Universidad Politecnica de Catalunya
Facultad de Ingenyeria

Escola Tecnica Superior d'Enginyeria de Telecomunicacio' de Barcelona

Proyecto de final de carrera:

Forecasting of Localization in Spain

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Sommario

Introduction

The objective of this study is the forecasting of the trend of logistic curve of location services, from data expressed as Hits /month, provided by the Spanish Genasys.

Genasys is a leading provider of location services for mobile devices and solutions for the management of geographic data.

In order to build the logistic curve of localization services, has been studied the performance of the logistic curves of the Internet and Mobile.

Specifically, we used logistic curves of the global Internet, the Internet in different countries such as Saudi Arabia, the United States and Norway, the network Global Mobile-Wireless and mobile network in Saudi Arabia.

Starting from the study of the three basic parameters that characterize the logistic equation of the curves known, we constructed the potential developments of the curve of location services. It was then traced a logistic curve of location services for each of the curves, whose performance is known and which was identified logistic equation that best describes it.

At each stage, the choice of the best logistic curve was made considering the standard error and using the Chi Square test, a test of hypothesis testing used in statistics.

The project is divided into six chapters.

The study of the logistic curve of location services in relation to Internet and mobile networks is done in the *first chapter*, which lists in detail the considerations on the relationship between the logistic curve, the object of prediction, and each of the logistic curve notes.

In this part, we have found the equations of the logistic curve through the calculation of the parameters that characterize them.

The choice of the best curve occurred even at this stage considering the standard error and using the Chi-square test.

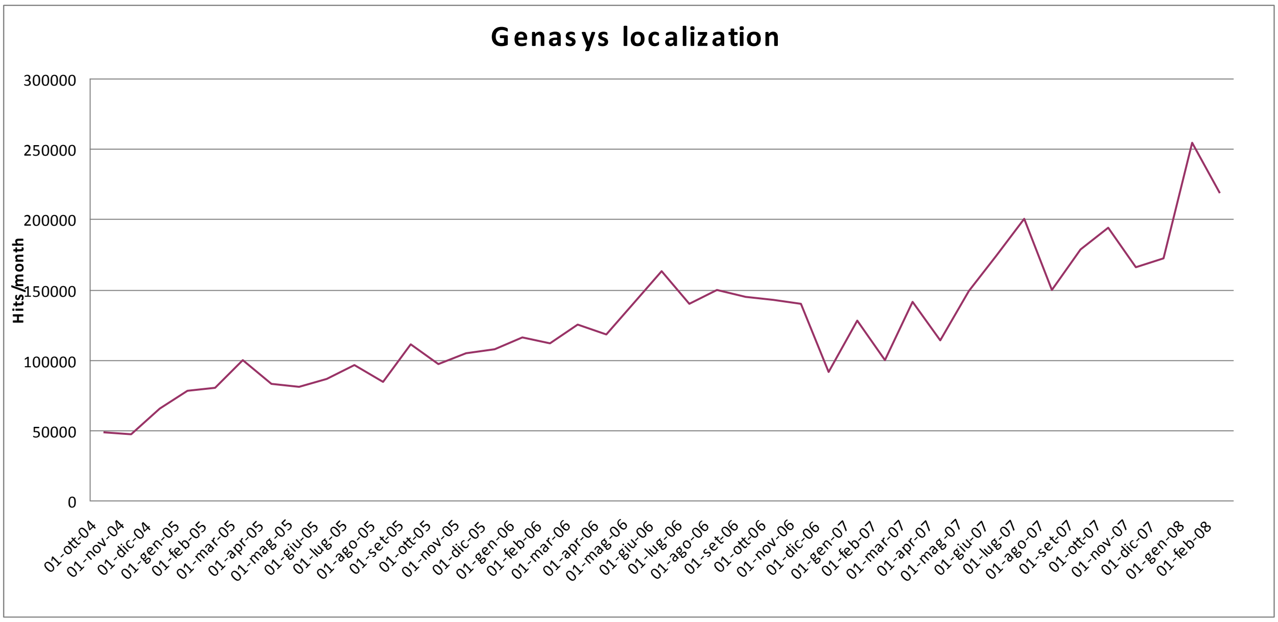
In the *second chapter*, we made comparisons between the different logistic curves used in the analysis and we made the prediction of the trend of logistic curve of location services.

An overview of the results achieved with this study, is shown in the *third* and last *chapter*, where we set the tables that show the start, end and saturation time and the values of the rise time of each of the logistic curve analysis.

1 Logistic Curve Analysis

In this chapter we will try to make on an analysis on the possible shape of the curve logistics of localization systems relying on the performance of logistic curves already known.

We tried to build the logistic curve of location services relying on data provided by a Spanish company called Genasys.



Genasys is a leading provider of solutions and services based on the location of mobile units and solutions for the management of geographic data.

Genasys offers a combination of experience, products and services that are common to these areas of activity, while maintaining a remarkable level of innovation and a strong commitment to customer satisfaction.

In particular, this analysis is based on logistic curves:

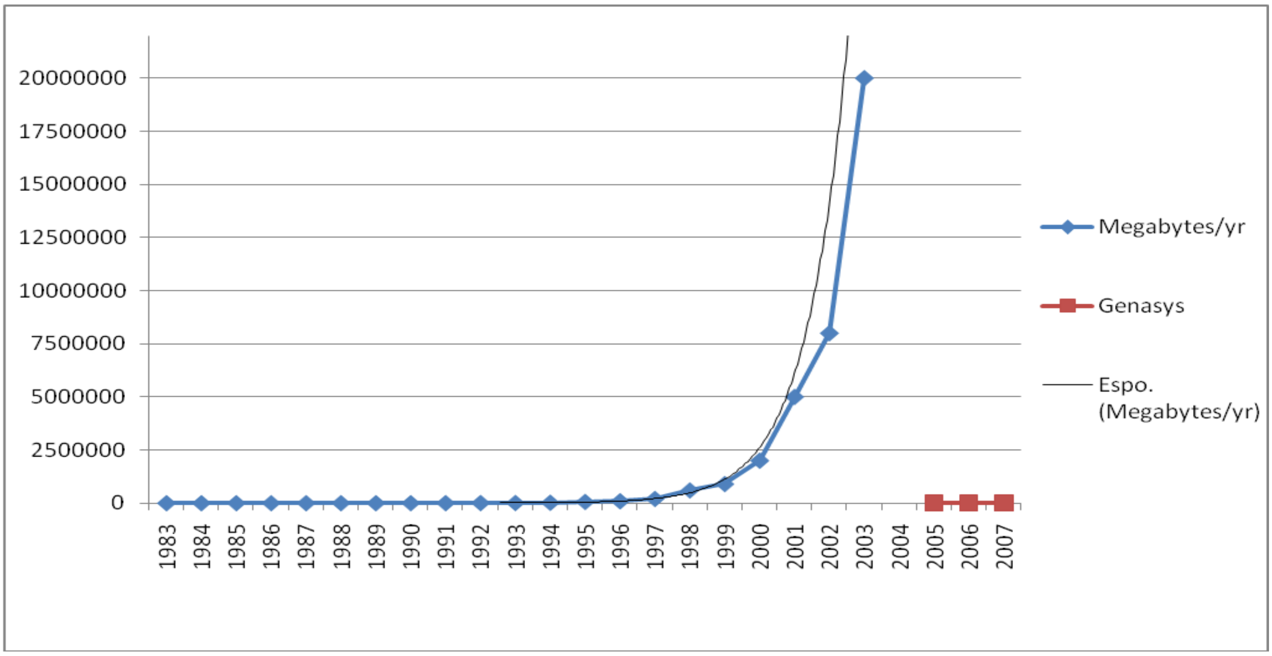
- Internet in the world
- Mobile worldwide
- Internet in Saudi Arabia
- Mobile Saudi Arabia
- Internet in U.S.
- Internet in Norway

1.1 Internet in the world

1.1.1 Considerations on the relationship between the logistic curves of the Internet and location services

At this stage we are dedicated to research and reports of possible links that may exist between the logistic curve for Internet service and that of location services.

Comparing these curves in the same graph we can see that there is a "time shift" between the beginning of the development of internet services and early development of location services.

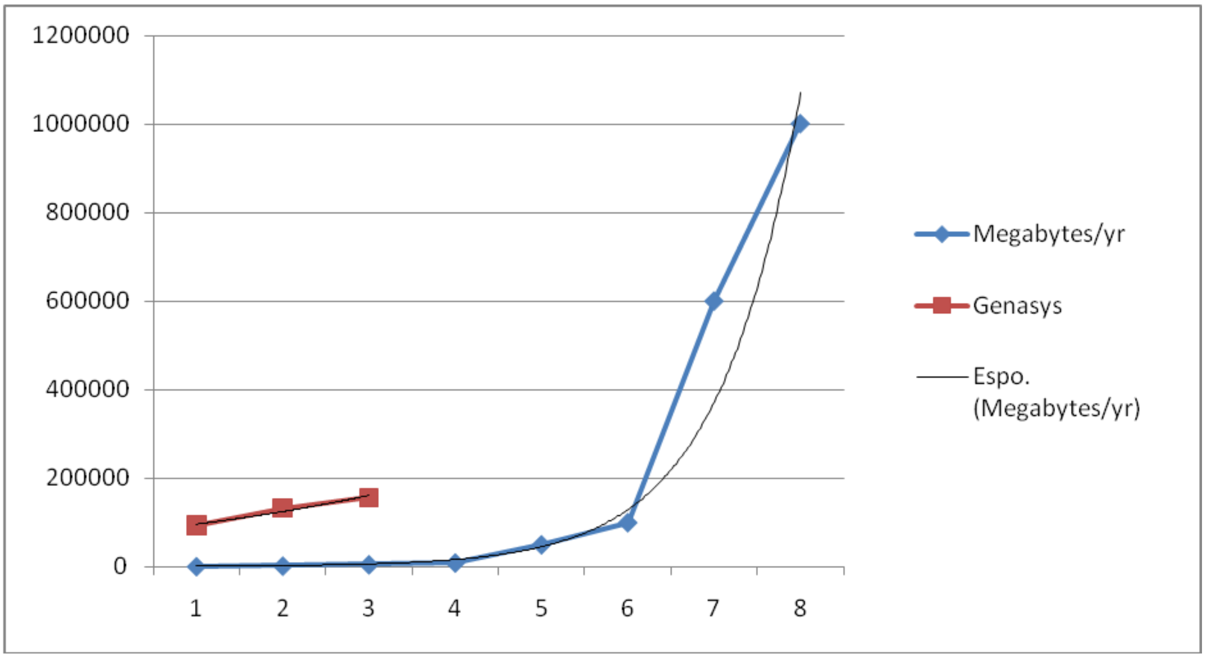


With the data at our disposal we can make a finer analysis on existing relationships.

With regard to the location services available data are those related to the first 3 years of development (2005 to 2007), unlike Internet service for which data are related to a time span of nearly 20 years. Such availability may be related to the fact that location services are a recent development and distribution, unlike the Internet that saw its birth in the early 80's and for which the information is clearer and more precise. Currently, the Internet service is one of the most widespread and deeply rooted in the world.

To make a more detailed analysis, we identify the similarities and differences between the two curves in their early years of development. In this way we can examine what happened during the introduction of such services.

To do this we insert in the same graph the two curves in parallel.



As can be seen from the figure, the two curves have different trends in terms of volume of Megabytes and in terms of time.

The curve of location services, provided by the company's data Genasys, presents a curve with a slope greater than that of the Internet. The growth rate curve that represents the time courses of the Internet is lower than the curve location.

It’s difficult establish the relationship between the two growth trends, however, one can observe that the traffic volume reached by the location services in the third period takes place in the curve of growth of the Internet only in the sixth.

We can explain this rate of development focusing on the analysis of technologies on which the location is supported. In fact, the location-based services are developed on the Internet and mobile systems already rooted in the market, this allows rapid diffusion of technology. This was not possible in the years of development of the Internet where there was still the “era of computers”. This scenario did not allow a rapid spread of the network.

1.1.2 Applications to find the best parameters for the logistic curve of the Internet

As a first step, we want to make a fitting with the logistic model of growth through transformation of variables (in order to be reduced to a linear equation) and then by linear regression. Recall that the logistic equation

$$y = K / (1 + a^* \exp(-bt))$$

becomes linear with the following transformation

$$\log(K/y - 1) = \log a - bt .$$

The data we suppose that the population limit (equilibrium) and K (remember that is the horizontal asymptote of the logistic function). Then we determine a and b by linear regression.

Since we are not aware of the limit to which the reach of the Internet traffic in Megabytes / yr go forward in search of that parameter. We have made the analysis on different values of K. We stopped our search when the value of the mean square made from the model was worse than that calculated in the previous system.

To make this process we assumed different values of K, we left the value of K = 9E+10

The idea is to calculate

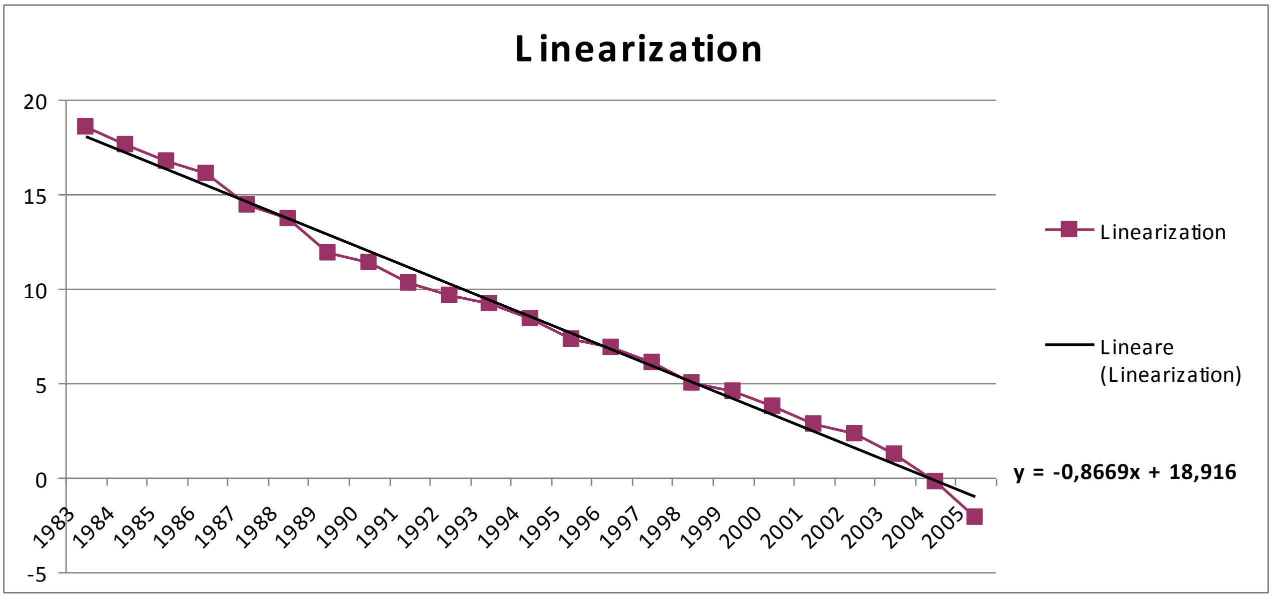
$\log(K/y - 1)$.

To this end we write in a cell of the Excel spreadsheet the following formula

`= LN(9E+10/B2-1).`

Where B2 is the first value of the function y. This calculation is done for all values of y available. Now draw the chart with "chart" entering the x-axis the days and y axis, the column data as soon as detected.

At the same time insert the regression “trendline" together with its equation.



Then

`log(a) = 18,916     a = 164102206    b = 0,8669`

We find the logistic curve

`y = 9E+10 / (1 + 164102206 *  exp (-0,8669 * t))`

Then we tried to optimize the choice of parameters using ''Solver'' (''Risolutore'') of Excel.

In another paper we have entered the data file according to the formula given by the logistic growth model

`y = K/(1 + a * exp (-b*t))`

To do this, we inserted the formula in column writing for example

`= I2 /( 1+( I3 * EXP(- I4 * A2)))`

Note that the parameters used are those included in Excel spreadsheet cells. Box I2 is that of the parameter K, I3 is the box for the parameter a, while the I4 box refers to the parameter b.

In particular these cells are introduced into the formula with the $ sign to make these cells remain fixed in the calculation. So we have:

```
= $I$2 /( 1+( $I$3 * EXP(-$I$4 * A2)))
```

Then calculate the square error by including in each row of column Excel spreadsheet formulas like

```
=(C2-B2)^2
```

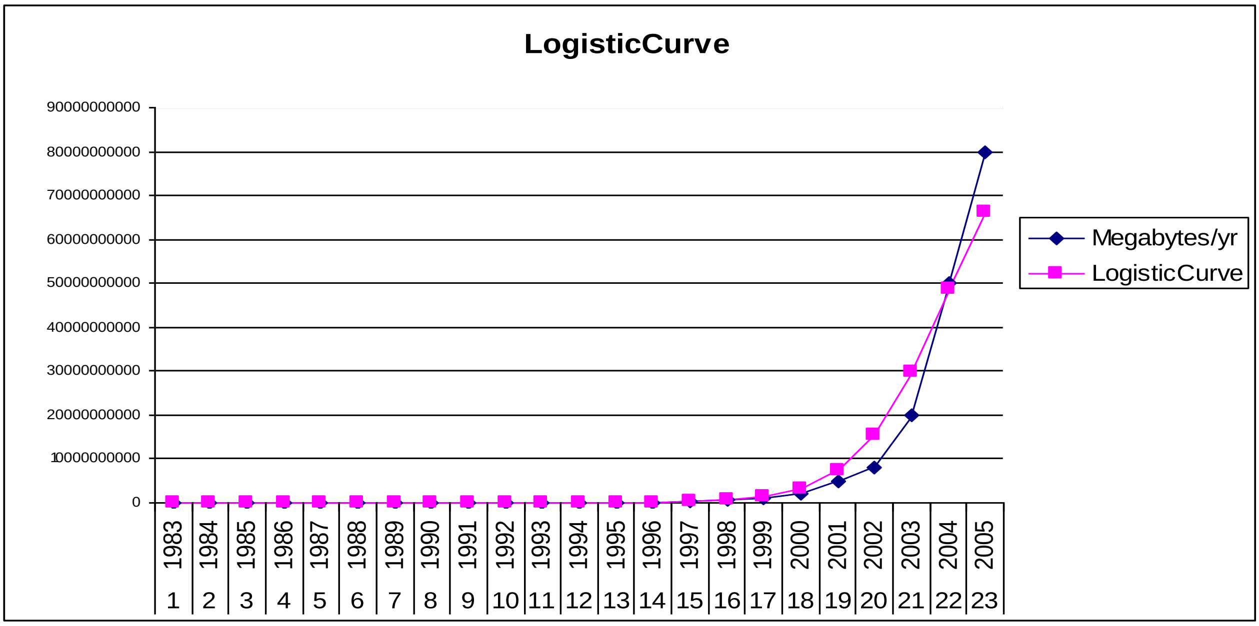
In this way we calculated the value of the mean square for the first period.

Using the "Solver" of Excel (found under the Tools menu, "Tools") optimize the parameters K, a and b by minimizing the sum of the standard deviations.

We set the "Solver" indicating that the parameters are optimized in the cells and I2-I4 (by entering the command, and then I2: I4 under "changing cells") and inserting the value of the sum of the standard deviations in cell the objective function to be minimized ("Target" cell D16), we start the "Solver".

The result of this simulation allows us to identify the best parameters a and b according to a saturation limit value set in the value K.

Finally, we can make a graph of the measured data and those calibrated.



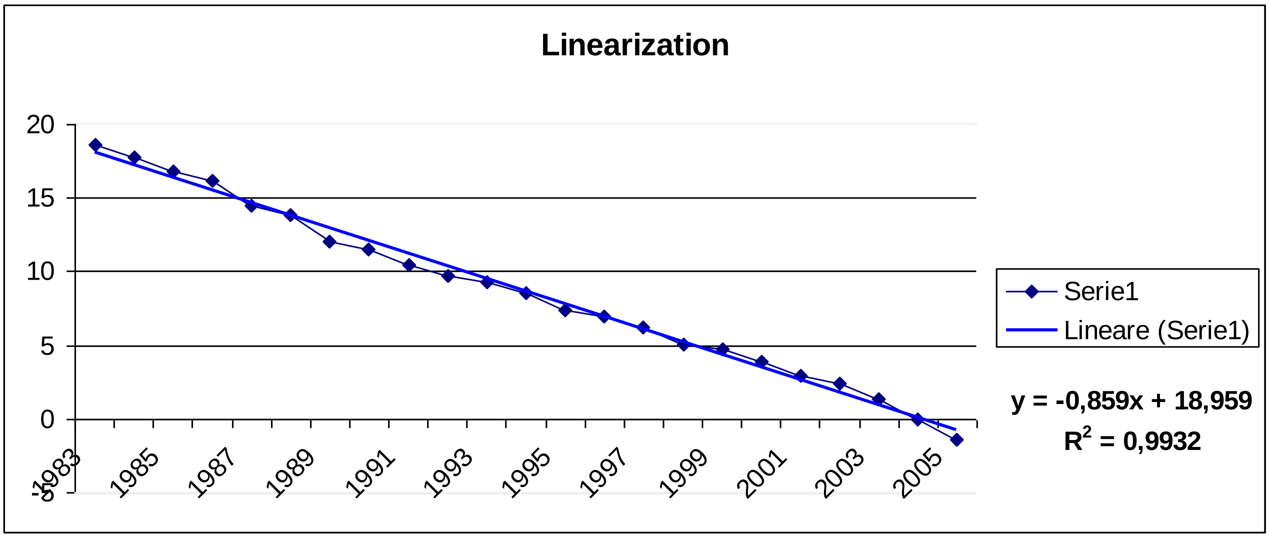
The optimized model has parameter values of:

k = 9E10   *a* = 164102206   *b* = 0,86694

For this model the value of the sum of squared errors is: 3,46857E+20

This procedure was performed for different values of K.

- For *K*= *1E+11* we have the following linearized model:



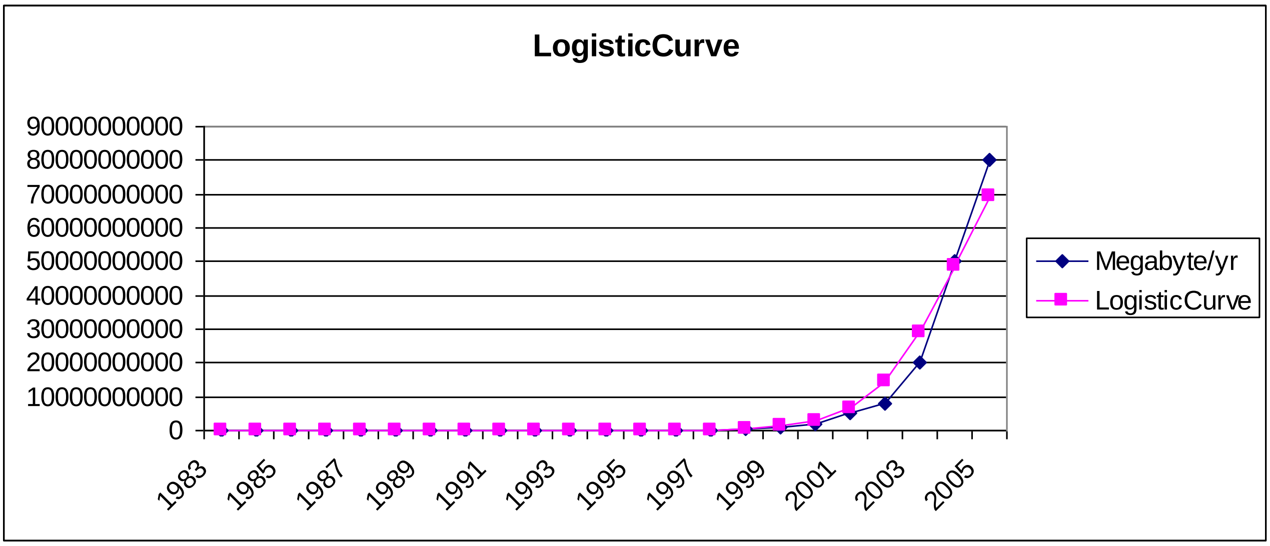
Then

log(*a*) = 18,959   *a* = 171312511   *b* = 0,859

We find the logistic curve

y = 1E+11 / (1 + 171312511 * exp (-*0,859* * *t*))

While using the solver we have the following graph of the measured data and those calibrated.



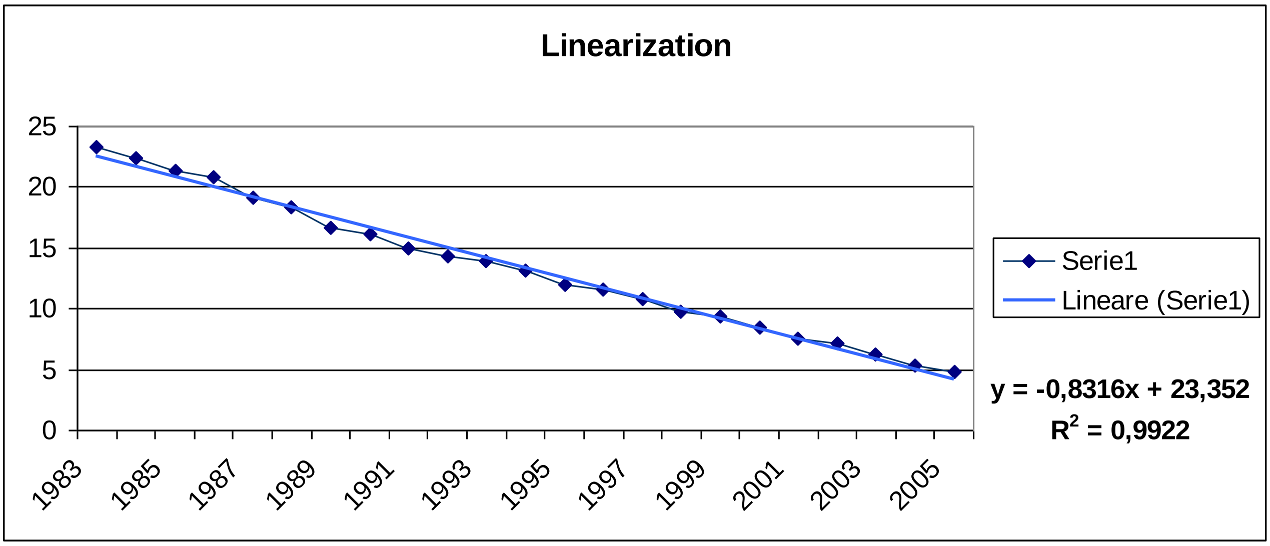
The optimized model has parameter values of:

`k = 1E11` *a* `= 171312511` *b* `= 0,85987`

For this model the value of the sum of squared errors is: `2,41141E+20`

The value just examined is less than the value calculated with the above K, then proceed in the search for good parameters going to increase the value of K.

- Per *K*= *1E+13* we have the following linearized model:



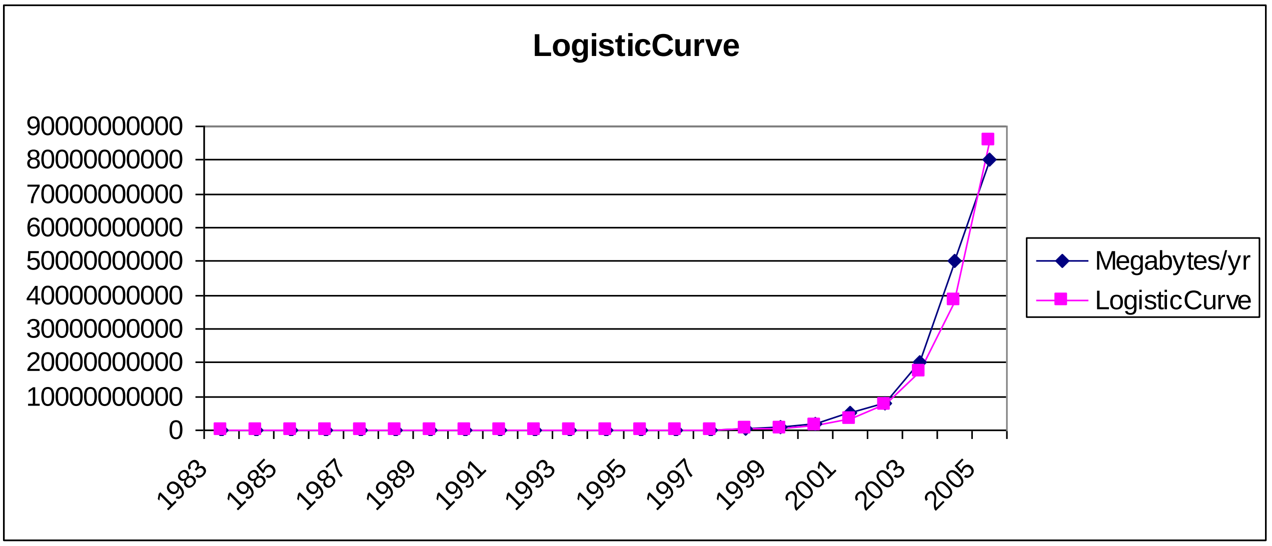
Then

`log(a) = 23,352     a = 13856219082    b = 0,8316`

We find the logistic curve

`y = 1E+13 / (1 + 13856219082 *  exp (-0,8316 * t))`

While using the solver we have the following graph of the measured data and those calibrated.



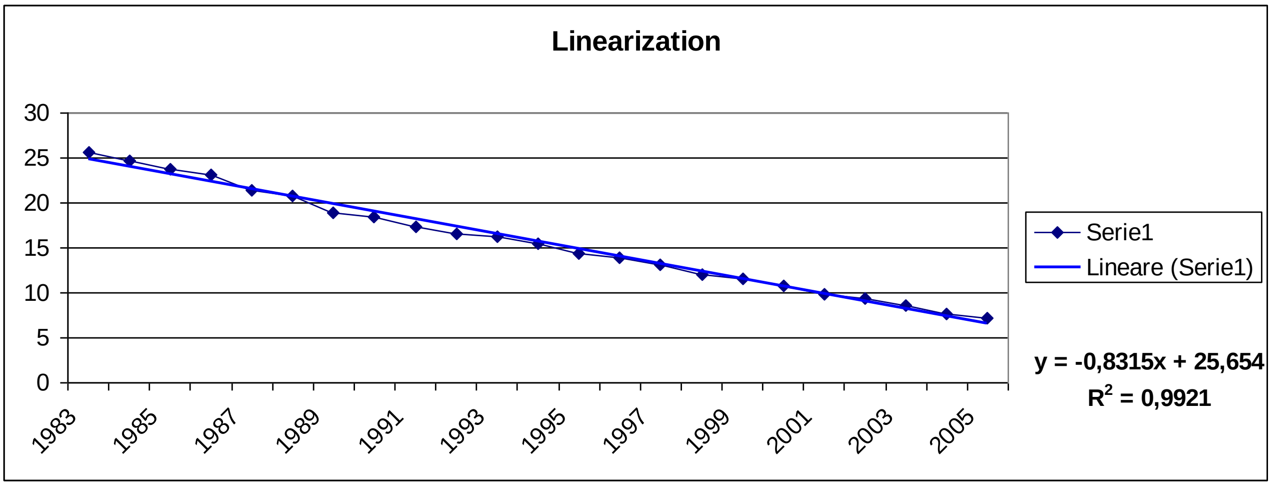
The optimized model has parameter values of:

`k = 1E13     a = 13856219082    b = 0,808714457`

For this model the value of the sum of squared errors is: 1,79469E+20

The value just examined is less than the value calculated with the above K, then proceed in the search for good parameters going to increase the value of K.

