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COORDINATION MECHANISMS IN SUPPLY CHAIN

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Abstract

This thesis attempts to analyze coordination mechanisms between producers and suppliers in a supply chain. Since the entities in a supply chain work usually independently each from the other, it is of crucial importance to develop coordination mechanisms so that they can set their objectives together and coordinate their activities to optimize the global system performance.

In the first part, some recent optimization models of coordination will be studied and discussed, like capacitated planning models for producers, lot-sizing models for suppliers and the ideal model with cooperation and sharing information from both parties.

The second part will consist in extending some of these coordination models to include some additional furthers. For example, some models will be extended to the case where more competing suppliers and producers are involved.

Then, a numerical investigation will also be undertaken to compare the different types of anticipation of the producer and their effects, compared to an ideal situation which performs a complete integration of the supplier into the producers model, a situation that is really difficult to apply in the real world.

Moreover, the same investigation through the extended models would be undertaken to test and validate these new models. There will be a comparison between three types of models, concerning the effect of incrementing the number of suppliers, the number of producers and the number of both of them.

Finally, it will be deduced that the best model for the producer would be having a reactive anticipation of the supplier’s model, that is create a closer coordination which will benefit not only the producer but also the supplier. Although it is difficult to implement compared with the other two models and the information about the supplier is difficult to find, it would have a really good improvement on the final cost nearly as good as the ideal model.

Related to the multiple parties, it has been concluded that in general terms the best model is the one with an individual producer and supplier. But looking more in detail there are some improvements in the total cost when the producer is divided by smaller producers, having less productivity each one. In that case, and depending on the features of the supplier, it may be some improvements on the total cost of the supply chain.

Contents

1- Symbols

Indices and index sets:

$t \in (1, \dots, T)$ Period

$i \in (1, \dots, I)$ Component

$j \in (1, \dots, J)$ Job

$l \in (1, \dots, L)$ Order

J_l index set of jobs customer order l

P_j set of immediate predecessor jobs of job j

$p \in (1..P)$ Producers

$s \in (1..S)$ Suppliers

Variables:

Y_t	Capacity manpower in period t
Y_t^+	Increase in capacity in period t
Y_t^-	Decrease in capacity in period t
Y_{tp}	Capacity manpower in period t for the producer p
Y_{tp}^+	Increase in capacity in period t for the producer p
Y_{tp}^-	Decrease in capacity in period t for the producer p
x_{jt}	Assembly indicator (x_{jt} =1 if job j is started at time t)
x_{jtp}	Assembly indicator (x_{jt} =1 if job j is started at time t in producer p)
q_{it}	Order quantity of component I at time t
y_t	Overtime in period t
y_{tp}	Overtime in period t for the producer p
${I_{it}}^P$	Inventory of component i in period t (producer)
${I_{itp}}^P$	Inventory of component i in period t for producer p
Δ_j	Number of periods job j surpasses its due date
Δ_{jp}	Number of periods job j surpasses its due date in producer p
δ_{it}	Anticipated quantity of component i to be delivered in t
d_{it}	Quantity of component i delivered in t
d_{its}	Order quantity of component i in period t delivered by supplier s
d_{itps}	Order quantity of component i in period t ordered by the producer p to the supplier s.

d_{it}^{+}	Number of units component i which are ordered but not delivered on time t
d_{it}^{-}	Number of units component i which are delivered but not ordered on time t
I_{it}^{S}	Inventory of component i in period t (supplier)
I_{its}^{S}	Inventory of component i in period t of supplier s
z_{it}	Setup indicator (z_{it}=1 if component i is manufactured in period t)
z_{its}	Setup indicator (z_{it}=1 if component i is manufactured in period t by supplier s)
z_{itp}	Setup indicator (z_{it}=1 if component i is manufactured in period t for producer p)
z_{itps}	Setup indicator (z_{it}=1 if component i is manufactured by the supplier s in period t for producer p)
Q_{it}	Production amount of product i in period t
Q_{its}	Production amount of product i in period t by supplier s
Q_{itps}	Production amount of product i by supplier k in period t for the producer p
ΔC_{t}	Increase in capacity in period t
ΔC_{ts}	Increase in capacity in period t for supplier s

Constants:

c	Cost per capacity unit
c^{+}	Cost to increase the capacity by one unit
c^{-}	Cost to decrease the capacity by one unit
p_{i}	Purchase price of component i
$\bar{c}$	Overtime cost
h_{i}^{P}	Holding cost of component i for the producer
F	Penalty cost for exceeding the due date
$Y_{1}^{'}$	Initial capacity
Y^{+max}	Maximal increase in capacity
Y^{-max}	Maximal decrease in capacity
Y_{p}^{+max}	Maximal increase in capacity for the producer p
Y_{p}^{-max}	Maximal decrease in capacity for the producer p
a_{js}	Capacity consumption of job j in supplier s after its start

D_j	Duration of the job
D_{jp}	Duration of the job for the producer p
E_l	Due date of customer order l
E_{lp}	Due date of customer order l for the producer p
v_{ij}	Quantity of component i required to start job j
v_{ijp}	Quantity of component i required to start job j for the producer p
p_i	Purchase price of component i
h_i^S	Holding cost of component i for the supplier
s_i	Setup cost of component i
τ	Cost to increase the capacity by one unit
K	Penalty cost due to delayed delivery
ϵ	Penalty cost due to premature delivery
$I_1^{'}$	Initial inventory
q_{it}	Order quantity of component i in period t
c_i	Manufacturing coefficient of component i
C	Normal capacity
C_s	Normal capacity for the supplier s
α_t	% capacity available in period t
α_{ts}	% capacity available in period t for the supplier s
ΔC_t^{max}	Maximal increase in capacity
ΔC_{ts}^{max}	Maximal increase in capacity for the supplier s
M	Number representing infinite

2- Preface

2.1- Origin of the project

This thesis has its origin in the author's interest to increase the theoretical basis of supply chain management (SCM) acquired in the subjects of the university. The main objective was to have the maximum knowledge on the implementation of SCM in order to demonstrate an added value to logistic companies in the end of the career. The study of the Coordination Mechanisms in Supply Chain is the first step to start working deeper in this field and to have an overview of the optimization models in supply chain.

2.2- Motivation

Supply chain management has recently undergone a fast development in theory and practice. Due to globalization and competence, most of the companies have split their logistic into various independent units. These units usually are decision making factories within a network of material and information flows. Nowadays, many logistic-based approaches to supply chain management are still within the traditional realm of one central decision maker unit. But few approaches have been made on operational supply chain in more than one independent decision maker.

The idea to create a coordination mechanism based on the operational supply chain that can introduce multiple suppliers and multiple producers has become a great motivation to start a further study of new mathematical models that fit into real companies.

3- Introduction

3.1- Objectives

The main objectives of this project are to analyze coordination mechanisms between producers and suppliers in a supply chain. The thesis will start with a study of recent optimization models of coordination mechanisms in operational supply chain, more precisely, there will be an investigation of the coordination mechanisms increasing the complexity between the suppliers and the producers both possessing some private information. In particular, there will be undertaken an analysis of the effect of a possible disclosure of private information on the overall performance of the supply chain. Afterwards, an extension and improvement of these models will be undertaken to include some additional furthers just to increase the members involved to an arbitrary number, i.e., multiple suppliers and multiple producers.

3.2- Scope

This thesis is organized as follows. After a brief review of related literature and theoretical background on section 4, there will be a study of some specific models of decision in section 5, developing the producer's model with some kinds of anticipations and the supplier's model. Then, it will be a linkage between them in the ideal model that coordinates both models. In the next section 6, an extension of the ideal model will be undertaken, creating some other models including one producer with multiple suppliers, multiple producers with one supplier and the generic model with multiple producers and multiple suppliers. Finally, in the last section, these models are exposed in an extensive numerical investigation with AMPL, to test and validate the anticipations from the producers and to know the influence of incrementing some parties of the extended models.

4- Theoretical background

4.1- Supply chain management

A supply chain is a network of organizations, people, technology, activities, information and resources that are involved, through upstream and downstream linkages, in the different processes that produce value in the form of products and services in the hands of the ultimate consumer.[1]

In theory, supply chain seeks to match supply with demand and do so with the minimal inventory. To achieve this objective, coordination with channel partners is needed, which can be suppliers, intermediaries, third-party service providers, and customers. Another issue of interest of SCM is to fulfill customer demands through the most efficient use of resources, including distribution capacity, inventory and labor. Various aspects of optimizing the supply chain include liaising with suppliers to eliminate bottlenecks; sourcing strategically to strike a balance between lowest material cost and transportation and traditional logistics optimization to maximize the efficiency of the distribution side.

4.2- Coordination mechanisms in supply chain

A supply chain is a set of organizations that are involved in transforming raw materials to a final product. Usually, these organizations are separate and independent economic parties. Although it is known that a fully integrated solution should result in optimal system performance, it is not always the best interest of each entity in the system. Due to this effect, most of independent supply chain members usually prefer to optimize their individual objectives rather than the entire system ones. It is of crucial importance in supply chain management to develop mechanisms that can align the objectives of independent supply chain members and coordinate their decisions and activities so as to optimize system performance. Also coordination mechanisms have to identify an improvement compared to an initial uncoordinated solution and to include incentives to implement the improved solution.

[1] Christopher (2005, p.17)

There are some definitions on the literature for coordination mechanisms in the field of mechanism design, where a mechanism constitutes a framework that specifies the outcomes of decentralized parties depending on the actions undertaken by them:

> A coordination mechanism is a mechanism for which the implementation of the optimal strategies by decentralized, self-interested parties may lead to a coordinated outcome and neither violates the individual rationality of the participating parties nor the budget balance of the system.[2]

Therefore, we have a coordination mechanism that have to assure that individual rationality requires that no party is worse off by participating in the mechanism, i.e., that the profits of all participating parties are at least equal to their profits achieved in the default solution. Budget balance means that the payments of parties in the mechanism sum up to zero, i.e., that the mechanism does not require an outside subsidy. [3]

4.3- Hierarchical planning in supply chain

This thesis is based on hierarchical planning. This concept is focused on a supply link within a supply network consisting of a producer and a supplier. As showed in Figure 1, to meet external customer demand, the producer places orders for some necessary components with a supplier, which, contingent on the supplier's capacity situation is more or less correctly carried out. More precisely, the delivery of components might not match the ordered amount and might not be in time. However, for a longer horizon of 10 weeks, the total amount is fixed according to a general procurement contract and is delivered correctly.

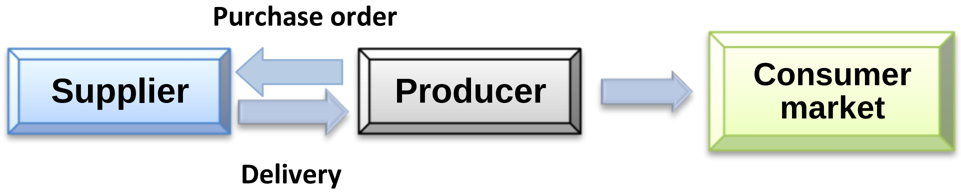

Figure 1. Supply link investigated

Hence, a relationship between producer and supplier is considered, with the producer being in the upper level and the supplier representing the lower level. Taking the position of the producer, we particularly assume that she is not fully informed of the supplier's decision situation, especially of the operational capacity conditions. To coordinate the link, we establish as a typical control measure a penalty cost for not delivering the correct amount in due time. That is, for given total external demand, we

[2] Mas-Colell et al. (1995, p. 857)
[3] Martin Albrecht (2010, p. 22)

are considering an operational short-term coordination of ordering and adjoin delivery decisions over a total horizon of T periods (see figure 2). Consequently, an order *q* represents a sequence of component orders, and the subsequent delivery d, tries to match such an order, possibly deviating from q by an amount of δ.

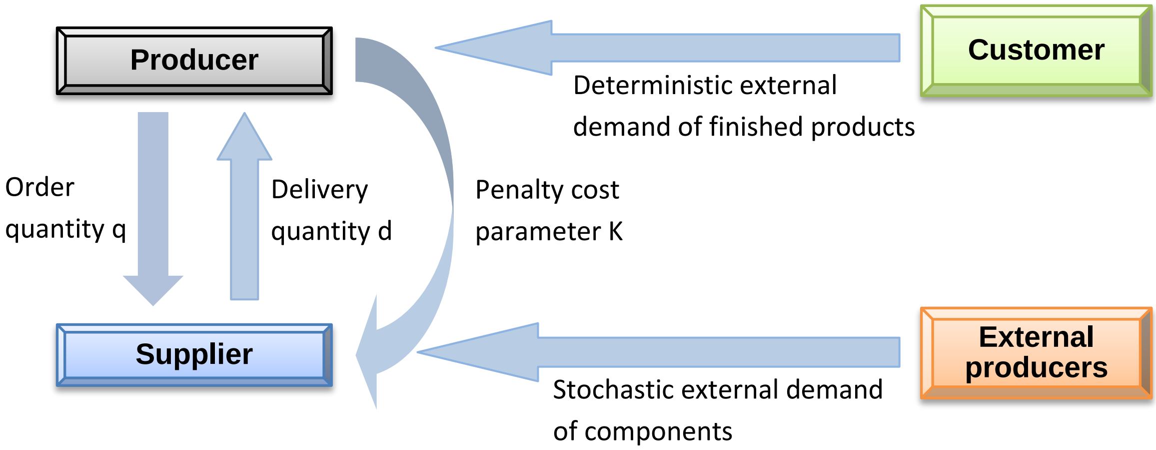

Figure 2. Producer- Supplier hierarchy

To put in more general terms, the top-down influence consists of a task-oriented instruction (q) and a control-oriented instruction (K). This latter instruction could be interpreted as a leadership activity and is exclusively employed to guarantee the correct execution.

Taking a closer look at the state of information, we reasonably assume that both parties maintain some privacy, i.e., they do not reveal all their data, which consequently results in an asymmetric information state. This situation, however, does not give rise to an opportunistic behavior; one would not capture essential features of a supply chain which is usually intended to establish a long-term trusting cooperation. Hence, we presume a team like behavior, i.e., all parties attempt to optimize a common goal. Still there exists private information, and it is one of our main concerns to quantify the effect of private information on the overall performance of the supply link.

Usually, a supply link has to be embedded into the entire supply network. As an example, the supplier might not have just one but several other producers he is in contact with. In capturing this influence of the full network, we describe the availability of the supplier's capacity as being stochastic. More, precisely, at the time when the producer places her purchasing order q with the supplier, the capacity is only stochastically known for both parties. Only later, when the delivery decision is to be made, the supplier is assumed to know all his commitments, and hence the capacity that is still available. This externally induced uncertainty adds to the lack of knowledge she already has with the unknown characteristics of the supplier.[4]

Finally, it is assumed that the basic idea of hierarchical planning is the separation of decisions according to their impact, e.g., on the profitability of the supply chain. The decisions at the upper levels, i.e., those with greater impact, are determined first and implemented as targets for the planning of the lower levels. Further important characteristics are the aggregation of data and decisions at the upper levels and the provision of feedback by the lower levels.

The main objective of hierarchical planning should be to regard team behavior: it not only assumes truthful information exchange, but also the willingness of parties to accept solutions inferior to the initial solution, provided that a system wide improvement is obtained.

[4] Hierarchical coordination mechanisms within the supply chain. C. Schneeweiss, K. Zimmer.

5- Preliminary models

This thesis is the improvement of a report from the European Journal of Operational Research in 2002 titled "Hierarchical coordination mechanisms within the supply chain". The paper analyses operational coordination mechanisms between a producer and a supplier within a supply chain having private local information. Based on these first models, we will extend them to find a model that can be applicable to all the possible supply chains whatever their sizes are, introducing multiple producers and multiple suppliers.

In the next section there is a description of the models used to create the coordination mechanisms in the supply chain. First of all, there is a presentation of the producer's and the supplier's mathematical models independently, and linking both models it can be represented the ideal mathematical model, which will benefit both parts as a team but only as a benchmark.

5.1- Producer's model

The producer's model consists on a capacitated project planning model, with due date penalty cost. More precisely, the company produces several products which are ordered by some external customers, and the components needed have to be delivered by the supplier.

