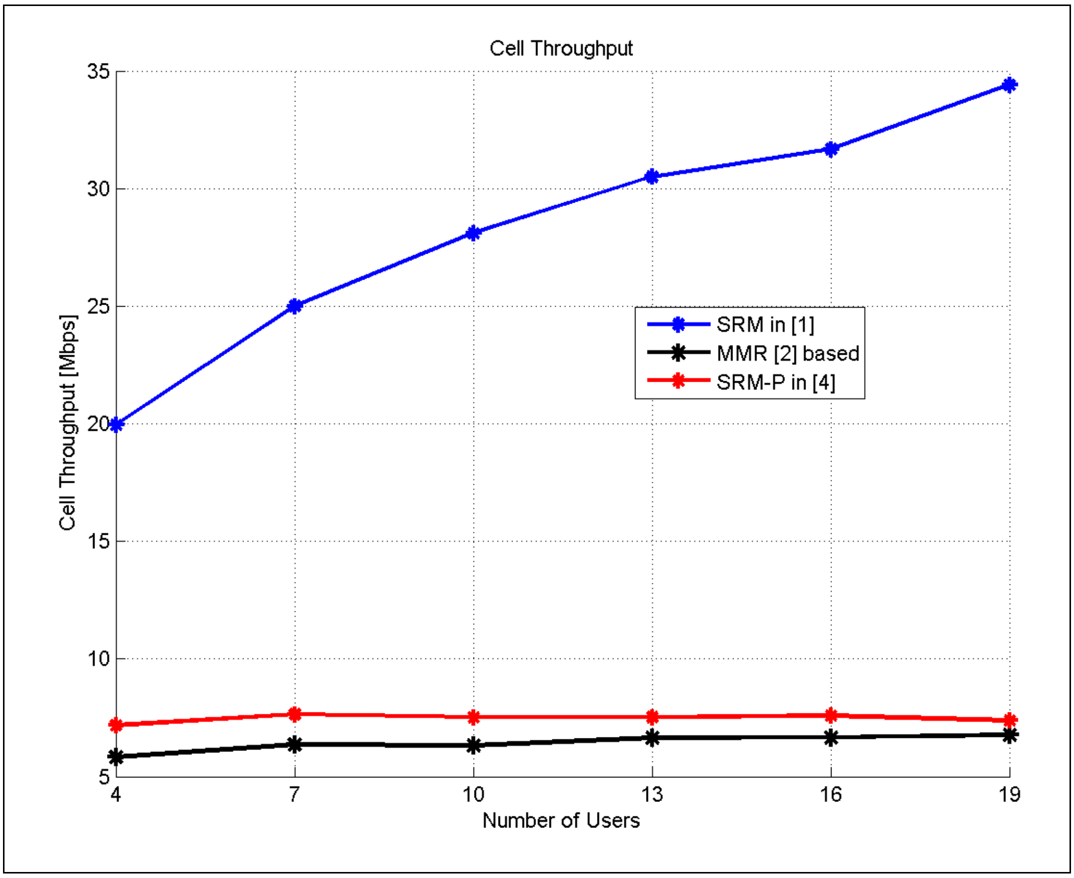

Figure 3.2: Cell Throughput

This great difference in the cell throughput between SRM and the others is the trade-off of having so low SFI resulting from **SRM** policy. As a principle applies that the more fairness you have, the less cell throughput can be achieved. By applying RM techniques in problems whose objective is to maximize the rate, then both the SFI and cell throughput, are reverse-dependent factors. The small difference in rate between **MMR** and **SRM-P**, is also vice-versa in terms of fairness for user load range up to 16 users; while for higher user loads SRM-P algorithm is clearly more efficient. It is interesting that for the fair algorithms (MMR and SRM-P) cell throughput remains roughly in the same levels, as the user load increases, while SRM algorithm keeps in increasing the cell throughput, because stronger channel gain users are becoming available as user load increases (multi-user diversity).

Users' Long Term Satisfaction is a metric that shows, at the end of a session, whether a user has received the rate they required. Note that, in order to make the system satisfaction-sensitive, the user rate requirements are increased as shown in Table 3.2, so that the differences of the algorithms are becoming obvious. If the requirements would have been low (quite lower than the ones used in this work), the satisfaction levels for the fair algorithms (**MMR**, and **SRM-P**) would have always been extremely high, while for the **SRM** algorithm would not have been made a great difference, so the user load impact would not have been obvious. The percentage of satisfied users can be visualized in the following Figure 3.3. LT-USI is normally low in SRM, and slightly decreasing as the number of users is increasing, while its characteristic is that its USI is insensitive to the rate requirements adjustment, means that no matter if the requirements are doubled or tripled, similar satisfaction will apply for any user load. On the other hand, given the high rate requirements apply, there is quite radical degradation of USI in the fair algorithms and the cross-point is close to 13 user- load for the chosen parameters. If the rate requirements were lower as normally, MMR and SRM-P would have been very close to one and the latter would have been performing better, since it is more efficient than MMR is; note that the results with normally lower rate requirements are not appearing here.

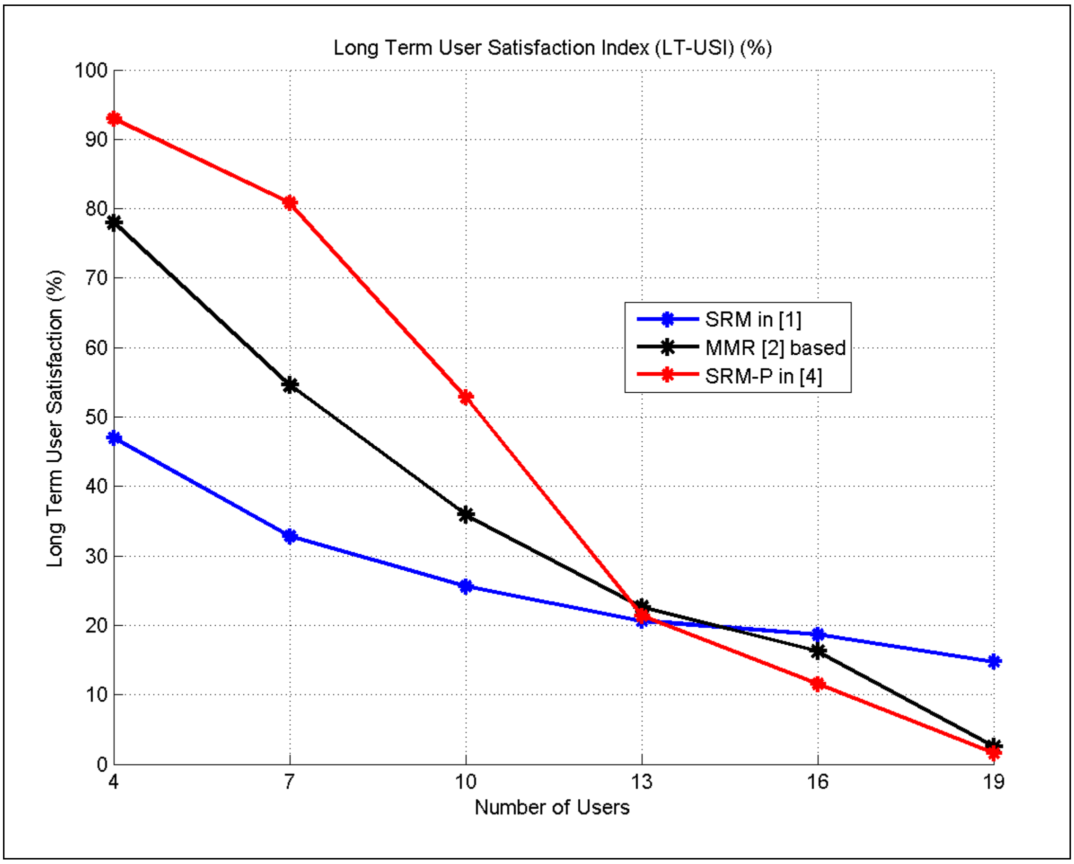

Figure 3.3: User Satisfaction Index (USI)

The following Figure 3.4.a depicts the rates that the users experienced, along with their rate requirements for comparison purposes. Rate requirements indicate user classes; in this case we have 3 classes of users (**gold**, **silver**, and **bronze**) having higher, medium and lower rate requirements respectively, according to Table 3.2 that indicates the possible rates and the probability distribution function that generates the requirements. For sake of simplicity, this plot represents a particular simulation of 7 users. In order to ease the observations, the blue bars represent the rate requirements of the users, and the other bars represent the rates that each user has been assigned by the algorithms. Note that in Figure 3.4.b all rates depicted are normalized, such as they all sum up to one, and clearly shows the exact rate proportion that each user has been assigned. Therefore, the closer to the blue bar are the algorithms' rates, the fairer they are (higher SFI).

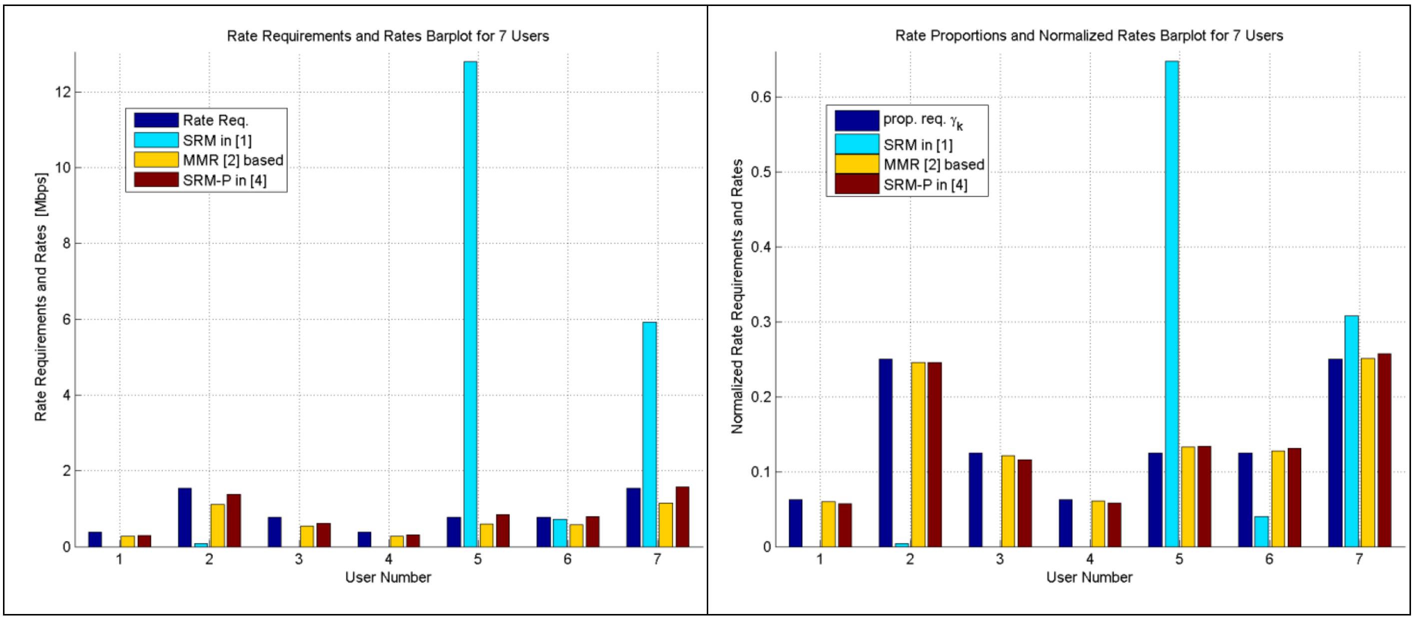

Figure 3.4: a. Rates and Rate Requirements for 7 users. b. Normalized values

Figure 3.4.a depicts the rates that have been assigned to the users along with their requirements, so conclusions only related to satisfaction can be made. On the other hand, Figure3.4.b shows only normalized values, so conclusions only related to fairness can be made. As expected, for this case of 7 users, in figure 3.4.a, **SRM-P** algorithm is the one that their assigned rates are closer to the rate requirements for all users, than in **MMR**. This is because LT-USI in SRM-P is higher than in MMR as observed previously in Figure 3.3. In figure 3.4.b, MMR algorithm is the one that their assigned normalized rates are closer to the rate proportions, than in SRM-P. This is because SFI in MMR is higher than in SRM-P as observed previously in SFI Figure 3.1. In both Figures 3.4.a and 3.4.b, assigned rates in **SRM** are quite far from the requirements, since the objective of the SRM algorithm is to maximize system's capacity and neither rate requirements, nor proportionalities are taken into account; thus the comparison for SRM in this Figure 3.4 is not very decent. Notice that the relative difference of the bars of different algorithms in Figure 3.4.a is not following the same form as in Figure 3.4.b since when normalizing we lose the absolute value information. This is due to the performance difference of the algorithms with respect to the total cell throughput, always by considering pure data rates.

The rates distribution is clearer in the following Figure 3.5, where the CDF of all the normalized rates assigned to the users is depicted. Note that only one user load case of 7 users is considered, as in the previous bar plot. As it can be seen, **MMR** algorithm's rates are more clearly step-wised. From the previous barplot, we have 3 classes of users (gold, silver, and bronze) having higher, medium and lower rate requirements respectively, according to Table 3.2. In Figure 3.5, CDF of MMR appears to have 3 steps, and tends to be discrete; the step values are roughly {0.0625, 0.125, 0.25}, and these proportions are the same as the rate requirement proportionalities. The 'step' observation is also obvious in the CDF of **SRM-P**, but not so clear, since the algorithm is roughly satisfying the proportionalities, in a less tight way than in MMR, and the range of the different assigned normalized rates is larger. In CDF of **SRM**, the graph is much curvier, which means that the range of the assigned rates is continuous.

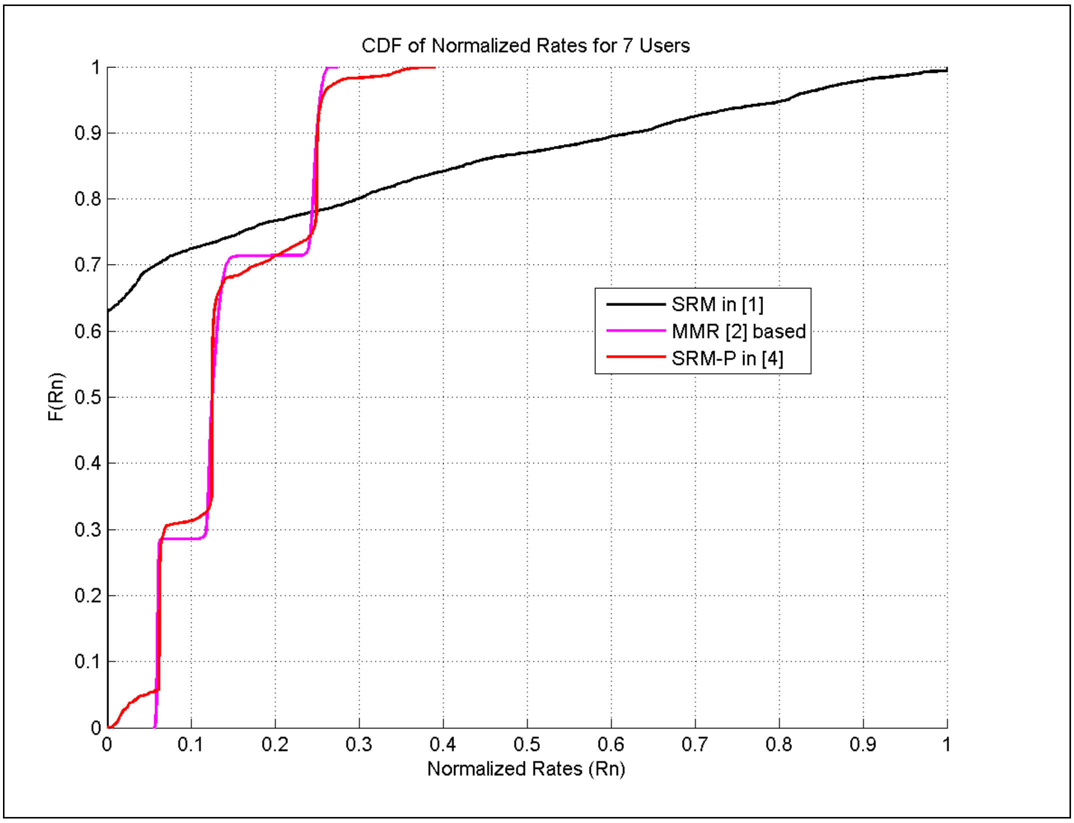

Figure 3.5: CDF of the assigned user rates (normalized)

Another important observation is that SRM curve starts from a point close to 0.62. It means that roughly 62% of the normalized rate samples (1000 TTIs x 7 users) are zero, and the great amount of users that are assigned no resources is verifying the theoretical expectation from section 3.1. In contrast, MMR starts from 0.06 with respect to the x-axis, which indicates that there are not any users without resources.