- For K= 1E+14 we have the following linearized model:



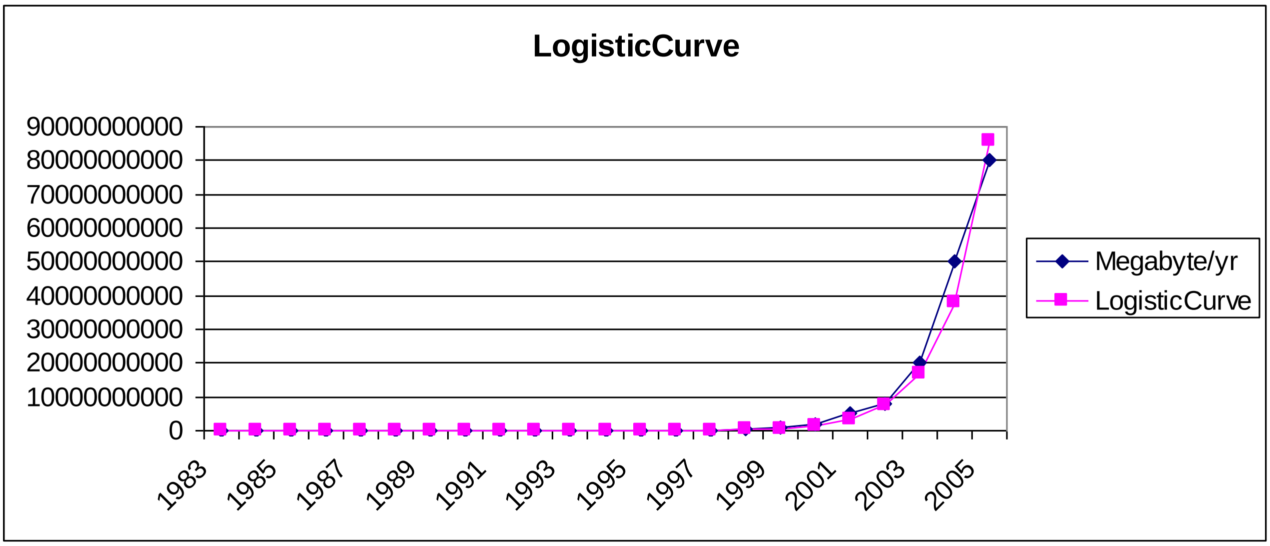
Then

log(*a*) = 25,654   *a* = 138481142767   *b* = 0,8315

We find the logistic curve

y = 1E+14 / (1 + 138481142767 * exp (-*0,8315* * *t*))

While using the solver we have the following graph of the measured data and those calibrated.



The optimized model has parameter values of:

k = 1E14   *a* = 138481142767   *b* = 0,808367601

For this model the value of the sum of squared errors is: 1,83655E+20

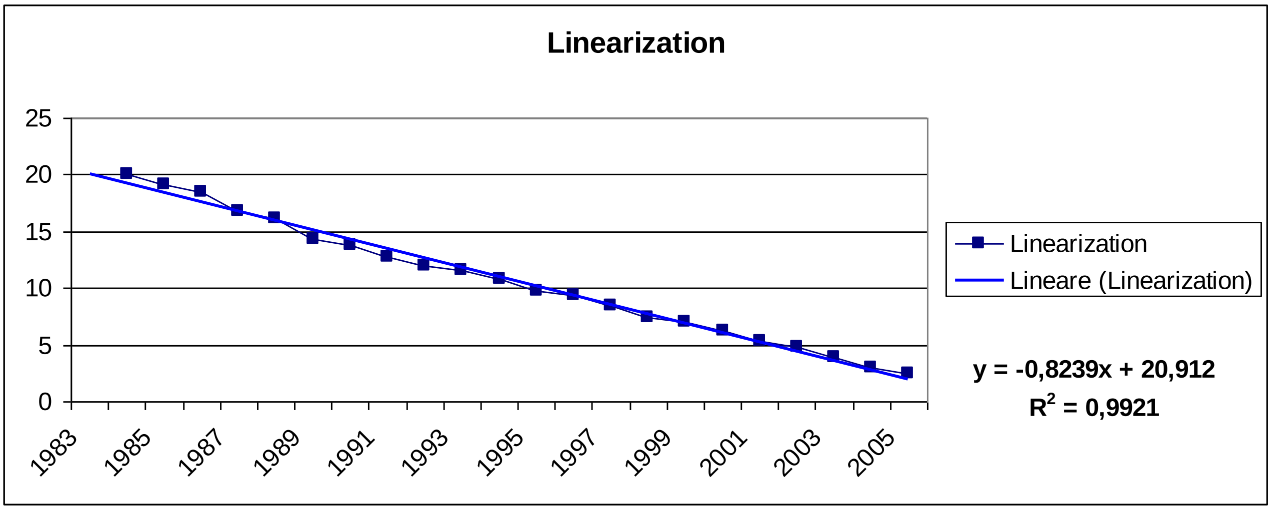
The value just examined is greater than the value calculated with the above K, then proceed in finding the optimal parameters are going to diminish the value of K.

The value of K will be taken within the range:

K < 1E14

At this point we choose a value of K <1E13 going to increase because we had a worsening of the value of the sum of squared errors.

- For *K*= *1E+12* we have the following linearized model:



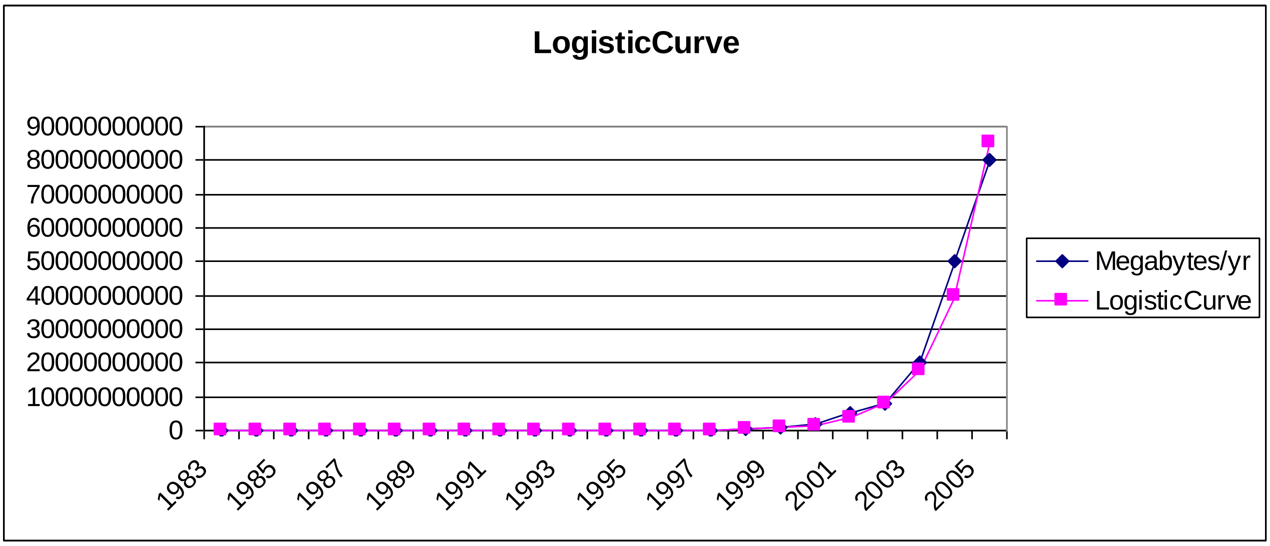
Then

log(*a*) = 20,912   *a* = 1207719853   *b* = 0,8239

We find the logistic curve

y = 1E+12 / (1 + 1207719853 * exp (*−0,8239 * t*))

While using the solver we have the following graph of the measured data and those calibrated.



The optimized model has parameter values of:

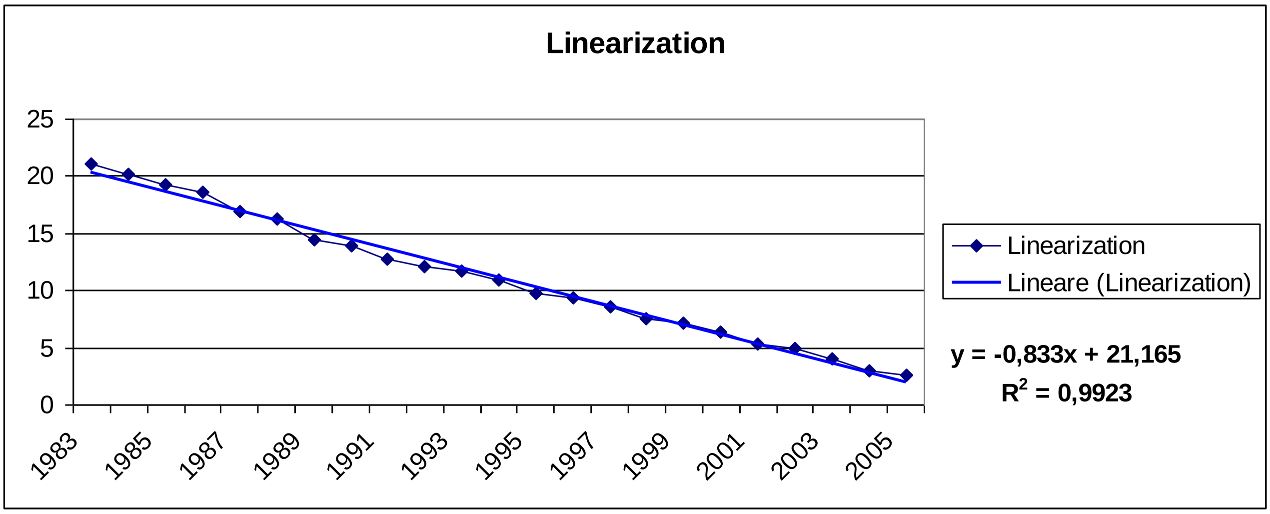
`k = 1E12` `a = 1401767812` `b = 0,812531084`

For this model the value of the sum of squared errors is: `1,39574E+20`

The value just examined is less than the previously calculated values for K, then we can say that this value of K may be the best.

We proceed in the search for good parameters going to increase the value of K.

- For *K*= *1,1111E+12* we have the following linearized model:



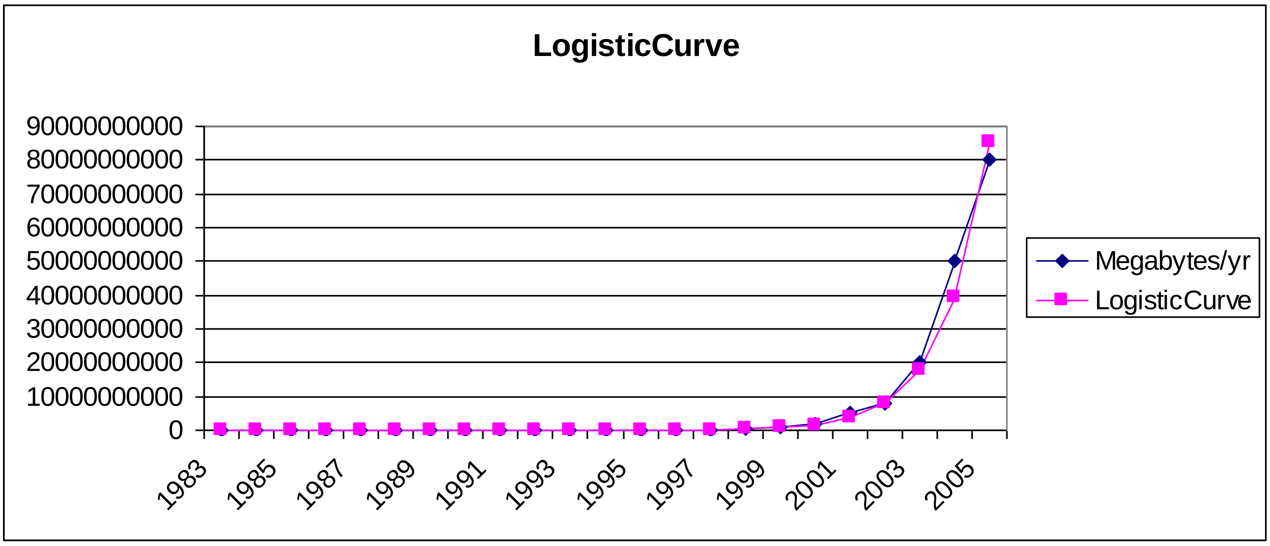
Then

log(*a*) = 21,165   *a* = 1555402202   *b* = 0,833

We find the logistic curve

y = 1,1111E+12 / (1 + 1555402202 * exp (-*0,833* * *t*))

While using the solver we have the following graph of the measured data and those calibrated.



The optimized model has parameter values of:

k = 1,1111E12   *a* = 1555402202   *b* = 0,812095061

For this model the value of the sum of squared errors is: 1,43833E+20

The value just examined is greater than previously calculated. At this moment we can say that the value of K above is the best value.

In summary:

K	*a*	*b*	*Somma ErroreQuadr*
9E10	164102206	0,86694	3,46857E+20
1E11	171312511	0,85987	2,41141E+20
1E13	13856219082	0,808714457	1,79469E+20

1E14	138481142767	0,808367601	1,83655E+20
1E12	1401767812	0,812531084	1,39574E+20
1,1111E12	1555402202	0,812095061	1,43833E+20

1.1.3 Chi-Square Test

f_0 and f_e are observed frequencies and expected frequencies.

H0 is the null hypothesis and H1 is the alternative hypothesis.

H0: There is a difference between the observed and expected frequencies.

H1: There is a difference between the observed and expected frequencies.

Test statistic:

$$\chi^2 = \sum \left[\frac{(f_o - f_e)^2}{f_e} \right]$$

The critical value is a value taken from a random variable chi-square with degrees of freedom (k-1), where k is the number of classes in which the random sample is grouped.

Step 1: Fix null hypothesis and alternative.

H0: There is agreement between the data.

H1: There is no agreement between the data.

In our case, the data are:

- volume of traffic in Megabytes / year of the Internet
- estimated data using the logistic equation.

Our goal is to apply the Chi Square Test to verify the consistency of the actual data and those estimated in terms of square error.

Step 2: Select the level of significance α.

Let $\alpha = 0.01$

The **level of significance** of a test is usually given by a test of hypothesis testing. In the simplest case is defined as the probability of accepting or rejecting the null hypothesis. The decision in this case is done using the p-value: if the value p (p-value) is less than the significance level, then the null hypothesis is rejected. The lower the p value, the more significant is the result.

Step 3: Select the test statistic

How to use the test $\chi 2$ statistics.

Step 4: H0 is rejected if the p-value is less than α = 0.01.

We calculate the test statistic at each logistic curve identified in the first part of the analysis:

1. *Logistic curve with K=9E+10*

Sum value χ^2 **= 10899425562**

This value was obtained by applying the formula:

$$\chi^2 = \sum \left[\frac{(f_o - f_e)^2}{f_e} \right]$$

Degrees of freedom = 23-1 = 22

The p (χ2>10899425562) = 0 has been calculated using the method *chi2cdf* (χ2 = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(10899425562, 22)
>> chi =
>>  0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of χ2 and then the p-value of 0 is less than the chosen significance level (α = 0.01). Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results

obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

2. *Logistic curve with K=1E+11*

Sum value χ^2 = 8328071512

Degrees of freedom = 23-1 = 22

The p (χ2>8328071512) = 0 has been calculated using the method *chi2cdf* (χ2 = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(8328071512,22)
>>  chi =
>>   0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of χ2 and then the p-value of 0 is less than the chosen significance level ($\alpha = 0.01$). Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

3. *Logistic curve with K=1E+13*

Sum value χ^2 **= 5823949746**

Degrees of freedom = 23-1 = 22

The p (χ2>5823949746) = 0 has been calculated using the method *chi2cdf* (χ2 = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(5823949746,22)
>> chi =
>>  0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of $\chi2$ and then the p-value of 0 is less than the chosen significance level ($\alpha = 0.01$). Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

4. *Logistic curve with K=1E+14*

Sum value χ^2 **= 6000313136**

Degrees of freedom = 23-1 = 22

The p ($\chi2$>6000313136) = 0 has been calculated using the method *chi2cdf* ($\chi2$ = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(6000313136,22)
>> chi =
>>   0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of $\chi2$ and then the p-value of 0 is less than the chosen significance level ($\alpha = 0.01$). Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

5. *Logistic curve with K=1E+12*

Sum value χ^2 **= 4245245095**

Degrees of freedom = 23-1 = 22

The p (χ2>4245245095) = 0 has been calculated using the method *chi2cdf* (χ2 = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(4245245095,22)
>> chi =
>>  0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of χ2 and then the p-value of 0 is less than the chosen significance level (α = 0.01). Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