For the producer, it is assumed that a set of orders are known on a period T. Each order contains a sequence of manufacturing jobs which, for each order *l* are assigned the same due date. In order to fulfill every customer, the producer has to place some orders to the supplier, who will procure her with the necessary components.

For the producer's model we will differentiate three types of anticipation and coordination. On the one extreme, the producer is not taking into account any feature of the supply chain (pure top-down hierarchy), whereas on the other extreme the supplier is fully integrated into the producer (ideal model) such that an anticipation disappears.

5.1.1- Pure top-down hierarchy (P)

This first model for the producer is an individual model that only takes care of optimizing its own benefit without taking into account any variables of the supplier. It means that in the pure top-down hierarchy the delivered amount d_{it} is identical to the

ordered amount. In this case, there is no anticipation function since there is no possible reaction to the producer's instruction.
The model would be represented as follows:

Objective function:

$[MIN]Z^P = \sum_{t=1}^{T}(cY_t + c^+Y_t^+ + c^-Y_t^-)$ (P1)

Constraints:

Capacity adaptation constraints:

$Y_{t+1} = Y_t + Y_t^+ - Y_t^-$ $\forall t = 1, \dots, T-1$ (P2)

$Y_1 = Y_1^{'}$ (P3)

$Y_t^+ \leq Y^{+max}$ $\forall t$ (P4)

$Y_t^- \leq Y^{-max}$ $\forall t$ (P5)

Capacity constraints:

$\sum_{j=1}^{J}\sum_{k=1}^{t} a_{j,t+1-k} x_{jk} \leq Y_t + y_t$ $\forall t$ (P6)

Network constraints:

$\sum_{t=1}^{T} x_{jt} = 1$ $\forall j$ (P7)

$\sum_{t=1}^{T}\left(t + D_j\right)x_{jt} \leq E_l + \Delta_j$ $\forall j \in Jl, \forall l$ (P8)

$\sum_{t=1}^{T}(t + D_h)x_{ht} \leq \sum_{t=1}^{T} t x_{jt}$ $\forall j, h \in P(j)$ (P9)

Material balance constraints:

$\sum_{j=1}^{J} v_{ij} x_{jt} \leq \delta_{it} + I_{it}$ $\forall i, t$ (P10)

$I_{i,t+1} = I_{it} + \delta_{it} - \sum_{j=1}^{J} v_{ij} x_{jt}$ $\forall i, t$ (P11)

$I_{i1} = 0$ $\forall i$ (P12)

Order delivery constraints:

$\delta_{it} = q_{it}$ $\forall i, t$ (P13)

Integrality and non-negativity constraints:

$Y_t, Y_t^+, Y_t^-, q_{it}, \delta_{it}, y_t, I_{it}, \Delta_j \geq 0$ $\forall i, t, j$ (P14)

$x_{jt} \in \{0,1\}$ $\forall j, t$ (P15)

Indices and index sets:

$t \in (1, \ldots, T)$ Period

$i \in (1, \ldots, I)$ Component

$j \in (1, \ldots, J)$ Job

$l \in (1, \ldots, L)$ Order

J_l index set of jobs customer order l

P_j set of immediate predecessor jobs of job j

Variables:

Y_t	Capacity manpower in period t
Y_t^+	Increase in capacity in period t
Y_t^-	Decrease in capacity in period t
x_{jt}	Assembly indicator (x_{jt} =1 if job j is started at time t)
q_{it}	Order quantity of component I at time t
y_t	Overtime in period t
I_{it}	Inventory of component i in period t (producer)
Δ_j	Number of periods job j surpasses its due date
δ_{it}	Anticipated quantity of component i to be delivered in t

Constants:

c	Cost per capacity unit
c^+	Cost to increase the capacity by one unit
c^-	Cost to decrease the capacity by one unit
p_i	Purchase price of component i
$\bar{c}$	Overtime cost
h_i^P	Holding cost of component i for the producer
F	Penalty cost for exceeding the due date
$Y_1^{'}$	Initial capacity
Y^{+max}	Maximal increase in capacity
Y^{-max}	Maximal decrease in capacity
a_{jk}	Capacity consumption of job j in period k after its start
D_j	Duration of the job
E_l	Due date of customer order l
v_{ij}	Quantity of component i required to start job j

The objective function will only consider capacity costs, as they are cheaper than overtime and holding costs, and the purchasing price remains constant so it does not affect the optimization.

Related to the constraints, they can be represented in five categories: capacity adaptation, capacity, network, material balance and order delivery.

The capacity adaptation is described in (P2) with a balance equation, the initial capacity amount is fixed in equation (P3) and the maximum values for the increases and decreases in the capacity are fixed in (P4) and (P5). Equation (P6), related also to the capacity, represents the consumption of every job when it starts until it finishes. In the network constraints, (P7) indicate that each job has to start only once. (P8) is related to the due date of each order, and (P9) determine the temporal network structure related to the predecessors of the jobs.

In material balance the constraint (P10) makes sure that a job j in time t can only be produced if the necessary components were made available for the supplier. Equation (P11) defines the material balance equation with initial condition (P12). Equation (P13) is the most interesting constraint. As the delivered amount is identical to the ordered amount, it can be assured that the supplier model is not taken into account, meaning that the model follows a pure top - down hierarchy. Finally, it is assumed the integrality and the non-negativity constraints in (P14) and (P15).

5.1.2- Non-reactive anticipation (NP)

In contrast to the pure top-down hierarchy, the non-reactive accounts for importance features of the supplier's model. This means that this model would try to guess some of the supplier's capacity constraints, extending the constraints of the last model with two new constraints, which would modify the order demand a little bit to adjust more to the supplier' capacity, but it is not anticipating the reaction of the supplier. Hence, the delivered amount d_{it} is no longer the ordered amount, so this constraint can be omitted, and substituted for a non-reactively anticipated set of relations (NP13 and NP14):

The next model would be described as follows:

Objective function:

$$[MIN]Z^P = \sum_{t=1}^{T}(cY_t + c^+Y_t^+ + c^-Y_t^-) \qquad \text{(NP1)}$$

Constraints:

Capacity adaptation constraints:

$Y_{t+1} = Y_t + Y_t^{+} - Y_t^{-}$	$\forall t = 1, \dots, T-1$	(NP2)
$Y_1 = Y_1^{'}$		(NP3)
$Y_t^{+} \leq Y^{+max}$	$\forall t$	(NP4)
$Y_t^{-} \leq Y^{-max}$	$\forall t$	(NP5)

Capacity constraints:

$\sum_{j=1}^{J} \sum_{k=1}^{t} a_{j,t+1-k} x_{jk} \leq Y_t + y_t$	$\forall t$	(NP6)

Network constraints:

$\sum_{t=1}^{T} x_{jt} = 1$	$\forall j$	(NP7)
$\sum_{t=1}^{T} \left(t + D_j\right) x_{jt} \leq E_l + \Delta_j$	$\forall j \in Jl, \forall l$	(NP8)
$\sum_{t=1}^{T} (t + D_h) x_{ht} \leq \sum_{t=1}^{T} t x_{jt}$	$\forall j, h \in P(j)$	(NP9)

Material balance constraints:

$\sum_{j=1}^{J} v_{ij} x_{jt} \leq \delta_{it} + I_{it}$	$\forall i, t$	(NP10)
$I_{i,t+1} = I_{it} + \delta_{it} - \sum_{j=1}^{J} v_{ij} x_{jt}$	$\forall i, t$	(NP11)
$I_{i1} = 0$	$\forall i$	(NP12)

Order delivery constraints:

$\sum_{i=1}^{I} c_i \delta_{it} \leq \alpha_t C + \Delta C_t$	$\forall t$	(NP13)
$\sum_{t=1}^{T} \tau \Delta C_t \leq AL$		(NP14)

Integrality and non-negativity constraints:

$Y_t, Y_t^{+}, Y_t^{-}, q_{it}, \delta_{it}, y_t, I_{it}, \Delta_j, \Delta C_t \geq 0$	$\forall i, t, j$	(NP15)
$x_{jt} \in \{0,1\}$	$\forall j, t$	(NP16)

New or modified variables:

ΔC_t	Increase in capacity in period t

New or modified variables constants:

C	Normal capacity
α_t	% capacity available in period t
ΔC_t^{max}	Maximal increase in capacity
τ	Cost to increase the capacity by one unit
AL	Aspiration level for extra capacity

The objective function will only consider again the capacity costs, as they are cheaper than other options.

New constraints (NP13) and (NP14) indicate estimations of the producer for the presumed capacity of the supplier. Hence, the supplier is taken into account by his presumed capacity and also by his aspiration level AL, which is related to the willing to build extra capacity. Then, the supplier is essentially represented by his capacity situation which seems not be an unreasonable assumption. Other constraints remain the same as in the model of pure top-down hierarchy (5.1.1).

5.1.3- Reactive anticipation (RP)

This model will try to react to the supplier's capacity creating an anticipation decision model. On this occasion, the producer will obtain the ordered amount anticipating the model for the supplier (see next section 5.2 for the supplier model). That is, the producer will introduce his data to an unreal supplier model, replacing the unknown parameters by their estimates. Then, the demand would be a task-oriented order quantity q_{it} joined with a control-oriented penalty cost K, which reflect the temporary situation the supply link is in. There will be only analyzed penalties for delayed deliveries but not for premature deliveries, due to the fact that premature deliveries only cause additional inventories for the producer, whereas delayed deliveries cause extremely unwanted stockouts and need therefore to be optimized. Similarly, optimization of purchasing costs is not of our concern.

Objective function:

$$[MIN]Z^P = \sum_{t=1}^{T}(cY_t + c^+Y_t^+ + c^-Y_t^-) + \sum_{t=1}^{T}\sum_{i=1}^{I} K\delta^+{}_{it} + \sum_{t=1}^{T}\bar{c}y_t + \sum_{t=1}^{T}\sum_{i=1}^{I} h_i^P I_{it} + \sum_{l=1}^{L} F \max_{j\in Jl} \Delta_j \qquad \text{(RP1)}$$

Constraints:

Capacity adaptation constraints:

$$Y_{t+1} = Y_t + Y_t^+ - Y_t^- \qquad \forall t = 1,\dots,T-1 \qquad \text{(RP2)}$$

$$Y_1 = Y_1^{'} \qquad \text{(RP3)}$$

$$Y_t^+ \leq Y^{+max} \qquad \forall t \qquad \text{(RP4)}$$

$$Y_t^- \leq Y^{-max} \qquad \forall t \qquad \text{(RP5)}$$

Capacity constraints:

$$\sum_{j=1}^{J}\sum_{k=1}^{t} a_{j,t+1-k}x_{jk} \leq Y_t + y_t \qquad \forall t \qquad \text{(RP6)}$$

Network constraints:

$\sum_{t=1}^{T} x_{jt} = 1$	$\forall j$	(RP7)
$\sum_{t=1}^{T} (t + D_j) x_{jt} \leq E_l + \Delta_j$	$\forall j \in Jl, \forall l$	(RP8)
$\sum_{t=1}^{T} (t + D_h) x_{ht} \leq \sum_{t=1}^{T} t x_{jt}$	$\forall j, h \in P(j)$	(RP9)

Material balance constraints:

$\sum_{j=1}^{J} v_{ij} x_{jt} \leq \delta_{it} + I_{it}$	$\forall i, t$	(RP10)
$I_{i,t+1} = I_{it} + \delta_{it} - \sum_{j=1}^{J} v_{ij} x_{jt}$	$\forall i, t$	(RP11)
$I_{i1} = 0$	$\forall i$	(RP12)

Order delivery constraints:

$\delta_{it} = q_{it}$	$\forall i, t$	(RP13)
$d_{i,t+1} + \delta_{i,t+1}^{+} - \delta_{i,t+1}^{-} = q_{it} + \delta_{it}^{+} - \delta_{it}^{-}$	$\forall i, t$	(RP14)
$d_{i1} + \delta_{i1}^{+} - \delta_{i1}^{-} = q_{i1}$	$\forall i$	(RP15)

Integrality and non-negativity constraints:

$Y_t, Y_t^{+}, Y_t^{-}, \ \delta_{it}, \delta^{+}{}_{it}, \delta^{-}{}_{it} y_t, I_{it}, \Delta_j, \Delta C_t \geq 0$	$\forall i, t, j$	(RP16)
$x_{jt} \in \{0,1\}$	$\forall j, t$	(RP17)

The objective function minimizes the capacity costs, holding costs and costs charged for not serving the customer in time. Specifically, in the first term there are the costs for the capacity manpower and the increases or decreases of the capacity depending on the period. The second term consists on minimizing the delayed deliveries. The third term consists on the costs for overtime in period T. The fourth term represents holding costs for components being stocked at the producer, and finally, the last term stands for costs the producer is charged for not serving her customers in due time. In this model only the delayed deliveries are optimized instead of purchasing costs or premature deliveries.

In order delivery constraints there are introduced three new equations. The first one relates the order demand of the producer with the order delivery optimized with the supplier model; hence q_{it} is not anymore a variable but a parameter (RP13). There is also introduced in this model the delayed and the premature deliveries caused by the supplier, in constraints RP14 and RP15.

To know how the parameter q_{it} is obtained, we will detail below the supplier model.

5.2- Supplier's model (S)

The supplier model is a linear lot-sizing model that considers setup costs but no setup times. This model tries to fulfill the producer's orders in time depending on the available capacity in each period.

Objective function:

$$[MAX]Z^S = \sum_{t=1}^{T}\sum_{i=1}^{I}\{p_i d_{it} - h_i^S I_{it}^S - s_i z_{it} - K\delta_{it}^+ - \epsilon\delta_{it}^-\} - \sum_{t=1}^{T}\tau\Delta C_t \qquad \text{(S1)}$$

Constraints:

Material balance constraints:

$$I_{i,t+1}^S = I_{it}^S + Q_{it} - d_{it} \qquad \forall i,t \qquad \text{(S2)}$$

$$I_{i1}^S = I_i^{'} \qquad \forall i \qquad \text{(S3)}$$

$$Q_{it} \le M z_{it} \qquad \forall i,t \qquad \text{(S4)}$$

Capacity constraints:

$$\sum_{i=1}^{I} c_i Q_{it} \le \alpha_t C + \Delta C_t \qquad \forall t \qquad \text{(S5)}$$

$$\Delta C_t \le \Delta C_t^{max} \qquad \forall t \qquad \text{(S6)}$$

Order delivery constraints:

$$d_{i,t+1} + \delta_{i,t+1}^+ - \delta_{i,t+1}^- = q_{it} + \delta_{it}^+ - \delta_{it}^- \qquad \forall i,t \qquad \text{(S7)}$$

$$d_{i1} + \delta_{i1}^+ - \delta_{i1}^- = q_{i1} \qquad \forall i \qquad \text{(S8)}$$

$$\sum_{t=1}^{T} d = \sum_{t=1}^{T} q \qquad \forall i \qquad \text{(S9)}$$

Integrality and non-negativity constraints:

$$Q_{it}, d_{it}, \delta_{it}^+, \delta_{it}^-, I_{it}^S, \Delta C_t \ge 0 \qquad \forall i,t \qquad \text{(S10)}$$

$$z_{it} \in \{0,1\} \qquad \forall i,t \qquad \text{(S11)}$$

Indices and index sets:

t ∈ (1, … , T)Period

i ∈ (1, … , I) Component

Variables:

d_{it} Quantity of component i delivered in t

I_{it}^{S}	Inventory of component i in period t(supplier)
z_{it}	Setup indicator (z_{it}=1 if component i is manufactured in period t)
d_{it}^{+}	Number of units component i which are ordered but not delivered on time t
d_{it}^{-}	Number of units component i which are delivered but not ordered on time t
ΔC_t	Increase in capacity in period t
Q_{it}	Production amount of product i in period t

Constants:

p_i	Purchase price of component i
h_i^{S}	Holding cost of component i for the supplier
s_i	Setup cost of component i
τ	Cost to increase the capacity by one unit
K	Penalty cost due to delayed delivery
ϵ	Penalty cost due to premature delivery
$I_1^{'}$	Initial inventory
q_{it}	Order quantity of component i in period t
c_i	Manufacturing coefficient of component i
C	Normal capacity
α_t	% capacity available in period t
ΔC_t^{max}	Maximal increase in capacity
M	Number representing infinite

The objective function represents the contribution margin for the supplier, consisting on maximizing the sales and reducing inventory, setup and penalty costs. In the function there are costs for the producer's penalty for not keeping the due date and costs for premature delivery. The last term stands for costs caused by a capacity extension.