Alternatively, the CDF of the normalized rates can be separated in groups, depending on the user class they belong to; the reason for displaying normalized rates instead of pure ones is that the group classes' separation is clearer and without so much ambiguity. In the next Figure 3.6 the CDF of the normalized rates per group can be seen. The conclusions previously mentioned can be verified according to the group separation for the fair algorithms (**MMR** and **SRM-P**). In **SRM** algorithm, only some of silver and golden group users are assigned all resources, while none user from bronze group is never assigned any resource. Note that this is not related to the SRM policy, and to none policy presented in this work. None policy included in this work differentiates among groups and the group generation function follows the mass distribution function of Table 3.2, and remains the same for all runs of the simulation. Even if the users change position between the runs, it may happen that some users are never transmitting when SRM is used. Figure 3.4 depicts the average rates of the users, and the bronze users 1 and 4 have no resources when SRM is used.

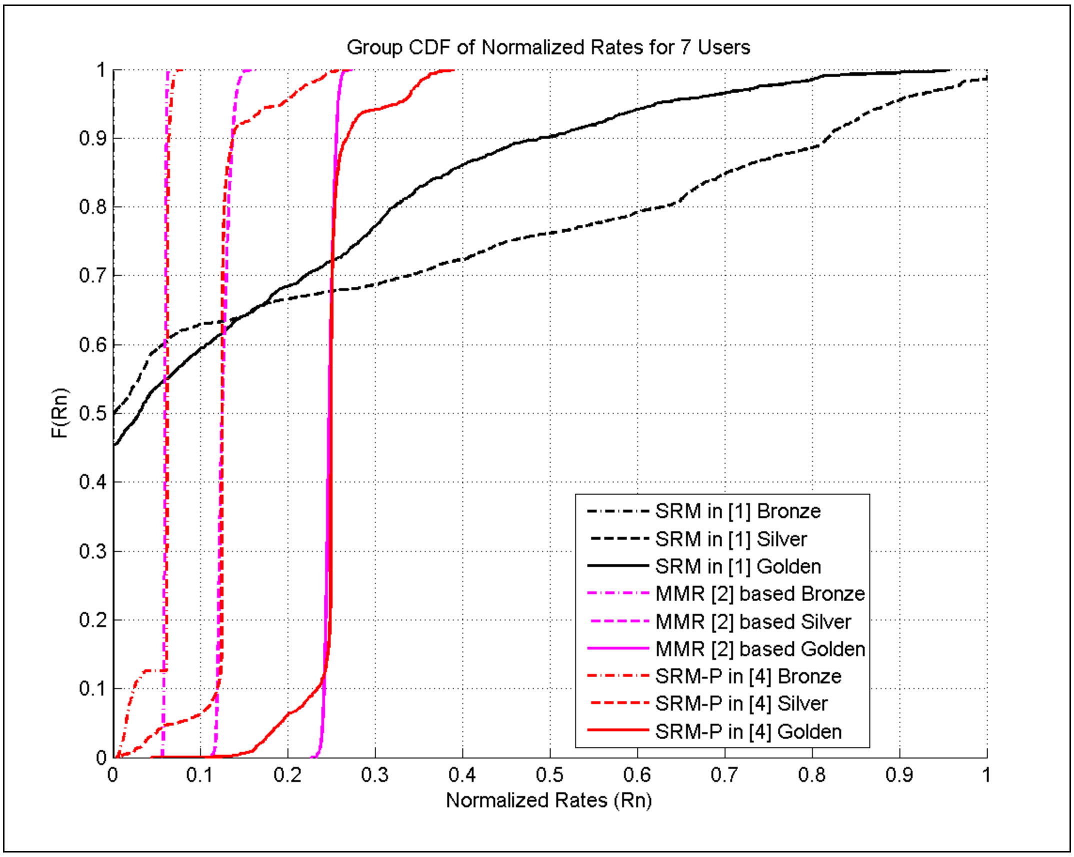

Figure 3.6: Per Group CDF of the assigned user rates (normalized)

Last, the following Table 3.3 depicts the computational time that each algorithm needed on average to perform the resource allocation.

Table 3.3. Average CPU Time (ms) vs Users

# users:	4	7	10	13	16	19
SRM	3.21	3.43	3.61	3.85	3.96	4.23
MMR	7.10	7.21	7.36	7.39	7.67	8.33
SRM-P	9.03	9.20	9.98	10.41	11.00	11.95

SRM algorithm appears to perform the fastest resource allocation, since the algorithm assigns each channel to the strongest user, while the fair algorithms are tracking user rates and then deciding to assign resources to the user that needs them the most in order to follow the required proportionalities. **SRM-P** sub-carrier Algorithm 3.4 is similar in concept with **MMR** Algorithm 3.3, so there is similar computational burden; the small difference appeared is due to the power allocation performed in SRM-P Algorithm 3.5, while in MMR equal power allocation is applied, whose computational time is considered zero. It has to be noted that even SRM is applying waterfilling as power allocation; its CPU time is still lower than MMR, which is applying EPA. The great difference is due to the dynamic sub-carrier allocation. Note that the computational time of these classical algorithms is not very sensitive with respect to the user load.

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4. System Fairness Adaptive Algorithms

In this chapter, the proposed fairness adaptive algorithms will be presented, along with their simulation results at the end of this chapter, including also the classical algorithms for both reference and comparison purposes. The different policies applied by each algorithm have a specific objective regarding the capacity vs fairness tradeoff that is already mentioned at the end of the first chapter. Metrics such as *User Satisfaction Index* (USI) and CPU time demands will be evaluated.

With these algorithms we will investigate the system fairness and capacity trade-off. As mentioned in previous chapters, these two metrics are reverse-dependent, means maximizing one result the other to be minimized. The problem of RRA is split in two procedures; first the *sub-carrier allocation algorithm* assigns sub-carriers to the users, and afterwards the *power allocation algorithm* is allocating which amount of power each sub-carrier is assigned. The algorithms presented here belong to the category of Rate Adaptive (RA) algorithms, since at every iteration; a rate maximization is attempted with a constraint on BS power.

Notation

In order to describe mathematically the problems that the algorithms are solving, the following notation will be used:

Table 4.1: Notations used

Symbols	
k	: *user index*, $k \in [1,2,..,K]$
K	: *total number of users*
n	: *subcarrier index*, $n \in [1,2,..,N]$
N	: *total number of subcarriers*
$p_{k,n}$	: *subcarriers' power w.r.t. user k*
$P_{\max}$	: *maximum BS transmitted power*
B	: *total bandwidth available*
$c_{k,n}$	: *subcarrier allocation indicator*, $c_{k,n} \in \{0,1\}$
$H_{k,n}$	: *effective subcarrier SNR*
$r_{k,n}$	: *subcarriers' rate w.r.t. user k*
R_k	: *users' rate vector*
γ_k	: *user rate proportionality constrains*, $\gamma_k \in [0,1]$
R_k^{req}	: *user rate requirements*
φ_k	: *user fairness index (UFI)*
Φ	: *system fairness index (SFI)*
Φ_t	: *system fairness target (SFT)*
Bullets	
$+$	: *advantages*
$-$	: *dissadvantages*
$\pm$	: *positive and / or negative*
o	: *neutral*

Notation notes:

1. $p_{k,n}, c_{k,n}$, and $r_{k,n}$ in the general case can be matrices with size KxN. Constraints are applied so that sub-carrier *n* is assigned to a maximum of one user *k*. The connection matrix $c_{k,n}$ shows whether a sub-carrier is assigned to user *k* or not, by having values 1 and 0 respectively. Thus each of $c_{k,n}$'s columns will sum up to 1.

2. $p_{k,n}$ in many cases inside the algorithms is used as vector p_n. Note that *k* user index is absent and the size of these vectors is 1xN. By using another vector, the *channel allocation* vector, which is the output of the *channel allocation algorithm,* we have the correspondences about which sub-carrier is assigned to which user, so the sub-carrier power vector p_n, along with the channel allocation vector contain all the information we need.

3. $H_{k,n}$ and $r_{k,n}$ can be found as H_n and r_n, respectively. What described in 2 applies in the same way. $H_{k,n}$ is the effective subcarrier SNR as defined in [4], including channel gains, noise power, and SNR gap.

4.1 Fairness based Sum Rate Maximization (FSRM) Problem

In the previous chapter the classical algorithms are described; SRM achieving maximum system capacity and very low SFI, while MMR achieving very high SFI and lower system capacity due to the proportionalities. SRM-P is achieving high rate proportionality fairness (e.g. SFI≈1), while maximizing capacity under the proportional constraints. So far the classical algorithms are performing so that the SFI is close to its extremes· either very low closing to $1/K$, or very high reaching 1. The objective of the following proposed algorithms is to maximize the system's capacity under the constraint of SFI to be equal to a target, the *System Fairness Target (SFT)*, which is the desired system fairness level, located in-between the mentioned extremes. The objective is to achieve a specific degree of fairness among users, and benefit the most in terms of system capacity. SFT possible values can be the same as SFI's, also limited to their extreme values $[1/K, 1]$.

Here the problem is Sum Rate Maximization under System Fairness Constraints (FSRM). System fairness is based on instantaneous rates, as described in Chapter 2. The problem formulation is the following:

Proposed problem formulation

$$
\begin{aligned}
\max_{c_{k,n},\, p_{k,n}} \quad & \frac{B}{N}\sum_{k}\sum_{n} c_{k,n}\cdot \log_2(1+p_{k,n}\cdot H_{k,n}) && (o)\\
\text{subject to} \quad & c_{k,n}\in\{0,1\} \quad \forall k,n && (c1)\\
& p_{k,n}\ge 0 \quad \forall k,n && (c2)\\
& \sum_{k} c_{k,n} = 1 \quad \forall k,n && (c3)\\
& \sum_{k}\sum_{n} p_{k,n} \le P_{\max} && (c4)\\
& \Phi = \Phi_t && (c5)
\end{aligned}
\qquad (4.1)
$$

The problem formulation differs from the one presented in section 3.3 only in the last constraint (c5). In the optimization problem (4.1), SFI has to be equal to SFT, while in the optimization problem (3.4) the instantaneous rates must follow the proportionality constraints. The proportionalities are still indirectly considered in problem (4.1), since they are used in the calculation of the SFI Φ. However, the rate proportionalities are relaxed when the resource allocation forces the SFI Φ to be equal to the desired SFT Φ_t. There is no point in including the rate proportionalities as an additional constraint, since by satisfying that constraint occurs SFI to be equal to 1, which in general is different than the desired SFT. Both constraints cannot be satisfied at the same time, since by including both constraints, no solution exists in the general case that $1/K < \Phi_t < 1$.

Problem 4.1 is not convex, since constraints (c1 and c3) are indicating that each sub-carrier has ($c_{k,n} = 1$) or does not have ($c_{k,n} = 0$) a connection with a user, and also it is assigned exclusively to one user at a time. Part of this problem is a binary integer programming (IP) assigning problem, and the domain of the objective function of the problem is not a convex set. Therefore the problem is not convex; this family of problems with bounded variables is classified as NP-hard problems. Further information can be found in Appendix A.

The proposed approach is based on heuristics of suboptimal solutions proposed by the authors [1,2,4,8] of the classical problems that are presented in chapter 3, but it is expanded and aims in a different objective, which is to also satisfy the SFI constraint.

4.2 System Fairness Adaptive Sum Rate Maximization Algorithm (FSRM)

This algorithm is a Sum Rate Maximization under SFI constraint (FSRM), and tries to solve the FSRM problem mentioned above. The proposed sub-carrier allocation algorithm is performed in two steps. The first step is based on Jang [1] (so currently we are in a max rate situation, along with poor SFI Φ_{SRM}); and in the second step sub-carrier reallocation is done, in order to increase SFI until some point near SFT (Φ_{FSRMsa}), so that the SFI constraint is roughly satisfied. This procedure can be visualized as the step 1 in the following Figure 4.1:

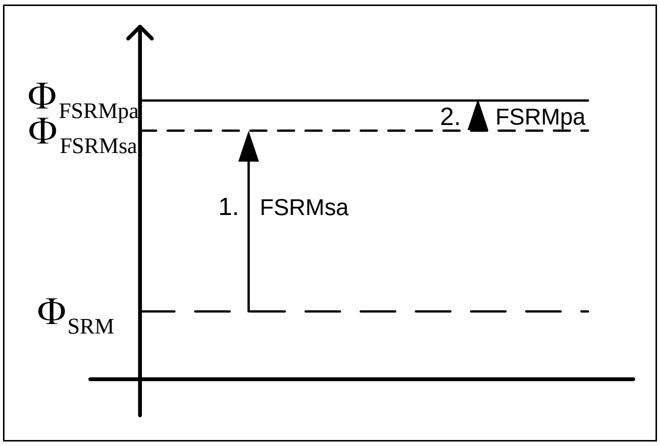

Figure 4.1: SFI approaching the desired fairness level (FSRM)

In the sub-carrier reallocation step, the algorithm is finding the user with the maximum rate, and its worst channel is removed, and given to the user that has the maximum gain in that particular sub-carrier. This procedure is done iteratively until the SFI roughly reaches SFT. This policy is considered as a *max rate* policy, since the chosen user is the one that could gain the most of that channel, under the constraint to increase the fairness of the system. The channel reallocation process is iterative until the SFI reaches the target SFT. Note the high involvement of the channel reallocation algorithm which accomplishes most of the desired procedure in increasing SFI. The proposed sub-carrier reallocation procedure is shown in the following Algorithm 4.1 (FSRMsa).

All notations used are explained in the Table 4.1 at the beginning of this chapter.

The proposed sub-carrier allocation algorithm pseudo code description is the following:

Initialization

$R_k = 0 \ \forall k$, $c_{k,n} = 0 \ \forall k,n$, $p = P_{max}/N$,

$k \in \Gamma = \{1,2,..,K\}$, *and* $n \in \Omega = \{1,2,..,N\}$

I. Apply Sub-carrier Allocation Algorithm 3.1 [Jang]

II. Sub-carrier Reallocation

calculate Φ according to eq.(2.14 and 2.15)

while $\Phi < \Phi_t$

$k1 = \arg\max_{k\in\Gamma}(R_k)$ find 1st user

$n1 = \arg\min_{n} |H_{k1,n}|$ find their worst channel

$c_{k1,n1} = 0$ remove the connection

$n2 = n1$ hand it in to user *k2*

$k2 = \arg\max_{k\in\Gamma\setminus\{k1\}} |H_{k,n2}|$ find 2nd user

$c_{k2,n2} = 1$ set the connection

$R_{k1} = R_{k1} - \frac{B}{N}\log_2(1 + p \cdot H_{k1,n1})$ update rate of user *k1*

$R_{k2} = R_{k2} + \frac{B}{N}\log_2(1 + p \cdot H_{k2,n2})$ update rate of user *k2*

recalculate Φ according to eq.(2.14 and 2.15)

end

end

Algorithm 4.1: Proposed Sub-carrier Allocation (FSRMsa) that increases SFI

In step I of Algorithm 4.1, the SRM sub-carrier allocation procedure is done by initially considering *equal power allocation* (EPA). Next in step II, the sub-carrier reallocation is applied so that FSRMsa algorithm is completed, also considering EPA.

Next, the power allocation algorithm is performed. The objective of the power allocation is to increase a little the SFI up to Φ_{FSRMpa}, so that the refinements left by the sub-carrier allocation algorithm are done; the involvement of the power allocation algorithm is not significant, since the sub-carrier allocation algorithm roughly met the constraints. Figure 4.1 illustrates this process as step 2.

The most straight forward way to increase the SFI is to find the user with maximum proportional rate, remove some resources from them (power only in this step), and then assign these resources to the user with minimum proportional rate. Here another policy will be applied; instead of assigning the resources to the user with minimum proportional rate, they will be assigned to the user that gains the maximum rate increment. So the assignment is done by re-allocating a considerably small *power fraction dp*. The algorithm is finding the max rate user to subtract power from. Then is searching through all the channels of all other users, and finds the channel that the maximum rate increment will be achieved by adding a *dp* power slice on it. This way the rate decrement is the minimum for the first user and the rate increment is the maximum for the second user, so this policy is a max rate policy. The procedure described is done iteratively until the SFI satisfactory reaches the SFT. The proposed power allocation algorithm is shown in the following Algorithm 4.2 (FSRMpa), and its policy is based on Han's power allocation algorithm [8].