6. *Logistic curve with K=1,11111E+12*

Sum value χ^2 **= 4404921055**

Degrees of freedom = 23-1 = 22

The p (χ2>4404921055) = 0 has been calculated using the method *chi2cdf* (χ2 = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(4404921055,22)
```

>> chi =

>> 0

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of $\chi2$ and then the p-value of 0 is less than the chosen significance level (α = 0.01). Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

In summary:

K	$\chi2$
9,00E+10	1,09E+10
1,00E+11	8,328E+09
1,00E+13	5,824E+09
1,00E+14	6E+09
1,00E+12	4,245E+09
1,11E+12	4,405E+09

The chi-square test was applied to test whether the logistic curve that best approximates the performance of the input data coincides with the one identified in the first phase of the study. According to the statistical hypothesis test, the observed data are significantly different from the actual data. However, the calculation of statistics shows that the lower value is in correspondence of *K = 1,00E+12*.

This coincides with what is assumed in the previous phase, namely that the logistic curve that best approximates the performance of the input data is described by the following equation:

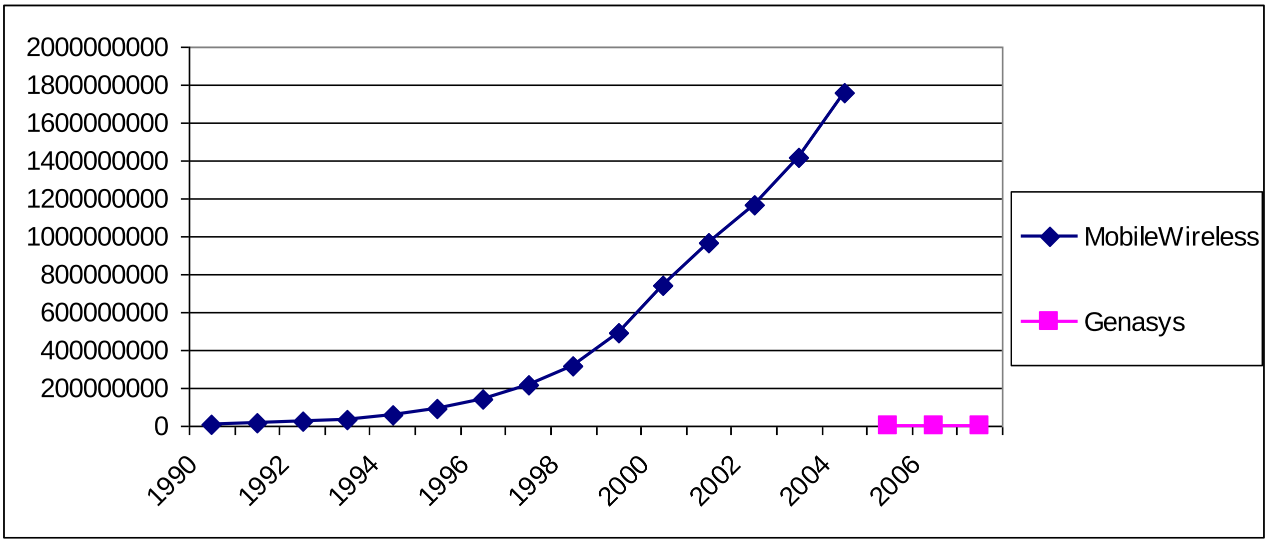
```
y = 1,00E+12 / (1 + 1401767812 *  exp (-0,812531084 * t)).
```

1.2 Mobile in the world

1.2.1 Considerations on the relationship between the logistic curves of the Mobile-Wireless and location services

At this stage we are dedicated to research and reports of possible links that may exist between the logistic curve for Mobile-Wireless users and that of location services.

Comparing these curves in the same graph we can see that there is a "time shift" between the curve for the number of registered users of Mobile-Wireless and the curve representing the development of location services.



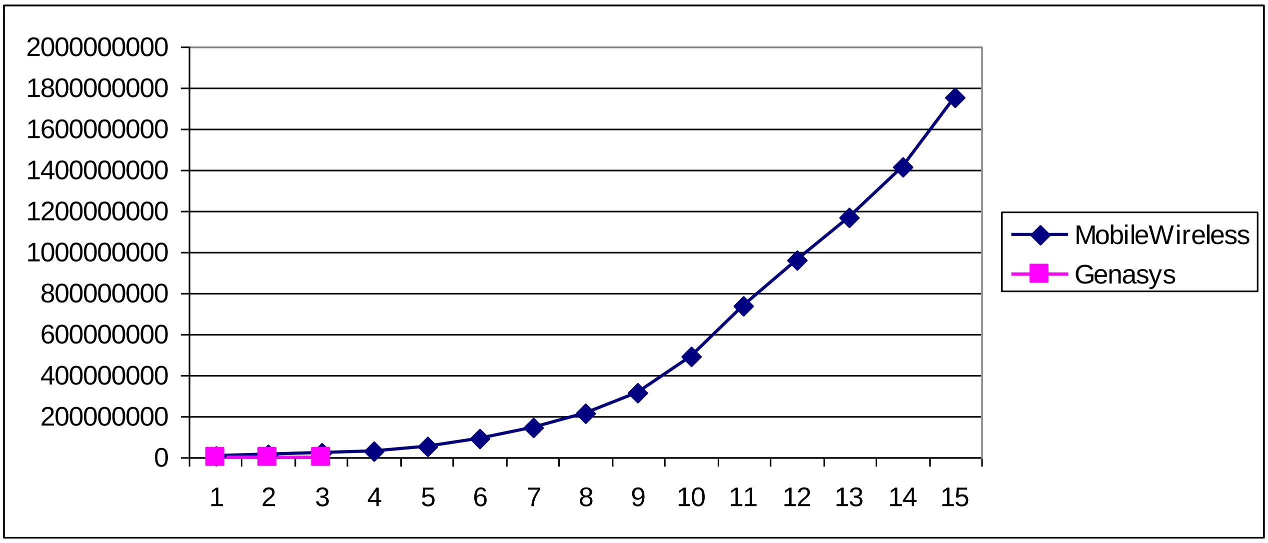
With the data at our disposal we can make a finer analysis on existing relationships.

With regard to the location services available data are those related to the first 3 years of development (2005 to 2007) in contrast to the Mobile-Wireless service whose data are related to a period of 15 years. Such availability may be related to the fact that location services are of recent development and distribution, as opposed to services Mobile-Wireless.

Today we are witnessing spectacular growth in the use of wireless communications. Innovative technological solutions using radio transmission are laying the foundations for a truly wireless world. One example in the wireless revolution is the astounding growth of mobile communications since the service was initially deployed. In 1990, there were only about 11 million mobile subscribers worldwide. This number increased to over 300 million by the end of 1998, and at the end of 2004 it had boomed to 1.75 billion, according to ITU statistics.

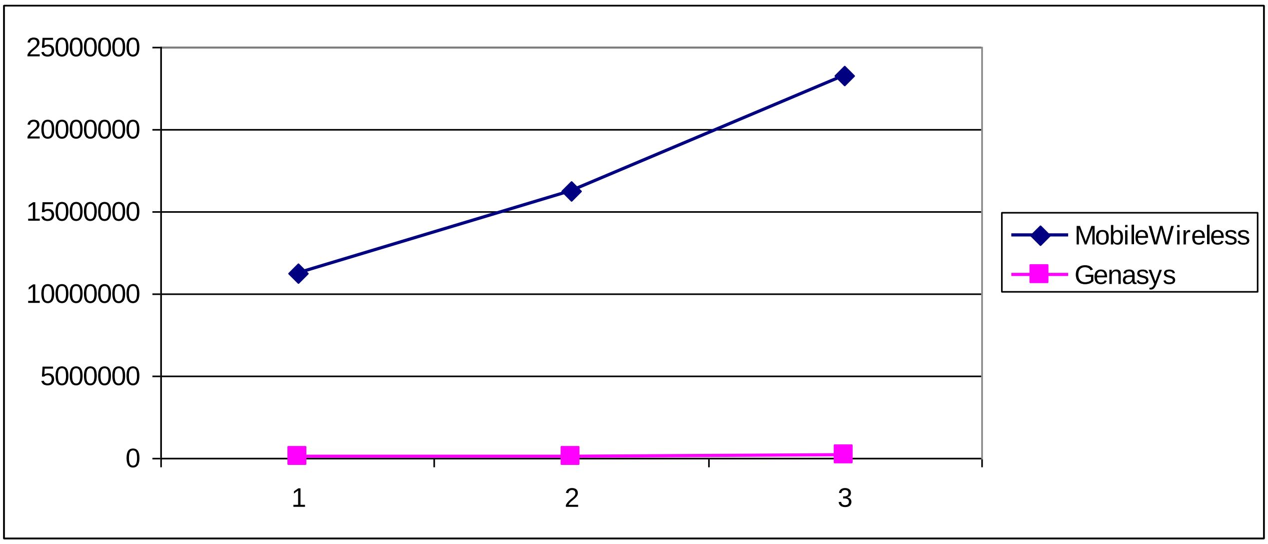
To make a more detailed analysis, we identify the similarities and differences between the two curves in their early years of development. In this way we can examine what happened during the introduction of such services.

To do this we insert the same graph the two curves in parallel.



At least in the first three years of analysis, the two curves have same trends. The two curves seem to overlap.

In reality, focusing on the trend of curves in the first three years, we verify that the location services have a slower pace than the curve of Mobile-Wireless. Perhaps because of the innovative technological solutions that use the radio over the years are having a strong and rapid development.



The curve of location services, provided by the company's Genasys, presents a curve with a slope less than that of Mobile-Wireless. The growth rate curve that represents the time trend in the number of registered users of Mobile-Wireless is more than the curve location.

1.2.2 Applications to find the best parameters for the logistic curve of the Mobile Wireless

As a first step, we want to make a fitting with the logistic model of growth through transformation of variables (in order to be reduced to a linear equation) and then by linear regression. Recall that the logistic equation

$$y = K / (1 + a* \exp(-bt))$$

becomes linear with the following transformation

$$\log(K/y - 1) = \log a - bt .$$

The data we suppose that the population limit (equilibrium) and K (remember that is the horizontal asymptote of the logistic function). Then we determine a and b by linear regression.

Since we are not aware of the extent to which value will reach the number of registered users of Mobile Wireless go forward in search of that parameter. We have made the analysis on different values of K. We stopped our search when the value of the mean square error made from the model was worse than that calculated in the previous system.

To make this process we assumed different values of K, we left the value of K = 1E+10

The idea is to calculate

$$\log(K/y - 1).$$

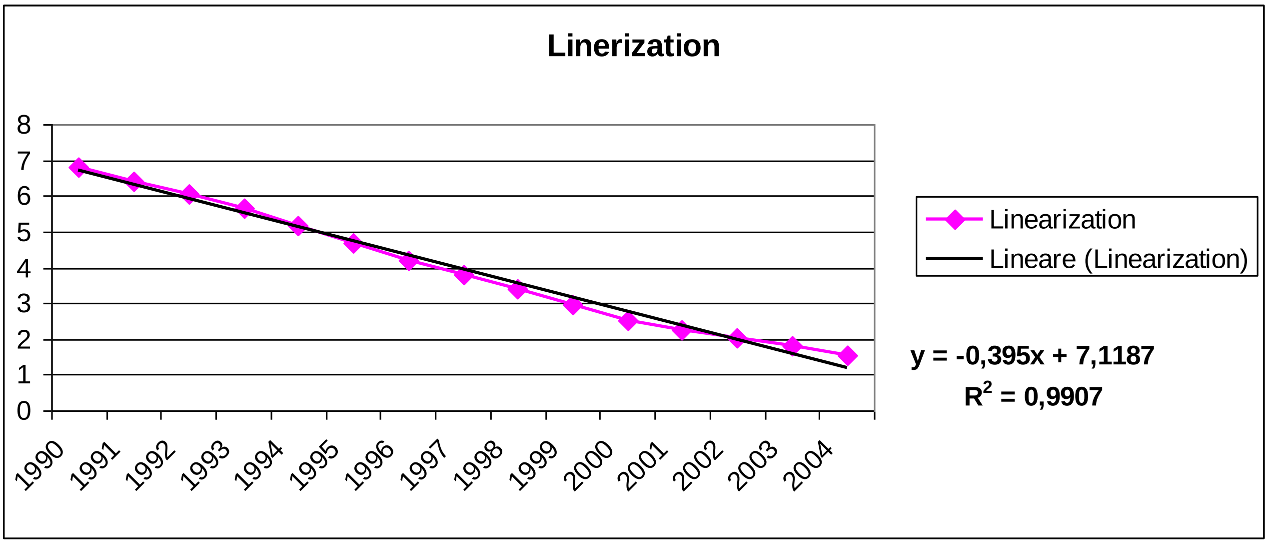
To this end we write in a cell of the Excel spreadsheet the following formula

= LN(1E+10/B2-1).

Where B2 is the first value of the function y. This calculation is done for all values of y available.

Now draw the chart with "chart" entering the x-axis the days and y axis, the column data as soon as detected.

At the same time insert the regression “trendline" together with its equation.



Then

`log(a) = 7,1187     a = 1234,8441    b = 0,395`

We find the logistic curve

`y = 1E+10 / (1 + 1234,8441 *  exp (-0,395 * t))`

Then we tried to optimize the choice of parameters using ''Solver'' (''Risolutore'') of Excel.

In another paper we have entered the data file according to the formula given by the logistic growth model

`y = K/(1 + a * exp (-b*t))`

To do this, we inserted the formula in column writing for example

`= I2 /( 1+( I3 * EXP(- I4 * A2)))`

Note that the parameters used are those included in Excel spreadsheet cells. Box I2 is that of the parameter K, I3 is the box for the parameter a, while the I4 box refers to the parameter b.

In particular these cells are introduced into the formula with the $ sign to make these cells remain fixed in the calculation. So we have:

`= $I$2 /( 1+( $I$3 * EXP(-$I$4 * A2)))`

Then calculate the square error by including in each row of column Excel spreadsheet formulas like

=(C2-B2)^2

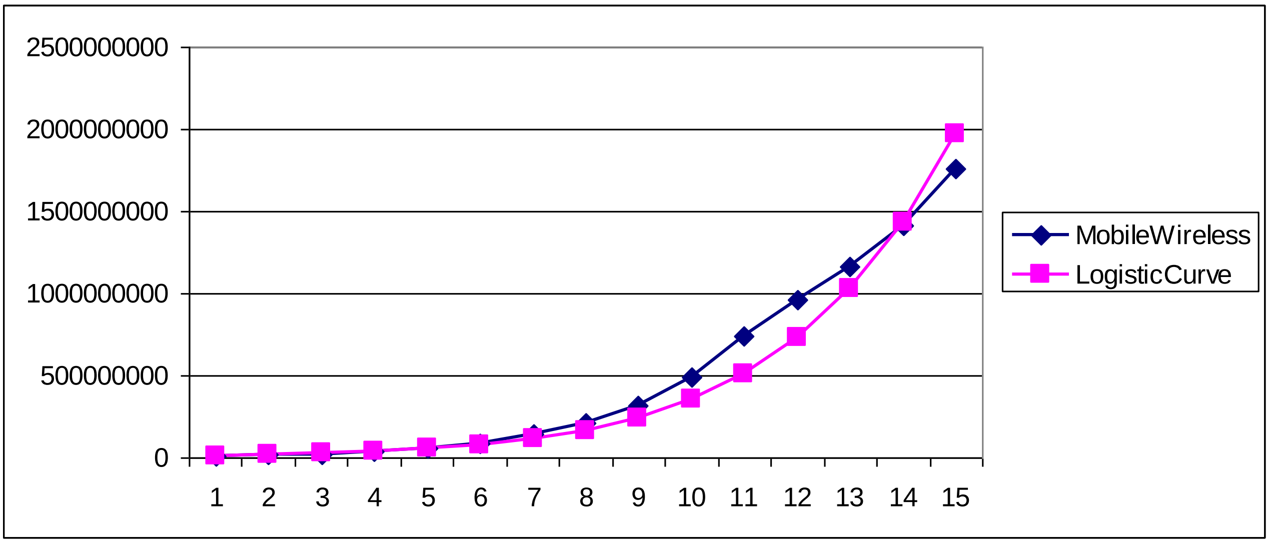
In this way we calculated the value of the mean square for the first period.

Using the "Solver" of Excel (found under the Tools menu, "Tools") optimize the parameters K, a and b by minimizing the sum of the standard deviations.

We set the "Solver" indicating that the parameters are optimized in the cells and I2-I4 (by entering the command, and then I2: I4 under “changing cells") and inserting the value of the sum of the standard deviations in cell the objective function to be minimized ("Target" cell D16), we start the "Solver".

The result of this simulation allows us to identify the best parameters a and b according to a saturation limit value set in the value K.

Finally, we can make a graph of the measured data and those calibrated.



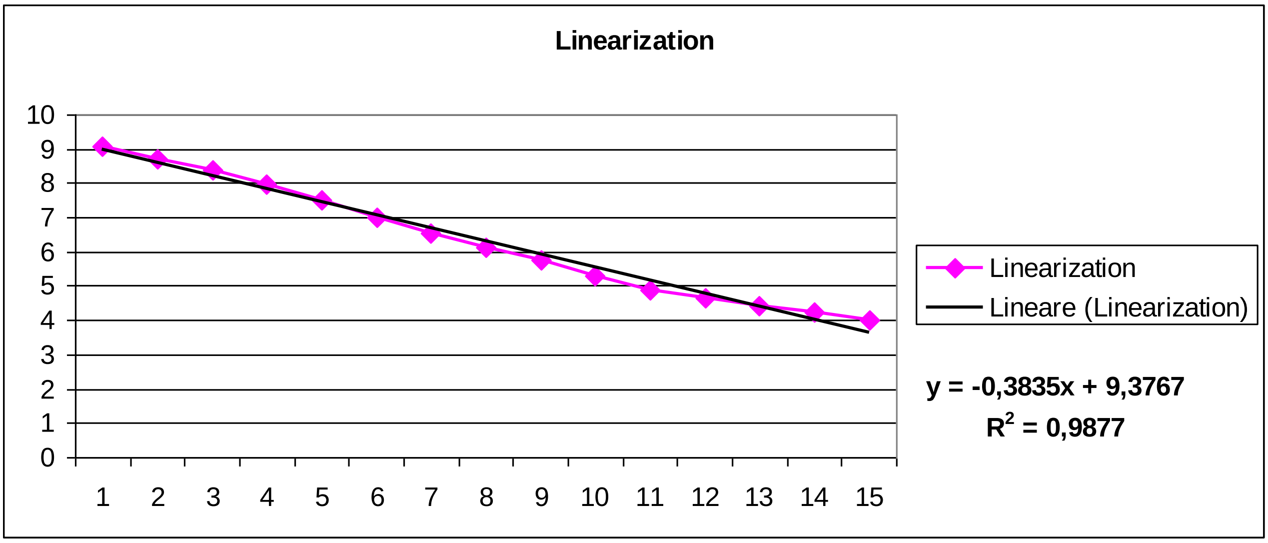
The optimized model has parameter values of:

k = 1E+10 *a* = 1234,844 *b* = 0,381012

For this model the value of the sum of squared errors is: 2,04709E+17

This procedure was performed for different values of K.

- For $K=$ `1E+11` we have the following linearized model:



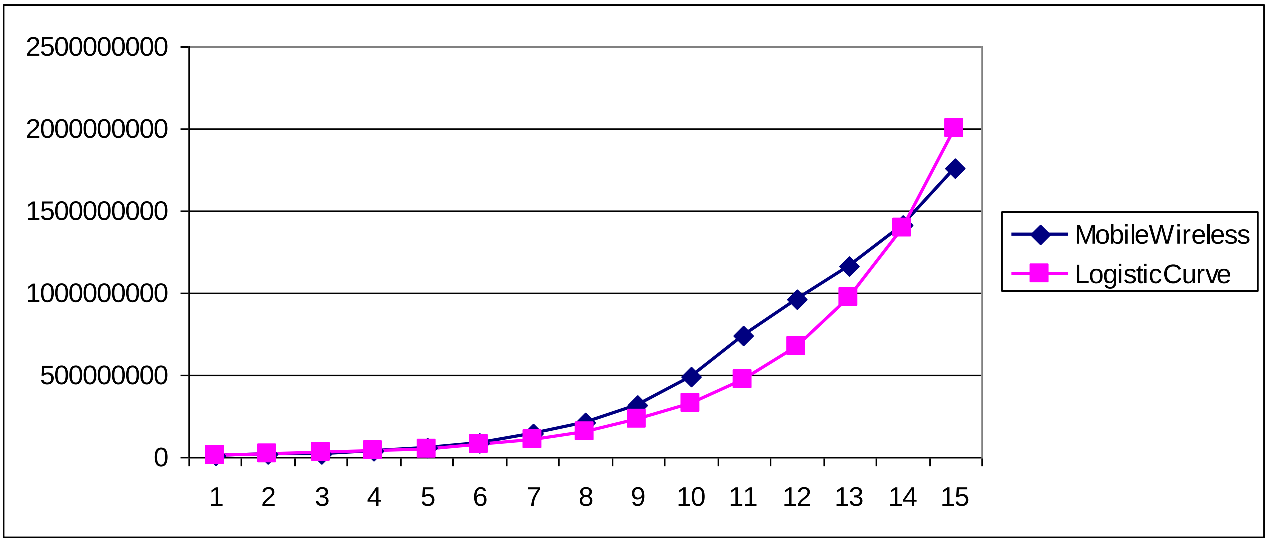
Then

`log(a) = 9,3767     a = 11809,977    b = 0,3835`

We find the logistic curve

`y = 1E+11 / (1 + 11809,977 *  exp (-0,3835 * t))`

While using the solver we have the following graph of the measured data and those calibrated.



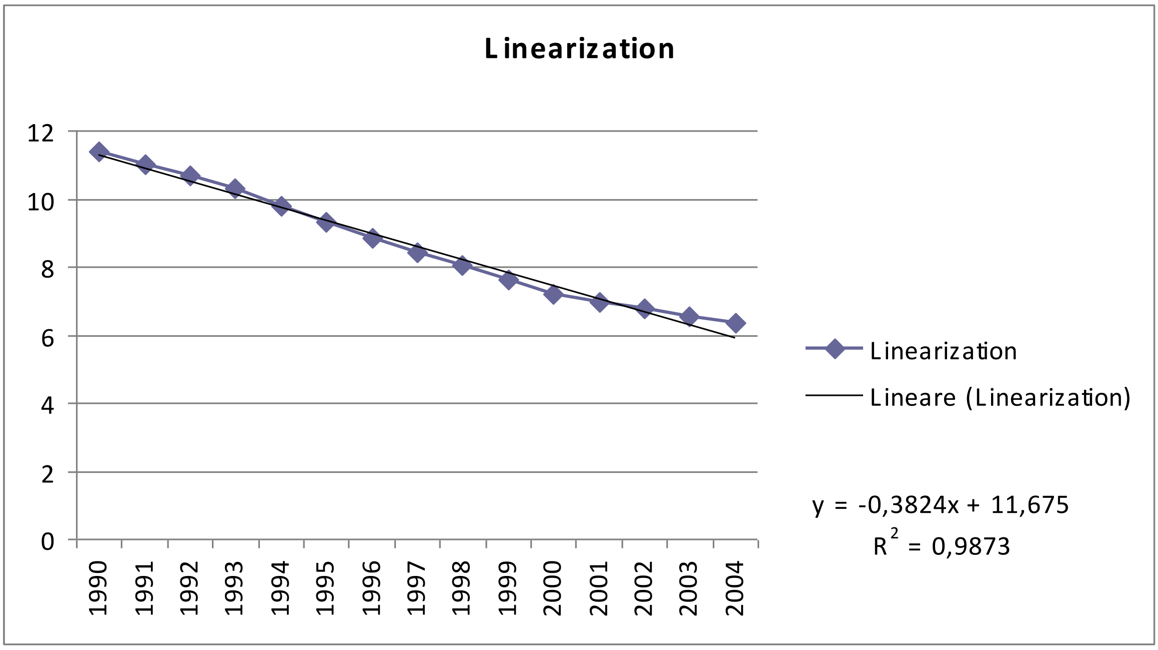
The optimized model has parameter values of:

`k = 1E+11     a = 11809,98   b = 0,365765`

For this model the value of the sum of squared errors is: `2,92936E+17`

The value just examined is greater than the value calculated with the previous K, we proceed in finding the optimal parameters are going to increase the value of K. Let's see what happens.

- Per $K = 1E+12$ we have the following linearized model:



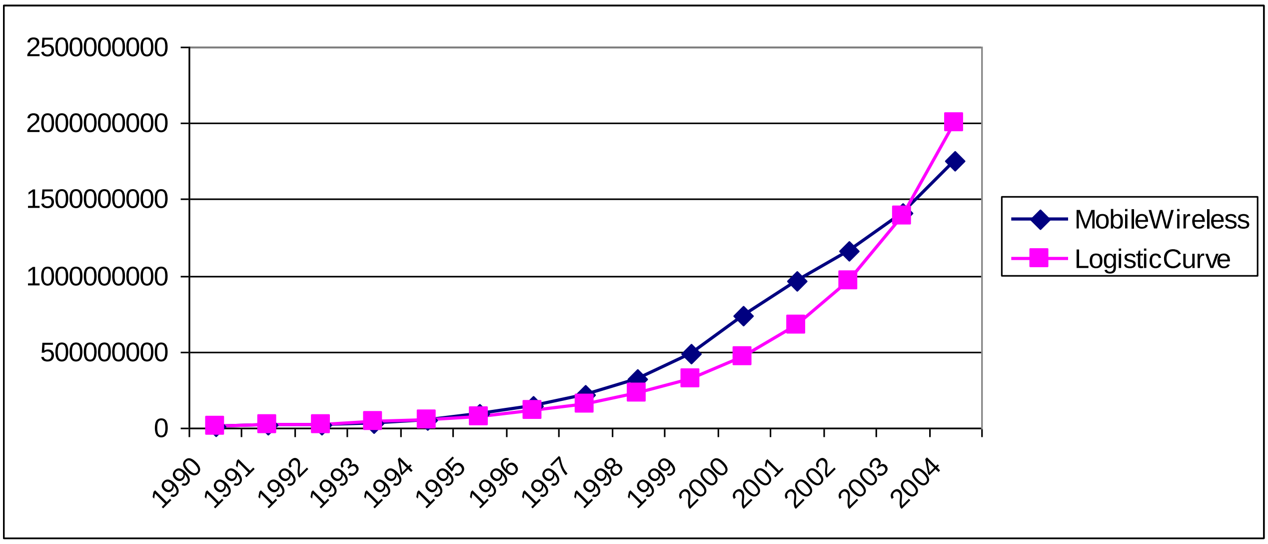
Then

$\log(a) = 11{,}675 \quad a = 117595 \quad b = 0{,}3824$

We find the logistic curve

$y = 1E+12 / (1 + 117595 * \exp(-0{,}3824 * t))$

While using the solver we have the following graph of the measured data and those calibrated.



The optimized model has parameter values of:

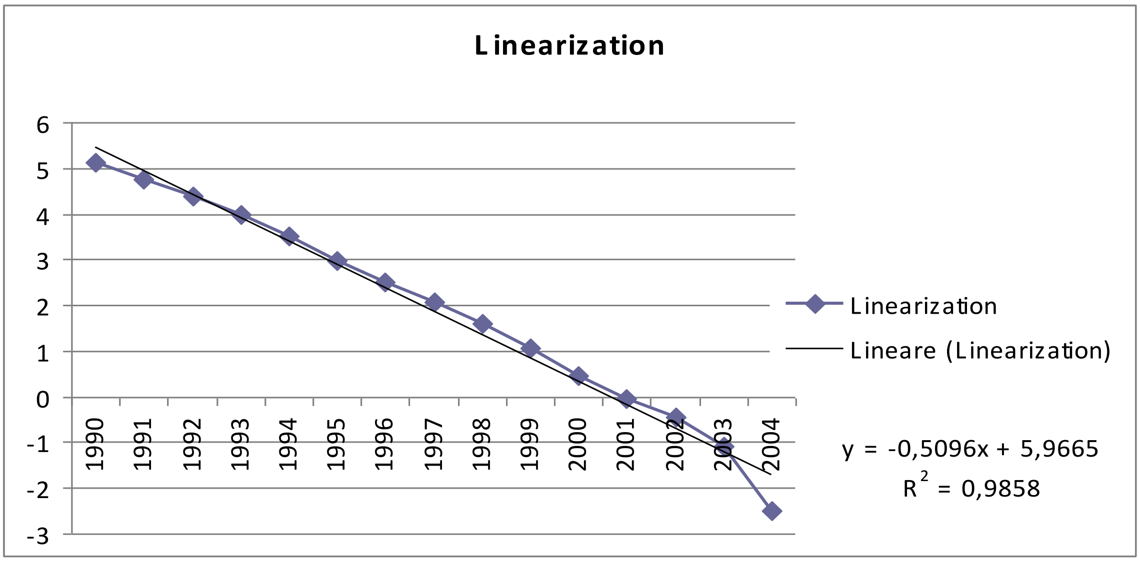
`k = 1E12` *a* `= 117595` *b* `= 0,364344`

For this model the value of the sum of squared errors is: 3,01874E+17

Given that the value is greater than just examined the computed value with the K = 1E10, proceed in the search for good parameters going to diminish the value of K. The value of K will be taken within the range: K < 1E10

At this point we choose a K value between 1E10 and last values of the set of input data. We check if the sum of squared errors decreases.

- For *K*= *1,9E+09* we have the following linearized model:



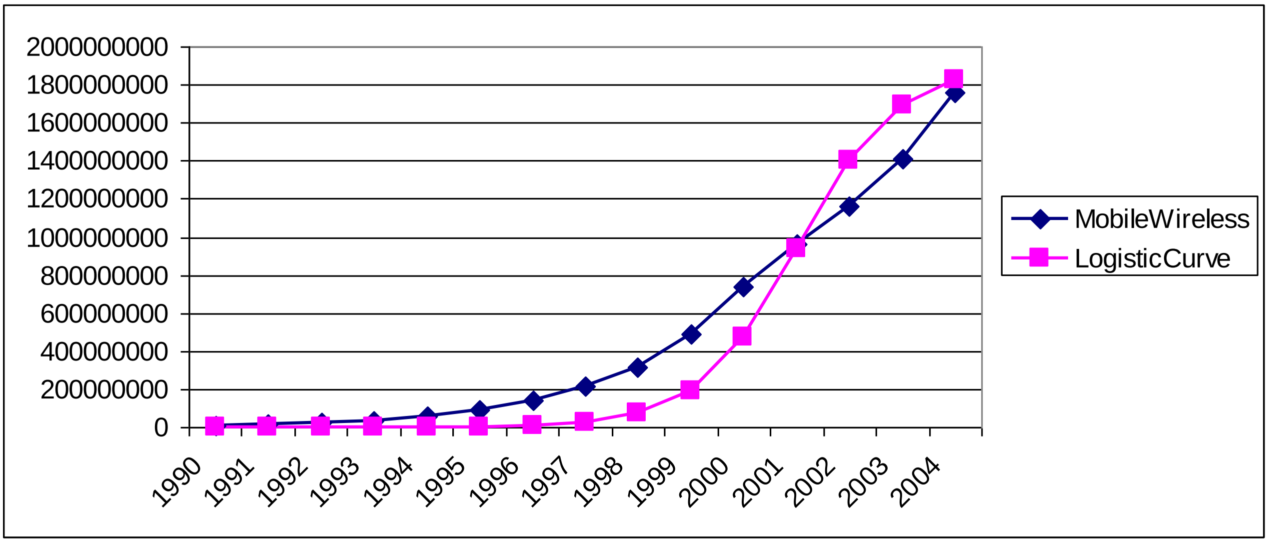
Then

`log(a) = 5,9665     a = 390138    b = 0,5096`

We find the logistic curve

`y = 1,9E+09 / (1 + 390138 *  exp (-0,5096 * t))`

While using the solver we have the following graph of the measured data and those calibrated.



The optimized model has parameter values of:

`k = 1,9E+09     a = 390.138    b = 1,070328`

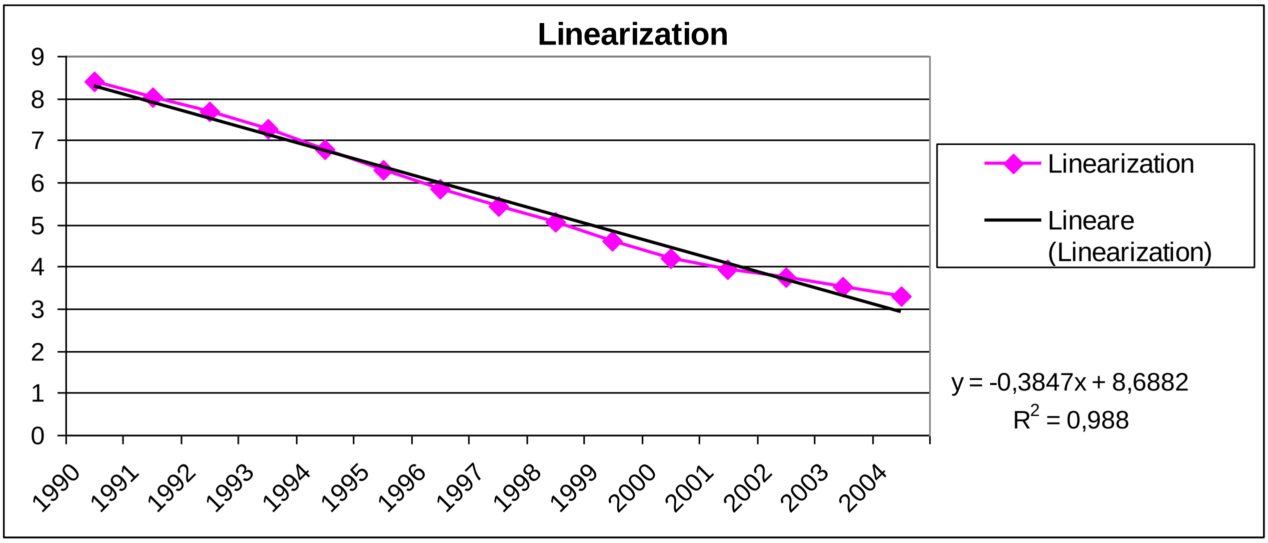
For this model the value of the sum of squared errors is: 4,28359E+17.

The value just examined is greater than that obtained at k = 1E10. We proceed in the search for good parameters going to increase the value of K in the following range:

1E10 < K < 1E11

At this point we choose a value of K <1E11 going to increase because we had a worsening of the value of the sum of squared errors.

- For *K*= *5E+10* we have the following linearized model:



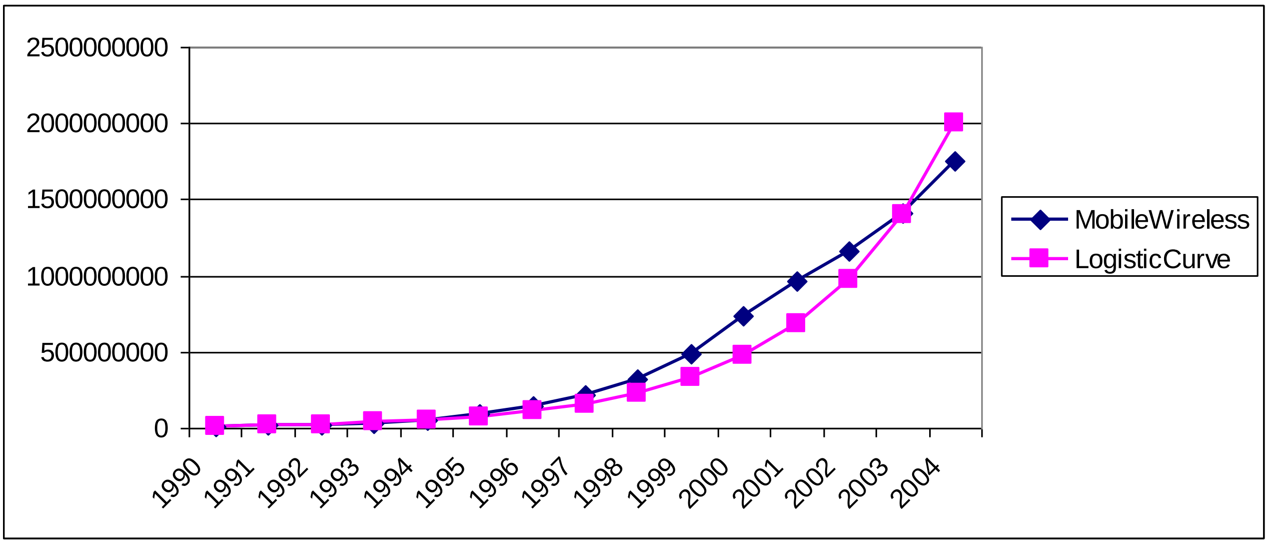
Then

log(*a*) = 8,6882   *a* = 5932,5   *b* = 0,3847

We find the logistic curve

y = 5E+10 / (1 + 5932,5 * exp (-*0,3847* * *t*))

While using the solver we have the following graph of the measured data and those calibrated.



The optimized model has parameter values of:

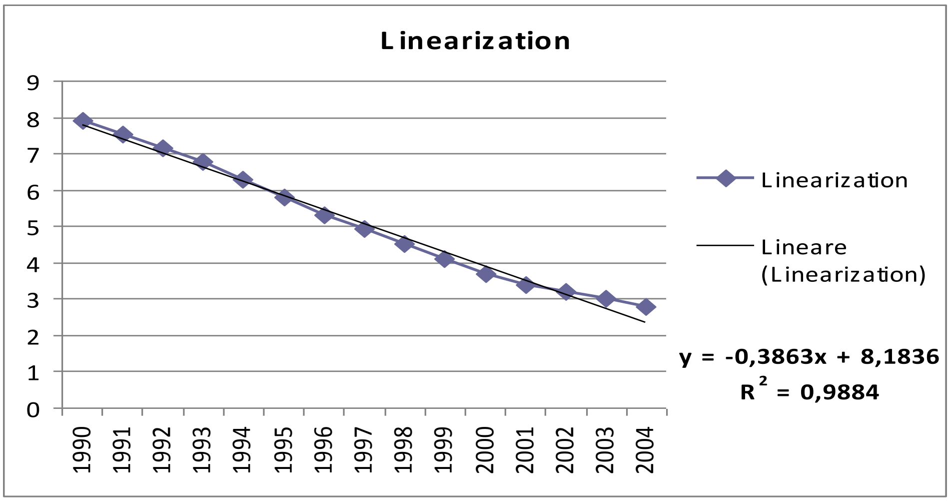
K = 5E+10   *a* = 5932,5   *b* = 0,367355

For this model the value of the sum of squared errors is: 2,82977E+17

The resulting number is greater than the value obtained by setting K = 1E+10, diminish the value of K, which will be included in the following range:

1E+10 < K < 5E+10

- For $K = 3E+10$ we have the following linearized model:



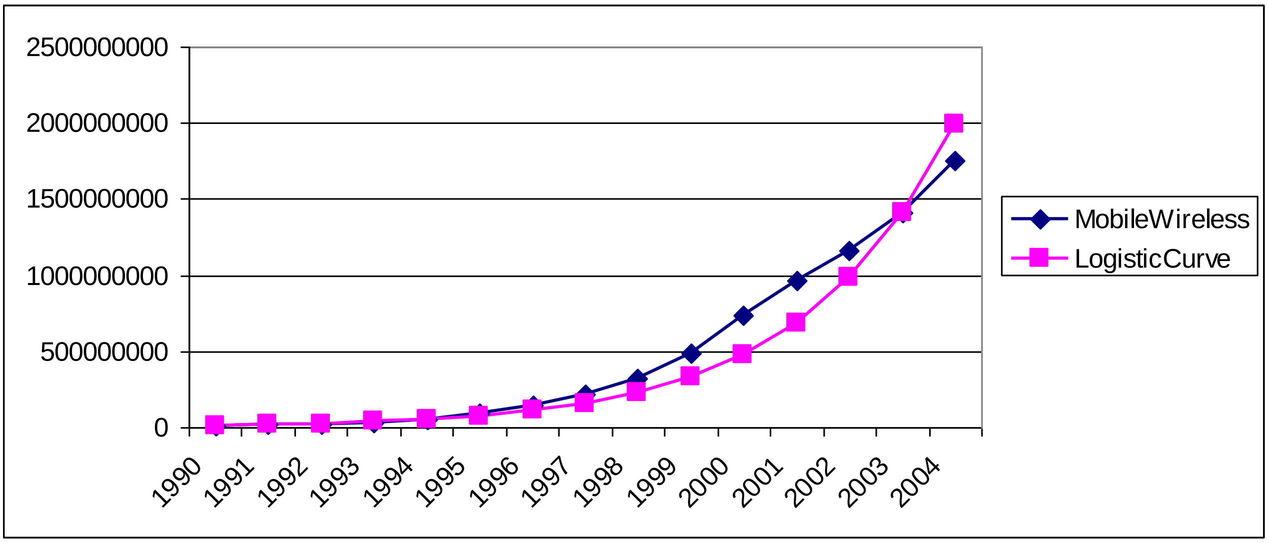
Then

`log(a) = 8,1836     a = 3581,73    b = 0,3863`

We find the logistic curve

`y = 3E+10 / (1 + 3581,73 *  exp (-0,3863 * t))`

While using the solver we have the following graph of the measured data and those calibrated.



The optimized model has parameter values of:

K= 3E+10 *a* = 3581,73 *b* = 0,369508

For this model the value of the sum of squared errors is: 2,69716E+17.

The value just examined is greater than previously calculated. At this moment we can say that the value of K=1E+10 is the best value.

In summary:

K	*a*	*b*	*Sum Square Error*
1,00E+10	1234,844	0,381012	2,05E+17
1,00E+11	11809,98	0,365765	2,93E+17
1,00E+12	117595	0,364344	3,02E+17
1,90E+09	390138	1,070328	4,28E+17
5,00E+10	5932,5	0,367355	2,83E+17
3,00E+10	3581,73	0,369508	2,70E+17

1.2.3 Chi-Square Test

f_0 and f_e are observed frequencies and expected frequencies.

H0 is the null hypothesis and H1 is the alternative hypothesis.

H0: There is a difference between the observed and expected frequencies.

H1: There is a difference between the observed and expected frequencies.

Test statistic:

$$\chi^2 = \sum \left[\frac{(f_o - f_e)^2}{f_e} \right]$$

The critical value is a value taken from a random variable chi-square with degrees of freedom (k-1), where k is the number of classes in which the random sample is grouped.

Step 1: Fix null hypothesis and alternative.

H0: There is agreement between the data.
H1: There is no agreement between the data.

In our case, the data are:
- mobile subscribers (wireless) in millions.
- estimated data using the logistic equation.

Our goal is to apply the Chi Square Test to verify the consistency of the actual data and those estimated in terms of square error.

Step 2: Select the level of significance α.
Let α = 0.01

The **level of significance** of a test is usually given by a test of hypothesis testing. In the simplest case is defined as the probability of accepting or rejecting the null hypothesis. The decision in this case is done using the p-value: if the value p (p-value) is less than the significance level, then the null hypothesis is rejected. The lower the p value, the more significant is the result.

Step 3: Select the test statistic
How to use the test χ2 statistics.

Step 4: H0 is rejected if the p-value is less than α = 0.01.
We calculate the test statistic at each logistic curve identified in the first part of the analysis:

1.Logistic curve with K=1E+10

Sum value χ^2 = ***327637768.1***

This value was obtained by applying the formula:

$$\chi^2 = \sum\left[\frac{(f_o - f_e)^2}{f_e}\right]$$

Degrees of freedom = 15-1 = 14

The p ($\chi 2$>*327637768.1*) = 0 has been calculated using the method *chi2cdf* ($\chi 2$ = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(327637768.1,14)
>> chi =
>> 0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of $\chi 2$ and then the p-value of 0 is less than the chosen significance level (α = 0.01). Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

1. *Logistic curve with K=1E+11*

Sum value χ^2 **= 499602927.2**

Degrees of freedom = 15-1 = 14

The p ($\chi 2$>499602927.2) = 0 has been calculated using the method *chi2cdf* ($\chi 2$ = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(499602927.2,14)
>> chi =
```

```
>>  0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of χ2 and then the p-value of 0 is less than the chosen significance level (α = 0.01). Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

2. *Logistic curve with K=1E+12*

Sum value χ^2 **= 517834715**

Degrees of freedom = 15-1 = 14

The p (χ2>517834715) = 0 has been calculated using the method *chi2cdf* (χ2 = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(517834715,14)
>> chi =
>> 0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of χ2 and then the p-value of 0 is less than the chosen significance level (α = 0.01). Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

3. *Logistic curve with K=1,9E+09*

Sum value χ^2 ***= 33458537282***

Degrees of freedom = 15-1 = 14

The p ($\chi2$>*33458537282*) = 0 has been calculated using the method *chi2cdf* ($\chi2$ = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(33458537282,14)
>> chi =
>> 0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of $\chi2$ and then the p-value of 0 is less than the chosen significance level (α = 0.01). Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

4. *Logistic curve with K=5E+10*

Sum value χ^2 ***= 479438782.8***

Degrees of freedom = 15-1 = 14

The p ($\chi2$>*479438782.8*) = 0 has been calculated using the method *chi2cdf* ($\chi2$ = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(479438782.8,14)
>> chi =
>>  0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of $\chi2$ and then the p-value of 0 is less than the chosen significance level (α = 0.01). Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results

obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

5. *Logistic curve with K=3E+10*

Sum value χ^2 = **452869945.6**

Degrees of freedom = 15-1 = 14

The p (χ2>452869945.6) = 0 has been calculated using the method *chi2cdf* (χ2 = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(452869945.6,14)
>> chi =
>>  0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of χ2 and then the p-value of 0 is less than the chosen significance level (α = 0.01). Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

In summary:

K	χ2
1,00E+10	327637768,1
1,00E+11	499602927,2
1,00E+12	517834715
1,90E+09	33458537282
5,00E+10	479438782,8
3,00E+10	452869945,6

The chi-square test was applied to test whether the logistic curve that best approximates the performance of the input data coincides with the one identified in the first phase of the study. According to the statistical hypothesis test, the observed data are significantly different from the actual data. However, the calculation of statistics shows that the lower value is in correspondence of *K = 1,00E+10*.

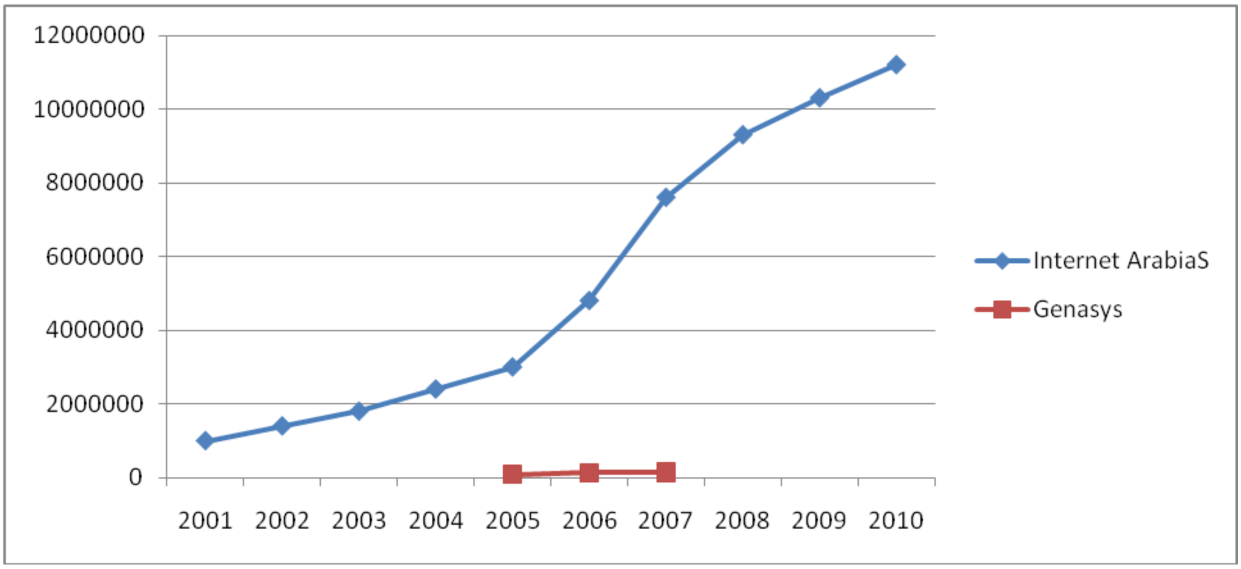
This coincides with what is assumed in the previous phase, namely that the logistic curve that best approximates the performance of the input data is described by the following equation:

$$y = 1{,}00E+10 / (1 + 1234{,}844 * \exp(-0{,}381012 * t)).$$

1.3 Internet in Saudi Arabia

1.3.1 Considerations on the relationship between the logistic curves of the Internet in Saudi Arabia and location services

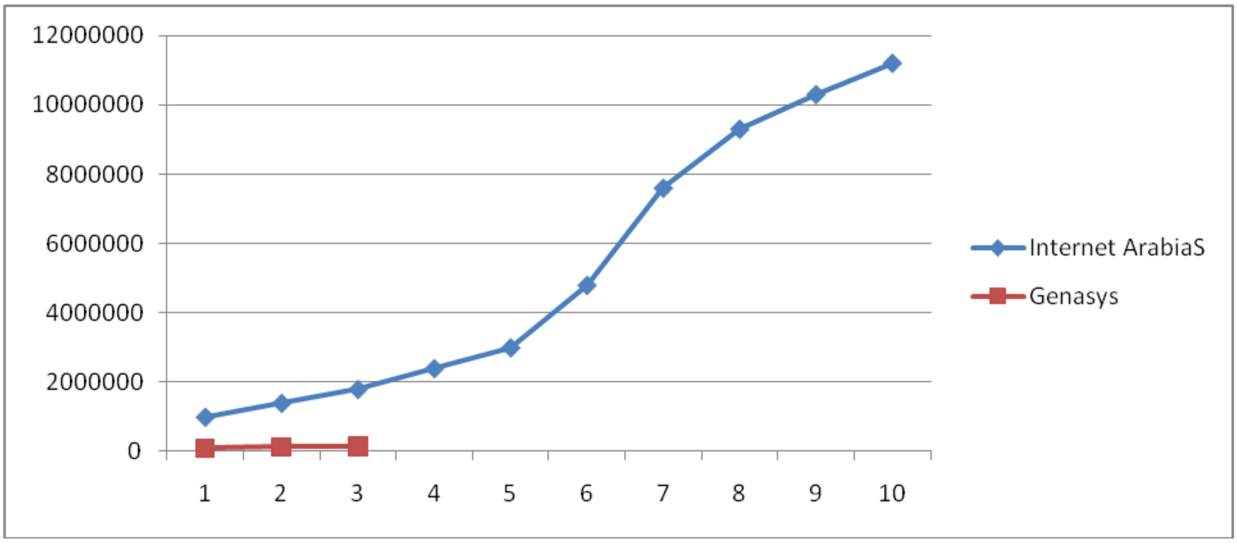
At this stage we are dedicated to research and reports of possible links that may exist between the logistic curve for Internet users in Saudi Arabia and that of location services. Comparing these curves in the same graph we can see that there is a "time shift" between the curve for the number of registered users of Internet in Saudi Arabia and the curve representing the development of location services.



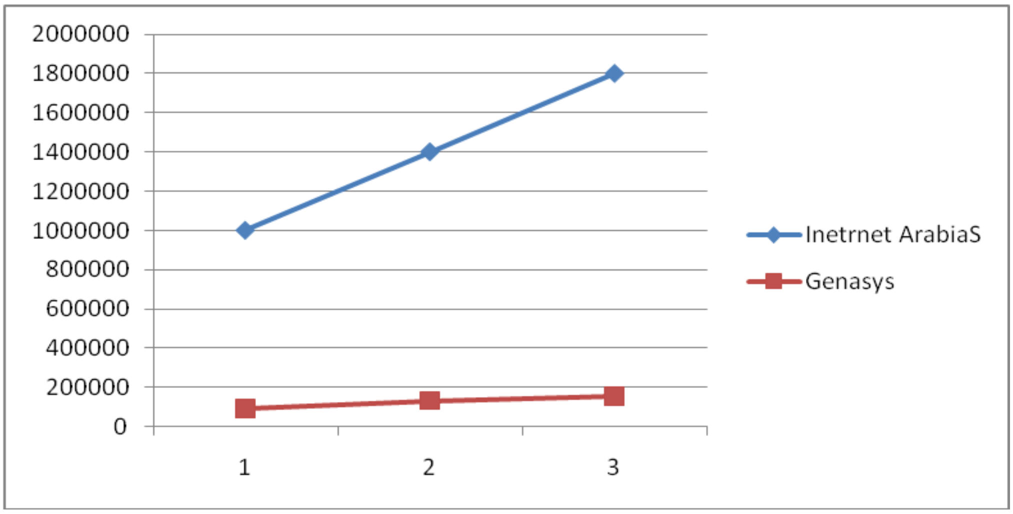
With the data at our disposal we can make a finer analysis on existing relationships.

With regard to the location services available data are those related to the first 3 years of development (2005 to 2007) in contrast to the Internet service whose data are related to a period of 10 years. Such availability may be related to the fact that location services are of recent development and distribution, as opposed to services Internet. Such availability may be related to the fact that location services are a recent development and distribution, unlike the Internet in Arabia Saudita that saw its birth in the 2000's and for which the information is clearer and more precise.

To make a more detailed analysis, we identify the similarities and differences between the two curves in their early years of development. In this way we can examine what happened during the introduction of such services.
To do this we insert the same graph the two curves in parallel.



At least in the first three years of analysis, the two curves have different trends. In reality, with particular attention to the evolution of the curves in the first three years, we can verify that the location services have a slower pace than the curve of the Internet in Arabia Saudita in terms of volume. We can also verify that the two curves have the different trend in terms of math performance.



The curve of location services, provided by Genasys, presents a curve with a slope almost different to that of the Internet in Saudi Arabia.

1.3.2 Applications to find the best parameters for the logistic curve of the Internet in Saudi Arabia

As a first step, we want to make a fitting with the logistic model of growth through transformation of variables (in order to be reduced to a linear equation) and then by linear regression. Recall that the logistic equation

$$y = K / (1 + a * \exp(-bt))$$

becomes linear with the following transformation

$$\log(K/y - 1) = \log a - bt \, .$$

The data we suppose that the population limit (equilibrium) and K (remember that is the horizontal asymptote of the logistic function). Then we determine a and b by linear regression.

Since we are not aware of the extent to which value will reach the number of registered users of Internet go forward in search of that parameter. We have made the analysis on different values of K. We stopped our search when the value of the mean square error made from the model was worse than that calculated in the previous system.

To make this process we assumed different values of K, we left the value of K = 1E+10

The idea is to calculate

log(*K/y* −1).

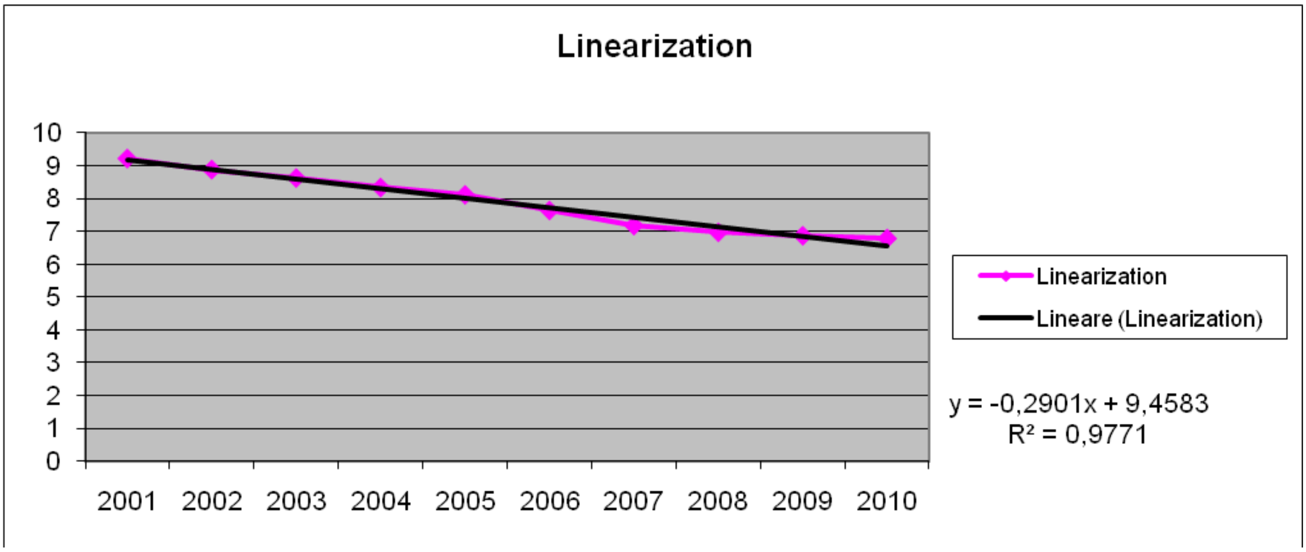
To this end we write in a cell of the Excel spreadsheet the following formula

= LN(1E+10/B2-1).

Where B2 is the first value of the function y. This calculation is done for all values of y available.

Now draw the chart with "chart" entering the x-axis the days and y axis, the column data as soon as detected.

At the same time insert the regression “trendline" together with its equation.



Then

log(*a*) = 9,4583   *a* = 12821,77   *b* = 0,2901

We find the logistic curve

y = 1E+10 / (1 + 12821,77 * exp (−0,2901 * *t*))

Then we tried to optimize the choice of parameters using ‘‘Solver’’ (‘‘Risolutore’’) of Excel.

In another paper we have entered the data file according to the formula given by the logistic growth model

y = *K*/(1 + *a* * *exp* (−*b***t*))

To do this, we inserted the formula in column writing for example