In this case there are three groups of constraints: material balance, capacity and order-delivery constraints. In the material balance constraints the inventory of the supplier is described (S2), as well as the initial inventory (S3). (S4) links the production variable Q to the setup variable z. The capacity constraints are described in (S5), where the production amount of component I will depend on the capacity of the supplier. This capacity also depends on a coefficient alpha, indicating the availability of the normal capacity in period t. This means that the producer doesn't know this value, as it remains as part of supplier's private information, and implies that there could be other producer's that are sharing the supplier's capacity, so the producers don't know which quantity of orders is available in each period t. Equations (S7) and (S8) represent the relationship between order and delivery, and (S9) takes account that total order material is ultimately to be delivered. Finally, it is assumed the integrality and the non-negativity constraints in (P14) and (P15).

5.3- Ideal model (ID)

The ideal model, which will be used as a benchmark, consists on the integration of the supplier in the producer's model to consider them as a one decision maker. This situation can allow a team without any information asymmetry, so the estimations are not necessary and the full capacity and other features of the supplier are known for the producer.

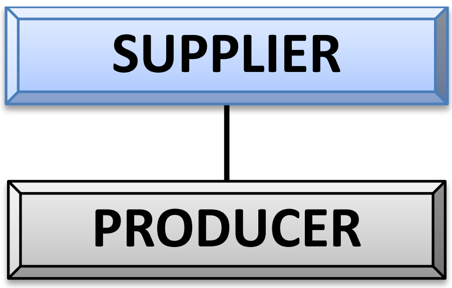

Figure 3. Scheme of a supply chain with one supplier and one producer

Objective function:

$$[MIN]Z^I = Z^P + Z^S = \sum_{t=1}^{T}(cY_t + c^+Y_t^+ + c^-Y_t^-) + \sum_{t=1}^{T}\bar{c}y_t + \sum_{t=1}^{T}\sum_{i=1}^{I} h_i^P I_{it} + \sum_{l=1}^{L} F \max_{j\in Jl} \Delta_j + \sum_{t=1}^{T}\sum_{i=1}^{I}\left(h_i^S I_{it} + s_i z_{it}\right) + \sum_{t=1}^{T}\tau\Delta C_t \quad \text{(ID1)}$$

Constraints:

Capacity adaptation constraints:

$$Y_{t+1} = Y_t + Y_t^+ - Y_t^- \quad \forall t = 1,\dots,T-1 \quad \text{(ID2)}$$

$$Y_1 = Y_1' \quad \text{(ID3)}$$

$$Y_t^+ \le Y^{+max} \quad \forall t \quad \text{(ID4)}$$

$$Y_t^- \le Y^{-max} \quad \forall t \quad \text{(ID5)}$$

Capacity constraints:

$\sum_{j=1}^{J}\sum_{k=1}^{t} a_{j,t+1-k} x_{jk} \leq Y_t + y_t$	$\forall t$	(ID6)
$\sum_{i=1}^{I} c_i Q_{it} \leq \alpha_t C + \Delta C_t$	$\forall t$	(ID7)
$\Delta C_t \leq \Delta C_t^{max}$	$\forall t$	(ID8)

Network constraints:

$\sum_{t=1}^{T} x_{jt} = 1$	$\forall j$	(ID9)
$\sum_{t=1}^{T} (t + D_j) x_{jt} \leq E_l + \Delta_j$	$\forall j \in Jl, \forall l$	(ID10)
$\sum_{t=1}^{T} (t + D_h) x_{ht} \leq \sum_{t=1}^{T} t x_{jt}$	$\forall j, h \in P(j)$	(ID11)

Material balance constraints:

$\sum_{j=1}^{J} v_{ij} x_{jt} \leq d_{it} + I_{it}$	$\forall i, t$	(ID12)
$I_{i,t+1} = I_{it} + d_{it} - \sum_{j=1}^{J} v_{ij} x_{jt}$	$\forall i, t$	(ID13)
$I_{i1} = 0$	$\forall i$	(ID14)
$I_{i,t+1}^S = I_{it}^S + Q_{it} - d_{it}$	$\forall i, t$	(ID15)
$I_{i1}^S = I_i^{'}$	$\forall i$	(ID16)
$Q_{it} \leq M z_{it}$	$\forall i, t$	(ID17)

Integrality and non-negativity constraints:

$Y_t, Y_t^+, Y_t^-, d_{it}, y_t, I_{it}, \Delta_j, I_{it}^S, Q_{it}, \Delta C_t \geq 0$	$\forall i, t, j$	(ID18)
$x_{jt} \in \{0,1\}, z_{it} \in \{0,1\}$	$\forall j, i, t$	(ID19)

Indices and index sets:

$t \in (1, \ldots, T)$ Period

$i \in (1, \ldots, I)$ Component

$j \in (1, \ldots, J)$ Job

$l \in (1, \ldots, L)$ Order

J_l index set of jobs customer order l

P_j set of immediate predecessor jobs of job j

Variables:

Y_t	Capacity manpower in period t
Y_t^+	Increase in capacity in period t
Y_t^-	Decrease in capacity in period t
x_{jt}	Assembly indicator (x_{jt} =1 if job j is started at time t)

y_t	Overtime in period t
I_{it}	Inventory of component i in period t (producer)
Δ_j	Number of periods job j surpasses its due date
d_{it}	Order quantity of component i in period t
I_{it}^S	Inventory of component i in period t(supplier)
z_{it}	Setup indicator (z_{it}=1 if component i is manufactured in period t)
ΔC_t	Increase in capacity in period t
Q_{it}	Production amount of product i in period t

Constants:

c	Cost per capacity unit
c^+	Cost to increase the capacity by one unit
c^-	Cost to decrease the capacity by one unit
p_i	Purchase price of component i
$\bar{c}$	Overtime cost
h_i^P	Holding cost of component i for the producer
F	Penalty cost for exceeding the due date
Y_1'	Initial capacity
Y^{+max}	Maximal increase in capacity
Y^{-max}	Maximal decrease in capacity
a_{jk}	Capacity consumption of job j in period k after its start
D_j	Duration of the job
E_l	Due date of customer order l
v_{ij}	Quantity of component i required to start job j
h_i^S	Holding cost of component i for the supplier
s_i	Setup cost of component i
τ	Cost to increase the capacity by one unit
I_1'	Initial inventory
c_i	Manufacturing coefficient of component i
C	Normal capacity
α_t	% capacity available in period t
ΔC_t^{max}	Maximal increase in capacity
M	Number representing infinite

In the ideal model, the delivered amount is no longer anticipated for the producer but explicitly optimized. Also the equation (P13) becomes obsolete, as the anticipation function for the delivery is not necessary.

The objective function of the unified model describes the minimization of the total costs for both parties including: capacity costs for the producer, overtime costs for the producer, holding costs for the producer, costs for delayed deliveries for the producer, holding costs for the supplier, manufacturing costs for the supplier and capacity costs for the supplier.

The constraints are shared in the producer and supplier previous models, so they have been already explained in the individual models.

In the ideal model, decisions are taken altogether with producer and supplier, so the benefit is considering the complete supply chain. This also means that each party individually will have a better economic benefit, as now it is known the information that was missing or estimated in the individual models, and then the chain will provide a better solution for each system.

6- Extended models

Concerning the supply chain structures, we have only taken into account one supplier and one producer. In the next study an extension of these models will be made, analyzing any possibility structure in the supply chain: multiple suppliers and one producer, multiple producers and one supplier and finally and arbitrary structure with multiple producers and multiple suppliers (Figure 4). In this section, only the ideal model will be extended and studied, considering that is the model that covers all important aspects of the other anticipations in one model.

Group	Parameter	Comment
Structure	1 – 1	Producer – Supplier
	1 – S	1 producer – Multiple suppliers
	P – 1	Multiple producers – 1 supplier
	P – S	Multiple producers – multiple suppliers

Figure 4. Table with different structures of the supply chain

6.1- Ideal model with one producer and multiple suppliers (IMS)

In the next section it is introduced multiple suppliers to the ideal model. This model tries to investigate the effect of smaller suppliers, each one with smaller capacities and features, compared to a bigger supplier like in the previous model, that is, what happens if the producer distributes her demand through multiple smaller suppliers rather than a bigger one. This case could be a good solution to the companies that have to work with large quantities and one supplier does not have enough capacity for delivering the total production.

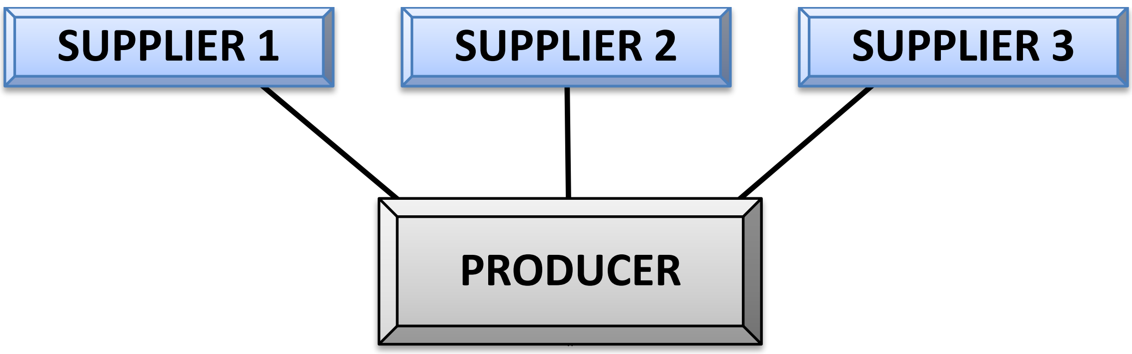

Figure 5. Scheme of a supply chain with multiple suppliers

Objective function:

$[MIN]Z^{IMS} =$

$$\sum_{t=1}^{T}(cY_t + c^{+}Y_t^{+} + c^{-}Y_t^{-}) + \sum_{t=1}^{T}\bar{c}y_t + \sum_{t=1}^{T}\sum_{i=1}^{I}h_i^P {I_{it}}^P + \sum_{l=1}^{L}F\max_{j\in Jl}\Delta_j + \sum_{t=1}^{T}\sum_{i=1}^{I}\sum_{s=1}^{S}\left(h_i^S I_{its}^S + s_i z_{itk}\right) + \sum_{t=1}^{T}\sum_{s=1}^{S}\tau\Delta C_{ts} \quad \text{(IMS1)}$$

Constraints:

Capacity adaptation constraints:

$$Y_{t+1} = Y_t + Y_t^{+} - Y_t^{-} \qquad \forall t = 1, \dots, T-1 \qquad \text{(IMS2)}$$

$$Y_1 = Y_1' \qquad \text{(IMS3)}$$

$$Y_t^{+} \le Y^{+max} \qquad \forall t \qquad \text{(IMS4)}$$

$$Y_t^{-} \le Y^{-max} \qquad \forall t \qquad \text{(IMS5)}$$

Capacity constraints:

$$\sum_{j=1}^{J}\sum_{k=1}^{t}a_{j,t+1-k}x_{jk} \le Y_t + y_t \qquad \forall t \qquad \text{(IMS6)}$$

$$\sum_{i=1}^{I}\sum_{s=1}^{S}c_i Q_{its} \le \sum_{s=1}^{S}\alpha_{ts}C_s + \Delta C_{ts} \qquad \forall t \qquad \text{(IMS7)}$$

$$\Delta C_{ts} \le \Delta C_{ts}^{max} \qquad \forall t, s \qquad \text{(IMS8)}$$

Network constraints:

$$\sum_{t=1}^{T}x_{jt} = 1 \qquad \forall j \qquad \text{(IMS9)}$$

$$\sum_{t=1}^{T}\left(t + D_j\right)x_{jt} \le E_l + \Delta_j \qquad \forall j \in Jl, \forall l \qquad \text{(IMS10)}$$

$$\sum_{t=1}^{T}(t + D_h)x_{ht} \le \sum_{t=1}^{T}t x_{jt} \qquad \forall j, h \in P(j) \qquad \text{(IMS11)}$$

Material balance constraints:

$$\sum_{j=1}^{J}v_{ij}x_{jt} \le \sum_{s=1}^{S}d_{its} + I_{it}^P \qquad \forall i, t \qquad \text{(IMS12)}$$

$$I_{i,t+1}^P = {I_{it}}^P + \sum_{s=1}^{S}d_{its} - \sum_{j=1}^{J}v_{ij}x_{jt} \qquad \forall i, t \qquad \text{(IMS13)}$$

$${I_{i1}}^P = 0 \qquad \forall i \qquad \text{(IMS14)}$$

$$\sum_{s=1}^{S}I_{i,t+1,s}^S = \sum_{s=1}^{S}I_{its}^S + Q_{its} - d_{its} \qquad \forall i, t \qquad \text{(IMS15)}$$

$$\sum_{s=1}^{S}I_{i1s}^S = I_i' \qquad \forall i \qquad \text{(IMS16)}$$

$$\sum_{s=1}^{S}Q_{its} \le \sum_{s=1}^{S}Mz_{its} \qquad \forall i, t \qquad \text{(IMS17)}$$

Order and delivery constraints:

$$\sum_{t=1}^{T}d_{i,t,p} = \sum_{t=1}^{T}Q_{i,t,p} \qquad \forall i, t, p \qquad \text{(IMSP18)}$$

Integrality and non-negativity constraints:

$$Y_t, Y_t^{+}, Y_t^{-}, d_{its}, y_t, {I_{it}}^P, \Delta_j, I_{its}^S, Q_{its}, \Delta C_{ts} \ge 0 \qquad \forall i, t, j, s \qquad \text{(IMS18)}$$

$x_{jt} \in \{0,1\}, z_{its} \in \{0,1\}$ $\forall j, i, t, s$ (IMS19)

New or modified indices and index sets:

$\mathrm{s} \in (1..\mathrm{S})$ Suppliers

New or modified variables:

d_{its}	Order quantity of component i in period t delivered by supplier s
z_{its}	Setup indicator (z_{it}=1 if component i is manufactured in period t by supplier s)
Q_{its}	Production amount of product i in period t by supplier s
I_{its}^{S}	Inventory of component i in period t of supplier s
ΔC_{ts}	Increase in capacity in period t for supplier s

New or modified constants:

C_s	Normal capacity for the supplier s
α_{ts}	% capacity available in period t for the supplier s
ΔC_{ts}^{max}	Maximal increase in capacity for the supplier s

There have been some changes in the model, as the new index set related to the set of suppliers. Related to the variables, all the ones concerning information about the suppliers now include the index s, to know the information for each supplier. Also the orders and the deliveries have changed, to know which quantity will produce each supplier for the producer. The constants related to the capacity of the suppliers have also changed to have every capacity included. In this model, a new constraint has been created, to adjust the order demand of the producer with the delivery amount of the supplier.

6.2- Ideal model with one supplier and multiple producers (IMP)

In the next model, we are working on a supply chain with one supplier and several producers. This model tries to investigate the effect of smaller producers, each one with smaller orders and jobs, compared to a bigger producer as it can be seen in the ideal model, that is, what happens if on supplier distributes the orders through multiple smaller producers rather than a bigger one. This could be the case when the same supplier has to distribute the products through a group of factories from the company that are located at various geographic points.