The proposed fairness adaptive power allocation algorithm pseudo code description is the following:

Initialization

$R_k = \sum_n c_{k,n} \frac{B}{N} \log_2(1 + p_{k,n} \cdot H_{k,n})$ calculate user rates

$k \in \Gamma = \{1,2,..,K\}$, *and* $i, n \in \Omega = \{1,2,..,N\}$

calculate Φ according to eq.(2.14 and 2.15)

while $\Phi - \Phi_t < \varepsilon$ ($\varepsilon = 10^{-3}$, *or any small tolerance*)

$k1 = \arg\max_{k \in \Gamma}(\frac{R_k}{\gamma_k})$ find 1st user

calculate all possible rate decrements for the channels of *k1*

$\Delta r_i^{dec} = B/N \log_2(1 + p_n \cdot H_{k1,n}) - B/N \log_2(1 + (p_n - dp) \cdot H_{k1,n}) \quad \forall n : c_{k1,n} == 1$

$n1 = \arg\min_i(\Delta r_i^{dec})$ find the channel that decreases rate the least

[user *k1:* channel *n1* will be removed a *dp* power slice, and will result a Δr_i^{dec} rate decrement]

calculate all possible rate increments of all other channels

$\Delta r_i^{inc} = B/N \log_2(1 + (p_n + dp) \cdot H_{k,n}) - B/N \log_2(1 + p_n \cdot H_{k,n}) \quad \forall k \neq k1 \; \forall n : c_{k,n} == 1$

$n2 = \arg\max_i(\Delta r_i^{inc})$ find the channel that increases rate the most

$k2 = k \; : \; c_{k,n2} == 1$ find the user that has been assigned *n2*

[user *k2:* channel *n2* will be added a *dp* power slice, and will result a Δr_i^{inc} rate increment]

$p_{n1} = p_{n1} - dp$ do the *dp* power transfer

$p_{n2} = p_{n2} + dp$

$R_{k1} = R_{k1} - \Delta r_i^{dec}$ update rate of user *k1*

$R_{k2} = R_{k2} + \Delta r_i^{inc}$ update rate of user *k2*

recalculate Φ according to eq.(2.14 and 2.15)

end

end

Algorithm 4.2: Proposed Adaptive Power Allocation (FSRMpa) that increases SFI

All algorithm acronyms can be found and explained in Table 4.3. For generality purposes, the proposed power allocation has been expanded with the proposed FSRM-Ppa in section 4.4, in order to be adaptive to the SFT in both directions (both increasing and decreasing SFI).

The basic steps of this proposed algorithm are the following:

- **Dynamic Sub-carrier Allocation (FSRMsa)**:
  i Sub-carrier allocation based on Jang [1]. Max rate, but very poor SFI, quite lower than SFT
  ii Sub-carrier reallocation: Significantly increases SFI by removing the worst sub-carrier of the max rate user ($k = \arg\max_{k \in Z}(R_k)$), and assign it to the user that has the highest gain on that sub-carrier, repeat (ii) until SFI ≈ SFT.

- **Adaptive Power (re-)Allocation (FSRMpa):**
  $$\min\ \Delta \mathrm{r}_{dec}\ \text{of prop.max rate user} \xrightarrow{dp} \max\ \Delta \mathrm{r}_{inc}\ \text{of all subcarriers}$$
  Subtract power dp from the channel that experiences the minimum rate decrement, of the user with the maximum proportional rate ($k = \arg\max_{k \in Z}(R_k / \gamma_k)$); and add it to the channel of any other user that results the maximum rate increment. Repeat until the fairness target SFT is met.

Summarizing the features of the proposed algorithm

+ SFI meets System Fairness Target (SFT)
+ Systems capacity is maximized, under the SFT constraint
– Some users have no resources; depending on how low is the chosen SFT
– The influence of power allocation is not significant, since sub-carrier allocation algorithm roughly satisfies the fairness constraint

4.3 System Fairness Adaptive Max Min Rate Algorithm (FMMR)

FMMR problem is a different problem than the Problem (4.1) presented here; is based on the MMR Problem (3.3), and in a similar way includes the additional constraint c5 of Problem (4.1). The simulation results selectively include this algorithm for completeness purposes. This algorithm is presented in detail in the thesis work [11], and here only its basic features will be remarked.

However, the policy applied here is based on MMR algorithm, presented in section 3.2 and it is expanded in order to become fairness-adaptive by adjusting the SFI as in section 4.2. The reader may also see the Figure 4.2, which describes the exact same SFI route approaching to the target SFT. This approach has also been implemented and compared with the rest algorithms. The basic steps of this proposed algorithm are the following:

- **Dynamic Sub-carrier Allocation (FMMRsa) that decreases SFI:**
  i Sub-carrier allocation based on Rhee [2]. Maximizing the min rate results extremely high SFI > SFT
  ii Sub-carrier reallocation: Significantly decreases SFI by removing the worst sub-carrier of the proportionally 2^{nd} max rate user, and assign it to the user with the maximum proportional rate, repeat (ii) until SFI $\approx$ SFT

- **Adaptive Power Allocation (FMMRpa) that decreases SFI:**

  $$\min\ \Delta r_{dec} \text{ of prop } 2^{nd} \text{max rate user} \xrightarrow{dp} \max\ \Delta r_{inc} \text{ of prop max rate user}$$

  Subtract power dp from the channel that experiences the minimum rate decrement, of the proportionally 2^{nd} max rate user; and assign it to the channel of the user with maximum proportional rate ($k = \arg\max_{k \in Z}(R_k/\gamma_k)$) that results the maximum rate increment.

  Repeat until the fairness target SFT is met.

Summarizing the features of the proposed algorithm

+ SFI meets System Fairness Target (SFT)
+ Protects minimum rate users by not subtracting resources from them
– Sub-carrier reallocation (step ii) policy is poor in terms of efficiency
– The influence of power allocation is not significant, since sub-carrier allocation algorithm roughly satisfies the constraints

4.4 System Fairness Adaptive Sum Rate Maximization with Proportionalities (FSRM-P)

As the previous algorithm, the same policy applies here as well, with the only difference that the classical sub-carrier allocation algorithm used as a basis is Wong's Algorithm 3.4, presented in chapter 3. The first step is based on Wong [4] (so currently we are in a high SFI Φ_{SRM-P} situation); and in the second step, a sub-carrier reallocation is done, in order to decrease the SFI down to $\Phi_{FSRM-Psa}$ (roughly the SFT is reached), so that the SFI constraint is roughly satisfied. This procedure can be visualized as the step 1 in the following Figure 4.2:

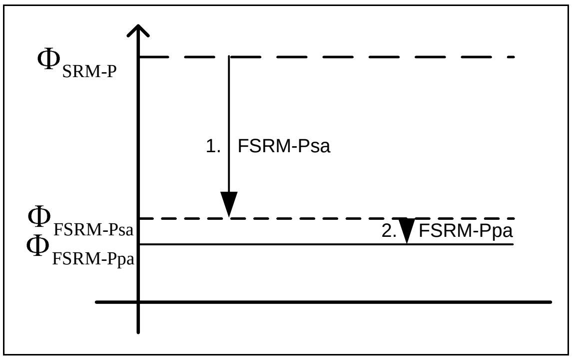

Figure 4.2: SFI approaching the desired fairness level (FSRM-P)

In the sub-carrier reallocation step, the algorithm is finding the user with the minimum proportional rate, and their worst channel is removed, and given to the user that has the maximum gain in that particular sub-carrier. This procedure is done iteratively until the SFI reaches SFT. The objective and the policy applied here is considered as a *max rate* policy, since the chosen user is the one that could gain the most of that channel, under the constraint to decrease the fairness of the system. Through the sub-carrier reallocation process it is possible that some users may run out of sub-carriers. However, this appears quite rare, since the final proportionally min rate users – after Wong's sub-carrier allocation – has been assigned quite many sub-carriers. Note that the most of the desired procedure is done through the channel allocation algorithm, thus its involvement is high. The proposed sub-carrier allocation algorithm is illustrated in the following Algorithm 4.3 (FSRM-Psa).

All notations used are explained in the Table 4.1 at the beginning of this chapter.

The proposed sub-carrier allocation algorithm pseudo code description is the following:

```
Initialization
    R_k = 0 ∀k , c_{k,n} = 0 ∀k,n , p = P_max/N ,
    k ∈ Γ = {1,2,..,K}, and n ∈ Ω = {1,2,..,N}

  I. Sub-carrier allocation Algorithm 3.4 [Wong]

 II. Sub-carrier Reallocation
   calculate Φ according to eq.(2.14 and 2.15)
   while Φ > Φ_t
     k1 = arg min_{k∈Γ}(R_k/γ_k)              find 1st user
     n1 = arg min_n |H_{k1,n}|                 find their worst channel
     c_{k1,n1} = 0                             remove the connection
     n2 = n1                                   hand it in to k2
     k2 = arg max_{k∈Γ\{k1}} |H_{k,n2}|        find 2nd user
     c_{k2,n2} = 1                             set the connection
     R_k1 = R_k1 - (B/N) log2(1 + p·H_{k1,n1}) update rate of user k1
     R_k2 = R_k2 + (B/N) log2(1 + p·H_{k2,n2}) update rate of user k2
     recalculate Φ according to eq.(2.14 and 2.15)
   end
end
```

Algorithm 4.3: Dynamic Sub-carrier Allocation (FSRM-Psa) that decreases SFI

Since subcarrier allocation Algorithm 3.3 results high SFI level but not close to 1, usually around SFI = 0.7, when fairness target has been set in higher values, Algorithm 4.3 should also increase fairness (notice that in Figure 3.1, SRM-P algorithm is Algorithm 4.3 followed by the corresponding power allocation Algorithm 4.4). For handling also these cases, Algorithm 4.3 has been expanded for increasing fairness based on FSRMsa Algorithm 4.1 when needed, so it becomes a bi-directional fairness dynamic sub-carrier allocation adaptive algorithm.

The sub-carrier allocation procedure could be expanded, in order to differently treat user groups instead of individual users as we considered so far. In that case, the groups are formed depending on the proportional rate requirements they have (i.e. users with same requirements belong to the same group). Therefore if we consider 3 classes of users (gold, silver, and bronze), we have 3 different levels of requirements, respectively. In this case the algorithm chooses to remove the worst channel of the worst proportional rate user ($k = \arg\min_{k\in Z}(R_k/\gamma_k)$) of the worst group (bronze) and assign it to the user with highest gain on that channel. Notice that this expanded case is not appearing in this thesis work.

In step I of Algorithm 4.3, the SRM-P sub-carrier allocation procedure is done by initially considering *equal power allocation* (EPA). Next in step II, the sub-carrier reallocation is applied, also considering EPA, so that FSRM-Psa algorithm is completed. Next, the power allocation algorithm is performed.

The objective of the power allocation is to decrease a little the SFI down to $\Phi_{\text{FSRM-Ppa}}$, so that the refinements left from the sub-carrier allocation algorithm are done; the involvement of the power allocation algorithm is not great, since the sub-carrier allocation algorithm roughly met the constraints. Figure 4.2 illustrates this process as step 2.

The power allocation algorithm should decrease SFI a little, so the SFT is reached. In order to decrease a little the SFI, there are many ways to achieve it, but we have to choose a *max rate* policy for this. A straight forward way to decrease the SFI is to find the user with the maximum proportional rate and to assign them more resources (power only in this step). So the assignment is done by assigning them a considerably small *power fraction dp* to the sub-carrier that results maximum rate increase to the user with the maximum proportional rate. After finding to whom and to which channel the *dp* goes, we are going to find from whom we will remove the same amount of power. The algorithm is searching all sub-carriers of all other users and finds the sub-carrier that results the minimum rate decrement if a *dp* is subtracted from it. This way the rate decrement is the minimum and the rate increment is the maximum. This procedure is done iteratively until the SFI satisfactory reaches the SFT. The proposed power allocation algorithm is shown in the following Algorithm 4.4 (FSRM-Ppa), and its policy is based on Han's power allocation algorithm [8].

The proposed power allocation algorithm pseudo code description is the following:

Initialization

$R_k = \sum_n c_{k,n} \frac{B}{N} \log_2(1 + p_{k,n} \cdot H_{k,n})$ calculate user rates

$k \in \Gamma = \{1, 2, .., K\}$, *and* $i, n \in \Omega = \{1, 2, .., N\}$

calculate Φ according to eq. (2.14 and 2.15)

while $\Phi - \Phi_t > \varepsilon$ ($\varepsilon = 10^{-3}$, *or any small tolerance*)

$k2 = \arg\max_{k \in \Gamma}(\frac{R_k}{\gamma_k})$ find 2nd user

calculate all possible rate increments for the channels of *k2*

$\Delta r_i^{inc} = B/N \log_2(1 + (p_n + dp) \cdot H_{k2,n}) - B/N \log_2(1 + p_n \cdot H_{k2,n}) \quad \forall i, n : c_{k2,n} == 1$

$n2 = \arg\max_i(\Delta r_i^{inc})$ find the channel that increases rate the most

[found everything about user *k2:* channel *n2* will be added a *dp* power slice and will result a Δr_i^{inc} rate increment]

calculate all possible rate decrements of all rest channels

$\Delta r_i^{dec} = B/N \log_2(1 + p_n \cdot H_{k,n}) - B/N \log_2(1 + (p_n - dp) \cdot H_{k,n}) \ \forall k \neq k2 \quad \forall n : c_{k,n} == 1$

$n1 = \arg\min_i(\Delta r_i^{dec})$ find the channel that decreases rate the least

$k1 = k \ : \ c_{k,n1} == 1$ find the user that has been assigned *n1*

```
      [found everything about user k1: channel n1 will be removed a
             dp power slice and will result a Δr_i^dec rate decrement]

      p_n2 = p_n2 + dp                     do the dp power transfer
      p_n1 = p_n1 − dp
      R_k2 = R_k2 + Δr_i^inc               update rate of user k2
      R_k1 = R_k1 − Δr_i^dec               update rate of user k1
      recalculate Φ according to eq.(2.14 and 2.15)
    end
end
```

Algorithm 4.4: Proposed Power Allocation (FSRM-Ppa) that decreases SFI

All algorithm acronyms can be found and explained in the following Table 4.3.

For generality purposes, the proposed power allocation has been expanded with the proposed FSRMpa in section 4.2, in order to be adaptive to the SFT in both directions (both increasing and decreasing SFI).

The basic steps of this proposed algorithm are the following:

- **Dynamic Sub-carrier Allocation (FSRM-Psa)**:
  - i Sub-carrier allocation based on Wong [4]. Maximizing rate with very high SFI, higher than SFT
  - ii Sub-carrier reallocation: Significantly decreases SFI by removing the worst sub-carrier of the user with the minimum proportional rate ($k = \arg\min_{k \in Z}(R_k / \gamma_k)$), and assign it to the user that has the highest gain on that sub-carrier, repeat (ii) until SFI $\approx$ SFT

- **Adaptive Power (re-)Allocation (FSRM-Ppa):**

  $$\text{min } \Delta\text{r}_{dec} \text{ of all subcarriers } \xrightarrow{dp} \text{max } \Delta\text{r}_{inc} \text{ of prop. max rate user}$$

  Subtract power *dp* from the channel that will experience the minimum rate decrement (consider all channels, except user with proportional max rate); and assign it to the channel of the user with proportional max rate ($k = \arg\max_{k \in Z}(R_k / \gamma_k)$) that results the maximum rate increment. Repeat until the fairness target SFT is met.

Summarizing the features of the proposed algorithm

+ SFI meets System Fairness Target (SFT)
+ Systems capacity is maximized, under the SFT constraint
− The influence of power allocation is not significant, since sub-carrier allocation algorithm roughly satisfies the constraints
− It is possible that some users run out of sub-carriers, after the sub-carrier reallocation process, depending on how low is the chosen SFT.

4.5 Simulation Results

In the following results, some performance metrics are presented, such as System Fairness Index (SFI), Cell Throughput, User Satisfaction, CDF, and a quantitative bar plot of the allocated rates to users, in order to present the behavior of the algorithms under certain circumstances, and different sets of user load and fairness levels. The simulations were performed in a Windows XP server 2003 x64 Intel Xeon machine with 4 cores fully occupied each by one simulation in parallel. 1000 TTIs (the fundamental time unit) are considered with 100 different user placements, all result 10.000 different channel realizations per algorithm per user load.