```
= I2 /( 1+( I3 * EXP(- I4 * A2)))
```

Note that the parameters used are those included in Excel spreadsheet cells. Box I2 is that of the parameter K, I3 is the box for the parameter a, while the I4 box refers to the parameter b.

In particular these cells are introduced into the formula with the $ sign to make these cells remain fixed in the calculation. So we have:

```
= $I$2 /( 1+( $I$3 * EXP(-$I$4 * A2)))
```

Then calculate the square error by including in each row of column Excel spreadsheet formulas like

```
=(C2-B2)ˆ2
```

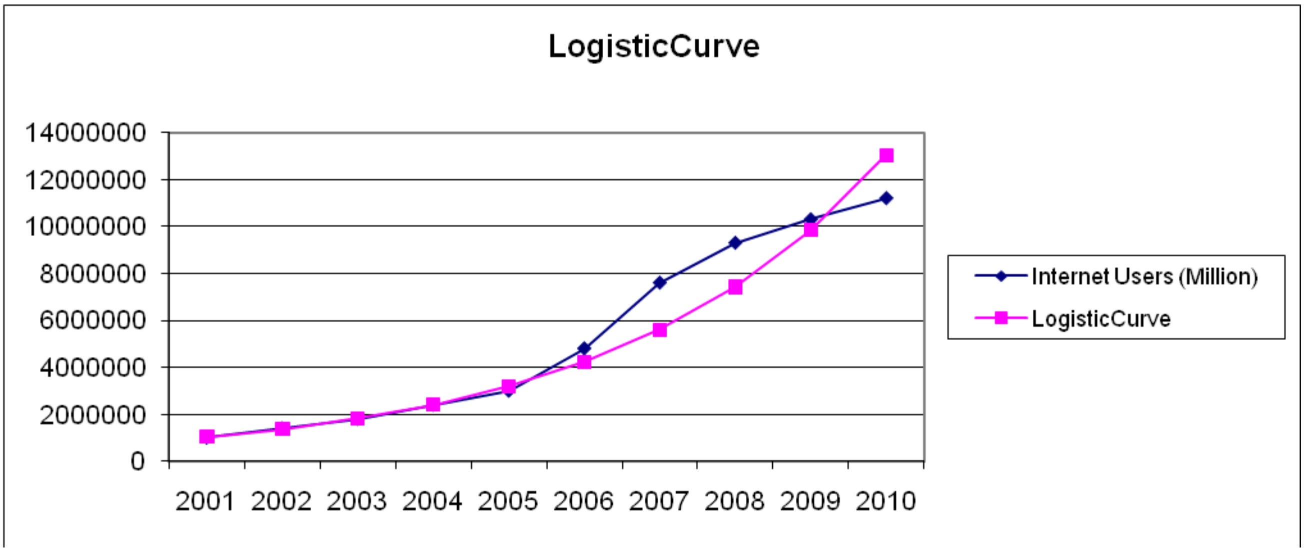
In this way we calculated the value of the mean square for the first period.

Using the "Solver" of Excel (found under the Tools menu, "Tools") optimize the parameters K, a and b by minimizing the sum of the standard deviations.

We set the "Solver" indicating that the parameters are optimized in the cells and I2-I4 (by entering the command, and then I2: I4 under “changing cells") and inserting the value of the sum of the standard deviations in cell the objective function to be minimized ("Target" cell D16), we start the "Solver".

The result of this simulation allows us to identify the best parameters a and b according to a saturation limit value set in the value K.

Finally, we can make a graph of the measured data and those calibrated.



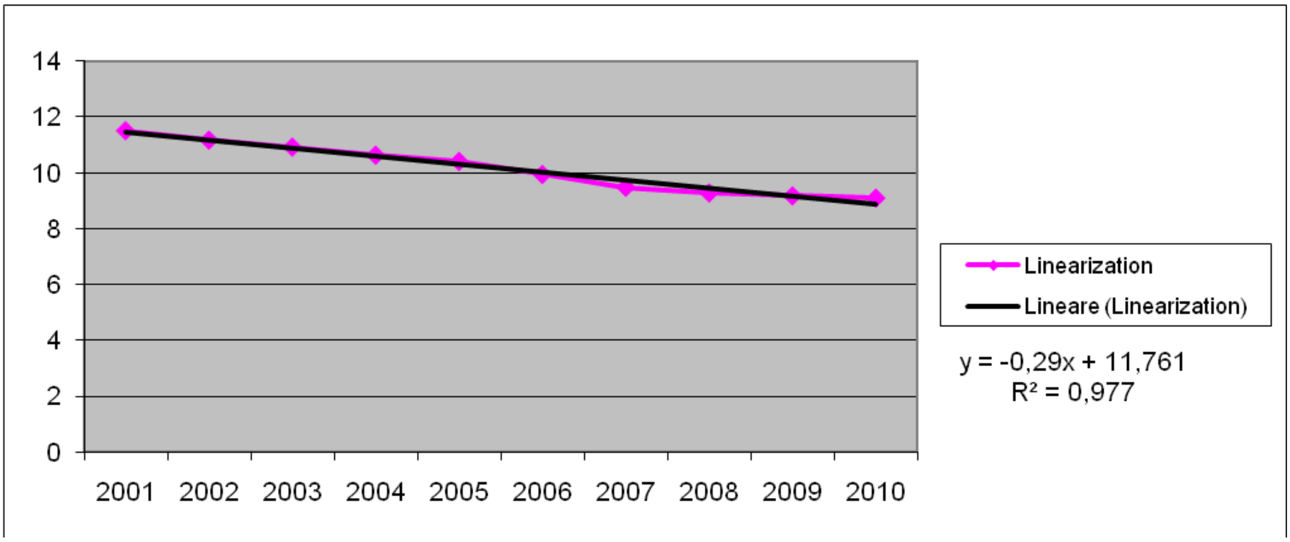
The optimized model has parameter values of:

k = 1E+10 *a* = 12821,77 *b* = 0,281848

For this model the value of the sum of squared errors is: 1,14589E+13

This procedure was performed for different values of K.

- For *K*= *1E+11* we have the following linearized model:



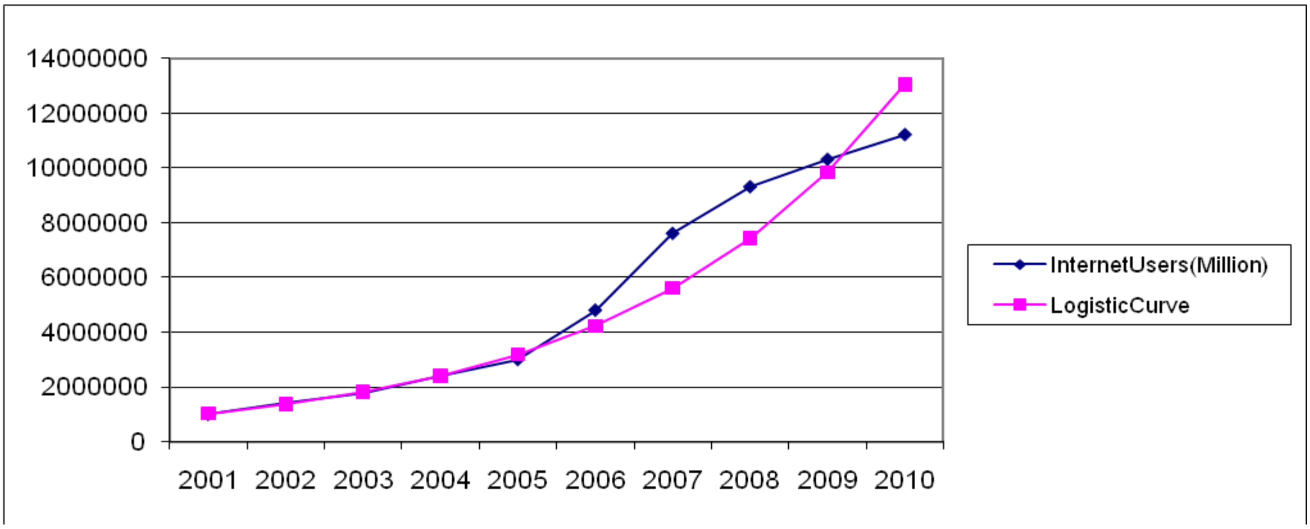
Then

log(*a*) = 11,761 *a* = 128155,5 *b* = 0,29

We find the logistic curve

y = 1E+11 / (1 + 128155,5 * exp (-0,29 * t))

While using the solver we have the following graph of the measured data and those calibrated.



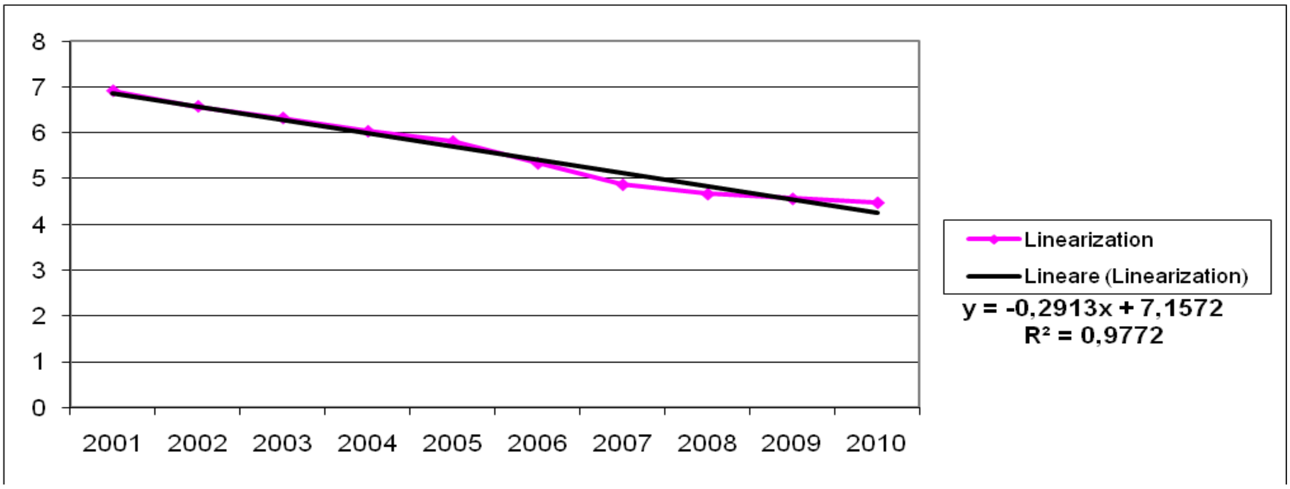
The optimized model has parameter values of:

k = 1E+11   *a* = 128155,5   *b* = 0,281684

For this model the value of the sum of squared errors is: 1,14693E+13

The assessed value is just less than the value calculated with the previous K, we proceed to find the optimal parameters are going to decrease further the value of K. Let's see what happens.

- Per *K*= *1E+09* we have the following linearized model:



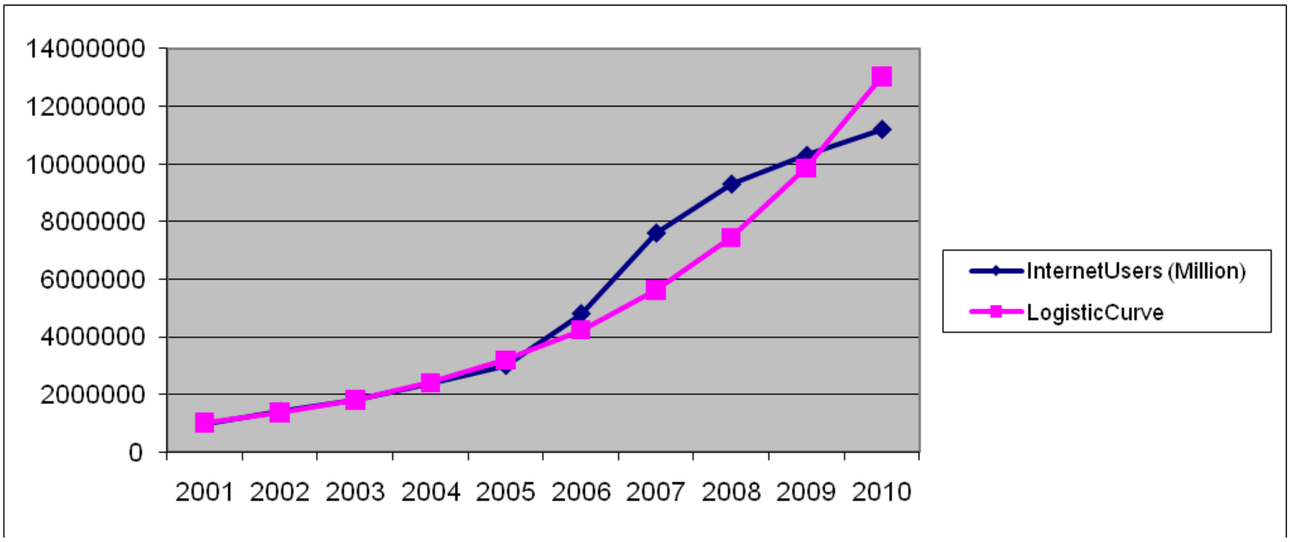
Then

log(*a*) = 7,1572 *a* = 1283,313 *b* = 0,*2913*

We find the logistic curve

y = 1E+09 / (1 + 1283,313* exp (-0,2913 * *t*))

While using the solver we have the following graph of the measured data and those calibrated.



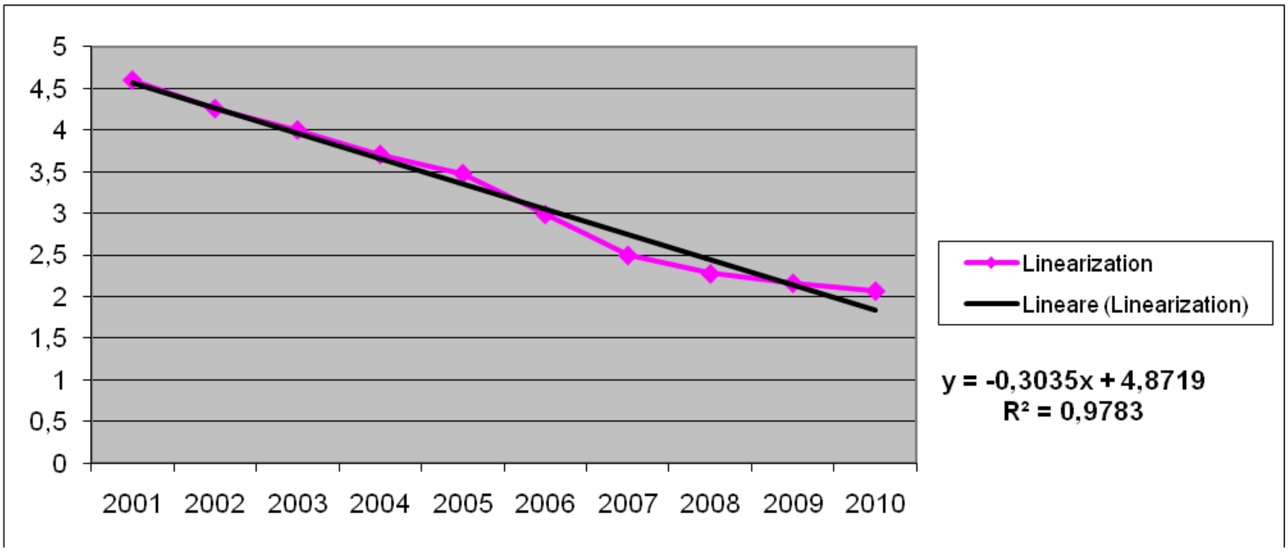
The optimized model has parameter values of:

k = 1E+09 *a* = 1283,313 *b* = 0,283061

For this model the value of the sum of squared errors is: 1,12978E+13

The assessed value is just less than the value calculated with the previous K, we proceed to find the optimal parameters are going to decrease further the value of K. Let's see what happens.

- For $K = 1\ E{+}08$ we have the following linearized model:



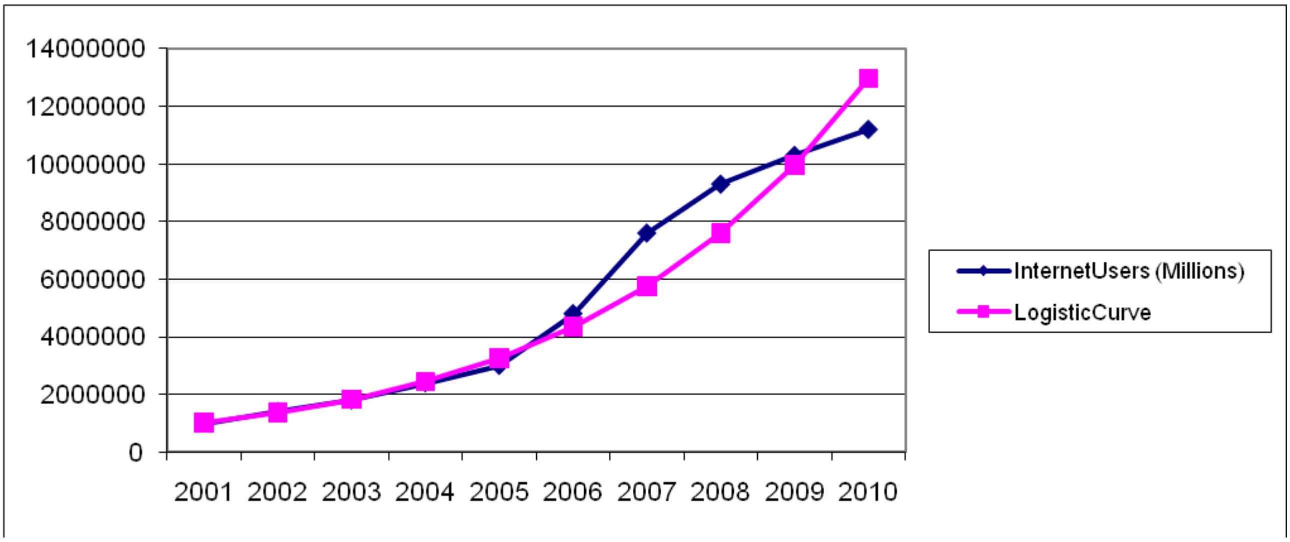
Then

$\log(a) = 4{,}8719 \quad a = 130{,}568 \quad b = 0{,}3035$

We find the logistic curve

$y = 1\ E{+}08\ /\ (1 + 130{,}568 * \exp(-0{,}3035 * t))$

While using the solver we have the following graph of the measured data and those calibrated.



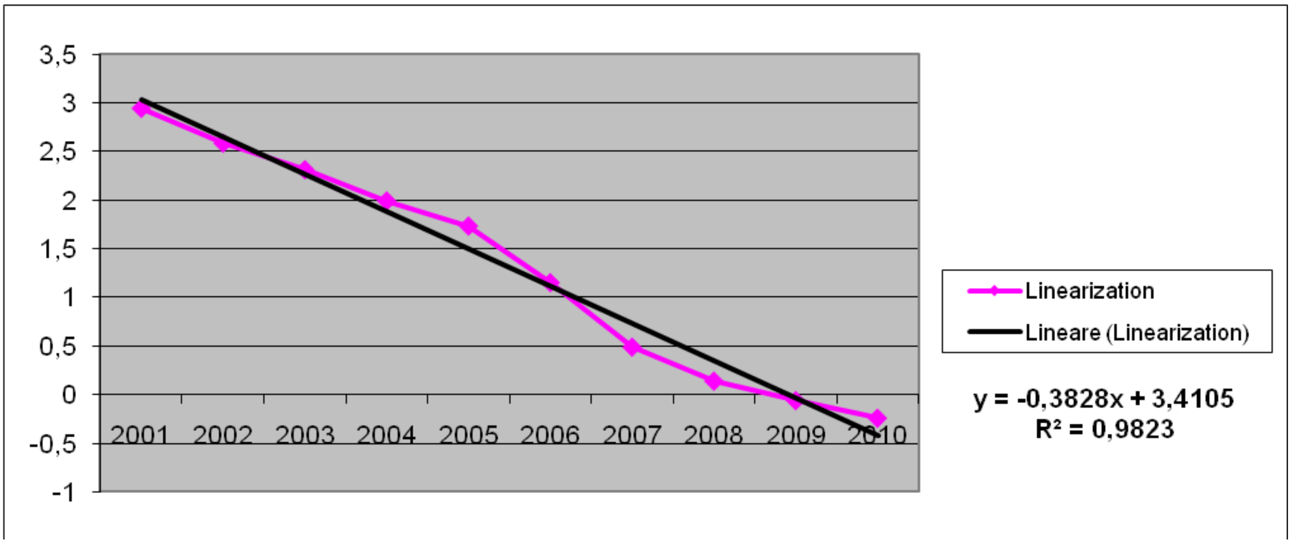
The optimized model has parameter values of:

k = 1 E+08   *a* = 130,5679   *b* = 0,296772

For this model the value of the sum of squared errors is: 9,76426E+12.

The assessed value is just less than the value calculated with the previous K, we proceed to find the optimal parameters are going to decrease further the value of K. Let's see what happens.

- For *K*= *2 E+07* we have the following linearized model:



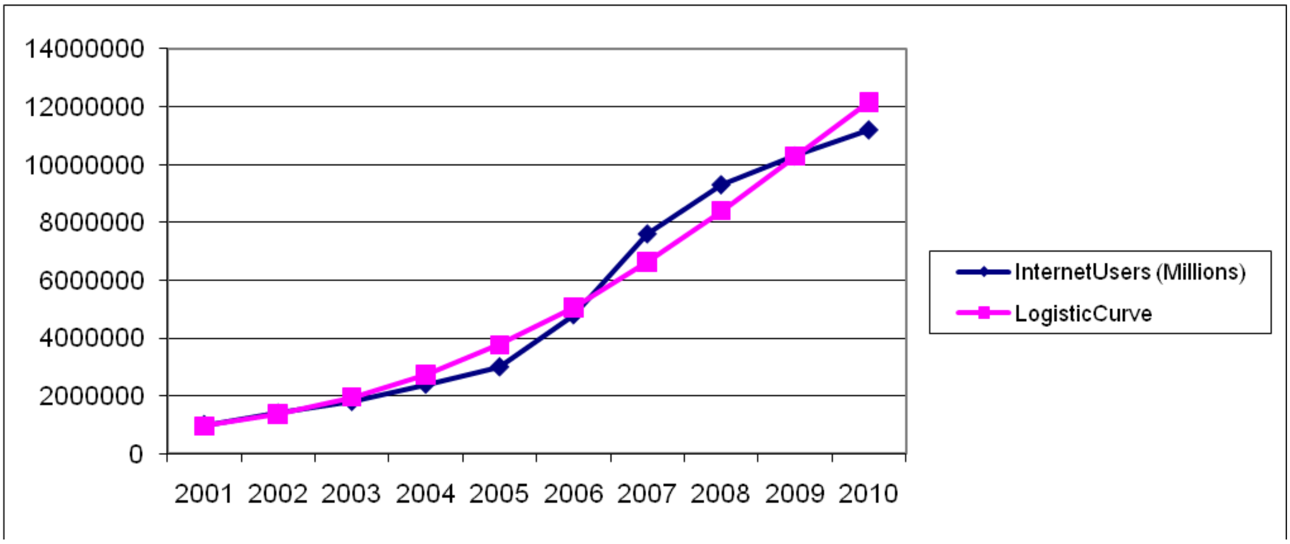
Then

log(*a*) = 3,4105   *a* = 30,28   *b* = 0,*3828*

We find the logistic curve

y = 2 E+07 / (1 + 30,28 * exp (−*0,3828* * *t*))

While using the solver we have the following graph of the measured data and those calibrated.



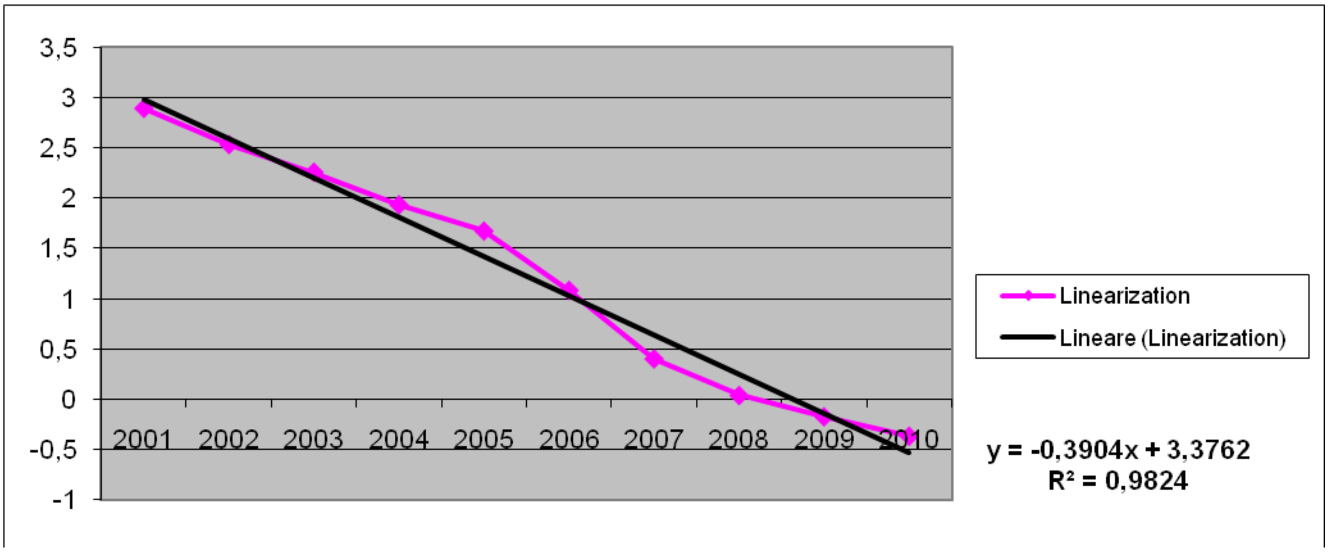
The optimized model has parameter values of:

`k = 19998926` *a* `= 28,81977` *b* `= 0,379848`

For this model the value of the sum of squared errors is: `3,45261E+12.`

The assessed value is just less than the value calculated with the previous K, we proceed to find the optimal parameters are going to decrease further the value of K. Let's see what happens.

- For *K*= *19000000* we have the following linearized model:



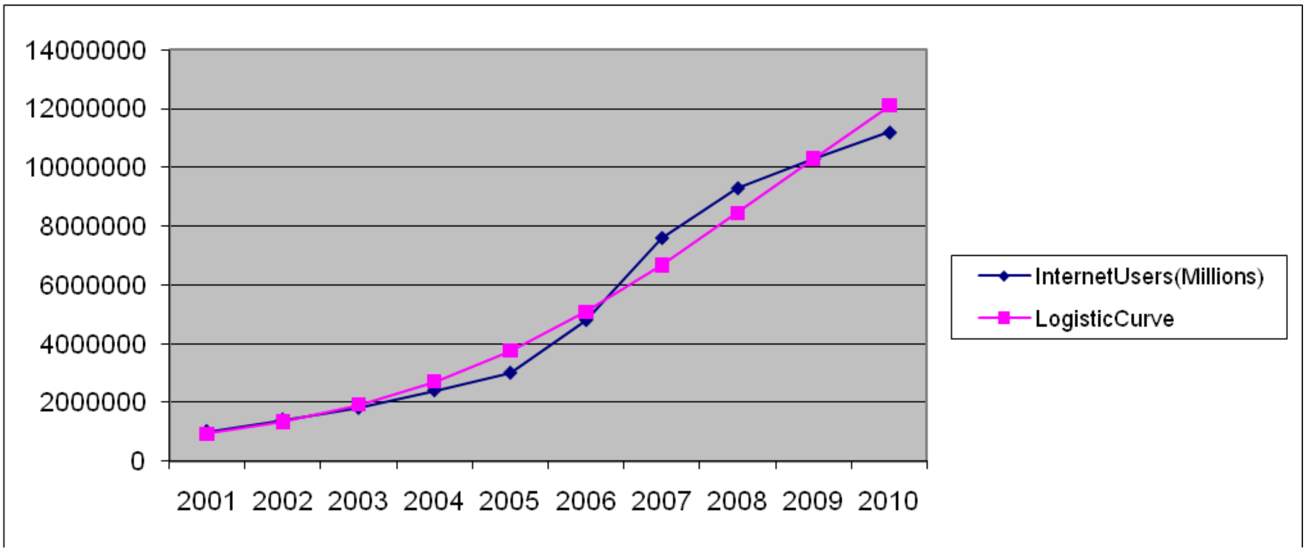
Then

log(*a*) = 3,3762 *a* = 29,26 *b* = 0,*3904*

We find the logistic curve

y = 19000000 / (1 + 29,26 * exp (−*0,3904* * *t*))

While using the solver we have the following graph of the measured data and those calibrated.



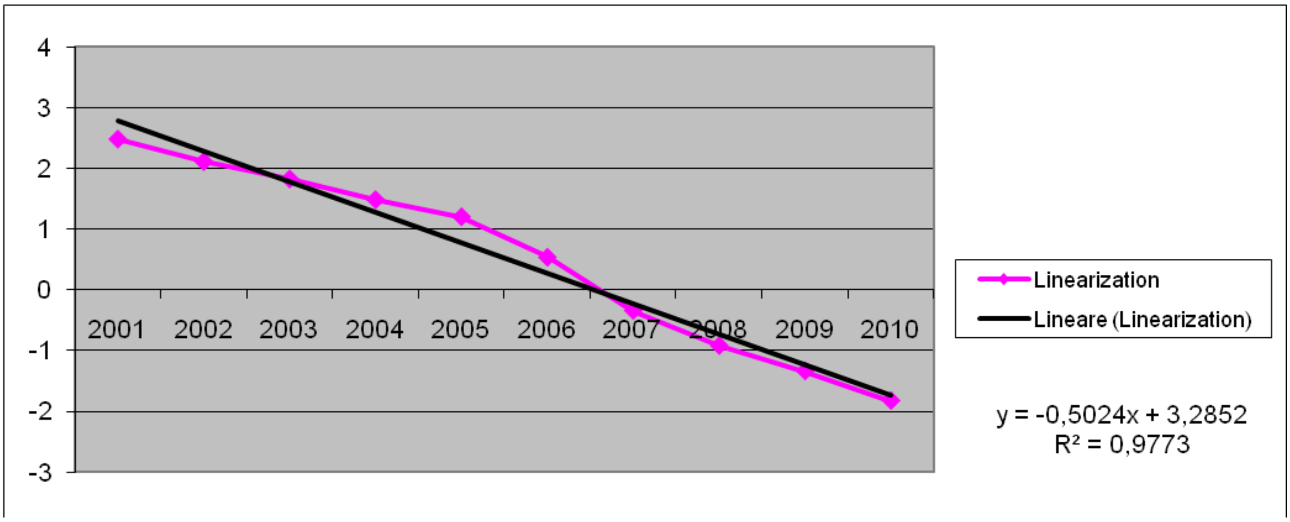
The optimized model has parameter values of:

k = 18998926 *a* = 28,87089 *b* = 0,392459

For this model the value of the sum of squared errors is: 3,18843E+12.

The assessed value is just less than the value calculated with the previous K, we proceed to find the optimal parameters are going to decrease further the value of K. Let's see what happens.

- For *K*= *13006411* we have the following linearized model:



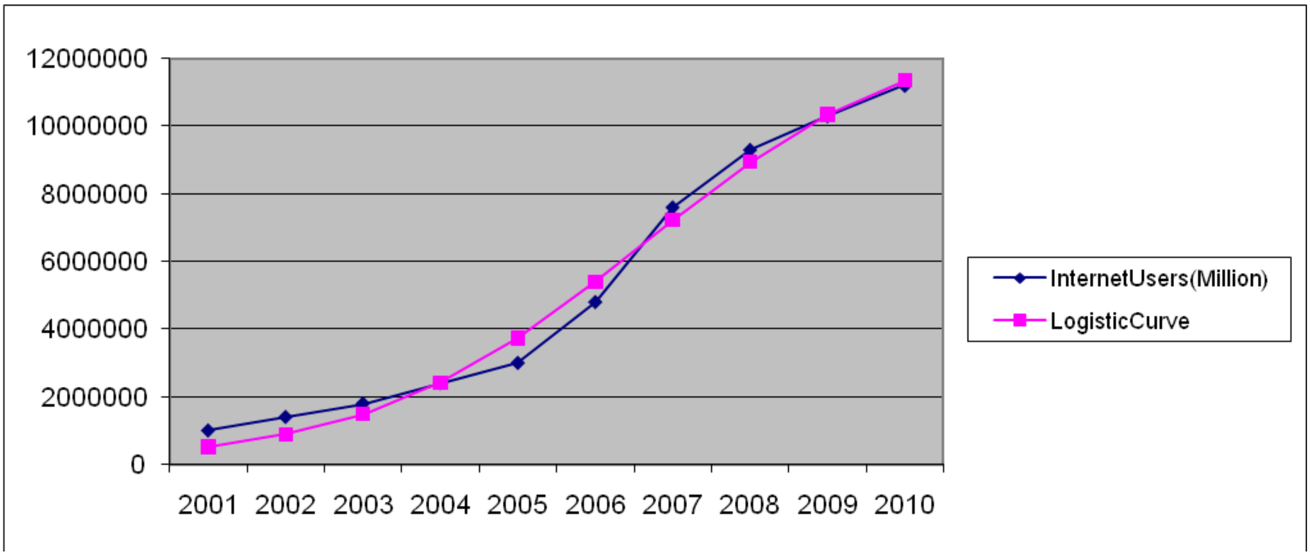
Then

log(*a*) = 3,2852 *a* = 26,714 *b* = 0,*5024*

We find the logistic curve

y = 13006411 / (1 + 26,714 * exp (-*0,5024* * *t*))

While using the solver we have the following graph of the measured data and those calibrated.



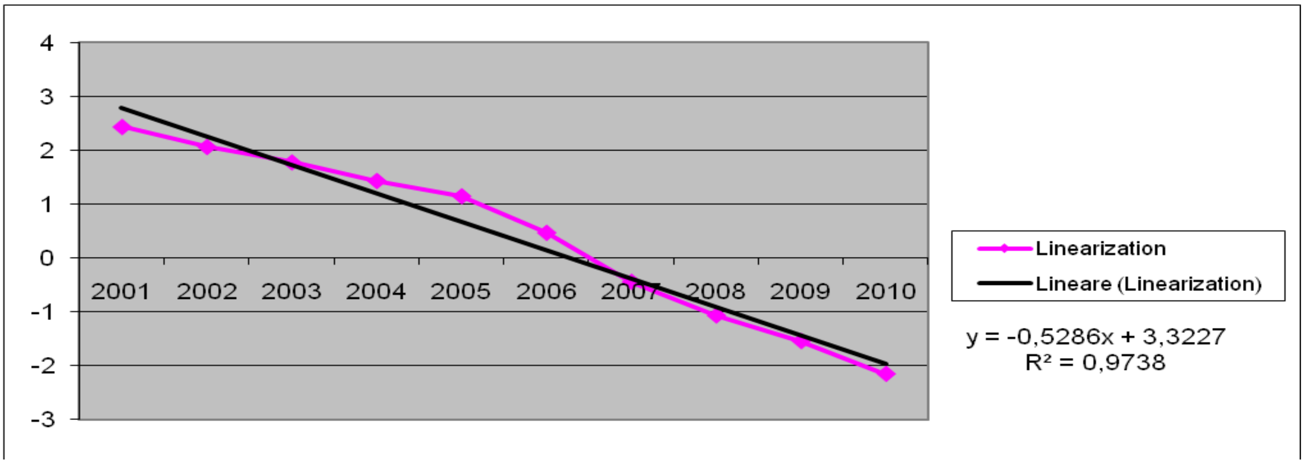
The optimized model has parameter values of:

k = 13006411 *a* = 42,13563 *b* = 0,566015

For this model the value of the sum of squared errors is: 1,764E+12.

The assessed value is just less than the value calculated with the previous K, we proceed to find the optimal parameters are going to decrease further the value of K. Let's see what happens.

- For *K*= *12500000* we have the following linearized model:



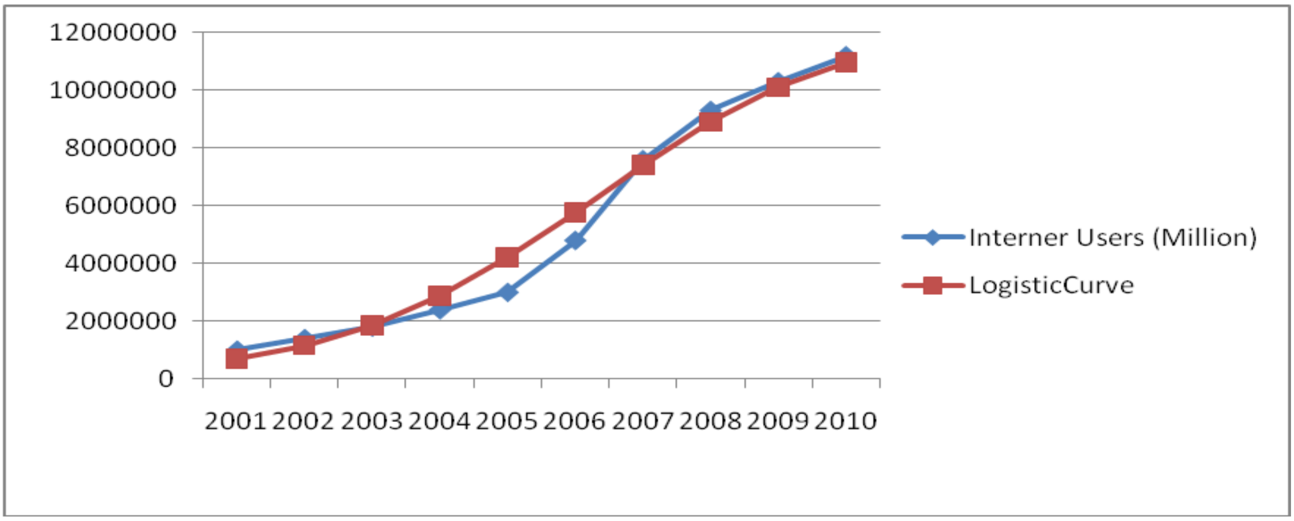
Then

`log(a) = 3,3227     a = 27,735    b = 0,5286`

We find the logistic curve

`y = 12500000 / (1 + 27,735 *  exp (-0,5286 * t))`

While using the solver we have the following graph of the measured data and those calibrated.



The optimized model has parameter values of:

`k = 12500000      a = 27,735    b = 0,5286`

For this model the value of the sum of squared errors is: `3,05794E+12`.

The assessed value is just greater than that obtained with K = 13006411.

The assessed value is just greater than previously calculated. At this point we can say that the value of K = 13006411 is the best value.

In summary:

K	*a*	*b*	*Sum Square Error*
1,00E+10	12822	0,28184798	2,05E+17
1,00E+11	1E+05	0,281684056	1,15E+13
1,00E+09	1283	0,28306097	1,13E+13
1,00E+08	4E+05	0,296772093	9,76E+12
2,00E+07	28,82	0,379847649	3,45E+12
1,90E+07	28,87	0,39245862	3,19E+12
13006411	42,14	0,566014816	1,764E+12
12500000	27,74	0,5286	3,05794E+12

1.3.3 Chi-Square Test

f_0 and f_e are observed frequencies and expected frequencies.
H0 is the null hypothesis and H1 is the alternative hypothesis.

H0: There is a difference between the observed and expected frequencies.
H1: There is a difference between the observed and expected frequencies.

Test statistic:

$$\chi^2 = \sum \left[\frac{(f_o - f_e)^2}{f_e} \right]$$

The critical value is a value taken from a random variable chi-square with degrees of freedom (k-1), where k is the number of classes in which the random sample is grouped.

Step 1: Fix null hypothesis and alternative.

H0: There is agreement between the data.

H1: There is no agreement between the data.

In our case, the data are:

- internet subscribers in millions.
- estimated data using the logistic equation.

Our goal is to apply the Chi Square Test to verify the consistency of the actual data and those estimated in terms of square error.

Step 2: Select the level of significance α.

Let $\alpha = 0.01$

The **level of significance** of a test is usually given by a test of hypothesis testing. In the simplest case is defined as the probability of accepting or rejecting the null hypothesis. The decision in this case is done using the p-value: if the value p (p-value) is less than the significance level, then the null hypothesis is rejected. The lower the p value, the more significant is the result.

Step 3: Select the test statistic

How to use the test $\chi 2$ statistics.

Step 4: H0 is rejected if the p-value is less than $\alpha = 0.01$.

We calculate the test statistic at each logistic curve identified in the first part of the analysis:

1. *Logistic curve with K=1E+10*

Sum value χ^2 ***= 1553048.129,9***

This value was obtained by applying the formula:

$$\chi^2 = \sum \left[\frac{(f_o - f_e)^2}{f_e} \right]$$

Degrees of freedom = 10-1 = 9

The p ($\chi 2 >$*1553048.129,9*) = 0 has been calculated using the method *chi2cdf* ($\chi 2$ = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(1553048.129,9)
>> chi =
>>  0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of $\chi 2$, which is equal to 21,67 (tabulated value), and then the p-value of 0 is less than the chosen significance level (α = 0.01).

Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

2. *Logistic curve with K=1E+11*

Sum value ***χ^2 = 1554762.731,9***

Degrees of freedom = 10-1 = 9

The p ($\chi 2 >$*1554762.731,9*) = 0 has been calculated using the method *chi2cdf* ($\chi 2$ = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(1554762.731,9)
>>  chi =
>>  0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of χ2, which is equal to 21,67 (tabulated value), and then the p-value of 0 is less than the chosen significance level (α = 0.01).

Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

3. *Logistic curve with K=1E+09*

Sum value χ^2 ***= 1527253.976,9***

Degrees of freedom = 10-1 = 9

The p (χ2>*1527253.976,9*) = 0 has been calculated using the method *chi2cdf* (χ2 = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(1527253.976,9)
>> chi =
>> 0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of χ2, which is equal to 21,67 (tabulated value), and then the p-value of 0 is less than the chosen significance level (α = 0.01).

Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results

obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

4. *Logistic curve with K=1E+08*

Sum value χ^2 ***= 1288287.471,9***

Degrees of freedom = 10-1 = 9

The p ($\chi2$>*1288287.471,9*) = 0 has been calculated using the method *chi2cdf* ($\chi2$ = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(1288287.471,9)
>> chi =
>> 0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of $\chi2$, which is equal to 21,67 (tabulated value), and then the p-value of 0 is less than the chosen significance level ($\alpha = 0.01$).

Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

5. *Logistic curve with K=2E+07*

Sum value χ^2 ***= 537661.4293,9***

Degrees of freedom = 10-1 = 9

The p ($\chi2$>*537661.4293,9*) = 0 has been calculated using the method *chi2cdf* ($\chi2$ = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(537661.4293,9)
>> chi =
>>  0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of $\chi2$, which is equal to 21,67 (tabulated value), and then the p-value of 0 is less than the chosen significance level (α = 0.01).

Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

6. *Logistic curve with K=1,9E+07*

Sum value χ^2 ***= 504458.2253,9***

Degrees of freedom = 10-1 = 9

The p ($\chi2$>504458.2253,9) = 0 has been calculated using the method *chi2cdf* ($\chi2$ = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(504458.2253,9)
>> chi =
>> 0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of χ2, which is equal to 21,67 (tabulated value), and then the p-value of 0 is less than the chosen significance level (α = 0.01).
Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

7. *Logistic curve with K=13006411*

Sum value χ^2 ***= 1035873.31,9***
Degrees of freedom = 10-1 = 9

The p (χ2>*1035873.31,9*) = 0 has been calculated using the method *chi2cdf* (χ2 = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(1035873.31,9)
>> chi =
>>  0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of χ2, which is equal to 21,67 (tabulated value), and then the p-value of 0 is less than the chosen significance level (α = 0.01).
Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

8. *Logistic curve with K=2E+07*

Sum value χ^2 ***= 774867.6281,9***

Degrees of freedom = 10-1 = 9

The p (χ2>774867.6281,9) = 0 has been calculated using the method *chi2cdf* (χ2 = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(774867.6281,9)
>> chi =
>>  0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of χ2, which is equal to 21,67 (tabulated value), and then the p-value of 0 is less than the chosen significance level (α = 0.01). Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

In summary:

K	*χ2*
1,00E+10	1553048,129
1,00E+11	1554762,731
1,00E+09	1527253,976
1,00E+08	1288287,471
19998926	537661,4293
18998926	504458,2253
13006411	1035873,31
1,25E+07	774867,6281

The chi-square test was applied to test whether the logistic curve that best approximates the performance of the input data coincides with the one identified in the first phase of the study. According to the statistical hypothesis test, the observed data are significantly different from the actual data. However, the calculation of statistics shows that the lower value is in correspondence of *K = 13006411*.

This coincides with what is assumed in the previous phase, namely that the logistic curve that best approximates the performance of the input data is described by the following equation:

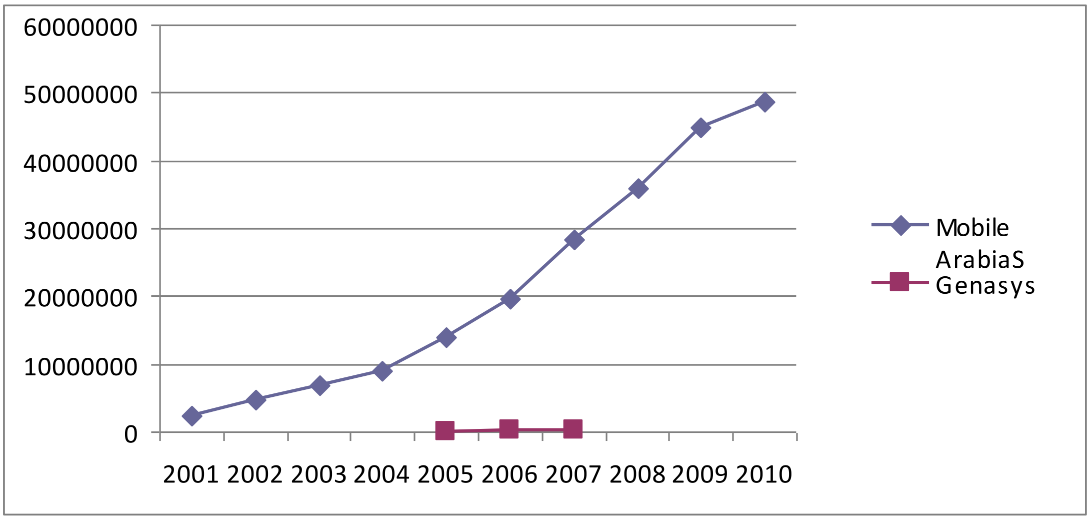
$$y = 13006411 / (1 + 42{,}13563 * \exp(-0{,}566015 * t)).$$

1.4 Mobile in Saudi Arabia

1.4.1 Cconsiderations on the relationship between the logistic curves of the Mobile in Saudi Arabia and location services

At this stage we are dedicated to research and reports of possible links that may exist between the logistic curve for mobile users in Saudi Arabia and that of location services.

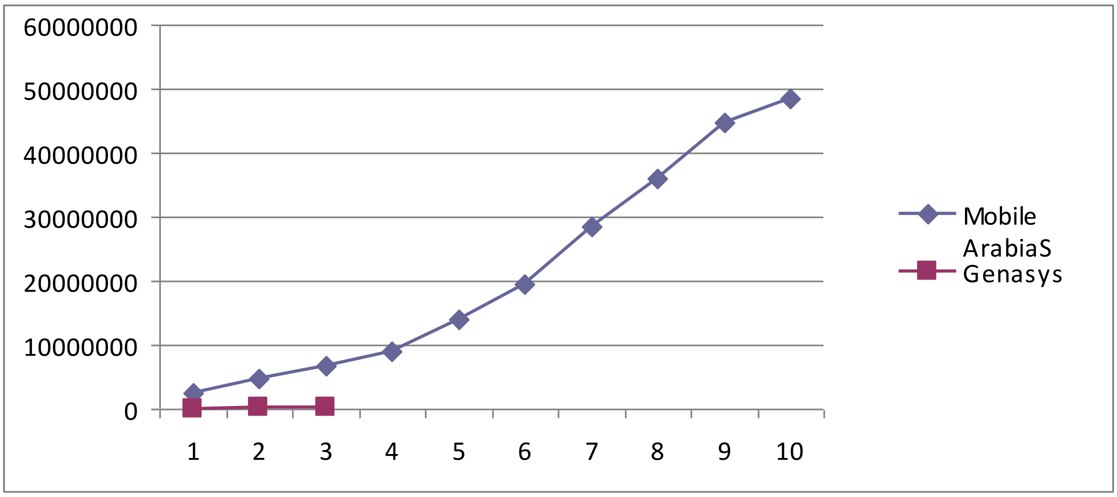
Comparing these curves in the same graph we can see that there is a "time shift" between the curve for the number of registered users of Mobile in Saudi Arabia and the curve representing the development of location services.



With the data at our disposal we can make a finer analysis on existing relationships.

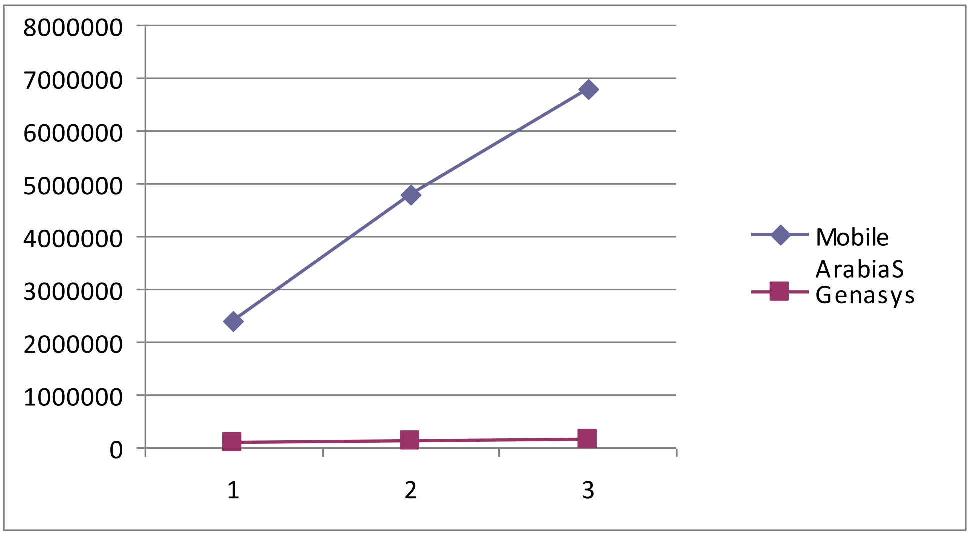
With regard to the location services available data are those related to the first 3 years of development (2005 to 2007) in contrast to the Internet service whose data are related to a period of 10 years. Such availability may be related to the fact that location services are of recent development and distribution, as opposed to services Mobile. Such availability may be related to the fact that location services are a recent development and distribution, unlike the mobile services for which the information is clearer and more precise.

To make a more detailed analysis, we identify the similarities and differences between the two curves in their early years of development. In this way we can examine what happened during the introduction of such services.
To do this we insert the same graph the two curves in parallel.



From the graph it is noted that the two curves have developed differently. The curve of mobile services in Saudi Arabia is growing faster than the curve on location services. In the third year the number of registered users of the mobile phone in Saudi Arabia is more than 6 million, while the volume reached by the location services is 155469 hits/year.

The difference is obvious when we examine the evolution of the two curves in the first three years.



The curve of location services, provided by the company's Genasys, presents a curve with a slope less than that of Mobile. The growth rate curve that represents the time trend in the number of registered users of Mobile is more than the curve location.

Comparing these trends, we may assume that the two curves evolve differently over time.

1.4.2 Applications to find the best parameters for the logistic curve of the Mobile in Saudi Arabia.

As a first step, we want to make a fitting with the logistic model of growth through transformation of variables (in order to be reduced to a linear equation) and then by linear regression. Recall that the logistic equation

$$y = K / (1 + a * \exp(-bt))$$

becomes linear with the following transformation

$$\log(K/y - 1) = \log a - bt .$$

The data we suppose that the population limit (equilibrium) and K (remember that is the horizontal asymptote of the logistic function). Then we determine a and b by linear regression.

Since we are not aware of the extent to which value will reach the number of registered users of Mobile go forward in search of that parameter. We have made the analysis on different values of K. We stopped our search when the value of the mean square error made from the model was worse than that calculated in the previous system.

To make this process we assumed different values of K, we left the value of K = 1E+10

The idea is to calculate

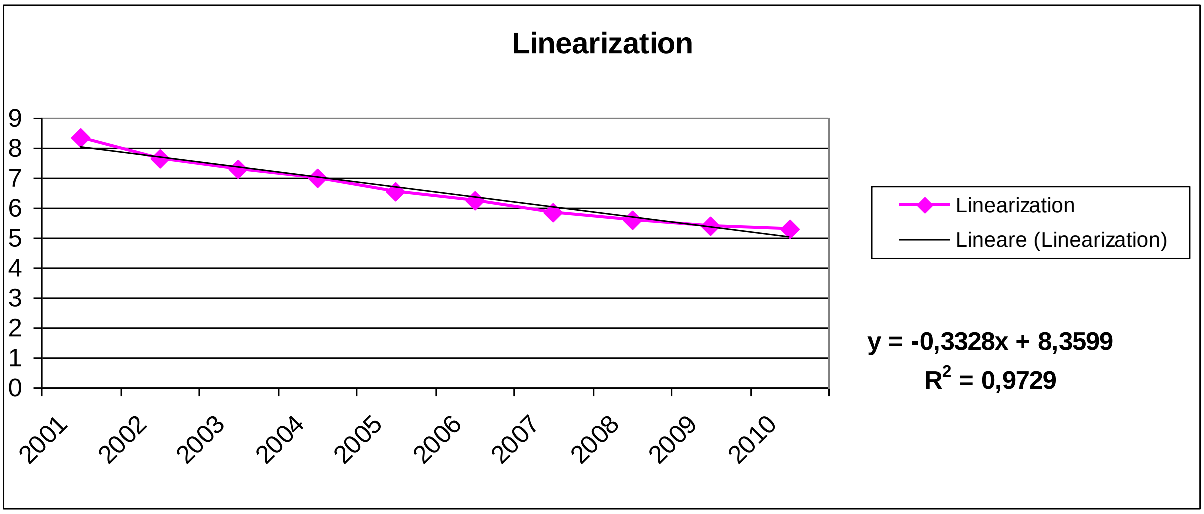
$$\log(K/y\ -1).$$

To this end we write in a cell of the Excel spreadsheet the following formula

= LN(1E+10/B2-1).

Where B2 is the first value of the function y. This calculation is done for all values of y available. Now draw the chart with "chart" entering the x-axis the days and y axis, the column data as soon as detected.

At the same time insert the regression "trendline" together with its equation.



Then

$\log(a) = 8{,}3599 \qquad a = 4250{,}96 \qquad b = 0{,}3328$

We find the logistic curve

$$y = 1E{+}10 / (1 + 4250{,}96 * \exp(-0{,}3328 * t))$$

Then we tried to optimize the choice of parameters using ''Solver'' (''Risolutore'') of Excel.

In another paper we have entered the data file according to the formula given by the logistic growth model

$$y = K/(1 + a * \exp(-b*t))$$

To do this, we inserted the formula in column writing for example