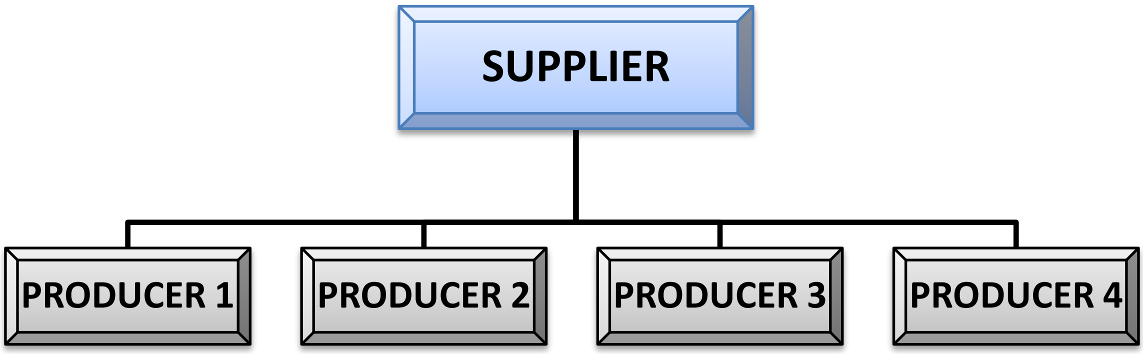

Figure 6. Scheme of a supply chain with one supplier and multiple producers

Objective function:

$$[MIN]Z^{IMP} = \sum_{t=1}^{T}\sum_{p=1}^{P}\left(cY_{tp} + c^{+}Y_{tp}^{+} + c^{-}Y_{tp}^{-}\right) + \sum_{t=1}^{T}\sum_{p=1}^{P}\bar{c}y_{tp} + \sum_{t=1}^{T}\sum_{i=1}^{I}\sum_{p=1}^{P}h_i^P I_{itp}{}^{P} + \sum_{l=1}^{L}F\max_{j\in Jl}\sum_{p=1}^{P}\Delta_{jp} + \sum_{t=1}^{T}\sum_{i=1}^{I}\left(h_i^S I_{it}{}^{S} + \sum_{p=1}^{P}s_i z_{itp}\right) + \sum_{t=1}^{T}\tau\Delta C_t \qquad \text{(IMP1)}$$

Constraints:

Capacity adaptation constraints:

$$Y_{t+1,p} = Y_{tp} + Y_{tp}^{+} - Y_{tp}^{-} \qquad \forall t = 1,\dots,T-1,p \qquad \text{(IMP2)}$$

$$Y_{1p} = Y_1^{'} \qquad \forall p \qquad \text{(IMP3)}$$

$$Y_{tp}^{+} \le Y^{+max} \qquad \forall t,p \qquad \text{(IMP4)}$$

$$Y_{tp}^{-} \le Y^{-max} \qquad \forall t,p \qquad \text{(IMP5)}$$

Capacity constraints:

$$\sum_{j=1}^{J}\sum_{k=1}^{t}\sum_{p=1}^{P}a_{j,p,t+1-k}x_{jkp} \le \sum_{p=1}^{P}Y_{tp} + y_{tp} \qquad \forall t \qquad \text{(IMP6)}$$

$$\sum_{i=1}^{I}\sum_{p=1}^{P}c_i Q_{itp} \le \alpha_t C + \Delta C_t \qquad \forall t \qquad \text{(IMP7)}$$

$$\Delta C_t \le \Delta C_t^{max} \qquad \forall t \qquad \text{(IMP8)}$$

Network constraints:

$$\sum_{t=1}^{T}x_{jtp} = 1 \qquad \forall j,p \qquad \text{(IMP9)}$$

$$\sum_{t=1}^{T}\left(t + D_{jp}\right)x_{jtp} \le E_{lp} + \Delta_{jp} \qquad \forall j \in Jl, \forall l, p \qquad \text{(IMP10)}$$

$$\sum_{t=1}^{T}\left(t + D_{hp}\right)x_{htp} \le \sum_{t=1}^{T}tx_{jtp} \qquad \forall j,h \in P(j) \qquad \text{(IMP11)}$$

Material balance constraints:

$\sum_{j=1}^{J}\sum_{p=1}^{P} v_{ijp}\, x_{jtp} \leq \sum_{p=1}^{P}(d_{itp} + I_{itp}{}^{P}) \qquad \forall i,t$ (IMP12)

$\sum_{p=1}^{P} I_{i,t+1,p} = \sum_{p=1}^{P}(I_{itp} + d_{itp} - \sum_{j=1}^{J} v_{ijp}\, x_{jtp}) \qquad \forall i,t$ (IMP13)

$I_{i1p} = 0 \qquad \forall i,p$ (IMP14)

$I_{i,t+1}^{S} = I_{it}^{S} + Q_{itp} - d_{itp} \qquad \forall i,t,p$ (IMP15)

$I_{i1}^{S} = I_{i}^{'} \qquad \forall i$ (IMP16)

$Q_{itp} \leq M z_{itp} \qquad \forall i,t,p$ (IMP17)

Integrality and non-negativity constraints:

$Y_{tp}, Y_{tp}^{+}, Y_{tp}^{-},\ d_{itp}, y_{tp}, I_{itp}, \Delta_{jp}, I_{it}^{S}, Q_{itp}, \Delta C_{t} \geq 0 \qquad \forall i,t,j,p$ (IMP18)

$x_{jtp} \in \{0,1\}, z_{itp} \in \{0,1\} \qquad \forall j,t,p$ (IMP19)

New or modified indices and index sets:

$\mathrm{p} \in (1..\mathrm{P})$ Producers

New or modified variables:

Y_{tp}	Capacity manpower in period t for the producer p
Y_{tp}^{+}	Increase in capacity in period t for the producer p
Y_{tp}^{-}	Decrease in capacity in period t for the producer p
x_{jtp}	Assembly indicator (x_{jt}=1 if job j is started at time t in producer p)
y_{tp}	Overtime in period t for the producer p
$I_{itp}{}^{P}$	Inventory of component i in period t for producer p
Δ_{jp}	Number of periods job j surpasses its due date in producer p
d_{itp}	Order quantity of component i in period t of the producer p
z_{itp}	Setup indicator (z_{it}=1 if component i is manufactured in period t for producer p)
Q_{itp}	Production amount of product i in period t for the producer p

New or modified constants

Y_{p}^{+max}	Maximal increase in capacity for the producer p
Y_{p}^{-max}	Maximal decrease in capacity for the producer p
a_{jkp}	Capacity consumption of job j in period k after its start for the producer p

D_{jp}	Duration of the job for the producer p
E_{lp}	Due date of customer order l for the producer p
v_{ijp}	Quantity of component i required to start job j for the producer p

As a new index set there can be found the group of producers. Related to the modified variables there are changes that affect all the variables that came from the producer's model. There are different capacities, overtimes and inventories for each producer. Also there are some changes in the order quantities form the producers (d_{itp}), which now takes into account the origin of each order according to the producer. About the production amount of the components that has to be manufactured (Q_{itp} , z_{itp}), now it is introduced also the destination of each production.

For the modified constants, now there are multiple capacities for the different producers, as well as related to the jobs taken in each factory: duration, capacity consumption and due dates for the orders in every producer.

6.3- Ideal model with multiple suppliers and multiple producers (IMSP)

Next model is the global ideal model that can fit with all type of supply chains, as it introduces multiple suppliers and multiple producers. This model can indicate the best distribution of the order quantities and deliveries, with an arbitrary structure of the chain. Essentially, this model tries to represent the behavior of smaller suppliers together with smaller producers, each one having smaller capacities, productivity and features, and tries to compare them with a bigger supplier and a bigger producer like in the ideal model.

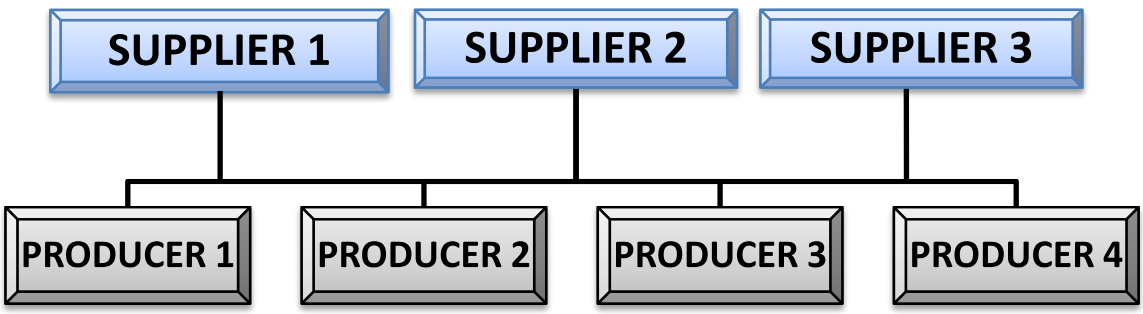

Figure 7. Scheme of a supply chain with an arbitrary distribution of suppliers and producers

Objective function:

$$[MIN]Z^{IMSP} =$$

$$\sum_{t=1}^{T}\sum_{p=1}^{P}\left(cY_{tp} + c^{+}Y_{tp}^{+} + c^{-}Y_{tp}^{-}\right) + \sum_{t=1}^{T}\sum_{p=1}^{P}\bar{c}y_{tp} + \sum_{t=1}^{T}\sum_{i=1}^{I}\sum_{p=1}^{P}h_{i}^{P}I_{itp}{}^{P} + \sum_{l=1}^{L}F\max_{j\in Jl}\sum_{p=1}^{P}\Delta_{jp} + \sum_{t=1}^{T}\sum_{i=1}^{I}\sum_{s=1}^{S}(h_{i}^{S}I_{its}^{S} + \sum_{p=1}^{P}s_{i}z_{itps}) + \sum_{t=1}^{T}\sum_{s=1}^{S}\tau\Delta C_{ts} \quad \text{(IMSP1)}$$

Constraints:

Capacity adaptation constraints:

$Y_{t+1,p} = Y_{tp} + Y_{tp}^{+} - Y_{tp}^{-}$ $\quad \forall t = 1,\ldots,T-1,p$ (IMSP2)

$Y_{1p} = Y_{1}^{'}$ $\quad \forall p$ (IMSP3)

$Y_{tp}^{+} \leq Y^{+max}$ $\quad \forall t,p$ (IMSP4)

$Y_{tp}^{-} \leq Y^{-max}$ $\quad \forall t,p$ (IMSP5)

Capacity constraints:

$\sum_{j=1}^{J}\sum_{k=1}^{t}\sum_{p=1}^{P}a_{j,p,t+1-k}x_{jkp} \leq \sum_{p=1}^{P}Y_{tp} + y_{tp}$ $\quad \forall t$ (IMSP6)

$\sum_{i=1}^{I}\sum_{p=1}^{P}\sum_{s=1}^{S}c_{i}Q_{itps} \leq \sum_{s=1}^{S}\alpha_{ts}C_{s} + \Delta C_{ts}$ $\quad \forall t$ (IMSP7)

$\Delta C_{ts} \leq \Delta C_{ts}^{max}$ $\quad \forall t,s$ (IMSP8)

Network constraints:

$\sum_{t=1}^{T}x_{jtp} = 1$ $\quad \forall j,p$ (IMSP9)

$\sum_{t=1}^{T}(t + D_{jp})x_{jtp} \leq E_{lp} + \Delta_{jp}$ $\quad \forall j \in Jl, \forall l, p$ (IMSP10)

$\sum_{t=1}^{T}(t + D_{hp})x_{htp} \leq \sum_{t=1}^{T}tx_{jtp}$ $\quad \forall j, h \in P(j)$ (IMSP11)

Material balance constraints:

$\sum_{j=1}^{J}\sum_{p=1}^{P}v_{ijp}x_{jtp} \leq \sum_{k=1}^{K}\sum_{p=1}^{P}(d_{itpk} + I_{itp}{}^{P})$ $\quad \forall i,t$ (IMSP12)

$\sum_{p=1}^{P}I_{i,t+1,p}{}^{P} = \sum_{p=1}^{P}I_{itp}{}^{P} + \sum_{s=1}^{S}d_{itps} - \sum_{j=1}^{J}v_{ijp}x_{jtp}$ $\quad \forall i,t$ (IMSP13)

$I_{i1p}{}^{P} = 0$ $\quad \forall i,p$ (IMSP14)

$I_{i,t+1,s}^{S} = I_{its}^{S} + Q_{itps} - d_{itps}$ $\quad \forall i,t,p,s$ (IMSP15)

$I_{i1s}^{S} = I_{i}^{'}$ $\quad \forall i,s$ (IMSP16)

$Q_{itps} \leq Mz_{itps}$ $\quad \forall i,t,p,s$ (IMSP17)

Integrality and non-negativity constraints:

$Y_{tp}, Y_{tp}^{+}, Y_{tp}^{-}, d_{itps}, y_{tp}, I_{itp}{}^{P}, \Delta_{jp}, I_{its}^{S}, Q_{itps}, \Delta C_{ts} \geq 0$ $\quad \forall i,t,j,p,s$ (IMSP19)

$x_{jtp} \in \{0,1\}, z_{itps} \in \{0,1\}$ $\quad \forall j,t,p,s$ (IMSP20)

New or modified indices and index sets:

p $\in (1..P)$ Producers

s $\in (1..S)$ Suppliers

New or modified variables:

Y_{tp}	Capacity manpower in period t for the producer p
Y_{tp}^{+}	Increase in capacity in period t for the producer p
Y_{tp}^{-}	Decrease in capacity in period t for the producer p
x_{jtp}	Assembly indicator (x_{jt}=1 if job j is started at time t in producer p)
y_{tp}	Overtime in period t for the producer p
${I_{itp}}^{P}$	Inventory of component i in period t for producer p
Δ_{jp}	Number of periods job j surpasses its due date in producer p
d_{itps}	Order quantity of component i in period t ordered by the producer p to the supplier s.
z_{itps}	Setup indicator (z_{it}=1 if component i is manufactured by the supplier s in period t for producer p)
Q_{itps}	Production amount of product i by supplier s in period t for the producer p
I_{its}^{S}	Inventory of component i in period t of supplier s
ΔC_{ts}	Increase in capacity in period t for supplier s

New or modified constants

Y_{p}^{+max}	Maximal increase in capacity for the producer p
Y_{p}^{-max}	Maximal decrease in capacity for the producer p
a_{jkp}	Capacity consumption of job j in period k after its start for the producer p
D_{jp}	Duration of the job for the producer p
E_{lp}	Due date of customer order l for the producer p
$v_{ij\,p}$	Quantity of component i required to start job j for the producer p
C_{s}	Normal capacity for the supplier s
α_{ts}	% capacity available in period t for the supplier s
ΔC_{ts}^{max}	Maximal increase in capacity for the supplier s

The global model (IMSP) integrates both producers and suppliers, as a mix of the previous models (IMS and IMP). Thus, the producer will know which is the best supplier to order the lot and the supply will deliver it on the best schedule possible.

Related to the changes on the model, there are the same indices, variables and constants as the previous models, except for the orders and the production amount of each component i, that now specify which producer is demanding the order and which supplier is producing the lot.

7- Numerical analysis

In this section, a numerical analysis will be undertaken to test and validate the initial models and the new extended models. The software used to solve all the models will be AMPL. AMPL is an acronym for "A Mathematical Programming Language", is an algebraic modeling language for describing and solving high-complexity problems for large-scale mathematical computation (i.e. large-scale optimization and scheduling-type problems). One particular advantage of AMPL is the similarity of its syntax to the mathematical notation of optimization problems. This allows for a very concise and readable definition of problems in the domain of optimization.[5]

7.1- Specification of the analysis

In order to analyze the models, we will specify all the data from each model.

The producer's model is a capacity model with a work load given by a sequence of 12 orders comprising altogether 28 jobs. The planning horizon is 10 periods. The structure of the orders and the jobs will be as follows:

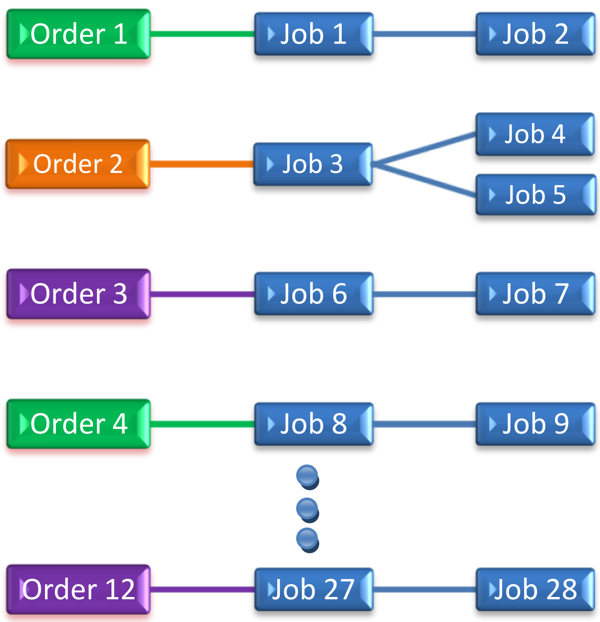

Figure 8. Structure of the orders

Figure 8 shows us that there are three types of orders. The first type (green) consists of 4 orders containing 2 jobs each order, the second type (orange) consists of 4 orders

[5] Robert Fourer, 2003.

but in this case each order has three jobs, two of them working in parallel. And the last type of orders (purple) consists also of 4 orders with 2 jobs each order. That makes a total of 12 orders and 28 jobs.