The main simulation parameters are shown in the following Table 4.2:

Table 4.2: Simulation Parameters

Parameter	Value
Number of cells	1 hexagonal
Maximum BS transmission power ($P_{\max}$)	1 W
Cell radius (R)	500 m
Mobile terminal speed	static
Carrier frequency	2 GHz
Number of sub-carriers (N)	192
Sub-carrier bandwidth (B/N)	15 KHz
Path loss attenuation (L_k)	using equation (2.5)
Log-normal shadowing std deviation (σ)	8 dB
Fast/Rayleigh fading	Typical Urban (TU)
AWGN power per sub-carrier ($N_0 \cdot B/N$)	-123.24 dBm
BER requirement	10^{-6}
Link adaptation	Continuous using equation (2.12)
Transmission Time Interval (TTI)	0.5 ms
Traffic model	Full buffer
User Proportionality Constraints	follow the probability mass function $\zeta_k = \begin{cases} 1 \ \textit{with probability}\ 0.5 & [320kbps] \\ 2 \ \textit{with probability}\ 0.3 & [640kbps] \\ 4 \ \textit{with probability}\ 0.2 & [1280kbps] \end{cases}$

Simulation parameter values are chosen such as most of the literature standard values for simulating the environment conditions. Other parameters, such as user rate requirements were multiplied by 5, and their values are as displayed on Table 4.2. These values are sufficient to see a difference in performance among the algorithms (i.e. to make the system satisfaction sensitive).

For avoiding any confusion from the numerous acronyms of the algorithms used, the reader may find them all organized in the following Table 4.3:

Table 4.3: Acronyms of Algorithms

Sum Rate Maximization (SRM)

- **SRM:** Sub-carrier Allocation + Waterfilling Power Allocation by Jang et al [1]

System Fairness adaptive SRM (FSRM)

- **FSRM apa:** FSRM with Adaptive Power Allocation only (SRM sub-carrier allocation by Jang [1] + FSRMpa)
- **FSRM dsa:** FSRM with Dynamic Sub-carrier Allocation only (FSRMsa + EPA)
- **FSRM joint:** FSRM with both Dynamic Sub-carrier Allocation (FSRMsa) + Adaptive Power Allocation (FSRMpa)

Max Min Rate (MMR)

- **MMR:** Sub-carrier allocation + Equal Power Allocation based on Rhee et al [2]

System Fairness adaptive MMR (FMMR)

- **FMMR apa:** FMMR with Adaptive Power Allocation only (MMR sub-carrier allocation by Rhee [2] + FMMRpa)
- **FMMR dsa:** FMMR with Dynamic Sub-carrier Allocation only (FMMRsa + EPA)
- **FMMR joint:** FMMR with both Dynamic Subcarrier Allocation (FMMRsa) + Adaptive Power Allocation (FMMRpa)

Sum Rate Maximization with rate Proportionalities (SRM-P)

- **SRM-P:** Sub-carrier Allocation + Power Allocation by Wong et al [4]

System Fairness adaptive SRM-P (FSRM-P)

- **FSRM-P apa:** FSRM-P with Adaptive Power Allocation only (SRM-P sub-carrier allocation by Wong[4] + FSRM-Ppa)
- **FSRM-P dsa:** FSRM-P with Dynamic Sub-carrier Alloc. only (FSRMsa + EPA)
- **FSRM-P joint:** FSRM-P with both Dynamic Sub-carrier Alloc (FSRM-Psa) + Adaptive Power Allocation (FSRM-Ppa)

---

short notation used:

pa	:	**power allocation**
apa	:	**adaptive power allocation**
sa	:	**sub-carrier allocation**
dsa	:	**dynamic sub-carrier allocation**
joint	:	**both dsa and apa**

Algorithm Acronyms Notes:

For sake of generality, as described in sections 4.2 and 4.4, **a unique** proposed power allocation algorithm is used**,** in order to **either increase, or decrease fairness**, depending on the target SFT.

4.5.1. System Capacity versus System Fairness Index versus User Load

The following Figure 4.3 comprises the overview of all joint fairness adaptive algorithms, including the non-fairness adaptive ones (classical), versus all Fairness Index range and versus the user load of 4 up to 19 users.

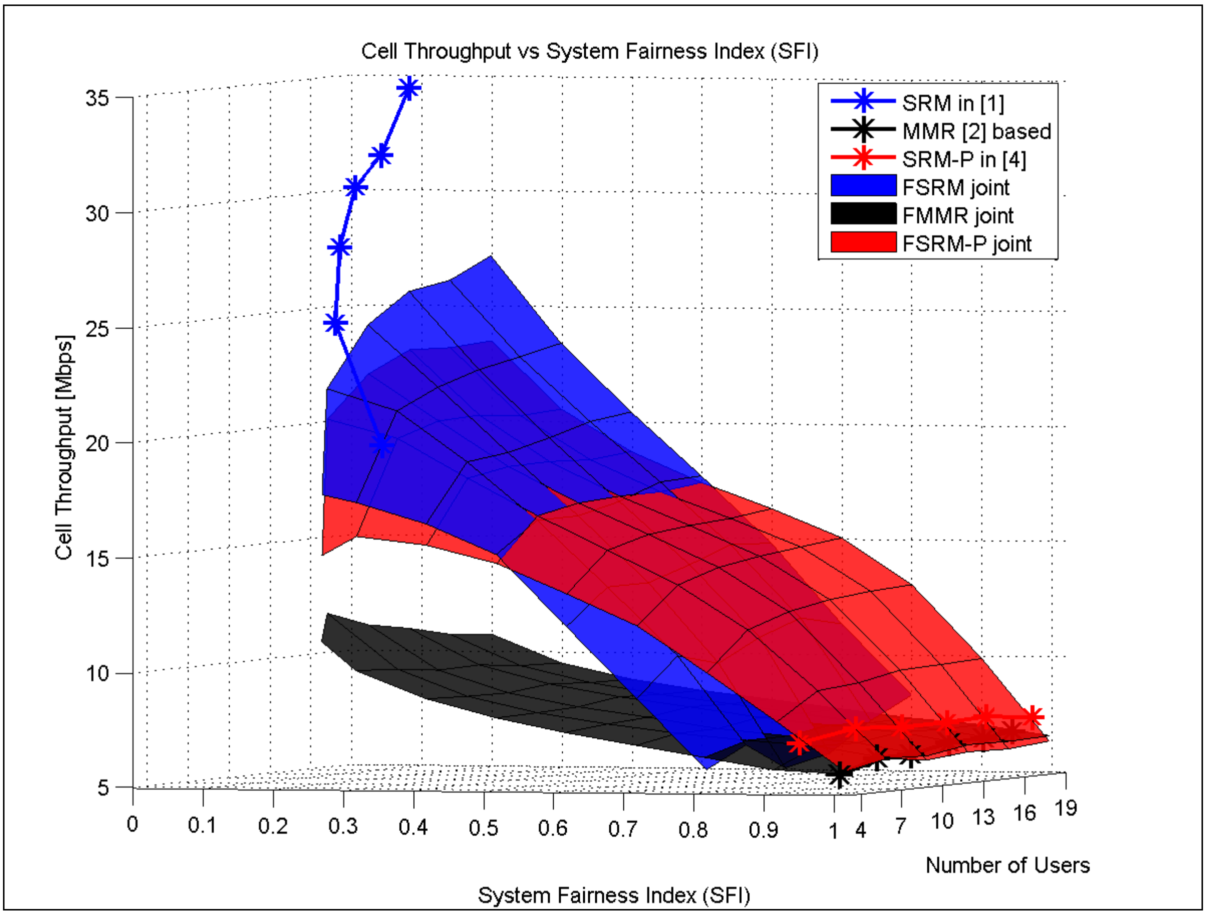

Figure 4.3: Cell Throughput versus Fairness versus Users

Classical algorithms are exactly as in Figure 3.1 and 3.2 combined and placed in the 3D plane according to their resulting throughput and fairness levels. Fairness Adaptive ones are also depicted as surfaces in order to study their performance and the corresponding Throughput – Fairness trade off from a macroscopic view. In the low fairness level plane, all algorithms appear to increase throughput by increasing the number of users, while in the high fairness plane appears to be more insensitive to the user load. In the previous chapter all metrics versus the user load have been studied for the classical algorithms, and as it is depicted in Figure 4.3, the **joint** fairness adaptive algorithms appear to converge in the extreme fairness planes where the classical ones are lying. **SRM** appears to increase the gap because it drops in lower SFI levels than 0.2, which consists the lowest simulated fairness target (SFT). **FMMR** due to its policy [11] to always assign resources to the best user, which is always the same one in both the sub-carrier and power allocation, the proportionalities are destroyed quite faster, so this reduces SFI in a more radical way – quite inefficient though, as shown in Figure 4.3. In all the following, the varieties of FMMR are intentionally not appearing in this work.

4.5.2. *System Capacity versus System Fairness Index*

In this section all fairness plane analysis is analyzed in the following by taking 3 slices of the 3D Figure 4.3 in low, medium, and high user load, also including all other varieties of the adaptive algorithms, which have been excluded from the 3D Figure 4.3 for viewing convenience.

The following Figure 4.4 is the overview of all algorithms, including the non-fairness adaptive ones (classical), for a low user load of 7 users. The system fairness target (SFT) of the fairness adaptive algorithms is set to 0.2 up to 1 with step 0.1.

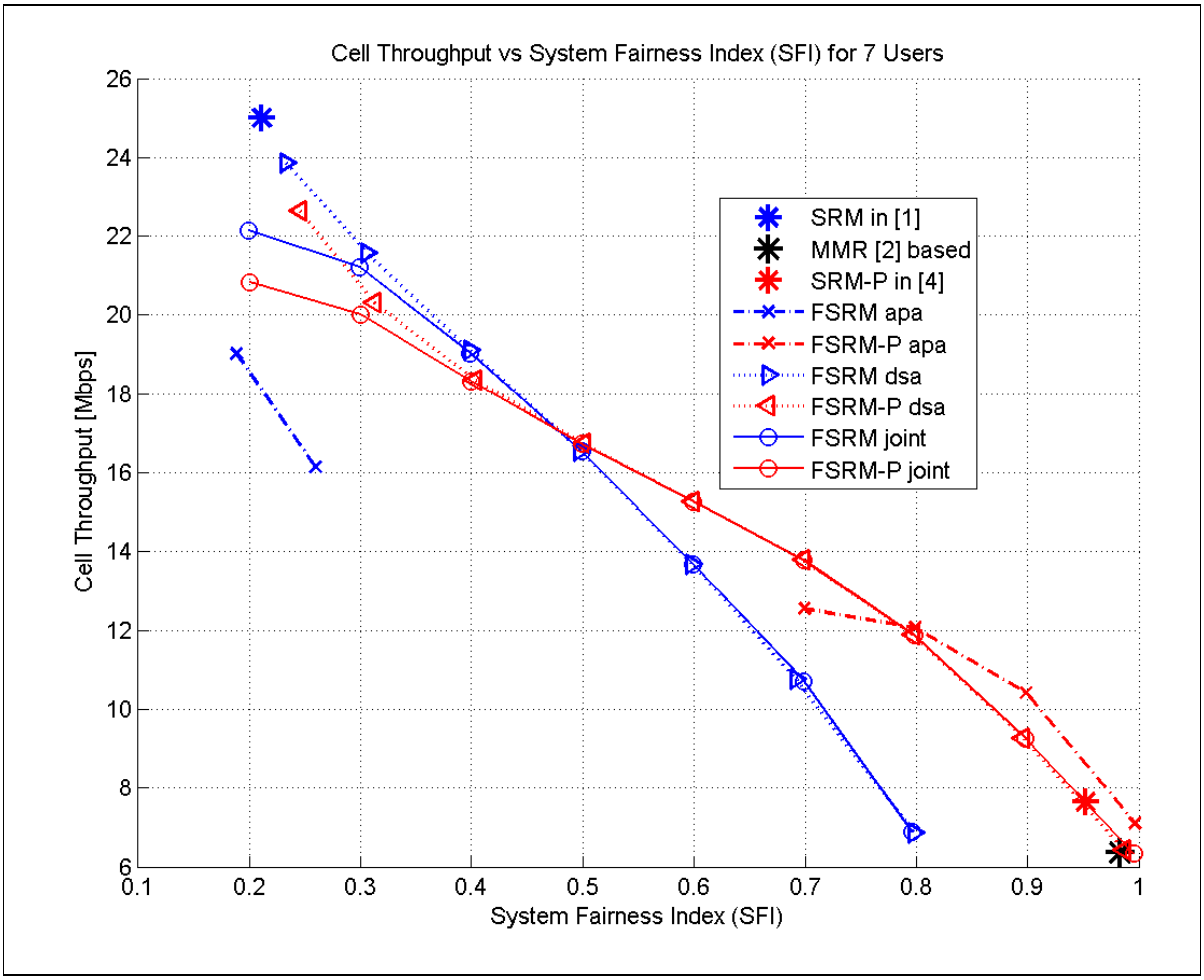

Figure 4.4: Cell Throughput versus System Fairness Index (SFI) for 7 users

Classical Algorithms

As discussed in the previous chapter 3, the classical algorithms are located in the two extreme sides of the systems fairness range as points, since are not fairness adaptive. **SRM** Algorithm, as expected, performs the best in terms of capacity, since it is performing purely capacity maximization, without considering any fairness; thus this value is related to the system's maximum capacity limit and it is considered unreachable by any other policy. On the other side, the other classical algorithms (MMR, SRM-P), that according to [3] they share the same objective, are located very close to SFI = 1, since their objective is to satisfy the rate proportionalities and that results the maximum fairness. The trade-off of the Capacity and the Fairness is obvious, since while significantly increasing fairness, capacity is reduced. As explained in the algorithms section, **SRM-P** compared with **MMR**, performs better in terms of capacity, but its SFI is slightly lower than MMR's. Thus, for the low user load of 7 users, the trade off is maintained and is not obvious which algorithm exploits better the resources; later on we will see clear advantage of SRM-P over MMR for higher user loads.

Fairness Adaptive Algorithms

Fairness Adaptive algorithms are the ones that are reaching a specific System Fairness Target, enabling SFI flexibility from the operator's side. FSRM algorithm is using as basis the SRM algorithm, thus is starting from low fairness levels; and by applying sub-carrier reallocation is roughly reaching the target. This route is obvious from the arrows of the sub-carrier allocation only **FSRM dsa** algorithm in Figure 4.4. Its SFI is roughly close to the target, while the corresponding joint is exactly meeting the target; this and any further change beyond the starting state of SRM causes a capacity reduction no matter if the SFI is increasing or decreasing. The differences between the **FSRM dsa** and the **FSRM joint** are greater in the extreme fairness levels (very low or very high) since there are more limitations due to the sub-carriers' allocation and it is not always possible to increase or decrease the SFI any longer. This impossibility is also appearing in the adaptive power allocation only- **FSRM apa** algorithm, in which for SFT = 0.3, in some cases is not possible to increase fairness any further, since there are no channels assigned to many users due to SRM sub-carrier allocation, and the maximum SFI is limited, as explained in Chapter 2 and in Annex III. In the remaining fairness range, the FSRM dsa roughly coincides with the joint, since the dsa algorithm has roughly satisfied the fairness target and later on, the extra power allocation step that is performed by the joint just applies small changes in order to exactly meet the target.

The same differences also apply to the **FSRM-P dsa** and **FSRM-P joint** algorithms for the lowest part of fairness, and in contrast to FSRM, there is capability of reaching any fairness level (from 1/K up to 1), since the algorithm begins the reallocations from high fairness state and all users are assigned sub-carriers. It is notable that the **FSRM-P dsa** algorithm, starting from high SFI (with the SRM-P algorithm), and by decreasing fairness down to the lowest extreme, tends to reach the maximum capacity of SRM. Also notable is that for a given SFT, the only apa algorithm (adaptive-power-allocation-only) that exceeds in capacity their corresponding dsa or joint is the **FSRM-P apa**, where successfully reaches the SFT of {0.8 0.9 and 1} and also achieves higher capacity, but as it will be shown in the following CPU time graph, it is inefficient, such as all power-allocation-only algorithms that try to cover a notable SFI gap and meet SFT.

As it can be seen in Figure 4.4, the dominating algorithm in the most unfair state is the SRM. For the range up to SFI = 0.5, FSRM is the dominating one, while for the remaining of the fairness range, FSRM-P is achieving higher rates. Last, for the fairest state, SRM-P is performing better. Up to this point the Capacity –

Throughput tradeoff is well seen, but in any case the overall performance conclusion should be also influenced by the following Figure 4.6, where the computational demand of each algorithm is depicted.

For **higher user loads** (13 and 19 users), as depicted in the following Figure 4.5, there are no significant variances in the Capacity versus Fairness plane.