```
= I2 /( 1+( I3 * EXP(- I4 * A2)))
```

Note that the parameters used are those included in Excel spreadsheet cells. Box I2 is that of the parameter K, I3 is the box for the parameter a, while the I4 box refers to the parameter b. In particular these cells are introduced into the formula with the $ sign to make these cells remain fixed in the calculation. So we have:

```
= $I$2 /( 1+( $I$3 * EXP(-$I$4 * A2)))
```

Then calculate the square error by including in each row of column Excel spreadsheet formulas like

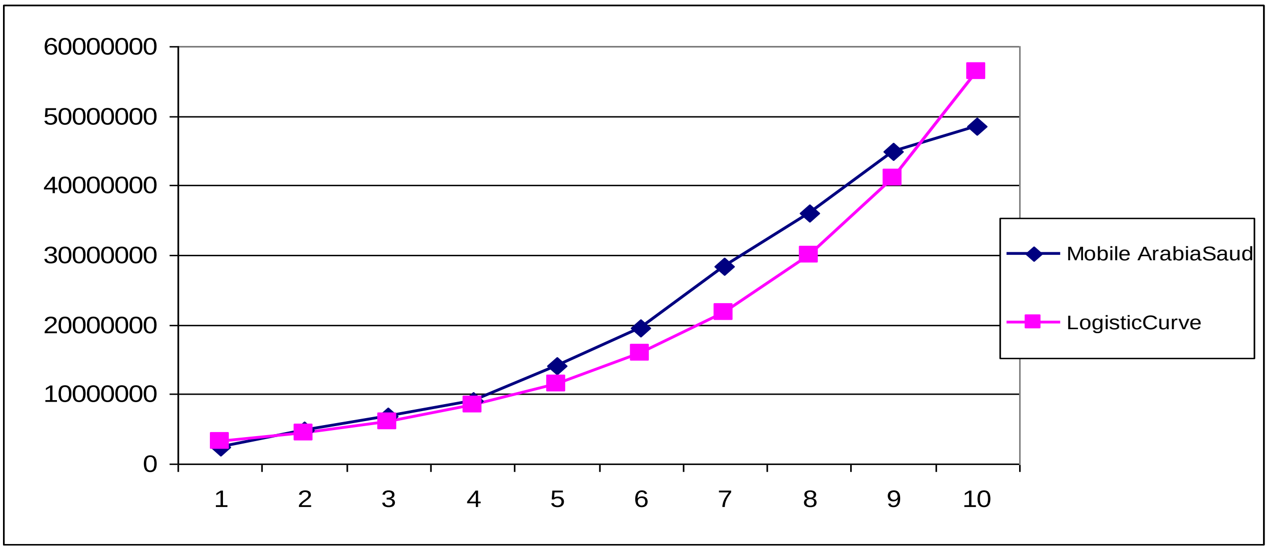
```
=(C2-B2)^2
```

In this way we calculated the value of the mean square for the first period. Using the "Solver" of Excel (found under the Tools menu, "Tools") optimize the parameters K, a and b by minimizing the sum of the standard deviations.

We set the "Solver" indicating that the parameters are optimized in the cells and I2-I4 (by entering the command, and then I2: I4 under “changing cells") and inserting the value of the sum of the standard deviations in cell the objective function to be minimized ("Target" cell D16), we start the "Solver".

The result of this simulation allows us to identify the best parameters a and b according to a saturation limit value set in the value K.

Finally, we can make a graph of the measured data and those calibrated.



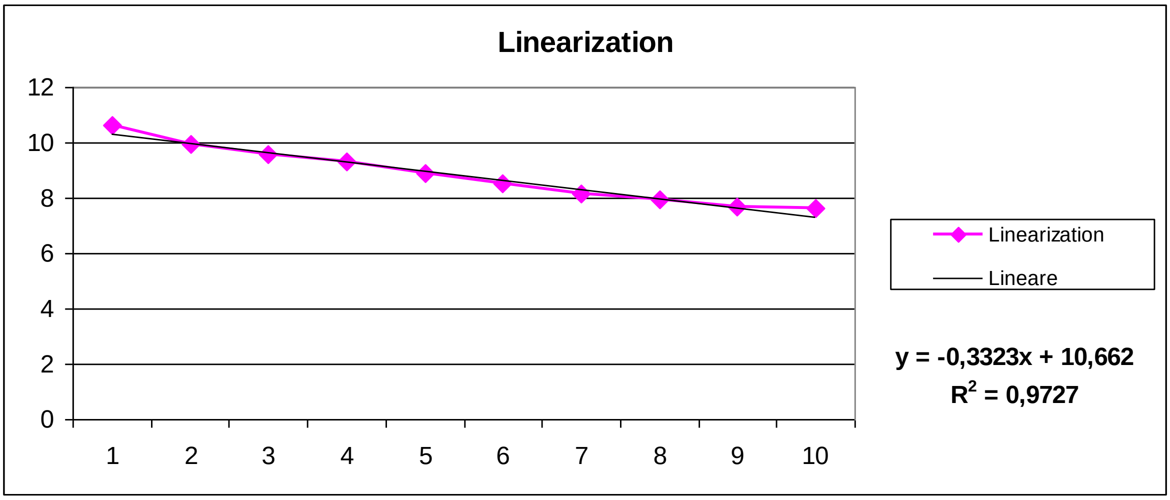
The optimized model has parameter values of:

k = 1E+10   *a* = 4250,96   *b* = 0,318234

For this model the value of the sum of squared errors is: 1,77091E+14

This procedure was performed for different values of K.

- For *K*= *1E+11* we have the following linearized model:



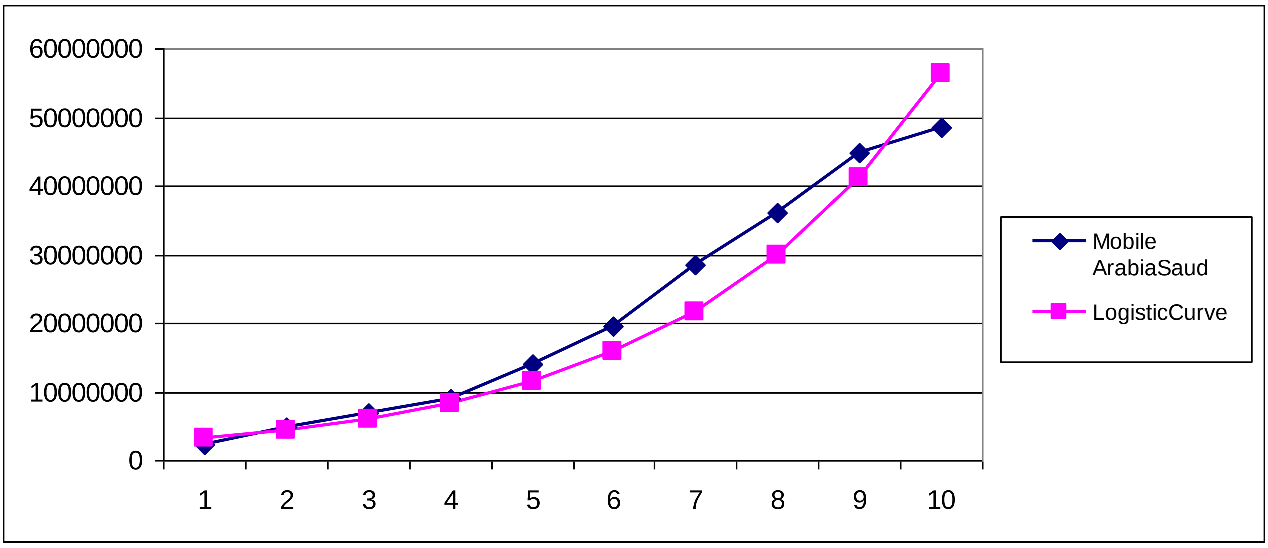
Then

log(*a*) = 10,662   *a* = 42701,96   *b* = 0,3323

We find the logistic curve

y = 1E+11 / (1 + 42701,96 * exp (-*0,3323* * *t*))

While using the solver we have the following graph of the measured data and those calibrated.



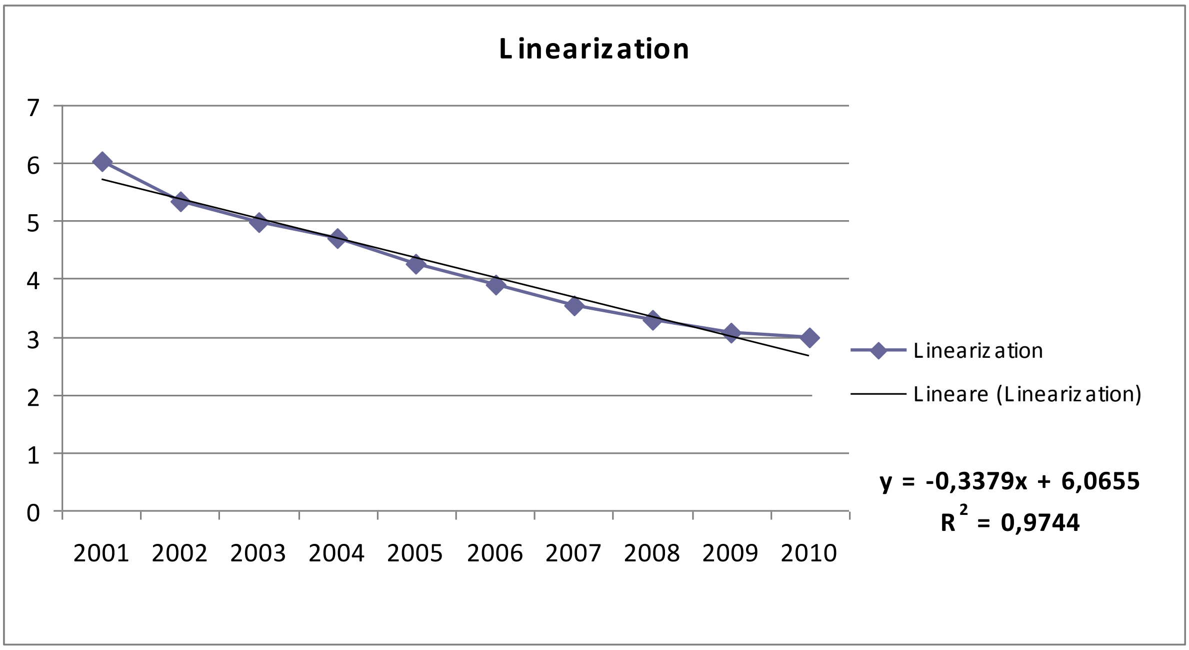
The optimized model has parameter values of:

k = 1E+11 *a* = 42701,96 *b* = 0,318239

For this model the value of the sum of squared errors is: 1,79879E+14

The value just examined is greater than the value calculated with the previous K, we proceed in finding the optimal parameters are going to decrease the value of K. Let's see what happens.

- Per *K*= *1E+09* we have the following linearized model:



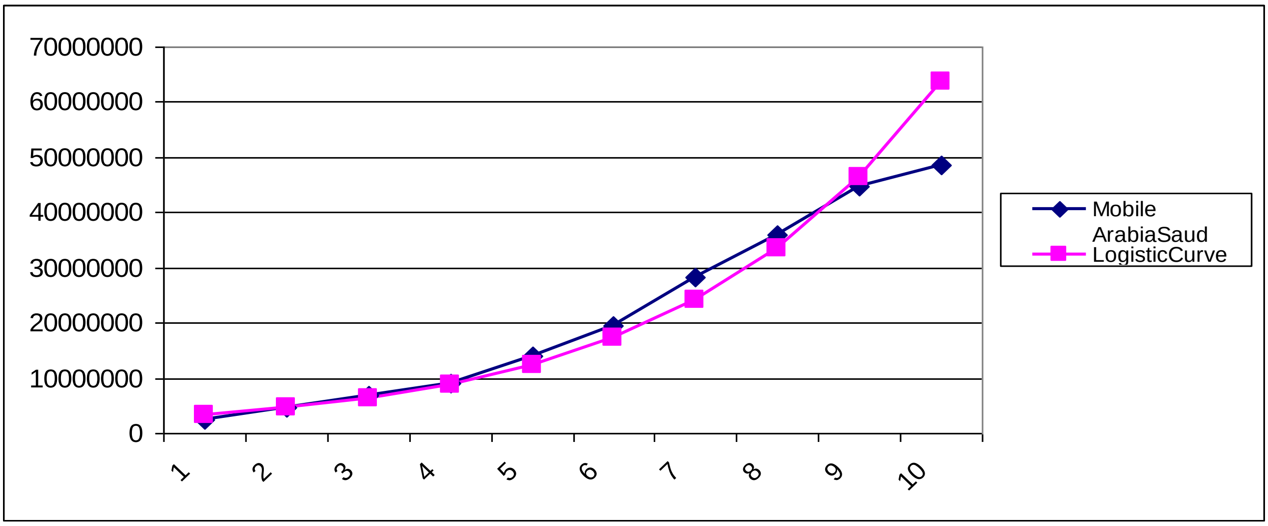
Then

log(*a*) = 6,0655 *a* = 430,738 *b* = 0,3379

We find the logistic curve

y = 1E+09 / (1 + 430,738 * exp (-0,3379 * *t*))

While using the solver we have the following graph of the measured data and those calibrated.



The optimized model has parameter values of:

k = 1E+09   *a* = 430,738   *b* = 0,3379

For this model the value of the sum of squared errors is: 2,65863E+14

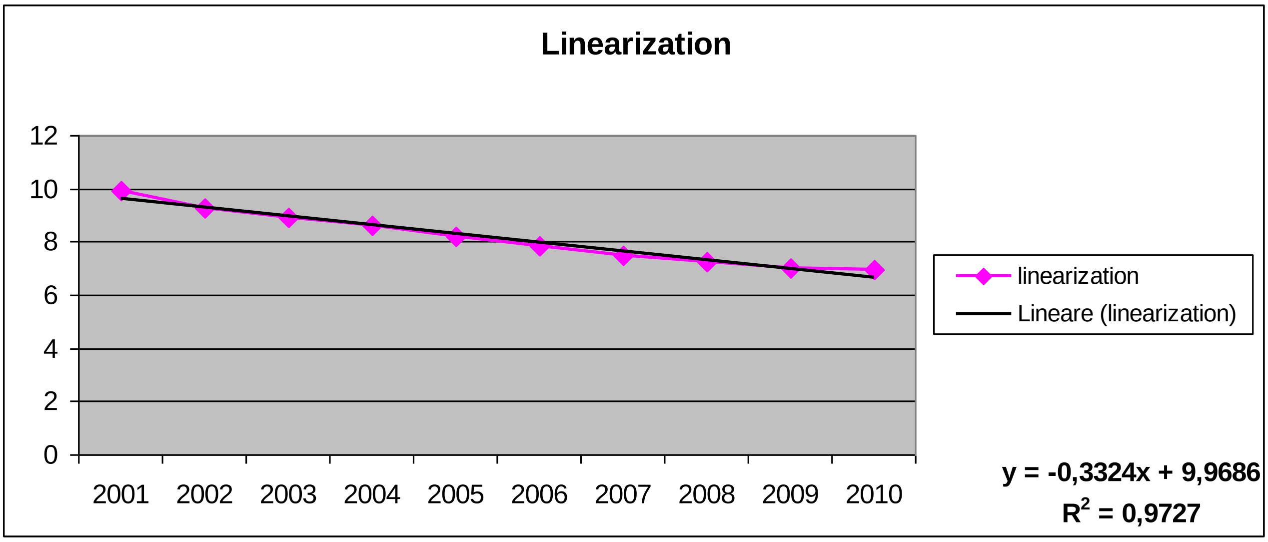
The value just examined is greater than that obtained at K= 1E+11. The value is also higher than the value obtained with K= 1E+10.

We proceed in the search for good parameters going to increase the value of K in the following range:

1 E+10 E <K <1 E +11

We take a value in this interval and check if the sum of squared errors decreases.

- For *K*= 5 *E+10* we have the following linearized model:



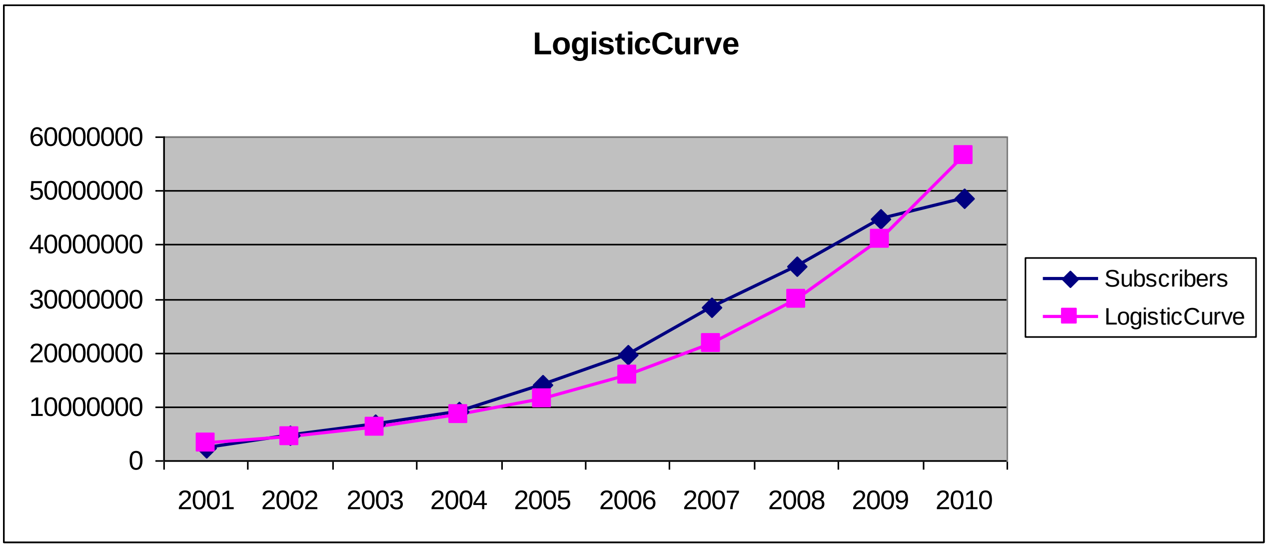
Then

`log(a) = 9,9686   a = 21345,6    b = 0,3324`

We find the logistic curve

`y = 5 E+10 / (1 + 21345,6 *  exp (-0,3324 * t))`

While using the solver we have the following graph of the measured data and those calibrated.



The optimized model has parameter values of:

`k = 5 E+10    a = 21345,6   b = 0,318265`

For this model the value of the sum of squared errors is: `1,79637E+14.`

The value just examined is greater than that obtained at K= 1E+10 but smaller than the value obtained at K=11.

We proceed in the search for good parameters going to increase the value of K in the following range:

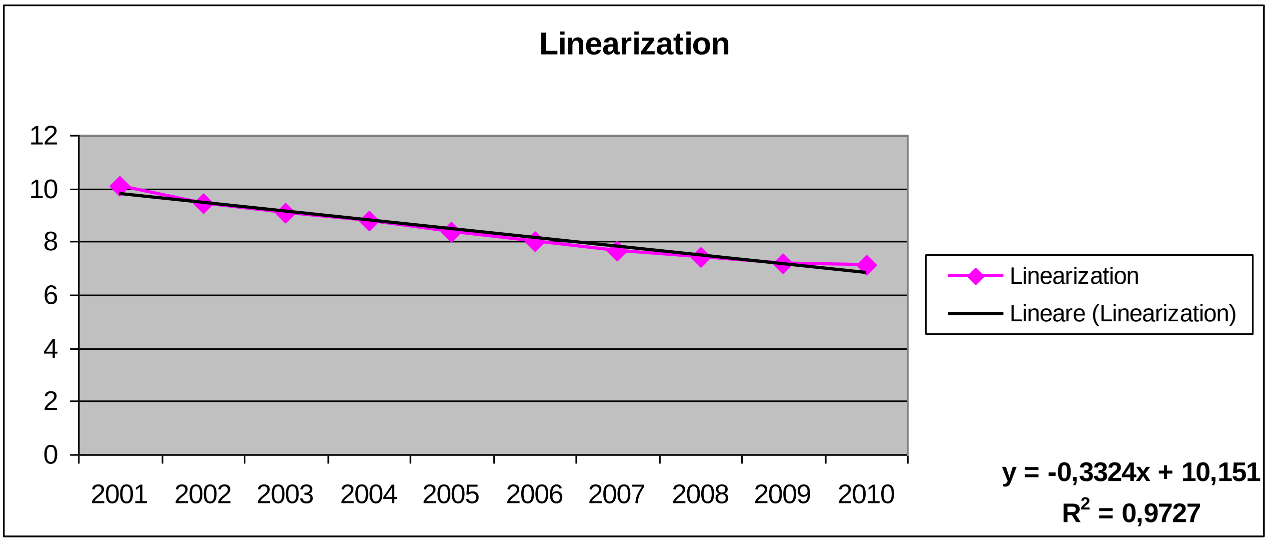
1 E+10 E <K <5 E +10

Or in the range:

5 E +10 <K < 1 E+10 E

By setting K in the first or second range of values, the error tends to grow. We show the statement by setting the value of K in the second interval.

- For $K=$ 6E+10 we have the following linearized model:



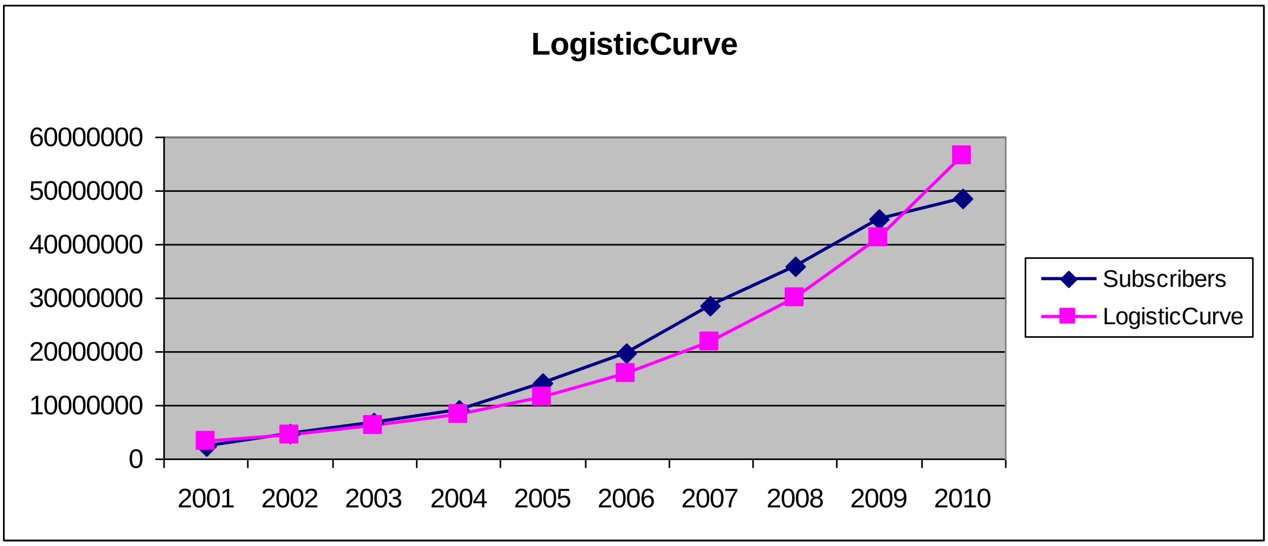
Then

log(*a*) = 10,151   *a* = 25616,7   *b* = 0,3324

We find the logistic curve

y = 6 E+10 / (1 + 25616,7 * exp (-0,3324 * *t*))

While using the solver we have the following graph of the measured data and those calibrated.



The optimized model has parameter values of:

k = 6 E+10 *a* = 25616,7 *b* = 0,318255

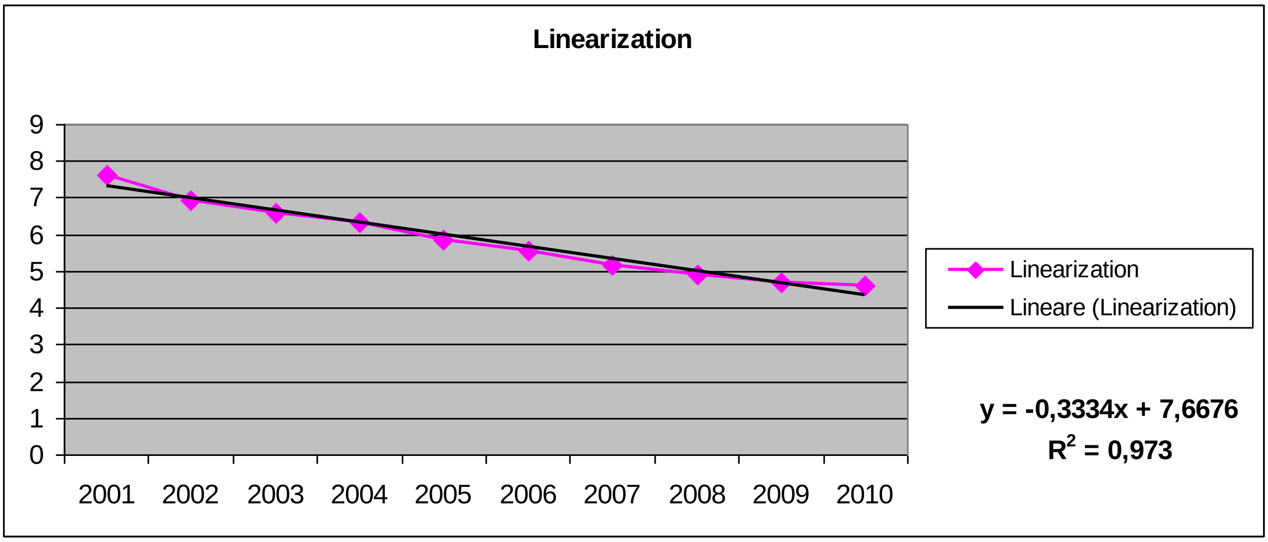
For this model the value of the sum of squared errors is: 1,79716E+14.

As stated previously, the error is greater than the value obtained at K = 1E+10 and K =5E+10. Perhaps the intervals considered above, are incorrect, so let the value of K that minimizes the error in a new range:

1E +09 <K < 1 E+10 E.

We take a value in this interval and check if the sum of squared errors decreases.

- For *K*= *5 E+09* we have the following linearized model:



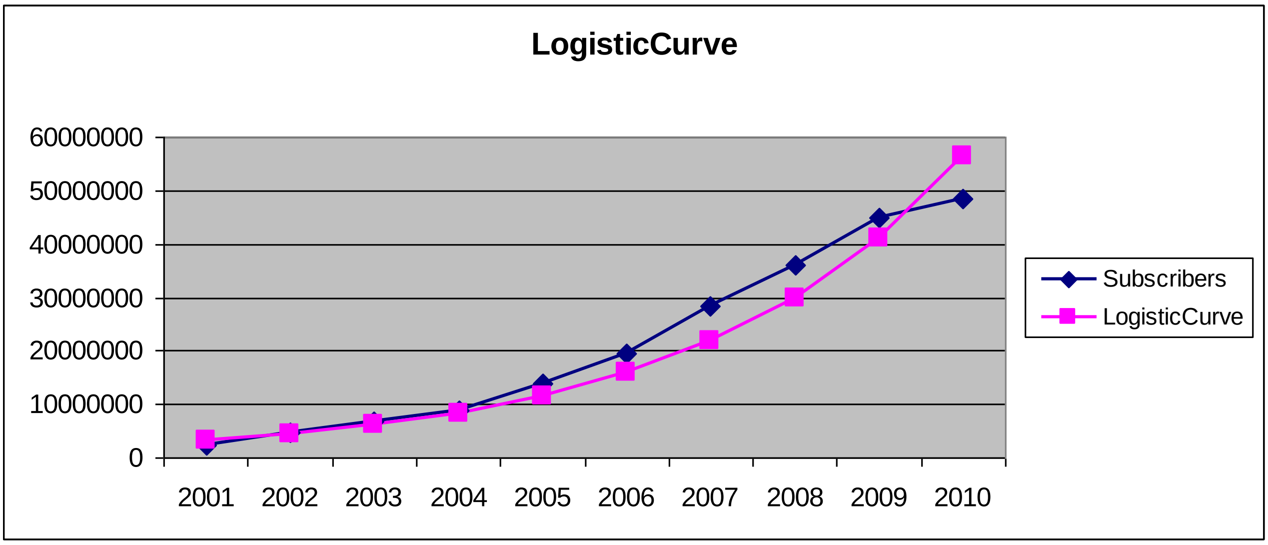
Then

log(*a*) = 7,6676 *a* = 2137,9 *b* = 0,*3334*

We find the logistic curve

y = 5 E+09 / (1 + 2137,9 * exp (*-0,3334* * *t*))

While using the solver we have the following graph of the measured data and those calibrated.



The optimized model has parameter values of:

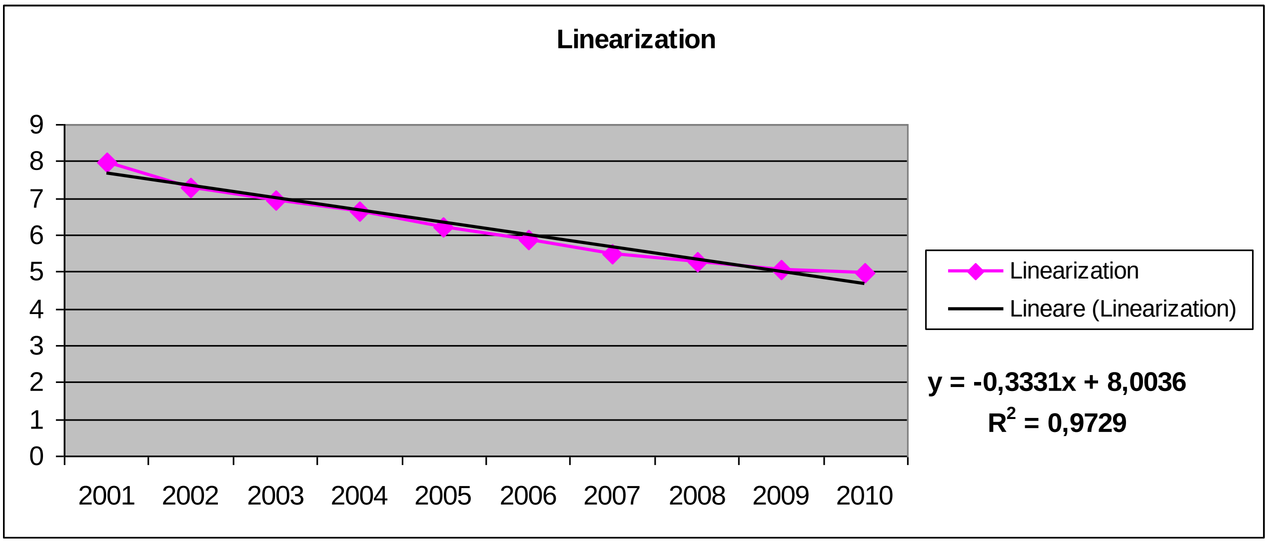
K = 5 E+09 *a* = 2137,9 *b* = 0,319396

For this model the value of the sum of squared errors is: 1,76981E+14.

The value of the error is less than that calculated at the K =1E+10.

We slightly increased the value of K, placing it equal to 7E+09. Let's see what happens.

- For *K*= *7E+09* we have the following linearized model:



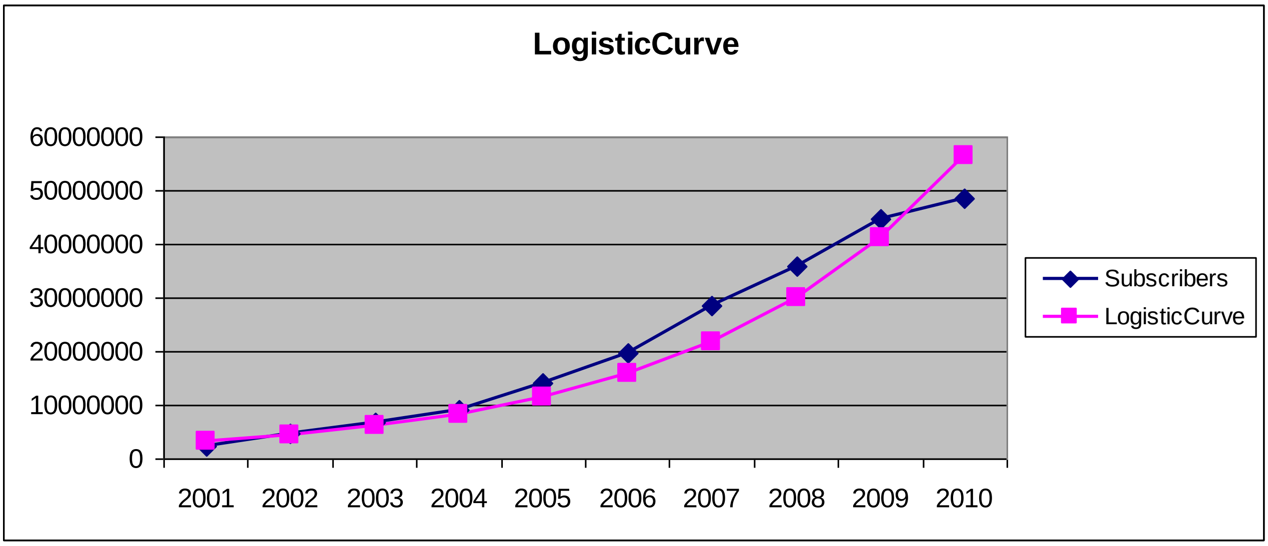
Then

log(*a*) = 8,0036 *a* = 2991,708 *b* = 0,*3331*

We find the logistic curve

$$y = 7\ E{+}09 / (1 + 2991{,}708 * \exp(-0{,}3331 * t))$$

While using the solver we have the following graph of the measured data and those calibrated.



The optimized model has parameter values of:

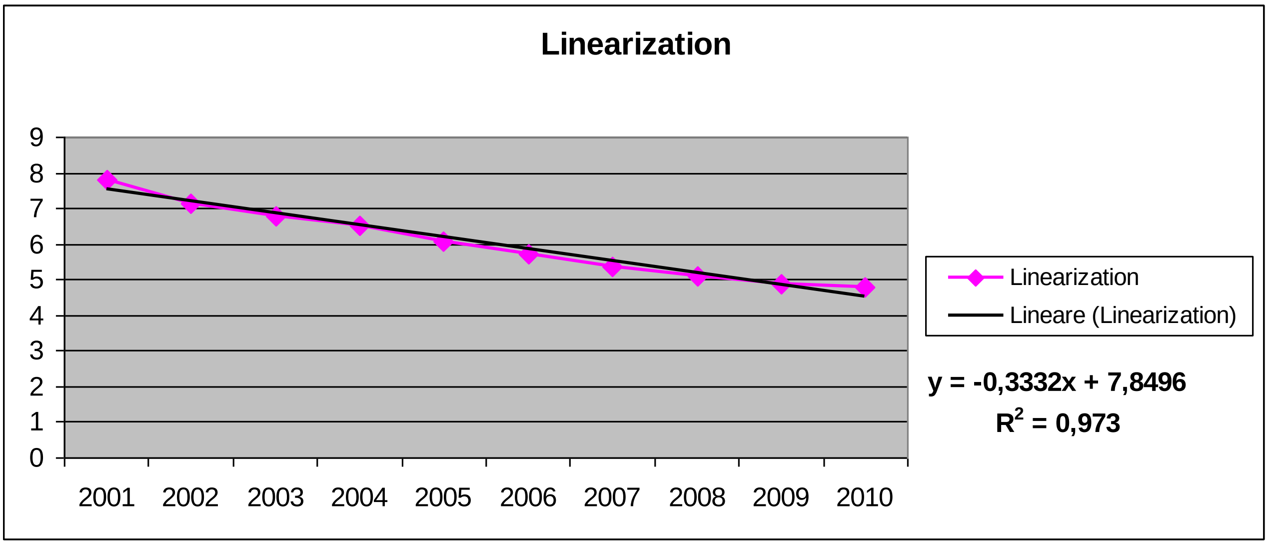
K = 7 E+09   *a* = 2991,708   *b* = 0,319041

For this model the value of the sum of squared errors is: 1,77838E+14.

The error is increased compared with that calculated at the K = 5E+09.

Let's try one last attempt, we decrease the value of K and fix a 6E+09. Let's see what happens.

- For *K*= *6E+09* we have the following linearized model:



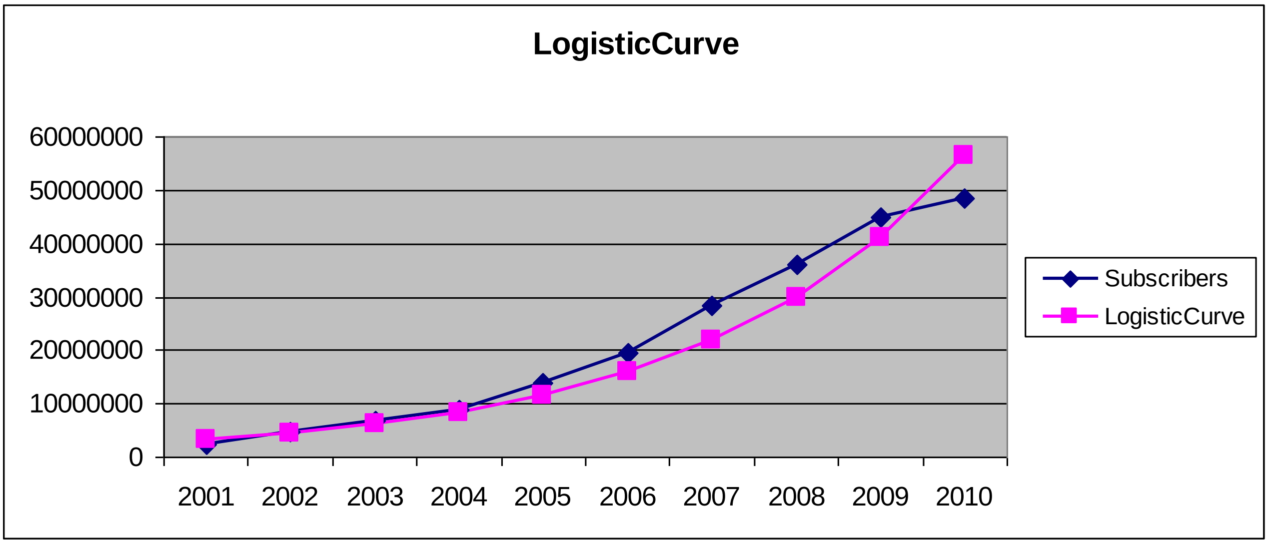
Then

`log(a) = 7,8496   a = 2564,8   b = 0,3332`

We find the logistic curve

`y = 6E+09 / (1 + 2564,8 *  exp (-0,3332 * t))`

While using the solver we have the following graph of the measured data and those calibrated.



The optimized model has parameter values of:

`K = 6E+09    a = 2564,8   b = 0,319188`

For this model the value of the sum of squared errors is: 1,77481E+14.

The value just examined is greater than previously calculated. At this moment we can say that the value of K=5E+09 is the best value.

In summary:

K	*a*	*b*	*Sum Square Error*
1,00E+10	4250,96	0,31823356	1,77091E+14
1,00E+11	42701,96	0,318238741	1,79879E+14
1,00E+09	430,738	0,3379	2,65863E+14
5,00E+10	21345,6	0,318264937	1,79637E+14
6,00E+10	25616,7	0,318255482	1,79716E+14
5,00E+09	2137,9	0,319396336	1,76981E+14
7,00E+09	2991,708	0,319041002	1,77838E+14
6,00E+09	2564,8	0,319188023	1,77481E+14

1.4.3 Chi-Square Test

f_0 and f_e are observed frequencies and expected frequencies.
H0 is the null hypothesis and H1 is the alternative hypothesis.

H0: There is a difference between the observed and expected frequencies.
H1: There is a difference between the observed and expected frequencies.

Test statistic:

$$\chi^2 = \sum \left[\frac{(f_o - f_e)^2}{f_e} \right]$$

The critical value is a value taken from a random variable chi-square with degrees of freedom (k-1), where k is the number of classes in which the random sample is grouped.

Step 1: Fix null hypothesis and alternative.
H0: There is agreement between the data.
H1: There is no agreement between the data.

In our case, the data are:
- mobile subscribers in millions.
- estimated data using the logistic equation.

Our goal is to apply the Chi Square Test to verify the consistency of the actual data and those estimated in terms of square error.

Step 2: Select the level of significance α.

Let $\alpha = 0.01$

The **level of significance** of a test is usually given by a test of hypothesis testing. In the simplest case is defined as the probability of accepting or rejecting the null hypothesis. The decision in this case is done using the p-value: if the value p (p-value) is less than the significance level, then the null hypothesis is rejected. The lower the p value, the more significant is the result.

Step 3: Select the test statistic