In the next section, there will be the data specified for the producer for each type, but is important to consider that it has to be repeated for the total number of orders.

Order type	1		2			3	
Number of orders	4		4			4	
Job	1	2	3	4	5	6	7
Predecessor	-	1	-	3	3	-	6
Number of component 1	10	20	0	20	0	10	10
Number of component 2	0	10	20	20	20	10	0
Duration	3	2	1	4	2	2	1
Due date	6	6	9	9	9	7	7

Table 1. Data of orders

a_{jt}	**Job j**						
	1	2	21	22	23	81	82
1	40	10	20	20	20	10	20
2	40	10		20	20	10	
3	40			20			
4				20			

Table 2. Capacity consumption of job j in period t after its start

Constants	**Value**
c	0,5 ($/h)
c^+	1($/h)
c^-	0,2($/h)
Y'_1	40 (h)
Y^{+max}	30(h)
Y^{-max}	30(h)
$\bar{c}$	2($/h)
F	60($/h)
h^p_1, h^p_2	2($/unit),2($/unit)
p_1,p_2	10($/unit),10($/unit)
K	1,5($/unit)
e	0,5($/unit)

Table 3. Data of producer's model

The supplier’s data will be specified as follows.

Constants	Value
p_1,p_2	10($/unit),10($/unit)
h^s_1, h^s_2	1($/unit),1($/unit)
s_1,s_2	1($), 1($)
τ	1,5($/h)
I'_1, I'_2	0 (units), 0 (units)
c_1,c_2	2(h/unit), 2(h/unit)
C	300(h)
ΔC^{max}_t	80(h)

Table 4. Data of supplier model

Period	1	2	3	4	5	6	7	8	9	10
alpha	0,62	0,20	0,44	0,14	0,30	0,22	0,22	0,22	0,50	0,50

Table 5. Expectation of the distribution variable alpha in period t

The most important fact in all the data is that a high value for penalty cost F for exceeding the due date has been chosen to make sure that delayed deliveries are really an issue and hence, coordination mechanisms are indeed effective.

For the new producers and suppliers created in the extended models the data is specified as follows.

There will be a total of 4 producers and/or 3 suppliers in the supply chain. In order to make a good comparison between models, the data chosen is the one dividing the original number by 4 or 3, depending if it is a parameter from the producer or from the supplier. In that way, the total amount of orders will be divided and distributed for each producer, so each one will have a total of 3 orders and 7 jobs (see figure 9).

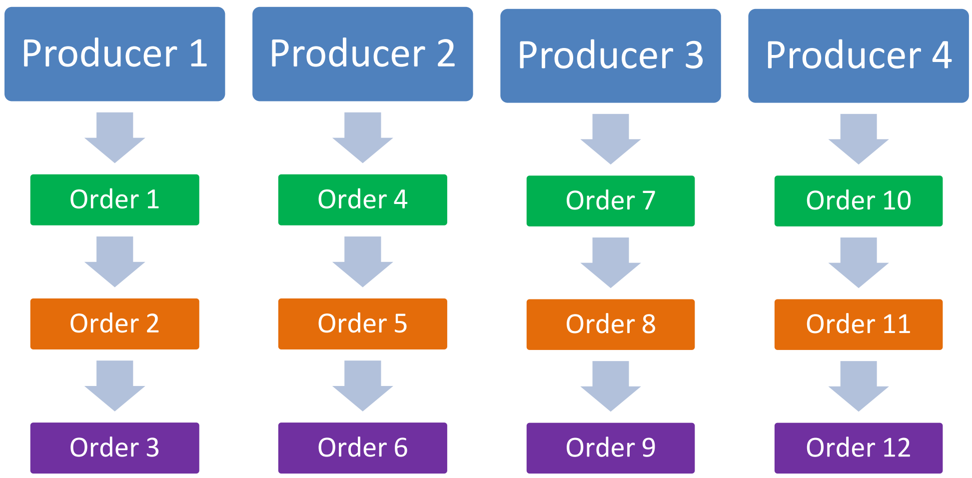

Figure 9. Distribution of the orders of the producers

The duration, predecessors, due dates, etc., and the parameters related with the orders will have the same value as mentioned before for the individual parties.
The values that change in these new models for the new producers will be the next:

Y'1	10 (h)
Y^{+max}	hours
Producer 1	7,5(h)
Producer 2	7,5(h)
Producer 3	7,5(h)
Producer 4	7,5(h)
Y^{-max}	hours
Producer 1	7,5(h)
Producer 2	7,5(h)
Producer 3	7,5(h)
Producer 4	7,5(h)

C	
Supplier 1	100
Supplier 2	100
Supplier 3	100
ΔCmaxt	
Supplier 1	26
Supplier 2	26
Supplier 3	26

Table 6. Data changed for new producers and suppliers

Period	1	2	3	4	5	6	7	8	9	10
Supplier 1- alpha	0,62	0,20	0,44	0,14	0,30	0,22	0,22	0,22	0,50	0,50
Supplier 2- alpha	0,62	0,20	0,44	0,14	0,30	0,22	0,22	0,22	0,50	0,50
Supplier 3- alpha	0,62	0,20	0,44	0,14	0,30	0,22	0,22	0,22	0,50	0,50

Table 7. ALpha values for the suppliers

In order to complete the *non reactive anticipation* and the *reactive anticipation* models, the following data will be used for their estimated values related to the supplier:

- **Non reactive anticipation:**

Constants	Value
τ	1,5($/h)
c_1, c_2	2(h/unit), 2(h/unit)
C	300(h)
AL	120(h)

Table 8. Data for the new constraints in non reactive anticipation

Period	1	2	3	4	5	6	7	8	9	10
Expected alpha	0,3	0,3	0,3	0,3	0,3	0,3	0,3	0,3	0,3	0,3

Table 9. Alpha expected for the non reactive anticipation model

- **Reactive anticipation:**

In order to react to the supplier's delivery, this model recreates a supplier model with their expected data, which will be:

Constants	Value
p_1,p_2	10($/unit),10($/unit)
h^s_1, h^s_2	1($/unit),1($/unit)
s_1,s_2	1($), 1($)
τ	1,5($/h)
I'_1, I'_2	0 (units), 0 (units)
c_1,c_2	2(h/unit), 2(h/unit)
C	300(h)
ΔC^{max}_t	100(h)

Table 10. Expected supplier data for the reactive anticipation model

Period	1	2	3	4	5	6	7	8	9	10
Expected alpha	0,3	0,3	0,3	0,3	0,3	0,3	0,3	0,3	0,3	0,3

Table 11. Expected alpha for the reactive anticipation model

To get the q, which is the parameter that gets the reaction to the supplier, it will be introduced the demand of the non reactive model to the estimated supplier model. Then, the supplier model will obtain his order delivery, which will be introduced to the reactive anticipation model in the equation:

$$\delta_{it} = q_{it}$$

This method is planned to get a good anticipative reaction to the supplier with a better performance.

7.2- Results

In this section, an extensive analytical investigation will be made, reproducing all the previous models in a spreadsheet and modifying some of the most important characteristics of the supplier one by one to get a sequential value with the program AMPL. The models will be compared in two ways; the first one will analyze the effect of different anticipation of the producers while the second one will analyze the ideal models with different number of producers and suppliers.

7.2.1- Total costs

The main result consists on analyze the total cost performance of the supply link with the different types of anticipative coordination and with the extended ideal models.
The total costs are obtained in optimizing the appropriate supply link for the specified data and in varying the alpha value to get the worst case and the best case, with some arbitrary values within the range of alpha. Then, taking the equation ID1 as a reference, all the values will be replaced for those obtained to get an impartial and adequate comparison between methods.
For different values of alpha, we decided to use arbitrary values within the range, which will be the following:

Alpha range / Period	0,1-0,2	0,2-0,3	0,3-0,4	0,4-0,5	0,5-0,6	0,6-0,7	0,7-0,8	0,8-0,9	1
1	0,11	0,21	0,31	0,41	0,51	0,61	0,71	0,81	1,00
2	0,18	0,28	0,38	0,48	0,58	0,68	0,78	0,88	1,00
3	0,10	0,20	0,30	0,40	0,50	0,60	0,70	0,80	1,00
4	0,13	0,23	0,33	0,43	0,53	0,63	0,73	0,83	1,00
5	0,17	0,27	0,37	0,47	0,57	0,67	0,77	0,87	1,00
6	0,20	0,30	0,40	0,50	0,60	0,70	0,80	0,90	1,00
7	0,20	0,30	0,40	0,50	0,60	0,70	0,80	0,90	1,00
8	0,20	0,30	0,40	0,50	0,60	0,70	0,80	0,90	1,00
9	0,15	0,25	0,35	0,45	0,55	0,65	0,75	0,85	1,00
10	0,15	0,25	0,35	0,45	0,55	0,65	0,75	0,85	1,00

Table 12. Arbitrary values assigned

The next figure gives the corresponding result of the variation of the parameter alpha and the effect that can cause to the models in each type of anticipation. This effect is due to the fact that the supplier is distributing the same components to other producers that are not taken into account by the producer; hence the capacity available is restricted. Looking at figure 10, the reactive anticipation is the closest to the ideal model, which is hardly applicable on the real world. Depending on how good the producer anticipated the supplier model, the total cost of the reactive model would slightly change up or down. In this case, the total cost can be improved, as the data used for the estimated supplier slightly defers on the real data. Then it can be assumed that for the closest data it can be obtained compared to the real supplier, the better solution you can get, and most similar to the ideal model solution. There is also clear the advantage of this model in front of the non reactive anticipation and the pure top-

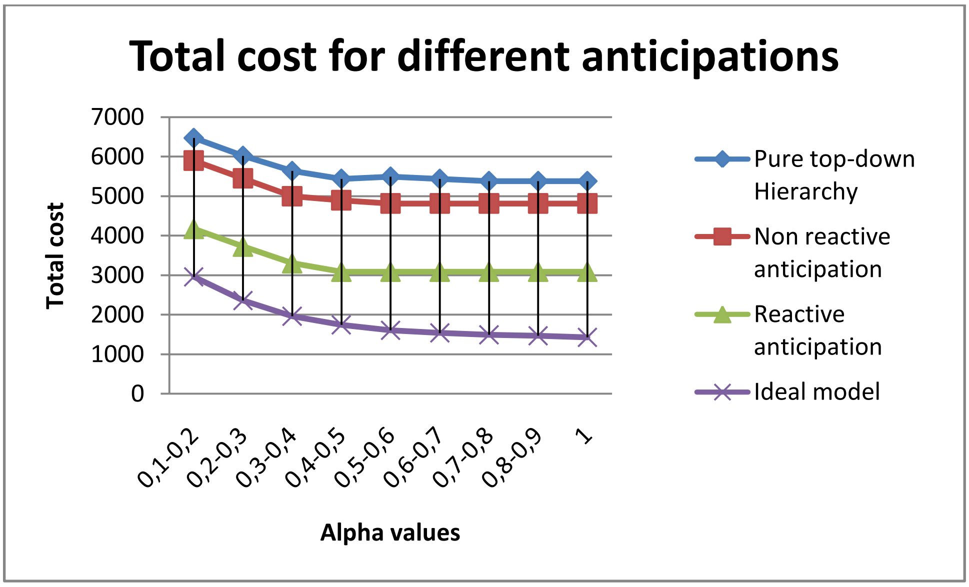

Figure 10. Total cost for different anticipations

down hierarchy. The pure top-down hierarchy model doesn't take into account any characteristics of the supplier or any possible reactions, so it is the worst model obtained as the total cost of the supply chain is really high. Non reactive anticipation model tries to take into account some important features of the supplier so it can get a better result than the pure top-down, but as it cannot react to the supplier deliveries the result cannot be as good as the reactive one. The most important of figure 10 is that it is showed the advantage of combining control-oriented and task-oriented coupling mechanisms.

For the extended models, in figure 11 there is a comparison with the ideal models depending if they have one or multiple parties.

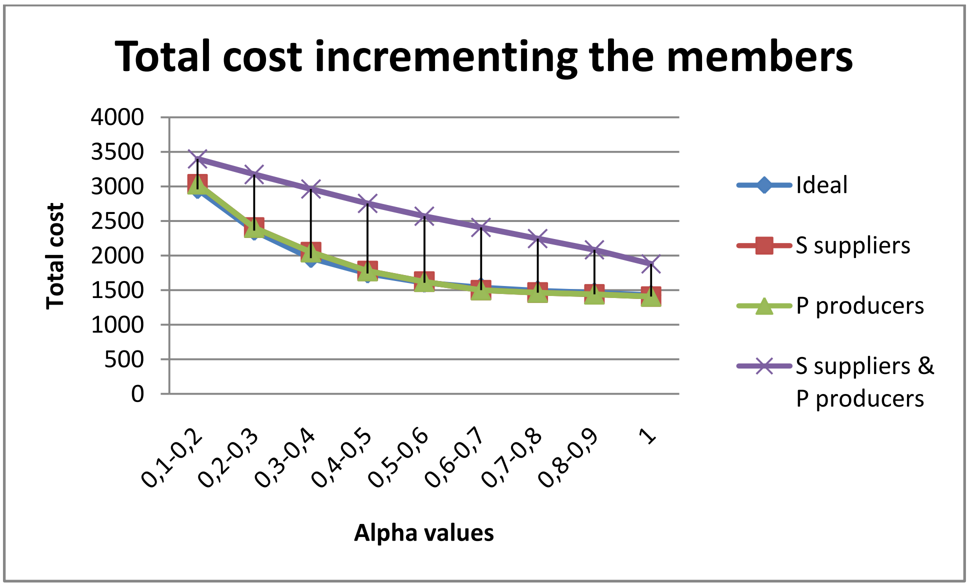

Figure 11. Total cost for ideal models with multiple producers/suppliers

As it can be observed in the results, the difference between incrementing only the suppliers, or incrementing only the producers is nearly unappreciated , as the models have the same reaction and performance than in the ideal model. In the case of multiple suppliers or multiple producers altogether, it can be seen that total cost is higher than the other models, even though the result improves with the extreme values of alpha.

It would be important to remark in this case, the values obtained in the model with an increment of the producers but with only one supplier. From alphas ranged higher than 60%, this model is slightly better than the ideal model, as you can see in the table 13.

Alpha range	0,1-0,2	0,2-0,3	0,3-0,4	0,4-0,5	0,5-0,6	0,6-0,7	0,7-0,8	0,8-0,9	1
Ideal	2960	2359	1964	1742	1608	1537	1494	1465	1427
S suppliers	2977	2375	1979	1758	1618	1545	1503	1471	1432
P producers	3030	2403	2048	1775	1621	1500	1464	1439	1407
Global	3396	3172	2962	2754	2569	2408	2246	2084	1881

Table 13. Numerical results in extended models with different range of alphas

7.2.2- Private knowledge about capacity expansion cost

Figure 12 gives the result for the pure top-down hierarchy, the non reactive anticipation, the reactive anticipation and the ideal model. The comparison of these models is performed in optimizing and evaluating total costs, using the variables optimized in equation ID1.