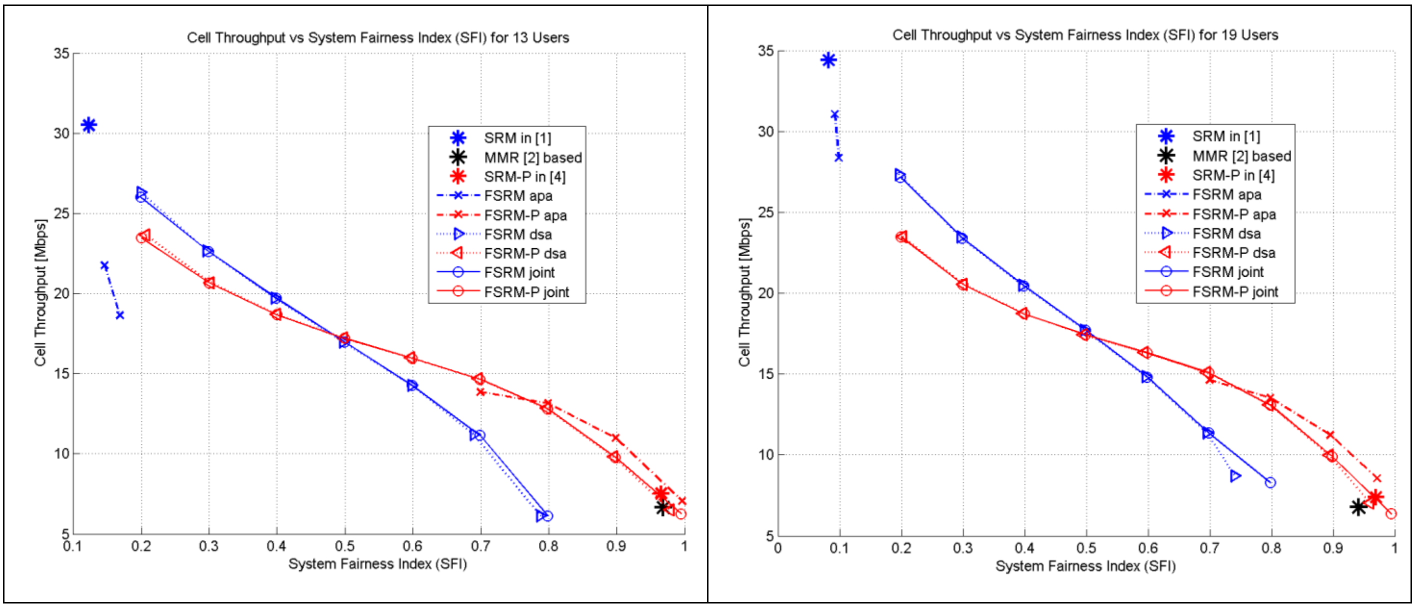

Figure 4.5: Cell Throughput versus System Fairness Index (SFI) for a. 13 and b. 19 users

The classical **SRM** significantly increases total cell throughput and also moves down to lower levels of fairness, since it is now closer to the limit of 1/K, where K is the number of users. Note that the value of SFI = 1/K is achievable if and only if one user is assigned all the resources, and all others remain connectionless. Also the limitations of the **FSRM apa** described previously is more obvious here for both cases of 13 and 19 users, since FSRM apa is unable to reach the SFT of 0.2 and 0.3. This is because the number of active users (i.e. the ones that have been assigned sub-carriers and do experience connection) when using SRM algorithm is not adequate to allow these higher fairness levels, so the result is the maximum feasible SFI. Last, the differences between the **FSRM-P dsa** and **FSRM-P joint,** in the low fairness levels of Figure 4.4 are not observable in Figure 4.5. The same apply for the **FSRM dsa** and **FSRM joint**, except for the fairest extreme case of the highest user load (19 users), where the dynamic sub-carrier allocation algorithm FSRM dsa fails to further increase SFI and meet the target.

In Figure 4.5, another small difference with respect to Figure 4.4, is that the classical **SRM-P** algorithm is clearly better than the **MMR**, since it performs better in both capacity and fairness metrics.

4.5.3. Average CPU Time versus System Fairness Index

Table 3.3 of Chapter 3 indicated that algorithms sensitivity is not significant regarding the number of users. Now, in order to compare the complexity of the fairness- adaptive algorithms, the measurements of the computational demand is depicted in the following Figure 4.6 as averages of the CPU time needed to complete the operation of each algorithm. Note that the y-axis scale is logarithmic and x-axis is still linear.

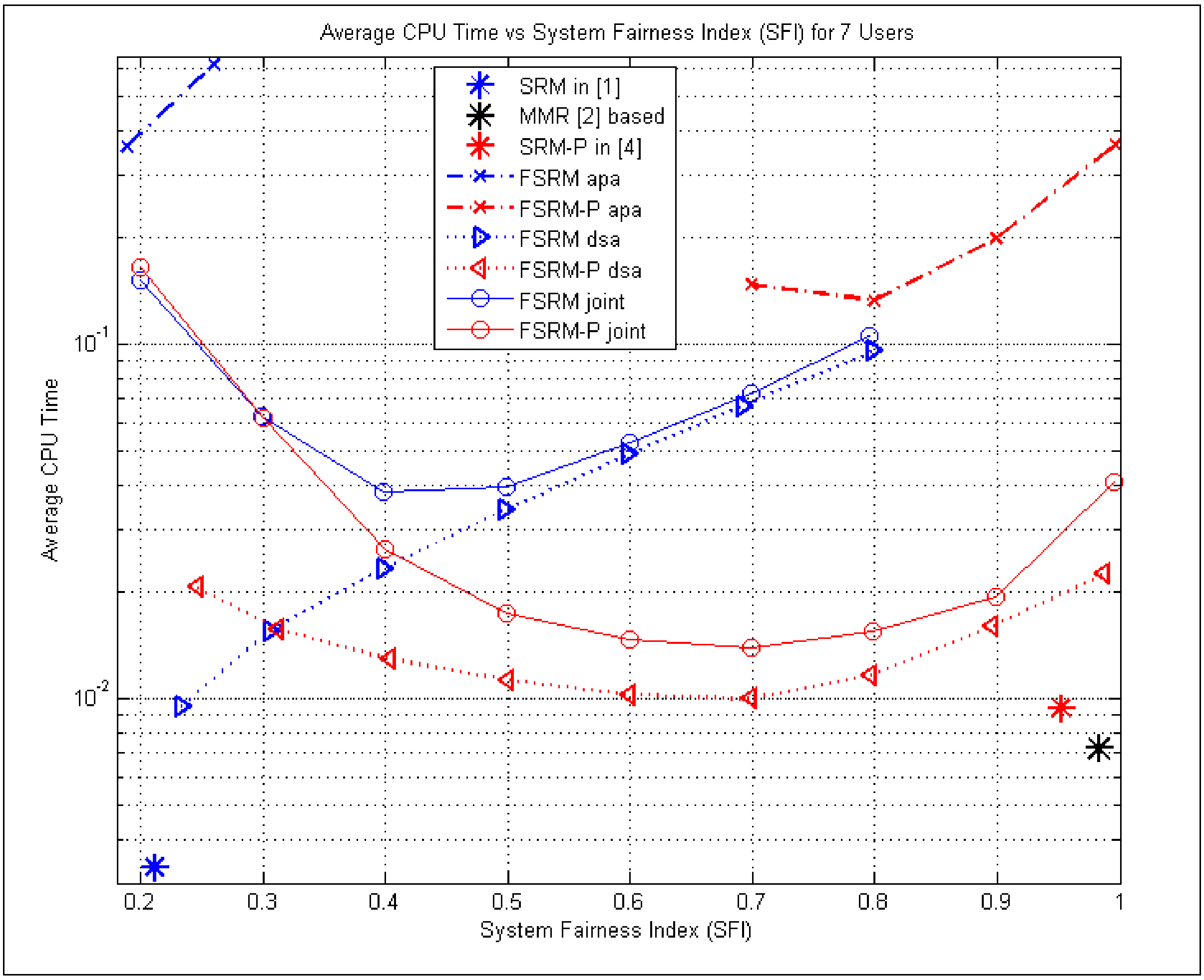

Figure 4.6: Average CPU time versus System Fairness Index (SFI) for 7 users

First observation is that all classical algorithms are quite computationally friendly compared to all fairness adaptive ones. The **SRM** is the overall faster one, since is only looking for the best channel gains and does the sub-carrier assignments to the corresponding users and then applies waterfilling for power allocation. The other classical algorithms **MMR** and **SRM-P** use more CPU time than SRM not only due to the calculation of the current user rates, but also due to the rate proportionalities tracking during both the sub-carrier and the power allocation procedures (MMR is performing only sub-carrier allocation).

For all fairness adaptive cases, of course the **joints** require more computational time than the respective dynamic sub-carrier allocation only ones (**dsa**'s), since the joints are additionally applying a fairness adaptive power allocation step. It is interesting that in both **FSRM** and **FSRM-P**, the time gap is lower for high SFI and gets larger for low SFI. This is justified by the fact that in unfair status (low SFI) the more efficient policy is to assign the best channel gain users more resources. In low SFI levels, the rate increments occurring by the dynamic sub-carrier allocation algorithm are higher and more radical, since any assignment of resources to the best channel gain users results maximum rate increments to the total capacity. In order for the joint algorithm to meet exactly the fairness target by performing the power allocation step, larger amounts of rate should be transferred from the small power reallocations, and it needs quite more power transfers from and to the weaker users in order to achieve the required rate transfers.

FSRM-P joint algorithm appears to be less computationally demanding close to 0.7, in the fairness plane, and it is justified from the fact that the resulting fairness level after the **SRM-Psa** sub-carrier allocation algorithm is around that value, so it is logical that it takes less time in this region, since it starts adjusting fairness from that point.

Both adaptive-power-allocation-only algorithms (**FSRM apa** and **FSRM-P apa**) are quite computationally inefficient, since they require more than 1 order of magnitude of more computational time than their dsa varieties, and 2 orders approximately more than the classical. They are so computationally inefficient even in the considered cases where SFT is close to their starting point, while for more distant SFTs the computational demand increases significantly.

Overall, by considering both computation demand and spectral efficiency the conclusion from a fairness perspective is that for **not strict** and **low SFT**, the **SRM** algorithm is by far the most efficient one, while for **SFT up to 0.3** the **FSRM dsa** approximately meets the desired SFT in less time, while for the rest fairer region (for **SFT > 0.3**) the **FSRM-P dsa** is preferred. Last for the fairer case (**SFT > 0.95**), **SRM-P** is the most efficient option. Thus, with 2 classical algorithms and their 2 corresponding dynamic sub-carrier allocation (dsa) algorithms the whole fairness plane is covered and the operator can switch in any fairness region they choose, while maximizing total cell throughput under the chosen SFT constraint.

4.5.4. User Satisfaction Index

In order to present satisfaction performance of the algorithms, it has to be noted that we should first choose a fairness target for the varieties of FSRM and FSRM-P algorithms, while the classical ones are not fairness adaptive and thus are fully described in one instance of the following graphs. For this, the fairness target SFT for the fairness adaptive algorithms has been set according to the following:

SFT {FSRM apa, FSRM-P apa, FSRM dsa, FSRM-P dsa, FSRM joint, FSRM-P joint} = { 0.2, 1, 0.6, 0.6, 0.6, 0.6}.

The following Figure 4.7 depicts the SFI level performed from those chosen SFT values.

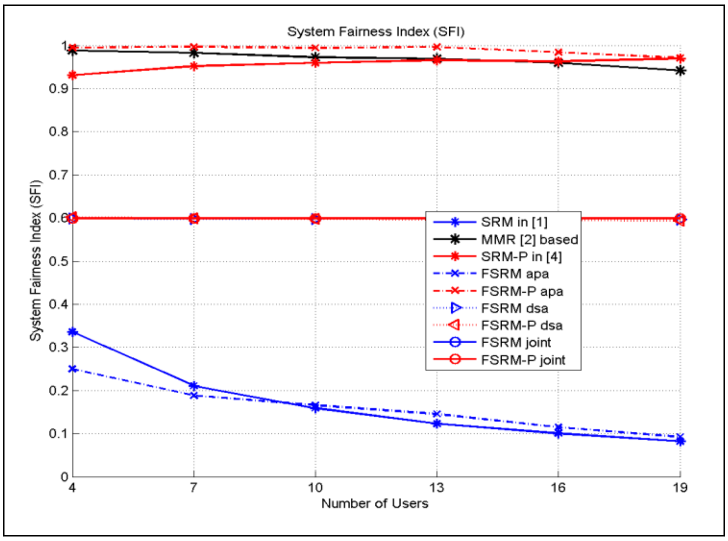

Figure 4.7: Fairness Index according to chosen target SFT

Note that as described in Anex III about apa limitations, **FSRM apa** algorithm while having a target SFT = 0.2, in the 4 users case the lowest SFI possible is ¼ = 0.25, while for higher user load of 10 users and above, is not possible to further increase fairness by only power reallocations. Thus, FSRM apa curve indicates the limits associated with the SRM subcarrier allocation. On the other side, **FSRM-P apa** SFT chosen equal to 1, as shown in Figure 4.7. The rest fairness adaptive dynamic sub-carrier allocation (**dsa**) algorithms and the **joints** runned under a SFT = 0.6 that has been successfully reached.

In the following Figure 4.8.a for the chosen SFT indicated above, the Long Term USI is depicted. Also note that the rate requirements are quite high so that the system becomes satisfaction sensitive and algorithms are more clearly performance distinguishable. First observation is that all **joint** algorithms are coincide with their corresponding dynamic sub-carrier allocation (**dsa**) only algorithms, since their minor impact is to meet more precisely the target, while dsa's are roughly meeting SFT. The most important observation is that these 4 algorithms (dsa and joints) are maintaining LT-USI in high levels while increasing user load and appear to be quite insensitive to user load compared to all other algorithms.

About the classical algorithms (**SRM**, **MMR**, and **SRM**-P) apply what discussed in Chapter 3, and are presenting here as a reference for the other algorithms. The most important observation is that SRM keeps their low level USI while increasing user load and requirements, while SRM-P and MMR are decreasing USI quite radical. Is worthy to be mentioned that **FSRM apa** is performing quite similar to SRM, while is unable to move far away from it, as shown in the Figure 4.7, while for the **FSRM-P apa** the small difference in SFI results in a small difference in terms of LT-USI.

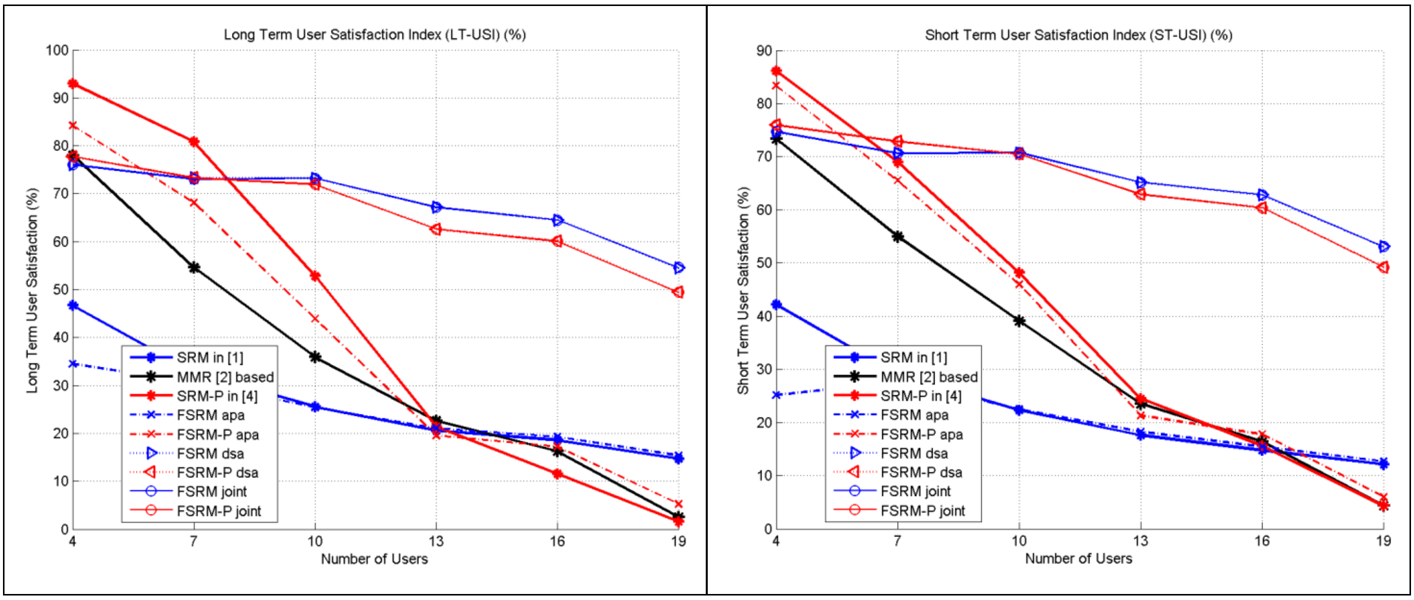

Figure 4.8: a. Long Term (LT-), b. Short Term (ST-) User Satisfaction Index (USI)

In Figure 4.8.b, Short Term USI is depicted. ST-USI is quite closer to the users' perspective and indicates a more objective metric about user satisfaction. As it can be seen after comparing Figure 4.8.a and 4.8.b, the similarities in the behavior of the algorithms are extremely high. Only difference is a small decrement / displacement in the y-axis, and is bounded to approximately 5%. Also the satisfaction gaps between closely performing algorithms has been slightly reduced.