How to use the test $\chi 2$ statistics.

Step 4: H0 is rejected if the p-value is less than α = 0.01.

We calculate the test statistic at each logistic curve identified in the first part of the analysis:

1. *Logistic curve with K=1E+10*

Sum value χ^2 ***= 6441546.15,9***

This value was obtained by applying the formula:

$$\chi^2 = \sum \left[\frac{(f_o - f_e)^2}{f_e} \right]$$

Degrees of freedom = 10-1 = 9

The p ($\chi 2$>6441546.15,9) = 0 has been calculated using the method *chi2cdf* ($\chi 2$ = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(6441546.15,9)
>> chi =
>>  0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of $\chi 2$, which is equal to 21,67 (tabulated value), and then the p-value of 0 is less than the chosen significance level (α = 0.01).

Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

2. *Logistic curve with K=1E+11*

Sum value ***χ^2 = 6571657.601,9***

Degrees of freedom = 10-1 = 9

The p (χ2>6571657.601,9) = 0 has been calculated using the method *chi2cdf* (χ2 = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(6571657.601,9)
>> chi =
>>  0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of χ2, which is equal to 21,67 (tabulated value), and then the p-value of 0 is less than the chosen significance level (α = 0.01).

Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

3. *Logistic curve with K=1E+09*

Sum value ***χ^2 = 5373852.688,9***

Degrees of freedom = 10-1 = 9

The p (χ2>5373852.688,9) = 0 has been calculated using the method *chi2cdf* (χ2 = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(5373852.688,9)
```

```
>> chi =
>>  0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of $\chi2$, which is equal to 21,67 (tabulated value), and then the p-value of 0 is less than the chosen significance level ($\alpha = 0.01$).

Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

4. *Logistic curve with K=5E+10*

Sum value $\boldsymbol{\chi^2}$ ***= 6560515.902,9***

Degrees of freedom = 10-1 = 9

The p ($\chi2$>6560515.902,9) = 0 has been calculated using the method *chi2cdf* ($\chi2$ = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(6560515.902,9)
>> chi =
>>  0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of $\chi2$, which is equal to 21,67 (tabulated value), and then the p-value of 0 is less than the chosen significance level ($\alpha = 0.01$).

Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

5. *Logistic curve with K=6E+10*

Sum value χ^2 ***= 6564136.099,9***

Degrees of freedom = 10-1 = 9

The p (χ2>6564136.099,9) = 0 has been calculated using the method *chi2cdf* (χ2 = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(6564136.099,9)
>> chi =
>> 0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of χ2, which is equal to 21,67 (tabulated value), and then the p-value of 0 is less than the chosen significance level (α = 0.01).

Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

6. *Logistic curve with K=5E+09*

Sum value χ^2 ***= 6443895.289,9***

Degrees of freedom = 10-1 = 9

The p (χ2>6443895.289,9)= 0 has been calculated using the method *chi2cdf* (χ2 = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(6443895.289,9)
```

```
>> chi =
>>  0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of $\chi 2$, which is equal to 21,67 (tabulated value), and then the p-value of 0 is less than the chosen significance level ($\alpha = 0.01$).

Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

7. *Logistic curve with K= 7E +09*

Sum value χ^2 ***= 6481534.55,9***

Degrees of freedom = 10-1 = 9

The p ($\chi 2$>6481534.55,9) = 0 has been calculated using the method *chi2cdf* ($\chi 2$ = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(6481534.55,9)
>> chi =
>>  0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of $\chi 2$ and then the p-value of 0 is less than the chosen significance level ($\alpha = 0.01$). Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

8. Logistic curve with K=6E+09

Sum value χ^2 ***= 6466003.894,9***

Degrees of freedom = 10-1 = 9

The p (χ2>6466003.894,9) = 0 has been calculated using the method *chi2cdf* (χ2 = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(6466003.894,9)
>> chi =
>> 0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of χ2, which is equal to 21,67 (tabulated value), and then the p-value of 0 is less than the chosen significance level (α = 0.01). Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

In summary:

K	χ2
1,00E+10	6441546,15
1,00E+11	6.571.658
1,00E+09	5.373.853
5,00E+10	6.560.516
6,00E+10	6.564.136
5,00E+09	6.443.895
7,00E+09	6481534,55
6,00E+09	6.466.004

The chi-square test was applied to test whether the logistic curve that best approximates the performance of the input data coincides with the one identified in the first phase of the study. According to the statistical hypothesis test, the observed data are significantly different from the actual data. However, the calculation of statistics shows that the lower value is in correspondence of K =5E+09.

This coincides with what is assumed in the previous phase, namely that the logistic curve that best approximates the performance of the input data is described by the following equation:

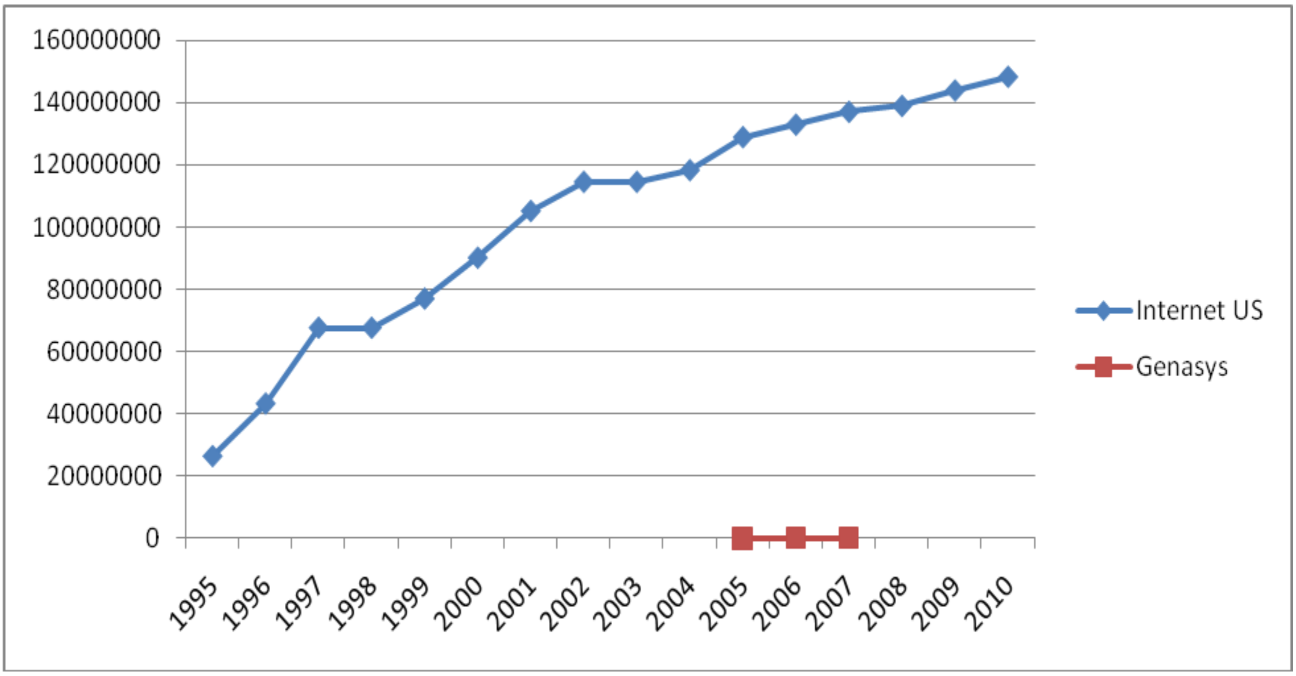
$$y = 5\text{ E+09} / (1 + 2137{,}9 * \exp(-0{,}319396 * t)).$$

1.5 Internet in Us

1.5.1 Considerations on the relationship between the logistic curves of the Internet in Us and location services

At this stage we are dedicated to research and reports of possible links that may exist between the logistic curve for Internet service in Us and that of location services.

Comparing these curves in the same graph we can see that there is a "time shift" between the beginning of the development of internet services and early development of location services.

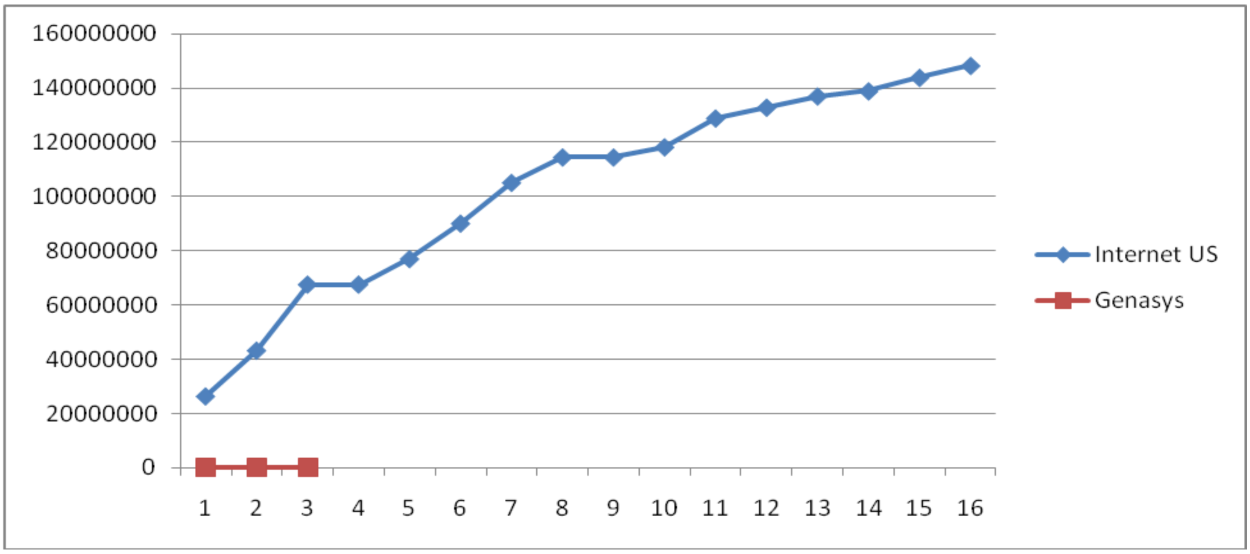


With the data at our disposal we can make a finer analysis on existing relationships.

With regard to the location services available data are those related to the first 3 years of development (2005 to 2007), unlike Internet service in Us for which data are related to a time span of nearly 16 years. Such availability may be related to the fact that location services are a recent development and distribution, unlike the Internet that saw its birth in Us in the early 90's and for which the information is clearer and more precise. Currently, the Internet service is one of the most widespread and deeply rooted in the world.

To make a more detailed analysis, we identify the similarities and differences between the two curves in their early years of development. In this way we can examine what happened during the introduction of such services.

To do this we insert in the same graph the two curves in parallel.

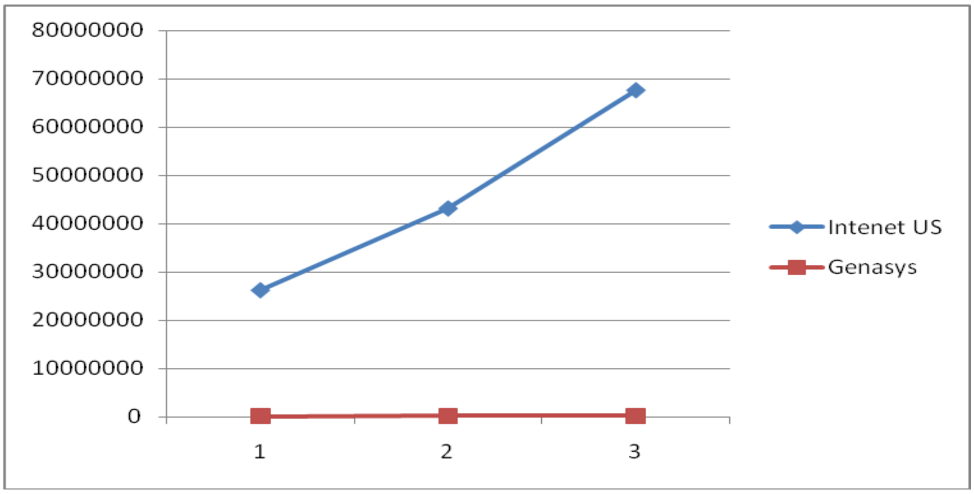


As can be seen from the figure, the two curves have different trends.

The curve of location services, provided by the company's data Genasys, presents a curve with a slope lower than that of the Internet. The growth rate of Internet curve is greater than the location curve .

It’s difficult establish the relationship between the two growth trends. Rate of growth of the Internet in the U.S. and very quick unlike the case for location systems.

With particular attention to the evolution of the curves in the first three years, we can verify that the location services have a slower pace than the curve of the Internet in Us. We can verify that the two curves haven’t the same trend in terms of math performance.



The curve of location services, provided by the company's data Genasys presents a curve with a slope almost different to that of the Internet in Us.

1.5.2 Applications to find the best parameters for the logistic curve of the Internet in Us

As a first step, we want to make a fitting with the logistic model of growth through transformation of variables (in order to be reduced to a linear equation) and then by linear regression. Recall that the logistic equation

$$y = K / (1 + a * \exp(-bt))$$

becomes linear with the following transformation

$$\log(K/y - 1) = \log a - bt .$$

The data we suppose that the population limit (equilibrium) and K (remember that is the horizontal asymptote of the logistic function). Then we determine a and b by linear regression.

Since we are not aware of the limit to which the reach of the Internet traffic in Megabytes / yr go forward in search of that parameter. We have made the analysis on different values of K. We stopped our search when the value of the mean square made from the model was worse than that calculated in the previous system.

To make this process we assumed different values of K, we left the value of K = 9E+10

The idea is to calculate

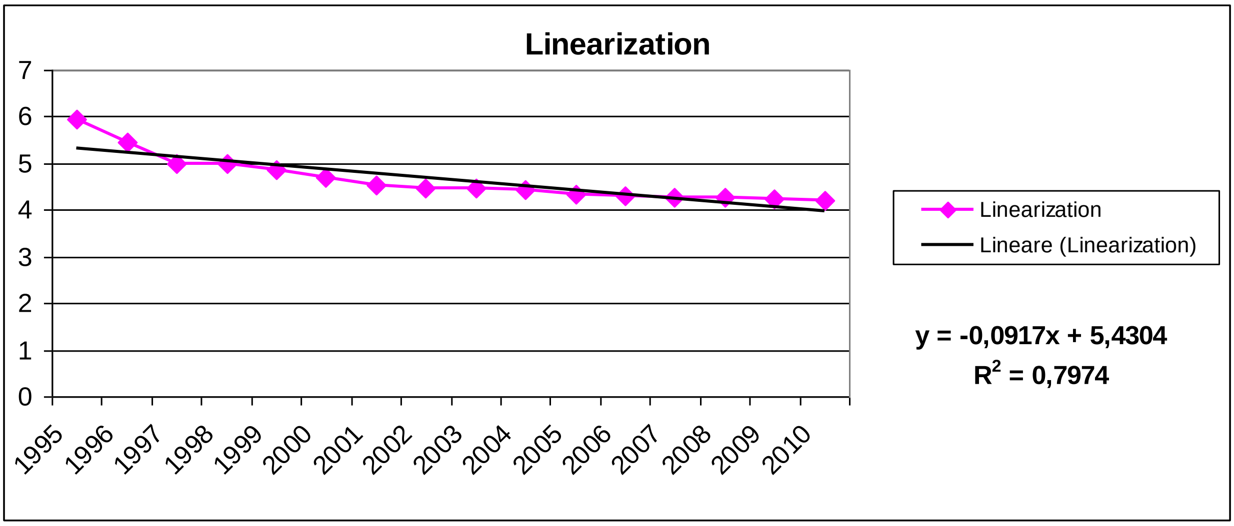
$\log(K/y - 1)$.

To this end we write in a cell of the Excel spreadsheet the following formula