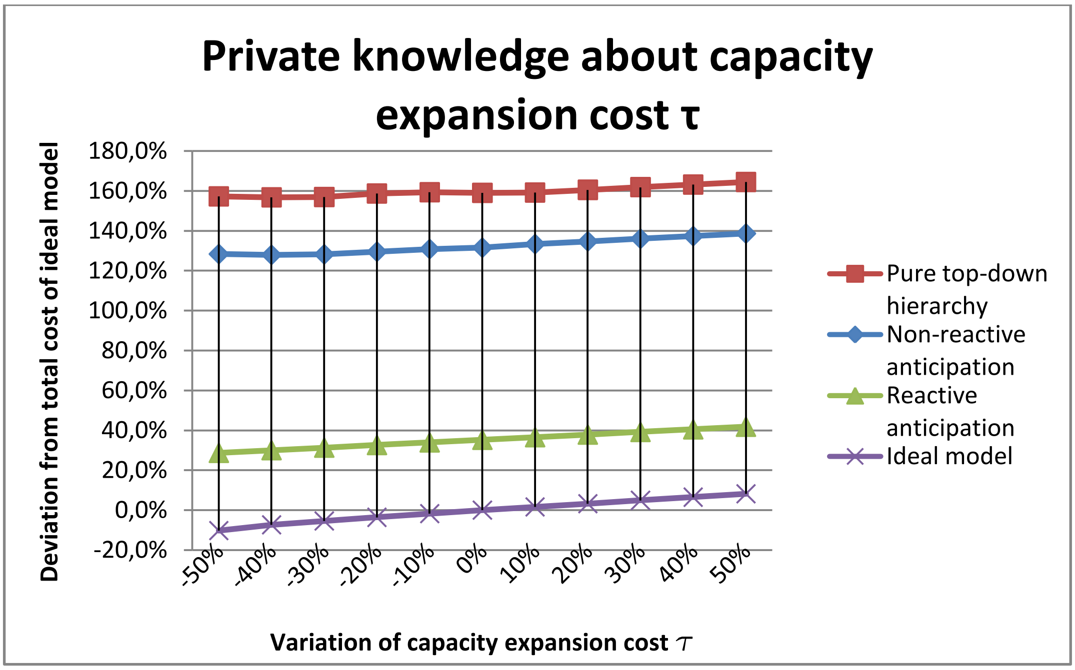

Figure 12. Private knowledge about capacity expansion cost

In this figure (Fig. 12), the expansion capacity cost is changing to higher or lower values, and the results compare the value get with the models to the deviation from the total cost of the ideal model in the initial value of τ=1,5.

As it is shown in Figure 12, results are improving in every model as the parameter value is lower. Hence, the deviation of anticipated model is better than the other two models but it is slightly worse than the ideal model.

For the extended models in the figure 13, it is found that incrementing de τ for the ideal model with multiple suppliers or multiple producers it is not a factor that affects the result, as the two models follows exactly the same deviation as the ideal model with individual parties. In the other hand, the model with various producers and suppliers altogether, has a deviated result from the others. Despite the bad results of the global model, it is getting some improvements with bigger τ, then it gets only a deviation from the other ideal models around 20%, in contrast of the deviation around 40% that it gets when τ are smaller.

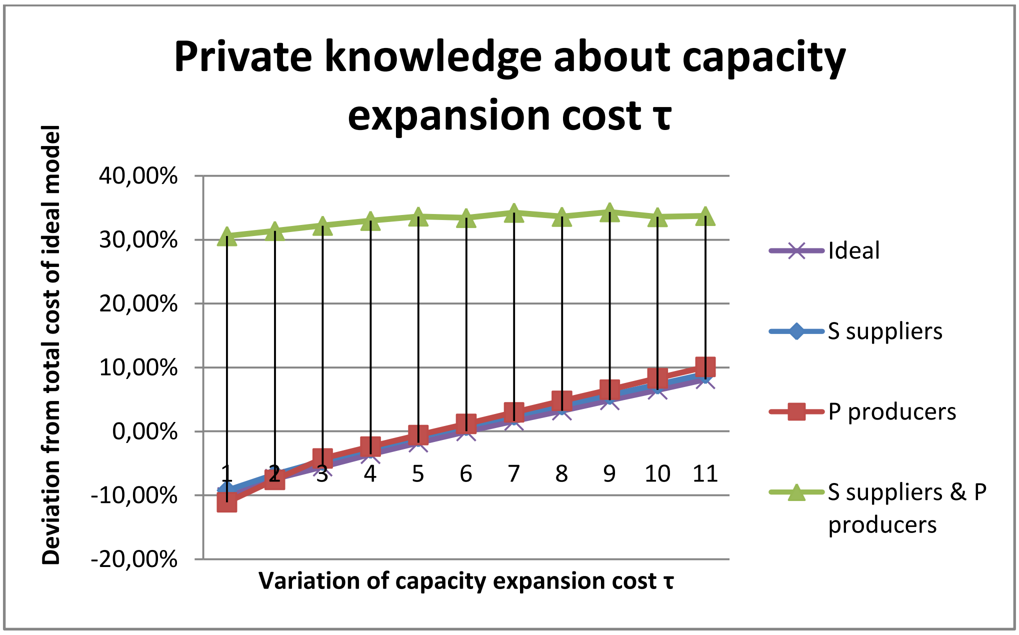

Figure 13. Private knowledge about capacity expansion cost for suppliers/producers

In table 14 it can be seen the numerical results for the extended models when there is a variation of the capacity cost τ. It is of importance to remark that some values from the model with multiple producers are again getting some better performance when the capacity expansion cost is reduced around a 40% or 50%. Then, there is an improvement of the total cost compared with the ideal model with only one producer.

Cost τ	-50%	-40%	-30%	-20%	-10%	0%	10%	20%	30%	40%	50%
Ideal	1974	2036	2078	2120	2160	2199	2235	2270	2306	2342	2378
%	-10,23	-7,41	-5,50	-3,59	-1,77	0,00	1,64	3,23	4,87	6,50	8,14
S suppliers	1996	2051	2097	2139	2179	2217	2253	2288	2324	2360	2396
%	-9,2	-6,7	-4,6	-2,7	-0,9	0,8	2,5	4,0	5,7	7,3	9,0
P producers	1955	2032	2106	2147	2186	2225	2265	2304	2343	2382	2421
%	-11,10	-7,59	-4,23	-2,36	-0,59	1,18	3,00	4,77	6,55	8,32	10,10
Global	2871	2889	2907	2924	2939	2934	2952	2938	2954	2937	2941
%	30,56	31,38	32,20	32,97	33,65	33,42	34,24	33,61	34,33	33,56	33,74

Table 14. Numerical results in extended models for different costs τ

7.2.3- Private knowledge about the supplier's capacity situation

The next results are considering the case that the normal supplier's capacity is not known by the producers. The more the supplier is committed to other producers, the less sure the producer can be to estimate its correct capacity. In figure 14, capacity will be varied taken into account that non reactive anticipation model and anticipated model have estimated a C=300(h). Again, the ideal model represents the best results and the pure top-down hierarchy the worst case. Between the two anticipation models is important to remark the benefit of having a reactive anticipation against having a non reactive one, because the improvement of the model is notorious. Additionally, the effect of having overestimations or underestimations of the capacity affects the total costs of these functions. For example, an overestimation of the capacity does not affect the performance of the anticipation while an underestimation results in higher costs.

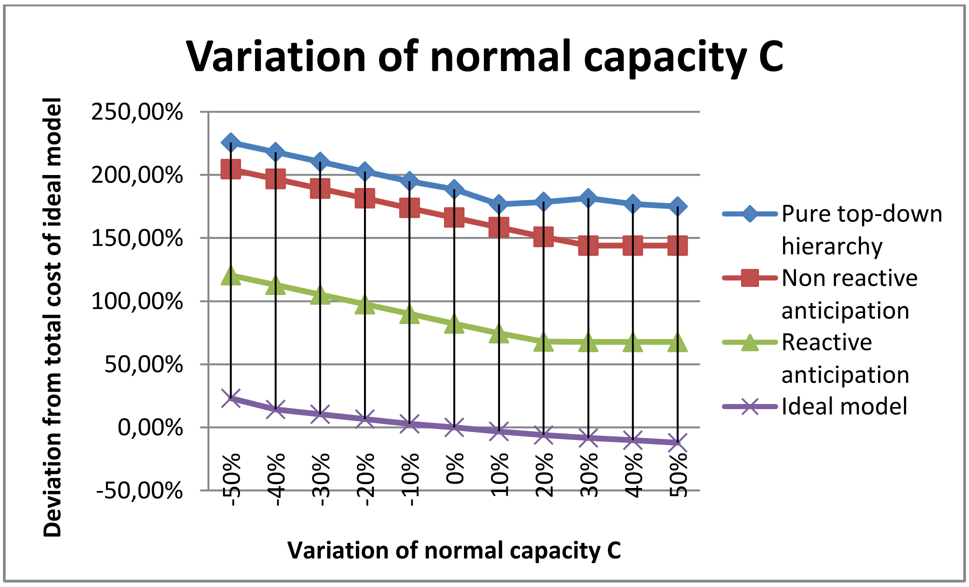

Figure 14. Variation of normal capacity for different anticipations

In the case of figure 15, it can be seen that the differences between the models with multiple parties and one individual, with the ideal model, are slightly different independently of the variation of the capacity. Again, we can see that the model with multiple parties in producers and suppliers is worse than the other models, but that he reacts different depending on the variation of the capacity. For example, for capacities reduced to a half, the global model only defers from the others around a 10%. But for capacities incremented to a half, then the total cost increments around a 40%.

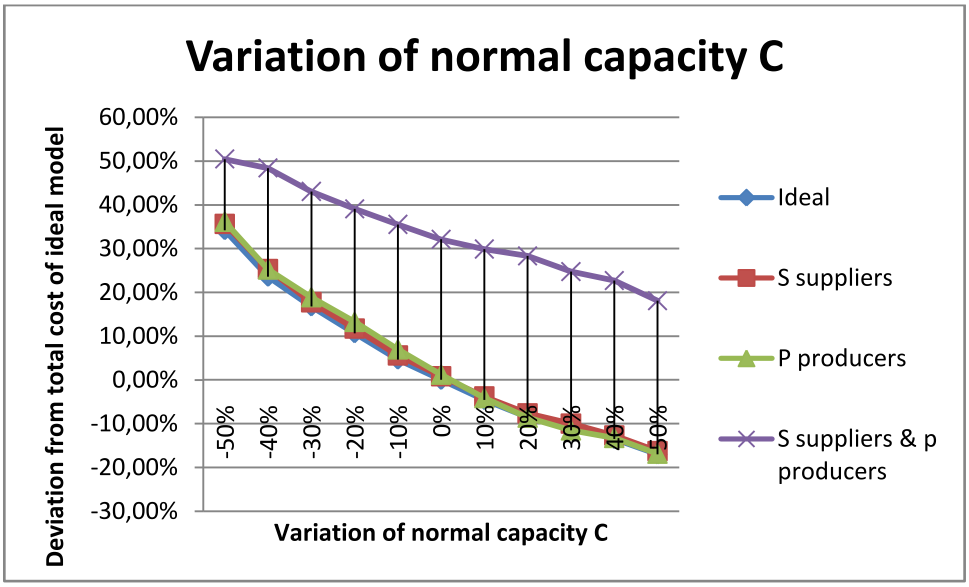

Figure 15. Variation of normal capacity for suppliers/producers

Finally, looking at the results in detail in table 16, it can be observed again that in two cases there is an improvement of the ideal model with multiple producers compared to the ideal with individual parties. Here, for capacities incremented around 20% or 30% multiple producers can have a better performance than the ideal model.

Capacity	-50%	-40%	-30%	-20%	-10%	0%	10%	20%	30%	40%	50%
Ideal	2951	2717	2568	2433	2301	2199	2098	2012	1963	1902	1825
%	34,20	23,56	16,78	10,64	4,64	0,00	-4,59	-8,50	-10,73	-13,51	-17,01
S suppliers	2982	2755	2589	2456	2321	2217	2115	2030	1979	1919	1841
%	35,61	25,28	17,74	11,69	5,55	0,82	-3,82	-7,69	-10,00	-12,73	-16,28
P producers	2993	2753	2614	2490	2352	2225	2107	2009	1945	1907	1828
%	36,11	25,19	18,87	13,23	6,96	1,18	-4,18	-8,64	-11,55	-13,28	-16,87
Global	3308	3263	3145	3058	2980	2905	2856	2822	2743	2698	2597
%	50,43	48,39	43,02	39,06	35,52	32,11	29,88	28,33	24,74	22,69	18,10

Table 15. Numerical results in extended models for different capacities

8- Concluding remarks

This thesis analyzed coordination mechanisms in supply chain. Using the theory of hierarchical planning it is presented different kinds of reactions for the producers, and finally it is obtained an ideal model that links the producer with the supplier, embracing the whole network. Then, an extension of the ideal model has been made to include more producers or more suppliers in the supply chain.

In total, it has been showed four types of anticipation: the pure top-down hierarchy, the non –reactive anticipation, the reactive anticipation and the ideal model, which is hardly applicable. For the pure top-down hierarchy, none of the features of the supplier are taken into account, which implies that only a task-oriented instruction is exerted. For the non-reactive anticipation, important characteristics of the supplier were considered, particularly his capacity situation and the ability to adapt his capacity. This type of anticipation, however, is still on a low level of integration, since it does not account for any reaction to the producer's action. Hence, as in the pure top-down case, one has only a task-oriented type of instruction showing a performance which, essentially, is not much better than the pure top-down hierarchy case. Only the reactive anticipation which fully takes into account the supplier's model results in a substantially improved outcome deviating only slightly from the ideal situation. Formally, this satisfactory performance is due to the combined effect of a task-oriented and a control-oriented coupling mechanism.

Clearly, the reactive anticipation is not that easy to calculate as compared with the pure top-down and the non-reactive anticipation coordination schemes. In addition, far more information needs to be known. On the other hand, the results do not differ too much from the ideal situation for which even more data is necessary and even more complex analysis must be performed. Hence for a situation in which the parties are willing closely to cooperate, the reactive anticipation can be considered as a reasonable scheme.

Three types of extended models have also been considered: the model with multiple suppliers which distribute to one producer, the model with multiple producers that order to the same supplier and the case when various producers order to a various suppliers. Taking the same amount of orders for all the producers and the same capacity for all the suppliers, we can conclude that the model with one producer and one supplier gives the best results in most of the cases, even though one producer – multiple suppliers and multiple producers – one supplier doesn't defer too much from the ideal results. In addition, multiple producers sometimes can give a better result than only one individual, depending on the capacity of the supplier and its parameters. On the other hand, working with smaller producers together with smaller suppliers it is not the best solution for the global supply chain performance as it can be seen in the results.

This thesis is focused on the operational performance of a supply chain using coordination mechanisms. So it would be important for a new contract between producer/s and supplier/s, to use these operational results for the negotiations. Comparing information costs with the advantages of a closer coordination would be a key issue not only favorable for the producer but for the supplier as well. In particular, in agreeing on a reactive anticipative coordination scheme one would choose optimal values for total costs and delayed deliveries.

Finally, it would be of major interest to study the introduction of multiple smaller producers, rather than a larger one, but considering the capacity and the features of the supplier to get the best optimized result in the total cost of the supply chain.

9- Appendices

The next section shows the extended models recreated with AMPL program, including the model and the data used. As they have been tested for different situations, the models that are next described may have some arbitrary data, depending on the situation last used.