Now, in order to go through user classes, the following Figure 4.9 depicts LT-USI for the three user classes: Bronze, Silver and Golden users respectively.

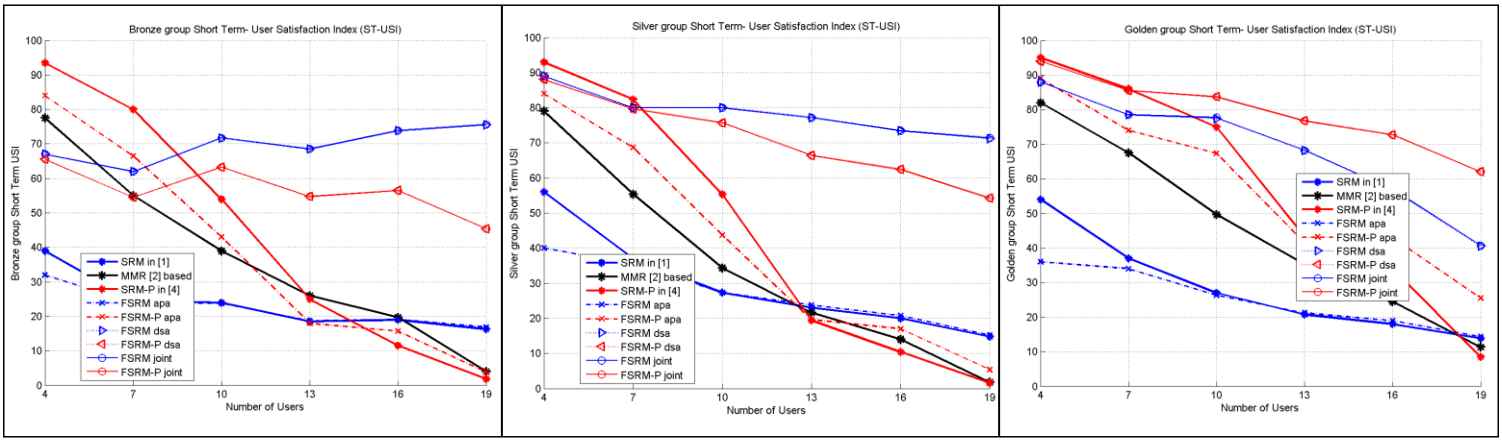

Figure 4.9: Per Group Long Term User Satisfaction Index (LT-USI)
a. Bronze group, b. Silver group, c. Golden group

In this more specific analysis the **joint** algorithms again coincide with their corresponding dynamic sub-carrier allocation (**dsa**) only algorithms, verifying the general observation made previously: that their minor impact is to meet more precisely the SFT, while the dsa's are roughly meeting SFT. All these 4 algorithms (dsa and joints) are maintaining LT-USI in high levels while increasing user load and appear to be somewhat insensitive to user load compared to all other algorithms.

General note: All the algorithms presented are only fairness-adaptive, i.e. their objective is to meet the chosen SFT target and they are not considering neither long, nor short term satisfaction objective. Considering the implementation and the objective of **all these algorithms**, an important notice is that they **are not user satisfaction-adaptive**, such as in [12].

Concluding with the USI versus user load, it can be said that the behavior is as depicted in Figure 4.8, and the amount of decrement varies depending on the rate requirements and the user load. In any case the behavior is monotonous and quite predictable.

The following Figure 4.10 depicts another interesting view of ST- and LT-USI versus fairness for 10 users.

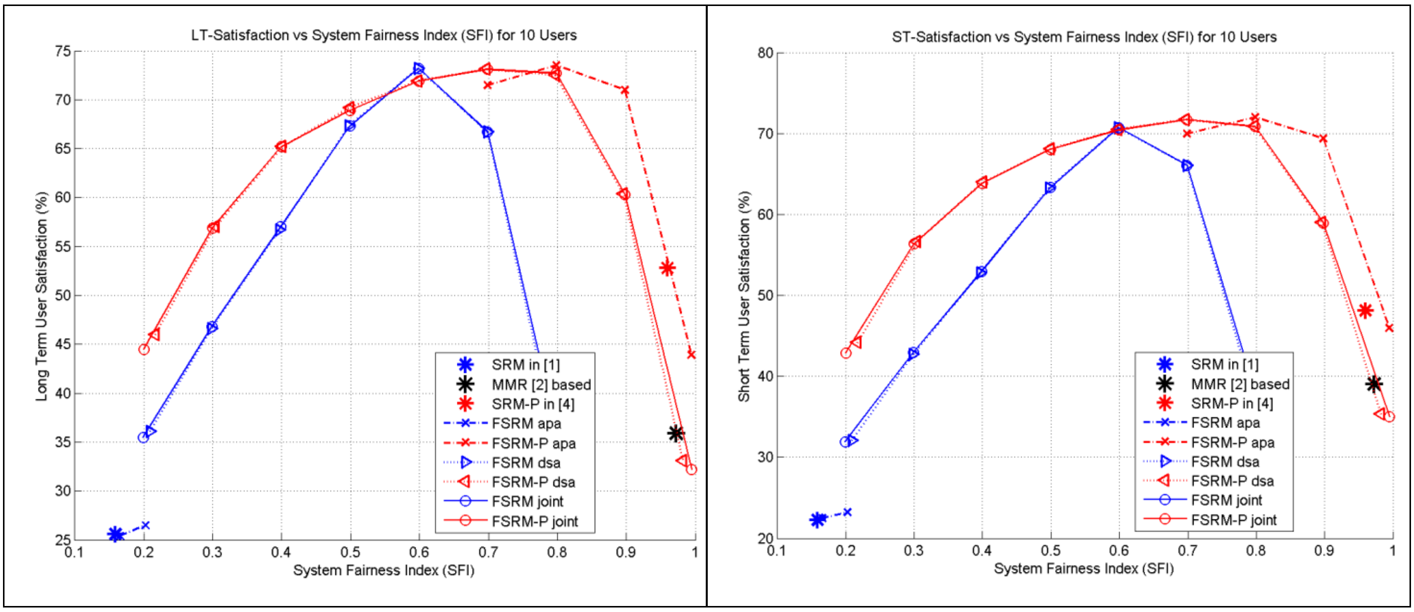

Figure 4.10: a. Long Term (LT-), b. Short Term (ST-) User Satisfaction Index (USI) versus System Fairness Index (SFI) for 10 users

As it can be seen in Figure 4.10.a, USI of **FSRM-P** is more wide-spreaded and almost always above **FSRM**, which is also true for other user loads. Furthermore, there is a SFI region that results maximum USI. Simulations show that this area of interest moves horizontally to the left when increasing user load, while it moves vertically by adjusting rate requirements. Therefore, given the users' rate requirements, network provider can choose the most efficient SFI to operate, while offering maximum satisfaction level to the users.

4.5.5. *Bar plot of the Rates*

In the following Figure 4.11.a is depicted how the rates have been distributed by the different algorithms to the users in the case of 10 users and chosen SFT = 0.6.

First observation is that both the **FSRM dsa** and **FSRM-P dsa** dynamic sub-carrier allocation algorithms are performing similar to their corresponding **joints**, all for the same chosen SFT equal to 0.6. In Figure 4.11.a pure rates distribution of FSRM algorithm appears to be quite better than FSRM, since the sum rate of all users is quite higher (also shown in Figures 4.4 and 4.5) and the distribution shown here indicates better efficiency for the SFRM-P algorithm. Notice that they all achieve SFT equal to 0.6, while they are distributing the rates (indirectly) in a different way. For example FSRM-P algorithm assigned users 1, 3, 4, and 8 less rate than their requirements, and in some cases less rate than FSRM. These users probably are weak users, since the system capacity of FSRM-P is quite higher than in the FSRM. Due to the selected policy, FSRMsa subtracts channels from the max rate user and not from the user with the max proportional rate, resulting many users assigned roughly equal rate, which is not the most efficient policy in terms of capacity; considering the general case of users with different rate requirements.

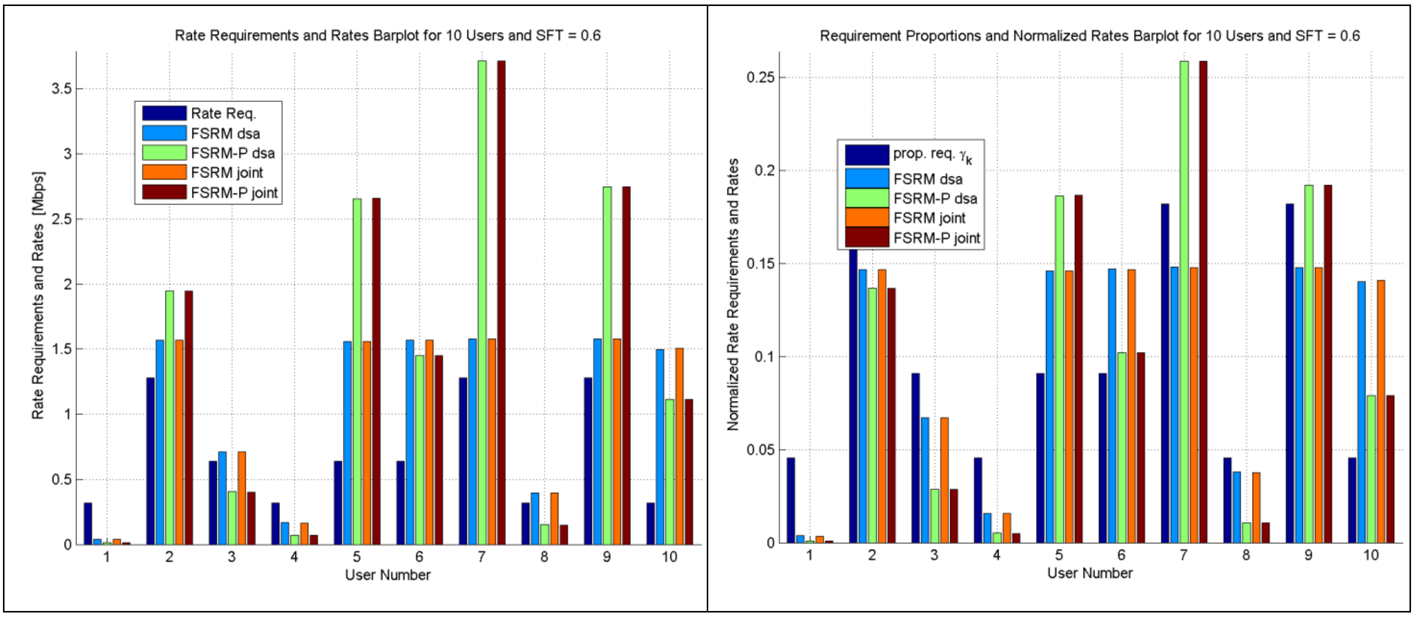

Figure 4.11.a: Bar plot of the Rates per user, b. Bar plot of the Normalized Rates

Notice that the relative difference of the bars of the algorithms in Figure 4.11.b is not following the same form as in Figure 4.11.a (check user 8), since when normalizing we lose the absolute value information. This is due to the performance difference of the algorithms with respect to the total cell throughput, always by considering pure data rates. Figure 4.11.a with pure rates is more associated with the satisfaction of the users (shows whether the requirements have been exceeded or not), while Figure 4.11.b is more associated with the fairness (shows how close to the proportional requirements are the rate proportions assigned by the algorithms).

4.5.6. CDF of the Rates

In the following Figure 4.12 the CDF of the rates is displayed for the case of 10 users and SFT = 0.6, in order to compare the fairness- adaptive algorithms under exactly the same requirements and targets. Figure 4.12 is also a more detailed way of viewing the information provided by Figure 4.11. For this case, all rate samples are included in the CDF. In the following, roughly speaking, only 2 curves are displayed instead of 4; this is because **joint** algorithms impact is trivial and they perform very similar to their corresponding **dsa**.

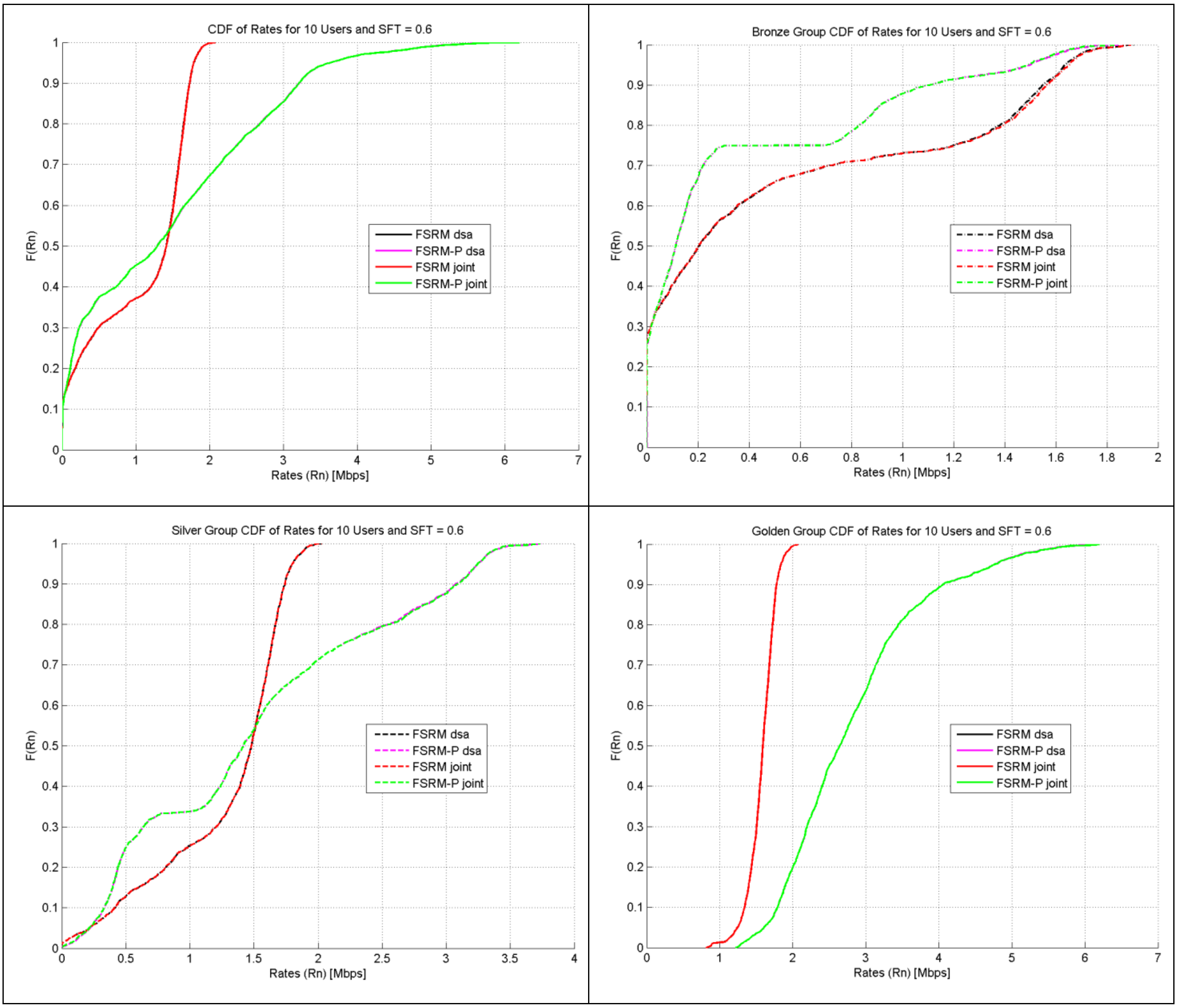

Figure 4.12.a: CDF of Rates for 10 users and SFT = 0.6,
b. Bronze group, c. Silver group, d. Golden group

In Figure 4.12.b it can be seen that 30% of the bronze users are assigning no resources from all **FSRM dsa** and **joint** and **FSRM-P dsa** and **joint**, while users from silver and golden groups are approximately always receiving resources. A clear advantage of FSRM-P for the golden class is depicted in Figure 4.12.d, where all golden users are assigned more than 1 Mbps up to 6 Mbps, while in FSRM are assigned roughly 1 Mbps up to 2 Mbps.

