Title: Cuadrados latinos y grafos de Moore
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Abstract

Entitled *Latin squares and Moore graphs*, this work combines two fields of mathematics quite peculiar. On one hand latin squares, almost as old as the ancient Grece and so little studied that most of the combinatorics publications only cite them in order to emphasize their applications in experimental design theory. And on the other hand, Moore graphs, a type so particular of graphs that they may only exist three, and only two of them are known, the last one still resisting to be found.

The Moore graph problem can be expressed in a very concise manner. It consists in finding regular graphs in which the distance between two of their vertices is at most 2. A graph of this type is called a Moore graph and it has been proven that they can only exist for degree values 3, 7 and possibly 57. The two first cases are known, while the third, which is known as *The Biggest Moore Graph*, hasn't been found yet.

The Moore graph problem belong to the extremal graph theory, and more specifically to the (Δ,D) problem. The (Δ,D) problem consists in finding graphs with the largest number of vértices for a given degree Δ and a diameter D. The (Δ,D) problem is of special importance because its implications in the design of net topologies and other questions like data alignment problems and cryptographic algorithms.

The main body of this work is divided into three chapters: the first offers an introduction very exhaustive to latin squares and presents some new results like a new expression for evaluating the number of latin squares; the second chapter introduces the (Δ,D) problem, exposes in detail the constructions of the Moore graphs of degree 3 and 7, and finally deals with *The Biggest Moore Graph*; and the third chapter describes the algorithms used for generating and enumerating latin squares, as well as the construction of Moore graphs through their adjacency matrix.

On the other side, this work also includes in electronic format a set of applications developed for generating latin squares and constructing Moore graphs. Among these applications, I must emphasize The Biggest Moore Graph, a program written in Visual Basic that makes the manipulation of the adjacency matrix of a Moore graph very visual, easy and efficient. It has been precisely thanks to this application that it has been possible to obtain a characterization of the adjacency matrix of *The Biggest Moore Graph*. In this characterisation appear, surprisingly, the latin squares, which have made it possible to reduce the size of the search for *The Biggest Moore Graph*.