A. Ideal model with one producer and multiple suppliers (IMS)

A.1. Model

```
# MODEL PARAMETERS
        param T >=0 integer;
        param I >=0 integer;
        param J >=0 integer;
        param L >=0 integer;
        param S >=0 integer;

#INDEX SETS
        set PERIOD := 1..T;
        set COMP := 1..I;
        set JOB := 1..J;
        set ORDER := 1..L;
        set JOBl {ORDER};
        set PRED {JOB};
        set PERIOD2 := 1..T+1;
        set SUPP := 1..S;

#DECISION VARIABLES
        var Y {t in PERIOD}>=0 ;
        var Yp {t in PERIOD}>=0 ;
        var Ym {t in PERIOD}>=0 ;
        var x {(j,t) in {JOB, PERIOD}}binary;
        var y {t in PERIOD}>=0 ;
        var Inv {(i,t) in {COMP, PERIOD}}>=0 ;
        var In {l in ORDER, j in JOBl[l]}>=0 ;
        var d {(i,t,k) in {COMP, PERIOD, SUPP}}>= 0 ;
        var hi {l in ORDER}>=0 ;
        var Invs {(i,t,k) in {COMP, PERIOD2, SUPP}}>= 0 ;
        var z {(i,t,k) in {COMP, PERIOD,SUPP}}binary;
        var INC {t in PERIOD, k in SUPP}>= 0 ;
        var Q {(i,t,k) in {COMP, PERIOD,SUPP}}>= 0 ;
#CONSTANTS
        param c>=0;
```

```
    param cp>=0;
    param cm>=0;
    param p{i in COMP} >=0;
    param co >=0;
    param hp {i in COMP} >=0;
    param pc>=0;
    param Yo>=0;
    param Ypmax>=0;
    param Ymmax>=0;
    param a{(j,k) in {JOB, PERIOD}}>=0;
    param D{j in JOB}>=0;
    param E{l in ORDER}>=0;
    param v{(i,j) in {COMP, JOB}}>=0;
    param hs {i in COMP} >=0;
    param s {i in COMP} >=0;
    param kc >=0;
    param MAX>=0;
    param Io {i in COMP}>=0;
    param cmc {i in COMP} >=0;
    param C {k in SUPP} >=0;
    param alfa {t in PERIOD, k in SUPP}>=0;
    param INCmax {t in PERIOD, k in SUPP}>=0;

#OBJECTIVE
    minimize objective: sum {t in PERIOD} (c*Y[t] + cp*Yp[t]+ cm*Ym[t]) + sum {t in
    PERIOD} co*y[t] + sum {t in PERIOD, i in COMP} hp[i] * Inv[i,t] + sum {l in
    ORDER} pc * hi[l]+ sum{t in PERIOD, i in COMP, k in
    SUPP}(hs[i]*Invs[i,t,k]+s[i]*z[i,t,k])+sum{t in PERIOD, k in SUPP}kc * INC[t,k];

#CONSTRAINTS
#CAPACITY ADAPTATION CONSTRAINTS
    subject to Capacity {t in 1..T-1}:
    Y[t+1]==Y[t]+Yp[t]-Ym[t];
    subject to Capacity2:
    Y[1]==Yo;
    subject to Capacity3 {t in PERIOD}:
    Yp[t]<=Ypmax;
    subject to Capacity4 {t in PERIOD}:
    Ym[t]<=Ymmax;

#CAPACITY CONSTRAINTS
    subject to Capacity5 {t in PERIOD}:
    sum{j in JOB, k in 1..t} a[j, t+1-k]*x[j,k]<=Y[t]+y[t];
    subject to Capacity6 {t in PERIOD, k in SUPP}:
    sum {i in COMP} cmc[i] * Q[i,t,k] <= alfa[t,k] * C[k] + INC[t,k];
    subject to Max_Capacity {t in PERIOD, k in SUPP}:
    INC[t,k] <= INCmax[t,k];
```

```
#NETWORK CONSTRAINTS
        subject to network1 {j in JOB}:
        sum{t in PERIOD} x[j,t]=1;
        subject to network2 {l in ORDER, j in JOBl[l]}:
        sum{t in PERIOD}(t+D[j])*x[j,t]<=E[l]+In[l,j];
        subject to network3 {j in JOB, h in PRED[j]}:
        sum{t in PERIOD}(t+D[h])*x[h,t]<=sum{t in PERIOD}t*x[j,t];

#MATERIAL BALANCE CONSTRAINTS
        subject to material1 {i in COMP, t in PERIOD}:
        sum{j in JOB}v[i,j]*x[j,t]<=sum{k in SUPP} d[i,t,k]+Inv[i,t];
        subject to material2 {i in COMP, t in 1..T-1}:
        Inv[i,t+1]==Inv[i,t]+sum{k in SUPP}d[i,t,k]- sum{j in JOB}v[i,j]*x[j,t];
        subject to material3 {i in COMP}:
        Inv[i,1]== 0;
        subject to Inventory {i in COMP, t in PERIOD, k in SUPP}:
        Invs[i,t+1,k] == Invs[i,t,k] + Q[i,t,k] - d[i,t,k];
        subject to In_INventory {i in COMP, k in SUPP}:
        Invs[i,1,k]== 0;
        subject to Production {i in COMP, t in PERIOD, k in SUPP}:
        Q[i,t,k] <= z[i,t,k]*MAX;

#OTHERS
        subject to DefineHi {l in ORDER, j in JOBl[l]}:
        hi[l]>=In[l,j];
        subject to Product {i in COMP, k in SUPP}:
        sum{t in PERIOD} d[i,t,k]==sum{t in PERIOD}Q[i,t,k];
```

A.2. Data of the model

```
param T := 10;
param I := 2;
param J := 28;
param L := 12;
param S := 3;
param c :=0.5;
param cp := 1;
param cm := 0.2;
param p:=
1 10
2 10;
```

```
param co := 2;
param hp :=
1 2
2 2;
param pc:= 60;
param Yo := 40;
param Ypmax:= 30;
param Ymmax:= 30;
param kc := 1.5;
param MAX := 1000000000000;
```

```
param v:    1    2    3    4    5    6    7    8    9    10
        11   12   13   14   15   16   17   18   19   20   21
        22   23   24   25   26   27   28:=
```

```
1    10   20   0    20   0    10   10   10   20   0    20
     0    10   10   10   20   0    20   0    10   10   10
     20   0    20   0    10   10
2    0    10   20   20   20   10   0    0    10   20   20
     20   10   0    0    10   20   20   20   10   0    0
     10   20   20   20   10   0;

param D:=

1    3
2    2
3    1
4    4
5    2
6    2
7    1
8    3
9    2
10   1
11   4
12   2
13   2
14   1
15   3
16   2
17   1
18   4
19   2
20   2
21   1
22   3
23   2
24   1
25   4
26   2
27   2
28   1
;

param E:=

1    6
2    9
3    7
4    6
5    9
6    7
7    6
8    9
9    7
10   6
11   9
12   7;

param a: 1 2 3 4 5 6 7 8 9 10:=
1    40   40   40   0    0    0    0    0    0    0
2    10   10   0    0    0    0    0    0    0    0
3    20   0    0    0    0    0    0    0    0    0
4    20   20   20   20   0    0    0    0    0    0
5    20   20   0    0    0    0    0    0    0    0
6    10   10   0    0    0    0    0    0    0    0
7    20   0    0    0    0    0    0    0    0    0
8    40   40   40   0    0    0    0    0    0    0
9    10   10   0    0    0    0    0    0    0    0
10   20   0    0    0    0    0    0    0    0    0
11   20   20   20   20   0    0    0    0    0    0
12   20   20   0    0    0    0    0    0    0    0
13   10   10   0    0    0    0    0    0    0    0
14   20   0    0    0    0    0    0    0    0    0
```

```
15      40      40      40      0       0       0       0       0       0       0
16      10      10      0       0       0       0       0       0       0       0
17      20      0       0       0       0       0       0       0       0       0
18      20      20      20      20      0       0       0       0       0       0
19      20      20      0       0       0       0       0       0       0       0
20      10      10      0       0       0       0       0       0       0       0
21      20      0       0       0       0       0       0       0       0       0
22      40      40      40      0       0       0       0       0       0       0
23      10      10      0       0       0       0       0       0       0       0
24      20      0       0       0       0       0       0       0       0       0
25      20      20      20      20      0       0       0       0       0       0
26      20      20      0       0       0       0       0       0       0       0
27      10      10      0       0       0       0       0       0       0       0
28      20      0       0       0       0       0       0       0       0       0;

set PRED[1] := ;
set PRED[2] := 1;
set PRED[3] := ;
set PRED[4] := 3;
set PRED[5] := 3;
set PRED[6] := ;
set PRED[7] := 6;
set PRED[8] :=;
set PRED[9] :=8;
set PRED[10] :=;
set PRED[11] :=10;
set PRED[12] :=10;
set PRED[13] :=;
set PRED[14] :=13;
set PRED[15] :=;
set PRED[16] :=15;
set PRED[17] :=;
set PRED[18] :=17;
set PRED[19] :=17;
set PRED[20] :=;
set PRED[21] :=20;
set PRED[22] :=;
set PRED[23] :=22;
set PRED[24] :=;
set PRED[25] :=24;
set PRED[26] :=24;
set PRED[27] :=;
set PRED[28] :=27;

set JOBI[1]:= 1 2;
set JOBI[2]:= 3 4 5;
set JOBI[3]:= 6 7;
set JOBI[4]:= 8 9;
set JOBI[5]:= 10 11 12;
set JOBI[6]:= 13 14;
set JOBI[7]:= 15 16;
set JOBI[8]:= 17 18 19;
set JOBI[9]:= 20 21;
set JOBI[10]:= 22 23;
set JOBI[11]:= 24 25 26;
set JOBI[12]:= 27 28;

param hs :=
1 1
2 1;
param s :=
1 1
2 1;
param Io :=
1 0
2 0;
param cmc :=
1 2
```

```
2 2;
param C :=
1 100
2 100
3 100;
```

```
param INCmax: 1 2 3:=
1 26    27      27
2 26    27      27
3 26    27      27
4 26    27      27
5 26    27      27
6 26    27      27
7 26    27      27
8 26    27      27
9 26    27      27
10 26   27      27;
```

```
param alfa : 1 2 3:=
1 0.71 0.71   0.71
2 0.78 0.78   0.78
3 0.70 0.70   0.70
4 0.73 0.73   0.73
5 0.77 0.77   0.77
6 0.80 0.80   0.80
7 0.80 0.80   0.80
8 0.80 0.80   0.80
9 0.75 0.75   0.75
10 0.75       0.75    0.75;
```

B. Ideal model with multiple producers and one supplier (IMP)

B.1. Model

```
#MODEL PARAMETERS
    param T >=0 integer;
    param I >=0 integer;
    param J >=0 integer;
    param L >=0 integer;
    param M >=0 integer;

#CONJUNTOS DE INDICES
    set PERIOD := 1..T;
    set COMP := 1..I;
    set JOB := 1..J;
    set ORDER := 1..L;
    set PROD := 1..M;
    set JOBl {ORDER, PROD};
    set PRED {JOB, PROD};
    set PERIOD2 := 1..T+1;


#DECISION VARIABLES
    var Y {t in PERIOD,m in PROD}>=0;
    var Yp {t in PERIOD, m in PROD}>=0;
    var Ym {t in PERIOD, m in PROD}>=0;
    var x {(j,t,m) in {JOB, PERIOD, m in PROD}}binary;
    var y {t in PERIOD, m in PROD}>=0;
```

```
        var Inv {(i,t,m) in {COMP, PERIOD, PROD}}>=0;
        var In {l in ORDER, m in PROD, j in JOBl[l,m]}>=0;
        var d {(i,t,m) in {COMP, PERIOD, PROD}}>= 0;
        var hi {m in PROD,l in ORDER}>=0;
        var Invs {(i,t) in {COMP, PERIOD2}}>= 0;
        var z {(i,t,m) in {COMP, PERIOD, PROD}}binary;
        var INC {t in PERIOD}>= 0;
        var Q {(i,t,m) in {COMP, PERIOD,PROD}}>= 0;

#CONSTANTS
        param c>=0;
        param cp>=0;
        param cm>=0;
        param p{i in COMP} >=0;
        param co >=0;
        param hp {i in COMP} >=0;
        param pc>=0;
        param Yo>=0;
        param Ypmax{m in PROD}>=0;
        param Ymmax{m in PROD}>=0;
        param a{(j,m, k) in {JOB, PROD, PERIOD}}>=0;
        param D{j in JOB, m in PROD}>=0;
        param E{l in ORDER, m in PROD}>=0;
        param v{(i,m,j) in {COMP, PROD, JOB}}>=0;
        param hs {i in COMP} >=0;
        param s {i in COMP} >=0;
        param kc >=0;
        param MAX>=0;
        param Io {i in COMP}>=0;
        param cmc {i in COMP} >=0;
        param C >=0;
        param alfa {t in PERIOD}>=0;
        param INCmax {t in PERIOD}>=0;

#OBJECTIVE
        minimize objective: sum {t in PERIOD, m in PROD} (c*Y[t,m] + cp*Yp[t,m]+
        cm*Ym[t,m]) + sum {t in PERIOD, m in PROD} co*y[t,m] + sum {t in PERIOD, i
        in COMP, m in PROD} hp[i] * Inv[i,t,m] + sum {l in ORDER, m in PROD} pc *
        hi[m,l]+ sum{t in PERIOD, i in COMP}(hs[i]*Invs[i,t]+s[i]*sum {m in PROD}
        z[i,t,m])+sum{t in PERIOD}kc * INC[t];

#CONSTRAINTS
#CAPACITY ADAPTATION CONSTRAINTS
        subject to Capacity {t in 1..T-1, m in PROD}:
        Y[t+1,m]==Y[t,m]+Yp[t,m]-Ym[t,m];
        subject to Capacity2 {m in PROD}:
        Y[1,m]==Yo;
        subject to Capacity3 {t in PERIOD, m in PROD}:
```

```
	Yp[t,m]<=Ypmax[m];
	subject to Capacity4 {t in PERIOD, m in PROD}:
	Ym[t,m]<=Ymmax[m];

#CAPACITY CONSTRAINTS
	subject to Capacity5 {t in PERIOD, m in PROD}:
	sum{j in JOB, k in 1..t} a[j, m, t+1-k]*x[j,k,m]<=(Y[t,m]+y[t,m]);
	subject to Capacity6 {t in PERIOD}:
	sum {i in COMP, m in PROD} cmc[i] * Q[i,t,m] <= alfa[t]*C + INC[t];
	subject to Max_Capacity {t in PERIOD}:
	INC[t] <= INCmax[t];

#NETWORK CONSTRAINTS
	subject to network1 {j in JOB, m in PROD}:
	sum{t in PERIOD} x[j,t,m]=1;
	subject to network2 {l in ORDER,  m in PROD, j in JOBl[l,m]}:
	sum{t in PERIOD}((t+D[j,m])*x[j,t,m])<=E[l,m]+In[l,m,j];
	subject to network3 {j in JOB,  m in PROD, h in PRED[j,m]}:
	sum{t in PERIOD}(t+D[h,m])*x[h,t,m]<=sum{t in PERIOD}t*x[j,t,m];

#MATERIAL BALANCE CONSTRAINTS

	subject to material1 {i in COMP, t in PERIOD, m in PROD}:
	sum{j in JOB}(v[i,m,j]*x[j,t,m])<=(d[i,t,m]+Inv[i,t,m]);
	subject to material2 {i in COMP, t in 1..T-1, m in PROD}:
	Inv[i,t+1,m]==(Inv[i,t,m]+d[i,t,m]- sum{j in JOB}v[i,m,j]*x[j,t,m]);
	subject to material3 {i in COMP, m in PROD}:
	Inv[i,1,m]== 0;
	subject to Inventory {i in COMP, t in PERIOD, m in PROD}:
	Invs[i,t+1] == Invs[i,t] + Q[i,t,m] - d[i,t,m];
	subject to In_INventory {i in COMP}:
	Invs[i,1]== 0;
	subject to Production {i in COMP, t in PERIOD, m in PROD}:
	Q[i,t,m] <=  z[i,t,m]*MAX;

#OTHERS
	subject to DefineHi {l in ORDER, m in PROD, j in JOBl[l,m]}:
	hi[m,l]>=In[l,m,j];
	subject to Product {i in COMP, m in PROD}:
	sum{t in PERIOD} d[i,t,m]==sum{t in PERIOD}Q[i,t,m];
```