General note: All the algorithms presented here are not treating groups. Therefore any difference appearing at this point is not related to the group that a user belongs, but to the channel gain that is experiencing and their proportionality constraints. The algorithms are deciding to reallocate resources depending on the channel condition of the users, and not on the group they belong to. Any group behavior is limited to the simulation example, is not general, only observations and none conclusion on groups should be made.

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5. Conclusions and Future Work

The objective of this thesis is to study ways to adjust system fairness levels of a typical OFDM system through both the subcarrier and the power allocation step, which both comprise the typical resource management process in an OFDM system. Different approaches of System Fairness Index (SFI) – Adaptive algorithms have been tested by varying policy and parameters in order to provide satisfactory levels of *Spectral Efficiency versus Fairness Tradeoff* with respect to all metrics (satisfaction, fairness, and capacity) and satisfy both sides; *operator* mainly with high spectral efficiency and *user* with high Satisfaction and User Fairness Index. Therefore this work focuses in the potential adjustment of the fairness level by the operator in order to balance the mentioned tradeoff and allow them to more fairly distribute satisfaction to the users; while giving more insight on the users' satisfaction, approaching the problem closer to their perspective.

5.1 Conclusions

By considering three classes of users (gold, silver, and bronze), different policies are applied by varying parameters in order to visualize performance tradeoffs throughout the whole range of SFI for different user loads. The dominating Dynamic Resource Allocation (DRA) algorithm varies depending on the system fairness level that the operator chooses. For unfair system state with both loose and very low SFT, the classical SRM algorithm is by far the best option, while for very fair system state, classical SRM-P dominates; both are strongly recommended to be chosen, since both of them have significantly low computational complexity, since the power allocations that they apply have linear complexity on the number of subcarriers; and these two cases are approaching the two extreme levels of SFI.

When the operator chooses an in-between-fair state, then the simulations show that the proposed fairness adaptive FSRM algorithm is the dominating one for SFI<0.3, while for the other fairer range (SFI≥0.3) FSRM-P algorithm is performing best. Considering the unfair state, as an overview, the dominating scheduling only FSRM dsa algorithm slightly differs and is preferable than the corresponding FSRM joint in terms of capacity, since the later algorithm additionally performs an adaptive power allocation in order to more precisely meet the fairness target, and therefore the computational complexity is greater. Exactly the same differences apply for the fairer part of the SFI domain, where the dominating FSRM-P dsa algorithm is preferable than the corresponding joint for the same reason. Concluding, for the in-between-fair state, the two dynamic subcarrier allocation-only SRM dsa and SRM-P dsa algorithms are recommended to be used by the operator when choosing SFT<0.3 and SFT≥0.3 respectively.

The conclusion for all approaches of adaptive power allocation-only algorithms is that they require huge computational resources, especially when large gap of system fairness should be covered. In addition, capacity is reducing a lot, compared to other approaches. Generally speaking, adaptive power allocation-only algorithms have none advantage, thus are not recommended to be used. However, exclusively in the SRM-P case and only where a small SFI gap should be covered (SFT>0.95), by applying the adaptive power allocation-only SRM-P apa algorithm, the capacity is higher than the classical SRM-P algorithm, but still the computational burden is quite high.

Golden users appear to be more satisfied with the FSRM-P algorithm in most cases. The same applies for the bronze and silver users, except from some cases of middle user load where FSRM offers more satisfaction. However, none of the policies is considering group treatment, so the group conclusions are only observations, are not general, and also depend on the rate requirements of the users. The dominating algorithm that covers most cases and offers both best spectral efficiency and better satisfaction is the FSRM-P.

As a general conclusion taken from simulation results, where rate requirements are high enough, so as the system becomes satisfaction-sensitive, better user satisfaction occurs most likely by using the algorithms that perform better in terms of capacity, given a reference system fairness target, and also considering computational efficiency.

5.2 Future Work

The algorithms implemented are fairness-adaptive only, means that no provision for user satisfaction has been considered for any specified time window. It means that instantaneous satisfaction in some cases unnecessarily may be quite large for some users, and also the additional dimension of time diversity has not been exploited. By introducing time-window diversity intuitively will result quite large improvement in the user satisfaction indexes, at a relatively smaller reduction in the total system throughput.

The algorithms could be expanded in order to treat user classes in a particular way. Priorities and other policies can apply to benefit or not some user classes depending on the decisions made by the operator.

Hybrid policies while increasing or decreasing fairness, in terms of higher capacity should also be considered. In a policy that decreases fairness, such as the FSRM-Psa algorithm, the most capacity prosperous result should be chosen from either choosing to shut down some users, or by redistributing resources among all active users. Also, in the general case of users belonging in different classes, FSRMsa will be significantly improved if instead of removing sub-carriers from the max rate user to do so from the user with the maximum proportional rate.

The tradeoff between system fairness and capacity is a quite complex problem. For a given SFT, there are numerous user rate combinations that result the same SFI. The objective is to find which combination maximizes the optimization objective function (i.e. which combination results highest system capacity). One interesting approach, also described in Annex II, is to apply a best predefined user rate distribution that is known and results the desired SFI. This rate distribution should be the optimal one that fits better to the user channel gains so that results the most total throughput. This problem is then comprised of a search over limited instances and a linear subcarrier allocation algorithm, which is fast and accurate.

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ANNEX I. Convexity Analysis

Definition of a *convex function*: For an objective function to be convex, the domain of the function must be a convex set. If it is a convex set, according to the second order condition ([1] p71), the 2nd order derivative of the objective function should be positive.

For a twice differentiable function f with convex domain:

f is convex if and only if $\nabla^2 f(x) \geq 0, \forall x \in domf$

Definition of a *convex set*: A set S, where $S \in \{1, N\}$, a set of channels in our case, belong to an $\Re^n$ space is convex if $\forall x_1, x_2 \in S$, $\theta \in [0,1]$, holds that $\theta x_1 + (1-\theta) x_2 \in S$ as well, means any line segment between two points of the set S also belongs to the set S ([1] p23).

Since in all Resource Management (RM) problems mentioned the domain S of the objective function is a summation over an integer set of channels, which is not a convex set, therefore the RM problem is not convex [2,3].

The summation mentioned above holds under the constraint that each channel $\forall x \in S$ should be assigned to one user at most. By relaxing this constraint (by adding an auxiliary variable), convexity is proved by [3] in Appendix I. The *connection indicator* $c_{k,n}$ is 1 if there is a connection, otherwise is 0.

If scheduling is performed then the problem of SRM is dealt with the method of Lagrange multipliers ([2] eq9). The solution of the problem yields a waterfilling method proved in [1] p245 and [3].

In our case, the proportional rate constraints $R_1 : R_2 : ... : R_K = \gamma_1 : \gamma_2 : ... : \gamma_K$ are taken into consideration, note that these ratios are non-linear, and the following problem (3.4), presented in section 3.3 is formulated as:

Initial problem formulation

$$
\begin{aligned}
\max_{c_{k,n},\, p_{k,n}} \quad & \frac{B}{N} \sum_k \sum_n c_{k,n} \cdot \log_2(1 + p_{k,n} \cdot H_{k,n}) && (o) \\
\text{subject to} \quad & c_{k,n} \in \{0,1\} \quad \forall k,n && (c1) \\
& p_{k,n} \geq 0 \quad \forall k,n && (c2) \\
& \sum_k c_{k,n} = 1 \quad \forall k,n && (c3) \\
& \sum_k \sum_n c_{k,n} p_{k,n} \leq P_{\max} && (c4) \\
& R_i : R_j = \gamma_i : \gamma_j \;\; \forall i,j \in \{1,..,K\}, i \neq j && (c5)
\end{aligned}
\qquad (3.4)
$$

In [3], the Lagrange multipliers technique is used which yields the optimal power allocation. According to [4], this is an NP-hard combinatorial optimization problem with non-linear constraints, and the computational complexity is such that it is highly improbable that polynomial time algorithms will be used to solve it optimally. The authors based in the assumptions of [5], made a simplification for the last constraint (c5) and introduced the predefined N_k the number of channels that users will be allocated. This way they satisfy constraint (c5) as the transformed one $N_1 : N_2 : ... : N_K = \gamma_1 : \gamma_2 : ... : \gamma_K$, implying that the amount of rate that a user may require will be proportional to the number of subcarriers they should be assigned. This way, after the subcarrier allocation, the objective (o) of the problem (3.4) is simplified into a maximization over *continuous* power variables:

SRM-P problem formulation

$$\begin{aligned} \max_{c_{k,n},\, p_{k,n}} \quad & \frac{B}{N}\sum_{k}\sum_{n}\log_2(1+p_{k,n}\cdot H_{k,n}) && (o) \\ \text{subject to} \quad & p_{k,n}\geq 0 \qquad \forall k,n && (c2) \\ & \sum_{k}\sum_{n} p_{k,n}\leq P_{\max} && (c4) \\ & R_i : R_j = \gamma_i : \gamma_j \ \forall i,j\in\{1,..,K\},\, i\neq j && (c5) \end{aligned} \qquad (3.4)$$

Note the absence of the subcarrier indicator $c_{k,n}$.

For sake of generality, in case all the unknown variables are integers (not the RM case), then the problem is an integer programming (IP) problem. IP problems are in many practical situations where bounded variables do exist. Binary integer programming (BIP) is the special case of integer programming where variables are required to be 0 or 1 (rather than arbitrary integers). This problem is also classified as NP-hard. The variables in scheduling are integer, since the connection indicator $c_{k,n}$ is either 0 or 1, which is not a convex set.

In the case where some of the unknown variables are integer and some real, the problem is called a mixed integer programming (MIP) problem. The variables in power allocation are real. These problems are classified in general also as NP-hard and there are computationally inefficient in most of the cases. However, some subclasses of IP and MIP problems, such as problems with totally unimodular constraint matrices and the right-hand sides of the constraints are integers, are quite efficient in terms of computational cost.

Conclusions:

The SRM and MMR problems are dealt as two-stage problems, first a heuristic scheduling and then a power allocation with the technique of Lagrange multipliers [2-4]. This policy is used in all problems found in the references. The initial forms of the SRM, MMR, and SRM-P, and in general all RM problems are NP-hard and non convex. So it is in the case that an extra constraint is added either for the rate proportionalities about the pre-mentioned problems, or for the SFT level for the problems of FSRM, FMMR, and FSRM-P in respect.

Therefore, even with this two-step approach, when scheduling and power allocation problems are split, even if a simplified suboptimal heuristic scheduling has been performed, the resulting power allocation problem can still be non convex. A general technique that is used in these kinds of problems is the use of the Lagrange multipliers. The resulting Lagrangian function is always a concave function even if the initial problem is not ([1] p216) and yields a lower bound on the optimal value ([1] eq5.15 and figure 5.2). If and only if the initial-primal problem is convex and Slater's condition holds then the duality gap is zero. Otherwise weak duality holds and the lower bound is less than the optimal value.

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ANNEX II. Fairness Analysis

In the following Figure IV.1, there are some possible values of SFI by altering the rates of 4 users. For simplicity, the system can provide 128 kbps and rates only from user 1 are showed, while all possible combinations of rate are distributed to the other 3 users, always summing up to 128 kbps, and by using a fairly small rate step. As it is expected, for very low SFI = 1/K, where K=4 users, all the rate is going to user 1, while others getting no connection. Then for higher SFI other users are assigned rate and very many combinations occurred. The other extreme happens when SFI=1 and all users get equal rates of 128 / 4 = 32 kbps each.

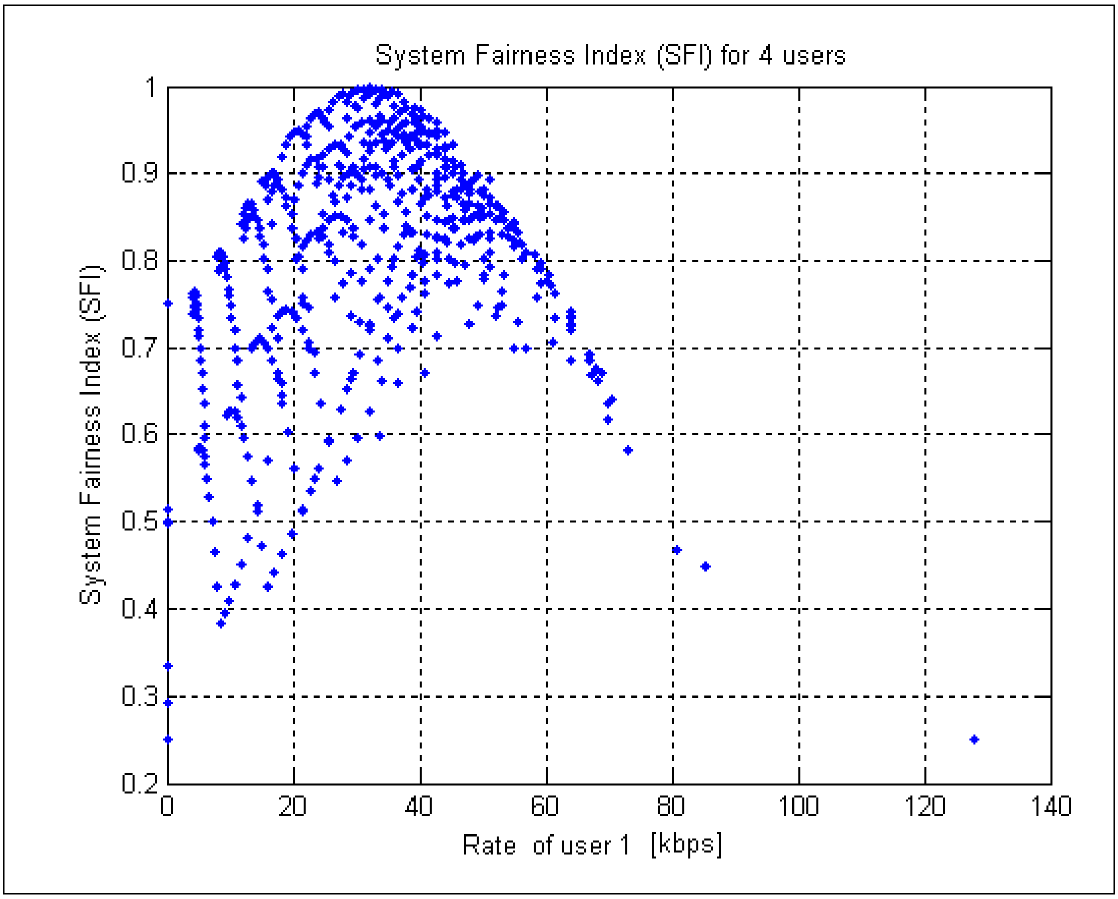

Figure IV.1: SFI vs Rate of user 1 out of 4

An approach for obtaining a specified fairness target, while maximizing total capacity

In [1-4] an interesting heuristic scheduling has been proposed, in order to keep the proportionalities, as described in chapter 2 about fairness and in Algorithm SRM-P as well. This scheduling is tracking the lowest proportional rate and assignes them resources, in order to equalize the predefined rate proportionalities. In this case we consider a "mask" or a rate distribution that fits in that mask. In this case the mask is exactly the predefined rate proportion γ_k, and the resulting fairness is very close to one, which is the highest possible SFT that can be applied.

As described before, for a given SFT<1 there are numerous combinations that result the desired SFT. One interesting approach to find which one results highest capacity is to brute force once all rate combinations for all fairness targets, and create some tables that will be quite large in general, but there are possible ways to significantly decrease the combinations. In order to obtain the rate distribution we need, only a reverse searching on the desired SFI is necessary. Once this happens, many matches will occur, all with roughly the same SFI; from all these occurrences, the rate distribution that is closer to the user normalized channel gain (UNCG), intuitively is more likely to be the one that results highest capacity. UNCG is the normalized channel gain for each user, as defined in [5],

$$\tilde{H}_k = H_k \Big/ \sum_{k=1}^{K} H_k \,,$$

where $H_k = \sum_{n=1}^{N} H_{k,n} / N$ is the relative channel gain for user *k*. By saying "distribution that is closer to..." means the distribution subtracted by the UNCG distribution results the minimum standard deviation. Once we obtain the predefined user rate distribution from the tables and apply it for a given SFI level. The complexity of this process is similar to the one proposed in [2] and [4]. It is fast since it is linear, and accurate that results SFI very close to the target SFT.

Key Assumption:

For a chosen SFT, the optimal rate distribution, which is unknown, if normalized results in the optimal normalized rate distribution. Latter distribution's deviation from the UNCG distribution is minimized. According to [1], sum rate maximization occurs when assigning the resources to best users. In this case, not only best users should be assigned resources, in order to meet SFT, but best users are assigned more resources than weaker ones in the most capacity efficient way.

References

[1] Jiho Jang, and Kwang Bok Lee; Transmit Power Adaptation for Multiuser OFDM Systems, IEEE Journal on Selected Areas In Communications, Vol. 21, No. 2, February 2003

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[3] Zukang Shen, Jeffrey G. Andrews, and Brian L. Evans; Adaptive Resource Allocation in Multiuser OFDM Systems With Proportional Rate Constraints, IEEE Transactions On Wireless Communications, Vol. 4, No. 6, November 2005

[4] Ian C. Wong, Zukang Shen, Brian L. Evans, and Jeffrey G. Andrews; A Low Complexity Algorithm for Proportional Resource Allocation in OFDMA Systems, SIPS 2004, 2004

[5] S. Han, S. Kim, E. Oh, and D. Hong; Adaptive Resource Allocation with Rate Proportionality Tracking in OFDMA Systems, IEEE 65th Vehicular Technology Conference, VTC2007-Spring, pp 3031 – 3035, 2007

ANNEX III. SRM apa limitations

As described in section 3.1, the SRM sub-carrier allocation algorithm in [1] is assigning channels to the users that experience best channel gains. Therefore far users always remain without connection. As noted in Chapter 2 in Fairness section, and also commented in Annex II, for a given system fairness target SFT, a minimum number of active users is required in order to be possible to achieve that particular SFT. In the simple example illustrated in Figure 2.4, where 3 users considered, we cannot obtain SFI > 2/3 when at least one user is inactive. This happens always when SRM sub-carrier allocation algorithm is applied and allows connection only to users closest to the BS and strongest in path gain. As a result only a small fairness region located in the unfair side can be achievable by applying any adaptive power allocation afterwards. In other words, by applying SRM sub-carrier allocation algorithm, SFI is extremely low, close to 1/K, where K is the total number of users (active + inactive). The maximum SFI that can be achieved by applying an adaptive power allocation is limited by the number of active users. Active users are the ones that experience connection; the ones that have been assigned at least one subcarrier and power greater than zero, so bits can be transmitted through their connection.

This very example is illustrated in the following Figure III.1, where the System Fairness is shown for all SRM approaches with SFT = 0.6 and different user loads, these approaches are described in detail in Chapters 3 and 4.

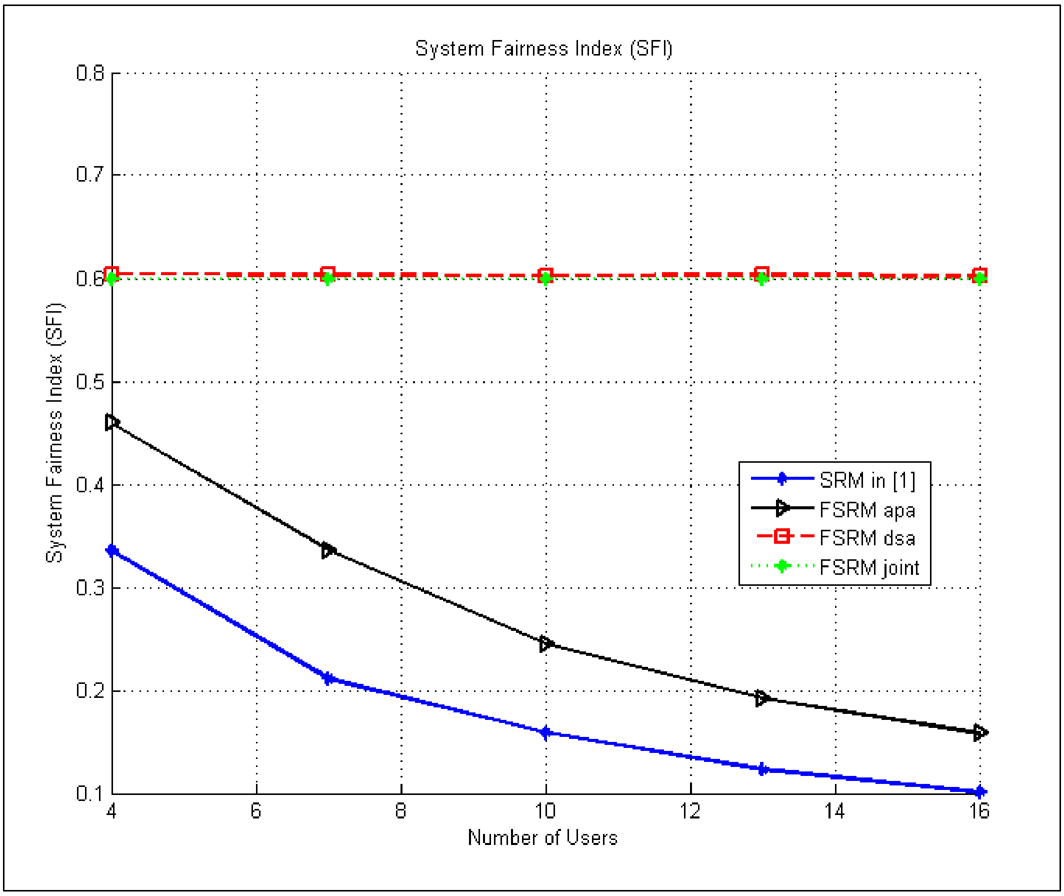

Figure III.1: System Fairness Index

As it can be seen in Figure III.1, given fairness target set to 0.6, the adaptive-power-allocation-only algorithm FSRM apa can only increase fairness around 0.1, by using the SRM sub-carrier allocation as described in the algorithms section. With the given channel allocation of SRM algorithm, the SFI is not possible to be increased any further by using any power allocation algorithm. This procedure of course is not efficient, since the subcarrier allocation of SRM is clearly giving connection only to the users that experience best channel conditions, and this objective should be followed in order to keep capacity in high levels. As it can be seen in the following Figure III.2, the F-SRM apa algorithm while performing in capacity quite lower than SRM (as expected), performs similar capacity as the dynamic subcarrier reallocation enabled algorithms F-SRM dsa and F-SRM joint, whose fairness level is quite higher, since they both manage to meet the target. Also the computational demand of F-SRM apa algorithm is more than one order of magnitude higher than the subcarrier reallocation enabled ones. Therefore, the conclusion for the apa algorithm is that has no benefit when the SFT is far from the very low fairness level provided by the SRM sub-carrier allocation algorithm.

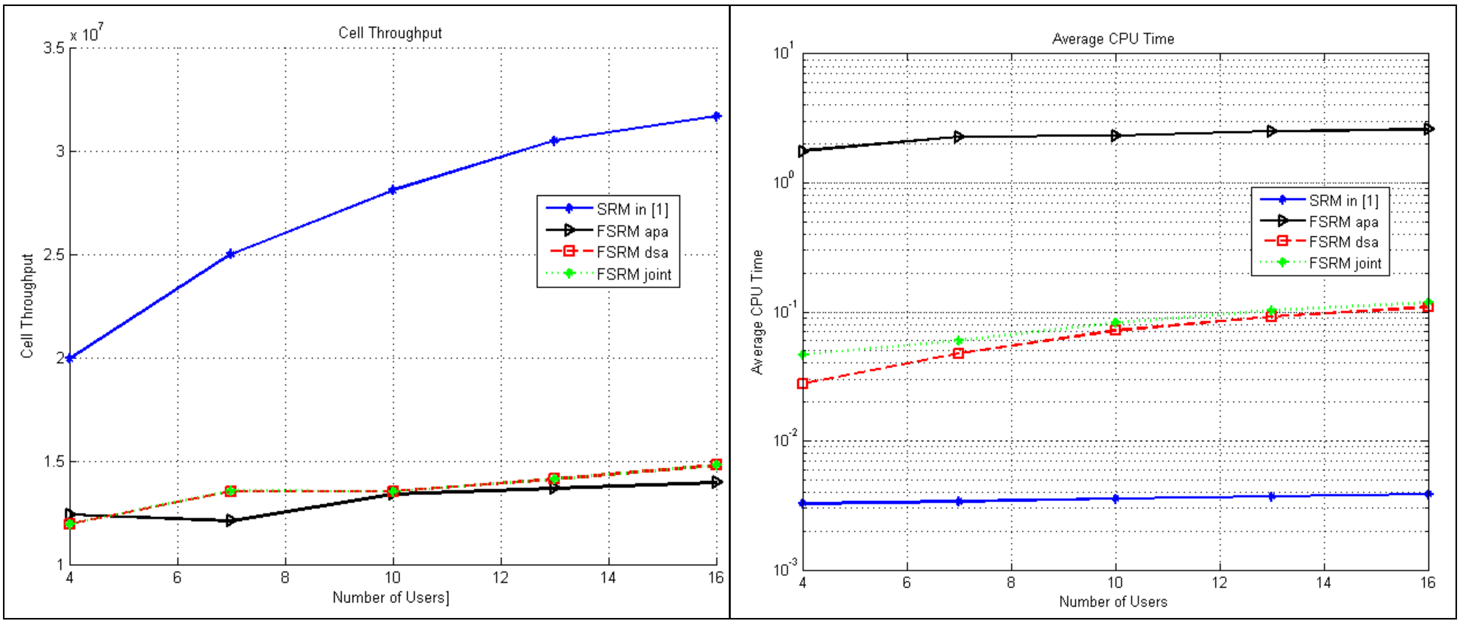

Figure III.2: Cell Throughput and Average CPU time

References

[1] Jiho Jang, and Kwang Bok Lee; Transmit Power Adaptation for Multiuser OFDM Systems, IEEE Journal on Selected Areas In Communications, Vol. 21, No. 2, February 2003

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List of Algorithms

List of Algorithm Acronyms

Sum Rate Maximization (SRM)

- **SRM:** Sub-carrier Allocation + Waterfilling Power Allocation by Jang et al [1]

System Fairness adaptive SRM (FSRM)

- **FSRM apa:** FSRM with Adaptive Power Allocation only (SRM sub-carrier allocation by Jang [1] + FSRMpa)
- **FSRM dsa:** FSRM with Dynamic Sub-carrier Allocation only (FSRMsa + EPA)
- **FSRM joint:** FSRM with both Dynamic Sub-carrier Allocation (FSRMsa) + Adaptive Power Allocation (FSRMpa)

Max Min Rate (MMR)

- **MMR:** Sub-carrier allocation + Equal Power Allocation based on Rhee et al [2]

System Fairness adaptive MMR (FMMR)

- **FMMR apa:** FMMR with Adaptive Power Allocation only (MMR sub-carrier allocation by Rhee [2] + FMMRpa)
- **FMMR dsa:** FMMR with Dynamic Sub-carrier Allocation only (FMMRsa + EPA)
- **FMMR joint:** FMMR with both Dynamic Subcarrier Allocation (FMMRsa) + Adaptive Power Allocation (FMMRpa)

Sum Rate Maximization with rate Proportionalities (SRM-P)

- **SRM-P:** Sub-carrier Allocation + Power Allocation by Wong et al [4]

System Fairness adaptive SRM-P (FSRM-P)

- **FSRM-P apa:** FSRM-P with Adaptive Power Allocation only (SRM-P sub-carrier allocation by Wong[4] + FSRM-Ppa)
- **FSRM-P dsa:** FSRM-P with Dynamic Sub-carrier Alloc. only (FSRMsa + EPA)
- **FSRM-P joint:** FSRM-P with both Dynamic Sub-carrier Alloc (FSRM-Psa) + Adaptive Power Allocation (FSRM-Ppa)

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short notation used:	**pa**	:	**power allocation**
	apa	:	**adaptive power allocation**
	sa	:	**sub-carrier allocation**
	dsa	:	**dynamic sub-carrier allocation**
	joint	:	**both dsa and apa**

Algorithm Acronyms Notes:
For sake of generality, as described in sections 4.2 and 4.4, **a unique** proposed power allocation algorithm is used**,** in order to **either increase, or decrease fairness**, depending on the target SFT.

List of Symbols

Symbols	
k	$:user\ index,\ k \in [1,2,..,K]$
K	$:total\ number\ of\ users$
n	$:subcarrier\ index,\ n \in [1,2,..,N]$
N	$:total\ number\ of\ subcarriers$
$p_{k,n}$	$:subcarriers'\ power\ w.r.t. user\ k$
$P_{\max}$	$:maximum\ BS\ transmitted\ power$
B	$:total\ bandwidth\ available$
$c_{k,n}$	$:subcarrier\ allocation\ indicator, c_{k,n} \in \{0,1\}$
$H_{k,n}$	$:effective\ subcarrier\ SNR$
$r_{k,n}$	$:subcarriers'\ rate\ w.r.t. user\ k$
R_k	$:users'\ rate\ vector$
γ_k	$:user\ rate\ proportionality\ constrains, \gamma_k \in [0,1]$
R_k^{req}	$:user\ rate\ requirements$
φ_k	$:user\ fairness\ index\ (UFI)$
Φ	$:system\ fairness\ index\ (SFI)$
Φ_t	$:system\ fairness\ target\ (SFT)$
Bullets	
$+$	$:advantages$
$-$	$:dissadvantages$
$\pm$	$:positive\ and\ /\ or\ negative$
o	$:neutral$

Notation notes:

1. $p_{k,n}, c_{k,n}$, and $r_{k,n}$ in the general case can be matrices with size KxN. Constraints are applied so that sub-carrier *n* is assigned to a maximum of one user *k*. The connection matrix $c_{k,n}$ shows whether a sub-carrier is assigned to user *k* or not, by having values 1 and 0 respectively. Thus each of $c_{k,n}$'s columns will sum up to 1.

2. $p_{k,n}$ in many cases inside the algorithms is used as vector p_n. Note that *k* user index is absent and the size of these vectors is 1xN. By using another vector, the *channel allocation* vector, which is the output of the *channel allocation algorithm*, we have the correspondences about which sub-carrier is assigned to which user, so the sub-carrier power vector p_n, along with the channel allocation vector contain all the information we need.

3. $H_{k,n}$ and $r_{k,n}$ can be found as H_n and r_n, respectively. What described in 2 applies in the same way. $H_{k,n}$ is the effective subcarrier SNR as defined in [4], including channel gains, noise power and SNR gap.

List of Abbreviations

3GPP	3rd Generation Partnership Project
AMC	Adaptive Modulation and Coding
APA	Adaptive Power Allocation
AWGN	Additive White Gaussian Noise
B3G	Beyond 3G
BER	Bit Error Rate
BS	Base Station
CDMA	Code Division Multiple Access
CPU	Central Processing Unit
DAB	Digital Audio Broadcasting
DSA	Dynamic Sub-carrier Allocation
DVB	Digital Video Broadcasting
EPA	Equal Power Allocation
FDMA	Frequency Division Multiple Acces
F-MMR	Fairness adaptive- Maximize the Minimum Rate
F-SRM	Fairness adaptive- Sum Rate Maximization
F-SRM-P	Fairness adaptive- Sum Rate Maximization under Proportional Rate Constraints
ICI	Inter-Channel Interference
IFFT	Inverse Fast Fourier Transform
IP	Integer Programming
ISI	Inter-Symbol Interference
LTE	Long Term Evolution
LT-USI	Long Term User Satisfaction Index
MA	Margin Adaptive
MAI	Multiple Access Interference
MMF	Maximum Minimum Fairness
MMR	Maximize the Minimum Rate
MR	Maximize Rate
MT	Mobile Terminal
NRT	Non Real Time
OFDM	Orthogonal Frequency Division Multiplexing

OFDMA	Orthogonal Frequency Division Multiple Access
PA	Power Allocation
PF	Proportional Fairness
QAM	Quadrature Amplitude Modulation
QoS	Quality of Service
QPSK	Quadrature Phase Shift Keying
RA	Rate Adaptive
RRA	Radio Resource Allocation
RRM	Radio Resource Management
SA	Sub-carrier Allocation
SES	Simple Exponential Smoothing
SFI	System Fairness Index
SFT	System Fairness Target
SNR	Signal to Noise Ratio
SRM	Sum Rate Maximization
SRM-P	Sum Rate Maximization under Proportional Rate Constraints
ST-USI	Short Term User Satisfaction Index
TDMA	Time Division Multiple Access
TTI	Transmission Time Interval
TU	Typical Urban
UDR	User Dissatisfaction Ratio
UFI	User Fairness Index
UMTS	Universal Mobile Telecommunications System
UNCG	User Normalized Channel Gain
USI	User Satisfaction Index
USR	User Satisfaction Ratio
VoIP	Voice over IP
WLAN	Wireless Local Area Network
WWW	World Wide Web