B.2. Data of the model

```
param T := 10;
param I := 2;
param J := 7;
param L := 3;
```

```
param M := 4;
param c :=0.5;
param cp := 1;
param cm := 0.2;
param p:=
1 10
2 10;
param co := 2;
param hp :=
1 2
2 2;
param pc:= 60;
param Yo := 10;

param Ypmax:=
1 7.5
2 7.5
3 7.5
4 7.5;
param Ymmax:=
1 7.5
2 7.5
3 7.5
4 7.5;
param kc := 1.5;
param MAX := 1000000000000;

param v:     1     2     3     4     5     6     7:=
1 1    10    20    0     20    0     10    10
2 1    0     10    20    20    20    10    0
1 2    10    20    0     20    0     10    10
2 2    0     10    20    20    20    10    0
1 4    10    20    0     20    0     10    10
2 4    0     10    20    20    20    10    0
1 3    10    20    0     20    0     10    10
2 3    0     10    20    20    20    10    0;

param D: 1 2 3 4:=

1      3     3     3     3
2      2     2     2     2
3      1     1     1     1
4      4     4     4     4
5      2     2     2     2
6      2     2     2     2
7      1     1     1     1;


param E:1 2 3 4:=
1      6     6     6     6
2      9     9     9     9
3      7     7     7     7;

param a: 1 2 3 4 5 6 7 8 9 10:=
1 1    40    40    40    0     0     0     0     0     0     0
2 1    10    10    0     0     0     0     0     0     0     0
3 1    20    0     0     0     0     0     0     0     0     0
4 1    20    20    20    20    0     0     0     0     0     0
5 1    20    20    0     0     0     0     0     0     0     0
6 1    10    10    0     0     0     0     0     0     0     0
7 1    20    0     0     0     0     0     0     0     0     0
```

```
1 2   40   40   40   0    0   0   0   0   0   0
2 2   10   10   0    0    0   0   0   0   0   0
3 2   20   0    0    0    0   0   0   0   0   0
4 2   20   20   20   20   0   0   0   0   0   0
5 2   20   20   0    0    0   0   0   0   0   0
6 2   10   10   0    0    0   0   0   0   0   0
7 2   20   0    0    0    0   0   0   0   0   0
1 3   40   40   40   0    0   0   0   0   0   0
2 3   10   10   0    0    0   0   0   0   0   0
3 3   20   0    0    0    0   0   0   0   0   0
4 3   20   20   20   20   0   0   0   0   0   0
5 3   20   20   0    0    0   0   0   0   0   0
6 3   10   10   0    0    0   0   0   0   0   0
7 3   20   0    0    0    0   0   0   0   0   0
1 4   40   40   40   0    0   0   0   0   0   0
2 4   10   10   0    0    0   0   0   0   0   0
3 4   20   0    0    0    0   0   0   0   0   0
4 4   20   20   20   20   0   0   0   0   0   0
5 4   20   20   0    0    0   0   0   0   0   0
6 4   10   10   0    0    0   0   0   0   0   0
7 4   20   0    0    0    0   0   0   0   0   0;

set PRED[1,1] := ;
set PRED[2,1] := 1;
set PRED[3,1] := ;
set PRED[4,1] := 3;
set PRED[5,1] := 3;
set PRED[6,1] := ;
set PRED[7,1] := 6;
set PRED[1,2] := ;
set PRED[2,2] := 1;
set PRED[3,2] := ;
set PRED[4,2] := 3;
set PRED[5,2] := 3;
set PRED[6,2] := ;
set PRED[7,2] := 6;
set PRED[1,3] := ;
set PRED[2,3] := 1;
set PRED[3,3] := ;
set PRED[4,3] := 3;
set PRED[5,3] := 3;
set PRED[6,3] := ;
set PRED[7,3] := 6;
set PRED[1,4] := ;
set PRED[2,4] := 1;
set PRED[3,4] := ;
set PRED[4,4] := 3;
set PRED[5,4] := 3;
set PRED[6,4] := ;
set PRED[7,4] := 6;

set JOBl[1,1]:= 1 2;
set JOBl[2,1]:= 3 4 5;
set JOBl[3,1]:= 6 7;
set JOBl[1,2]:= 1 2;
set JOBl[2,2]:= 3 4 5;
set JOBl[3,2]:= 6 7 ;
set JOBl[1,3]:= 1 2;
set JOBl[2,3]:= 3 4 5;
set JOBl[3,3]:= 6 7 ;
set JOBl[1,4]:= 1 2;
set JOBl[2,4]:= 3 4 5;
set JOBl[3,4]:= 6 7 ;

param hs :=
1 1
2 1;
param s :=
```

```
1 1
2 1;
param Io :=
1 0
2 0;
param cmc :=
1 2
2 2;
param C := 300;

param INCmax :=
1 80
2 80
3 80
4 80
5 80
6 80
7 80
8 80
9 80
10 80;

param alfa :=
1 0.62
2 0.20
3 0.44
4 0.14
5 0.30
6 0.22
7 0.22
8 0.22
9 0.50
10 0.50;
```

C. Ideal model with multiple suppliers and multiple producers (IMSP)

C.1. Model

```
#MODEL PARAMETERS
	param T >=0 integer;
	param I >=0 integer;
	param J >=0 integer;
	param L >=0 integer;
	param M >=0 integer;
	param S >=0 integer;

#INDEX SETS
	set PERIOD := 1..T;
	set COMP := 1..I;
	set JOB := 1..J;
	set ORDER := 1..L;
	set PROD := 1..M;
	set JOBl {ORDER, PROD};
	set PRED {JOB, PROD};
	set PERIOD2 := 1..T+1;
	set SUPP := 1..S;
```

```
#DECISION VARIABLES
	var Y {t in PERIOD,m in PROD}>=0;
	var Yp {t in PERIOD, m in PROD}>=0;
	var Ym {t in PERIOD, m in PROD}>=0;
	var x {(j,t,m) in {JOB, PERIOD, m in PROD}}binary;
	var y {t in PERIOD, m in PROD}>=0;
	var Inv {(i,t,m) in {COMP, PERIOD, PROD}}>=0;
	var In {l in ORDER, m in PROD, j in JOBl[l,m]}>=0;
	var d {(i,t,m,k) in {COMP, PERIOD, PROD,SUPP}}>= 0;
	var hi {m in PROD,l in ORDER}>=0;
	var Invs {(i,t,k) in {COMP, PERIOD2,SUPP}}>= 0;
	var z {(i,t,m,k) in {COMP, PERIOD, PROD,SUPP}}binary;
	var INC {t in PERIOD,k in SUPP}>= 0;
	var Q {(i,t,m,k) in {COMP, PERIOD,PROD, SUPP}}>= 0;

#CONSTANTS
	param c>=0;
	param cp>=0;
	param cm>=0;
	param p{i in COMP} >=0;
	param co >=0;
	param hp {i in COMP} >=0;
	param pc>=0;
	param Yo>=0;
	param Ypmax{m in PROD}>=0;
	param Ymmax{m in PROD}>=0;
	param a{(j,m, k) in {JOB, PROD, PERIOD}}>=0;
	param D{j in JOB, m in PROD}>=0;
	param E{l in ORDER, m in PROD}>=0;
	param v{(i,m,j) in {COMP, PROD, JOB}}>=0;
	param hs {i in COMP} >=0;
	param s {i in COMP} >=0;
	param kc >=0;
	param MAX>=0;
	param Io {i in COMP}>=0;
	param cmc {i in COMP} >=0;
	param C {k in SUPP} >=0;
	param alfa {t in PERIOD, k in SUPP}>=0;
	param INCmax {t in PERIOD, k in SUPP}>=0;

#OBJECTIVE
	minimize objective: sum {t in PERIOD, m in PROD} (c*Y[t,m] + cp*Yp[t,m]+
	cm*Ym[t,m]) + sum {t in PERIOD, m in PROD} co*y[t,m] + sum {t in PERIOD, i
	in COMP, m in PROD} hp[i] * Inv[i,t,m] + sum {l in ORDER, m in PROD} pc *
	hi[m,l]+ sum{t in PERIOD, i in COMP, k in SUPP}(hs[i]*Invs[i,t,k]+s[i]*sum {m in
	PROD} z[i,t,m,k])+sum{t in PERIOD,k in SUPP}kc * INC[t,k];

#CONSTRAINTS
```

```
#CAPACITY ADAPTATION CONSTRAINTS
	subject to Capacity {t in 1..T-1, m in PROD}:
	Y[t+1,m]==Y[t,m]+Yp[t,m]-Ym[t,m];
	subject to Capacity2 {m in PROD}:
	Y[1,m]==Yo;
	subject to Capacity3 {t in PERIOD, m in PROD}:
	Yp[t,m]<=Ypmax[m];
	subject to Capacity4 {t in PERIOD, m in PROD}:
	Ym[t,m]<=Ymmax[m];

#CAPACITY CONSTRAINTS
	subject to Capacity5 {t in PERIOD, m in PROD}:
	sum{j in JOB, k in 1..t} a[j, m, t+1-k]*x[j,k,m]<=(Y[t,m]+y[t,m]);
	subject to Capacity6 {t in PERIOD,k in SUPP}:
	sum {i in COMP} cmc[i] * sum{m in PROD} Q[i,t,m,k] <=alfa[t,k]*C[k] + INC[t,k];
	subject to Max_Capacity {t in PERIOD, k in SUPP}:
	INC[t,k] <= INCmax[t,k];

#NETWORK CONSTRAINTS
	subject to network1 {j in JOB, m in PROD}:
	sum{t in PERIOD} x[j,t,m]=1;
	subject to network2 {l in ORDER,  m in PROD, j in JOBl[l,m]}:
	sum{t in PERIOD}((t+D[j,m])*x[j,t,m])<=E[l,m]+In[l,m,j];
	subject to network3 {j in JOB,  m in PROD, h in PRED[j,m]}:
	sum{t in PERIOD}(t+D[h,m])*x[h,t,m]<=sum{t in PERIOD}t*x[j,t,m];

#MATERIAL BALANCE CONSTRAINTS
	subject to material1 {i in COMP, t in PERIOD, m in PROD}:
	sum{j in JOB}(v[i,m,j]*x[j,t,m])<=sum{k in SUPP}(d[i,t,m,k]+Inv[i,t,m]);
	subject to material2 {i in COMP, t in 1..T-1, m in PROD}:
	Inv[i,t+1,m]==sum{k in SUPP}(Inv[i,t,m]+d[i,t,m,k]- sum{j in JOB}v[i,m,j]*x[j,t,m]);
	subject to material3 {i in COMP, m in PROD}:
	Inv[i,1,m]== 0;
	subject to Inventory {i in COMP, t in PERIOD, k in SUPP, m in PROD}:
	Invs[i,t+1,k] == Invs[i,t,k] +Q[i,t,m,k] - d[i,t,m,k];
	subject to In_INventory {i in COMP, k in SUPP}:
	Invs[i,1,k]== 0;
	subject to Production {i in COMP, t in PERIOD, m in PROD, k in SUPP}:
	Q[i,t,m,k] <=  z[i,t,m,k]*MAX;

#OTHERS
	subject to DefineHi {l in ORDER, m in PROD, j in JOBl[l,m]}:
	hi[m,l]>=In[l,m,j];
```

C.2. Data of the model

```
param T := 10;
param I := 2;
param J := 7;
param L := 3;
param M := 4;
param S := 3;
param c :=0.5;
param cp := 1;
param cm := 0.2;
param p:=
1 10
2 10;
param co := 2;
param hp :=
1 2
2 2;
param pc:= 60;
param Yo := 10;
param Ypmax:=
1 7.5
2 7.5
3 7.5
4 7.5;
param Ymmax:=
1 7.5
2 7.5
3 7.5
4 7.5;
param kc :=1.5;
param MAX := 1000000000000;

param v:    1    2    3    4    5    6    7:=
1 1    10   20   0    20   0    10   10
2 1    0    10   20   20   20   10   0
1 2    10   20   0    20   0    10   10
2 2    0    10   20   20   20   10   0
1 4    10   20   0    20   0    10   10
2 4    0    10   20   20   20   10   0
1 3    10   20   0    20   0    10   10
2 3    0    10   20   20   20   10   0;

param D: 1 2 3 4:=

1    3    3    3    3
2    2    2    2    2
3    1    1    1    1
4    4    4    4    4
5    2    2    2    2
6    2    2    2    2
7    1    1    1    1;

param E:1 2 3 4:=

1    6    6    6    6
2    9    9    9    9
3    7    7    7    7;

param a: 1 2 3 4 5 6 7 8 9 10:=
1 1    40   40   40   0    0    0    0    0    0    0
2 1    10   10   0    0    0    0    0    0    0    0
```

```
3 1   20   0    0    0    0   0   0   0   0   0
4 1   20   20   20   20   0   0   0   0   0   0
5 1   20   20   0    0    0   0   0   0   0   0
6 1   10   10   0    0    0   0   0   0   0   0
7 1   20   0    0    0    0   0   0   0   0   0
1 2   40   40   40   0    0   0   0   0   0   0
2 2   10   10   0    0    0   0   0   0   0   0
3 2   20   0    0    0    0   0   0   0   0   0
4 2   20   20   20   20   0   0   0   0   0   0
5 2   20   20   0    0    0   0   0   0   0   0
6 2   10   10   0    0    0   0   0   0   0   0
7 2   20   0    0    0    0   0   0   0   0   0
1 3   40   40   40   0    0   0   0   0   0   0
2 3   10   10   0    0    0   0   0   0   0   0
3 3   20   0    0    0    0   0   0   0   0   0
4 3   20   20   20   20   0   0   0   0   0   0
5 3   20   20   0    0    0   0   0   0   0   0
6 3   10   10   0    0    0   0   0   0   0   0
7 3   20   0    0    0    0   0   0   0   0   0
1 4   40   40   40   0    0   0   0   0   0   0
2 4   10   10   0    0    0   0   0   0   0   0
3 4   20   0    0    0    0   0   0   0   0   0
4 4   20   20   20   20   0   0   0   0   0   0
5 4   20   20   0    0    0   0   0   0   0   0
6 4   10   10   0    0    0   0   0   0   0   0
7 4   20   0    0    0    0   0   0   0   0   0;

set PRED[1,1] := ;
set PRED[2,1] := 1;
set PRED[3,1] := ;
set PRED[4,1] := 3;
set PRED[5,1] := 3;
set PRED[6,1] := ;
set PRED[7,1] := 6;
set PRED[1,2] := ;
set PRED[2,2] := 1;
set PRED[3,2] := ;
set PRED[4,2] := 3;
set PRED[5,2] := 3;
set PRED[6,2] := ;
set PRED[7,2] := 6;
set PRED[1,3] := ;
set PRED[2,3] := 1;
set PRED[3,3] := ;
set PRED[4,3] := 3;
set PRED[5,3] := 3;
set PRED[6,3] := ;
set PRED[7,3] := 6;
set PRED[1,4] := ;
set PRED[2,4] := 1;
set PRED[3,4] := ;
set PRED[4,4] := 3;
set PRED[5,4] := 3;
set PRED[6,4] := ;
set PRED[7,4] := 6;

set JOBI[1,1]:= 1 2;
set JOBI[2,1]:= 3 4 5;
set JOBI[3,1]:= 6 7;
set JOBI[1,2]:= 1 2;
set JOBI[2,2]:= 3 4 5;
set JOBI[3,2]:= 6 7 ;
set JOBI[1,3]:= 1 2;
set JOBI[2,3]:= 3 4 5;
set JOBI[3,3]:= 6 7 ;
set JOBI[1,4]:= 1 2;
set JOBI[2,4]:= 3 4 5;
set JOBI[3,4]:= 6 7 ;
```

```
param hs :=
1 1
2 1;
param s :=
1 1
2 1;
param Io :=
1 0
2 0;
param cmc :=
1 2
2 2;
param C :=
1 150
2 150
3 150;

param INCmax: 1 2 3:=
1 26.6  26.6    26.6
2 26.6  26.6    26.6
3 26.6  26.6    26.6
4 26.6  26.6    26.6
5 26.6  26.6    26.6
6 26.6  26.6    26.6
7 26.6  26.6    26.6
8 26.6  26.6    26.6
9 26.6  26.6    26.6
10 26.6         26.6    26.6;

param alfa : 1 2 3:=
1 0.62 0.62 0.62
2 0.20 0.2 0.2
3 0.44 0.44 0.44
4 0.14 0.14 0.14
5 0.30 0.3 0.3
6 0.22 0.22 0.22
7 0.22 0.22 0.22
8 0.22 0.22 0.22
9 0.50 0.5 0.5
10 0.50 0.5 0.5;
```

10- References

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