```
= LN(1E+10/B2-1).
```

Where B2 is the first value of the function y. This calculation is done for all values of y available. Now draw the chart with "chart" entering the x-axis the days and y axis, the column data as soon as detected.

At the same time insert the regression “trendline" together with its equation.



Then

$\log(a) = 5{,}4304 \quad a = 228{,}24 \quad b = 0{,}0917$

We find the logistic curve

$$y = 1E+10 / (1 + 228{,}24 * \exp(-0{,}0917 * t))$$

Then we tried to optimize the choice of parameters using ``Solver'' (``Risolutore'') of Excel.

In another paper we have entered the data file according to the formula given by the logistic growth model

y = K/(1 + *a * exp* (*-b*t*))

To do this, we inserted the formula in column writing for example

```
= I2 /( 1+( I3 * EXP(- I4 * A2)))
```

Note that the parameters used are those included in Excel spreadsheet cells. Box I2 is that of the parameter K, I3 is the box for the parameter a, while the I4 box refers to the parameter b.

In particular these cells are introduced into the formula with the $ sign to make these cells remain fixed in the calculation. So we have:

```
= $I$2 /( 1+( $I$3 * EXP(-$I$4 * A2)))
```

Then calculate the square error by including in each row of column Excel spreadsheet formulas like

```
=(C2-B2)^2
```

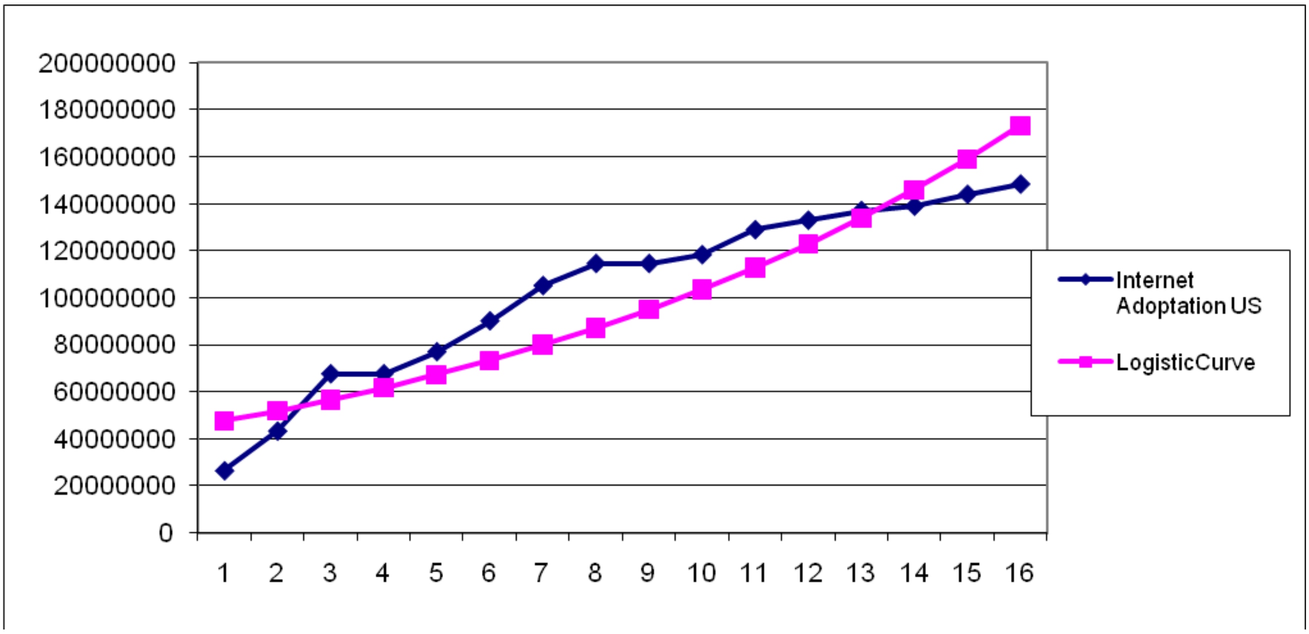
In this way we calculated the value of the mean square for the first period.

Using the "Solver" of Excel (found under the Tools menu, "Tools") optimize the parameters K, a and b by minimizing the sum of the standard deviations.

We set the "Solver" indicating that the parameters are optimized in the cells and I2-I4 (by entering the command, and then I2: I4 under “changing cells") and inserting the value of the sum of the standard deviations in cell the objective function to be minimized ("Target" cell D16), we start the "Solver".

The result of this simulation allows us to identify the best parameters a and b according to a saturation limit value set in the value K.

Finally, we can make a graph of the measured data and those calibrated.



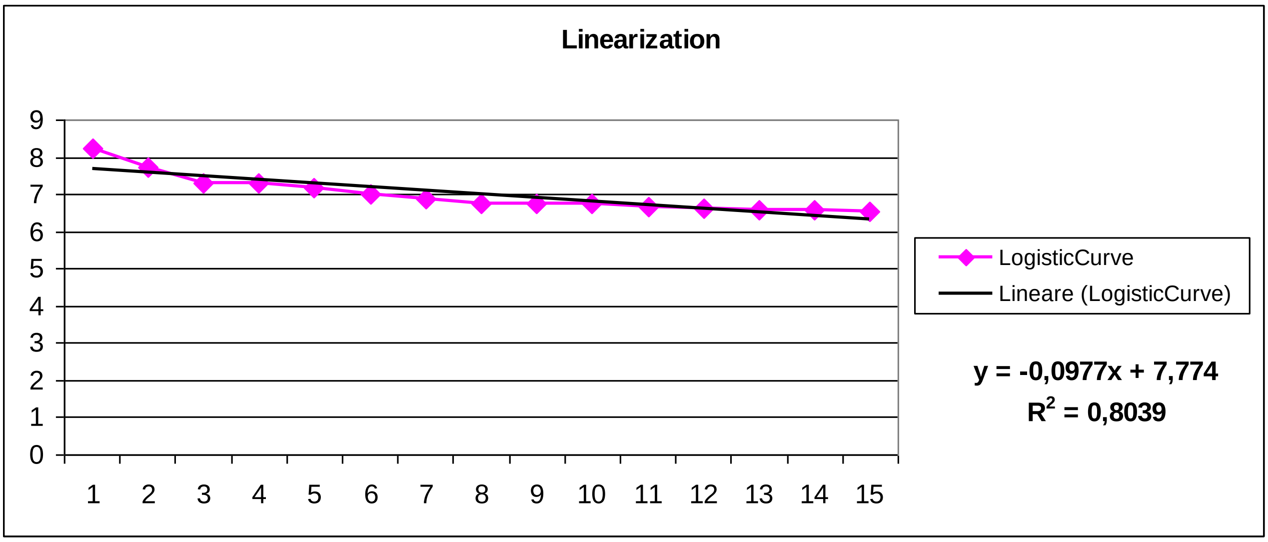
The optimized model has parameter values of:

`k = 1E10` *a* `= 228,24` *b* `= 0,086978`

For this model the value of the sum of squared errors is: `4,32028 E+15`

This procedure was performed for different values of K.

- For *K*= *1E+11* we have the following linearized model:



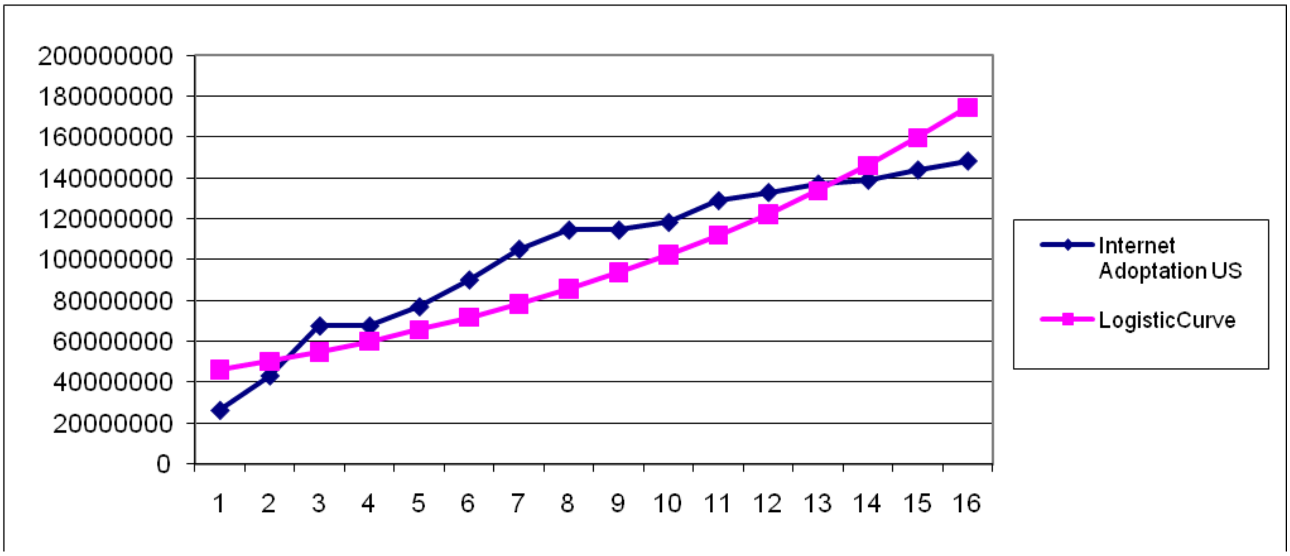
Then

log(*a*) = 7,774 *a* = 2377,964 *b* = 0,*0977*

We find the logistic curve

y = 1E+11 / (1 + 2377,964 * exp (-*0,0977* * *t*))

While using the solver we have the following graph of the measured data and those calibrated.



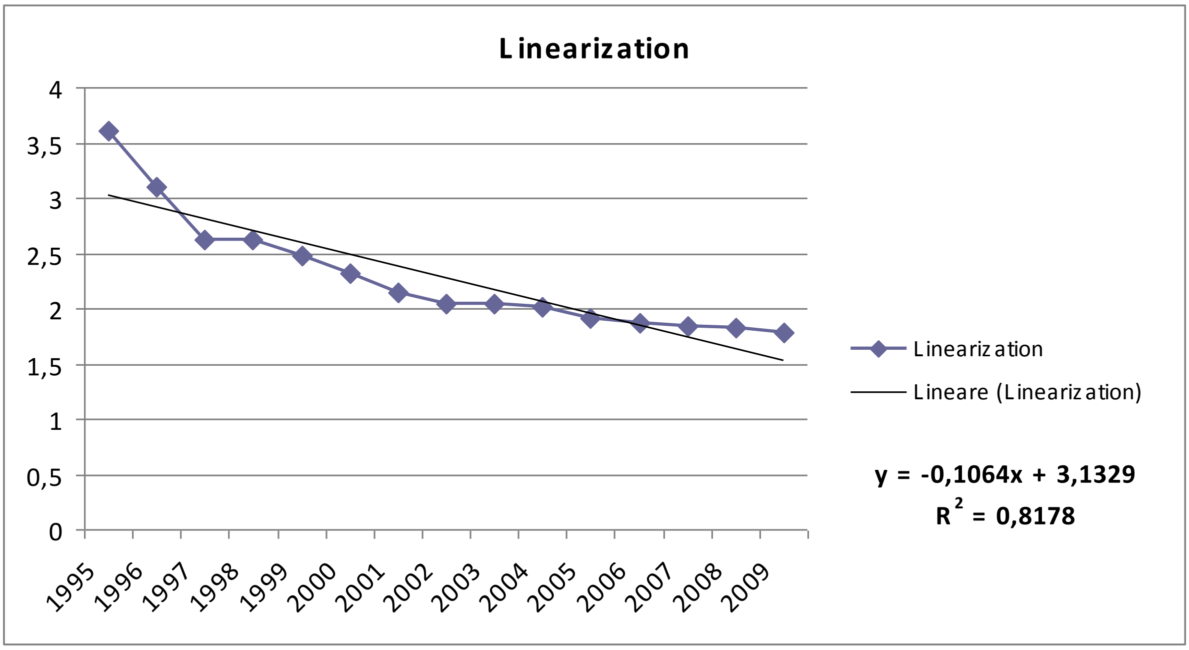
The optimized model has parameter values of:

k = 1E11 *a* = 2377,964 *b* = 0,*0*89046

For this model the value of the sum of squared errors is: 4,76584E+15

The value just examined is greater than the value calculated with the previous K, we proceed in finding the optimal parameters are going to decrease the value of K. Let's see what happens.

- Per *K*= *1E+09* we have the following linearized model:



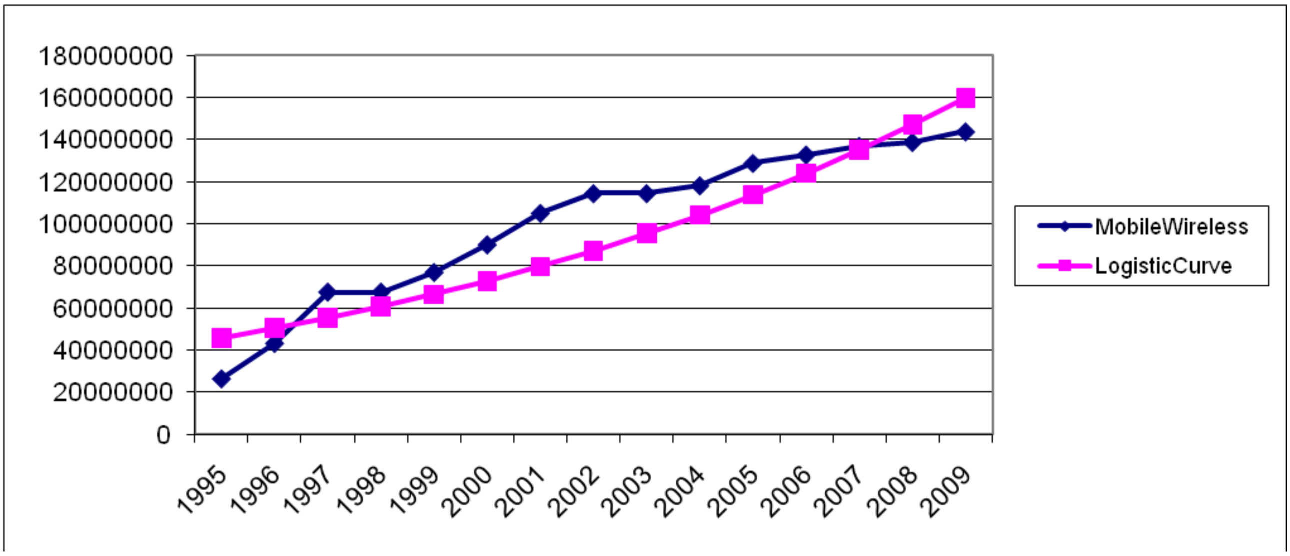
Then

log(*a*) = 3,1329   *a* = 22,94   *b* = 0,1064

We find the logistic curve

y = 1E+09 / (1 + 22,94 * exp (-*0,1064* * *t*))

While using the solver we have the following graph of the measured data and those calibrated.



The optimized model has parameter values of:

k = 1E09   *a* = 22,9255   *b* = 0,098131

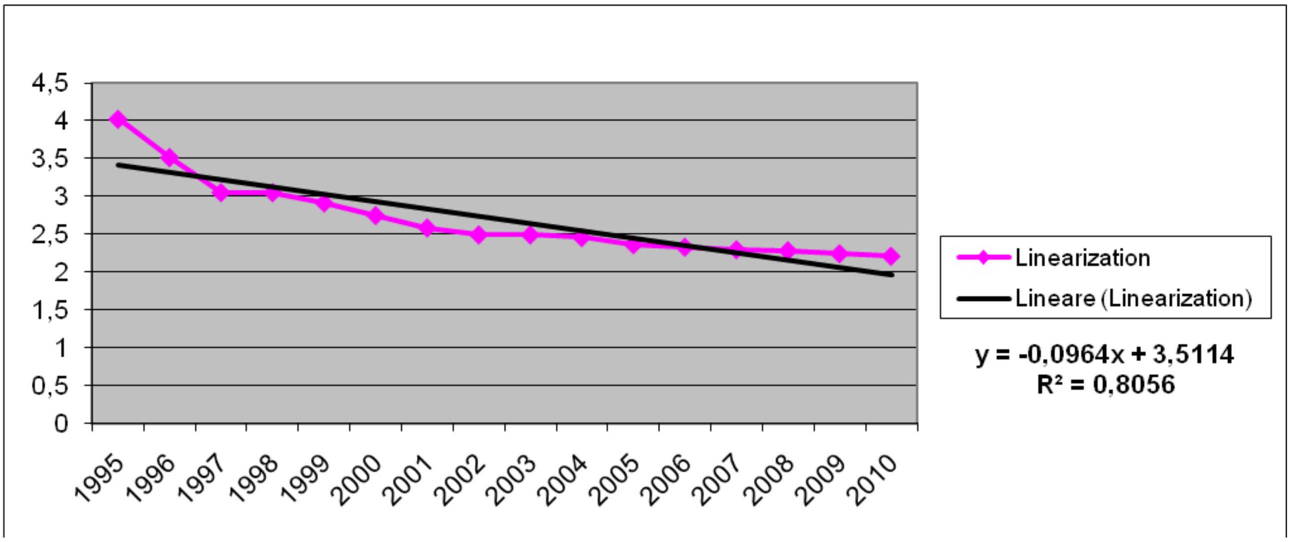
For this model the value of the sum of squared errors is: 1,79469E+20

The value is considered just below the value calculated with the previous K, then proceed to search for parameters of good intention of reducing the value of K.

Since the largest value of the data is of the order of E +09, then try to assess the value of K including the following range:

1 +09 E <K <1 E +10

- For *K*= *1,5E+09* we have the following linearized model:



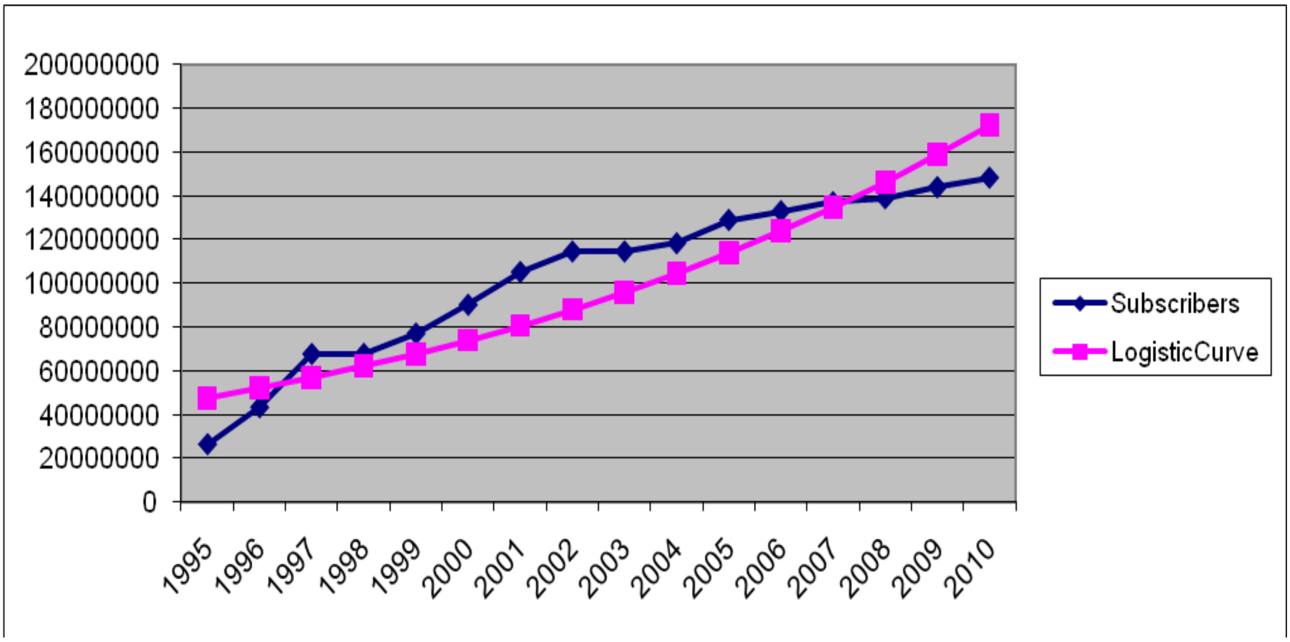
Then

log(*a*) = 3,5114 *a* = 33,495 *b* = 0,*0964*

We find the logistic curve

y = 1,5E+09 / (1 + 33,495 * exp (-*0,0964* * *t*))

While using the solver we have the following graph of the measured data and those calibrated.



The optimized model has parameter values of:

`k = 1,5 E09` *a* `= 33,46712` *b* `= 0,091837`

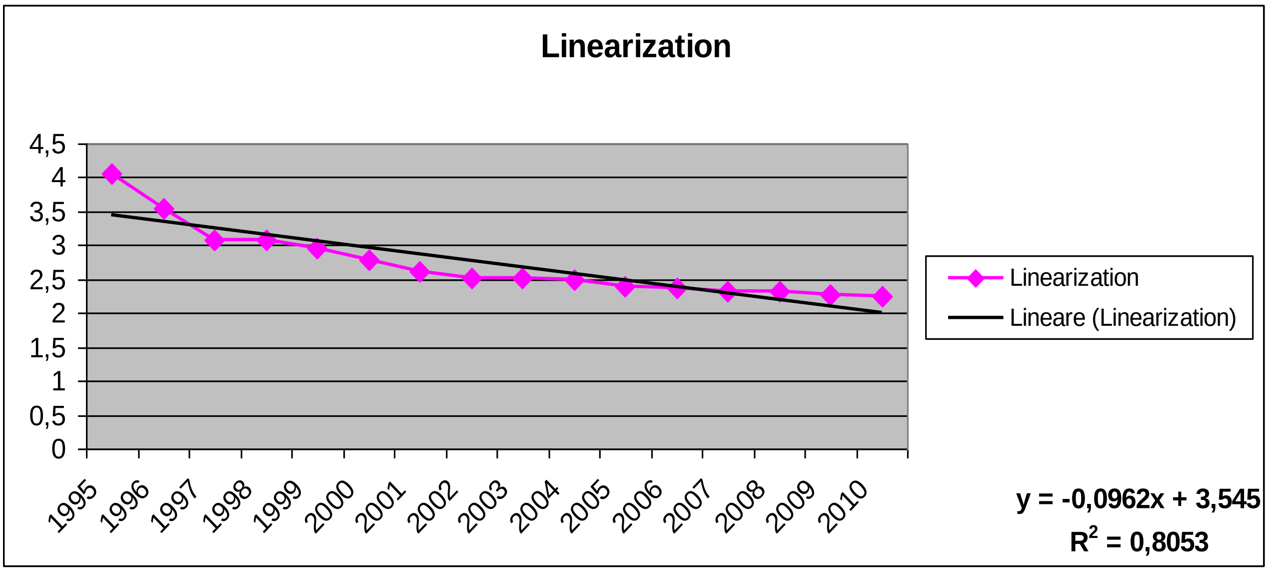
For this model the value of the sum of squared errors is: `4,04979E+15`

The assessed value is just less than the value calculated with the previous K, then proceed to find the optimal parameters are going to increase the value of K.

The value of K will be taken within the range:

1.5 E +09 <K <1 E +10

- For *K*= *1,55 E+09* we have the following linearized model:



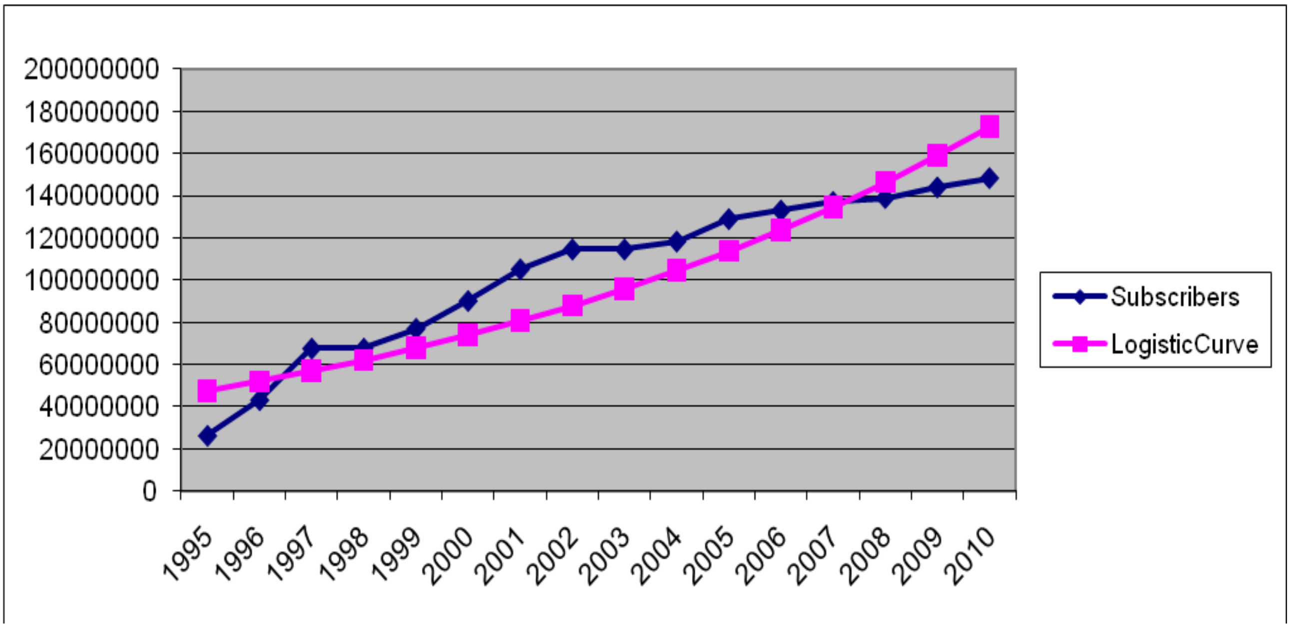
Then

log(*a*) = 3,545   *a* = 34,64   *b* = 0,*0962*

We find the logistic curve

y = 1,55 E+09 / (1 + 34,64 * exp (−*0*,*0962* * *t*))

While using the solver we have the following graph of the measured data and those calibrated.



The optimized model has parameter values of:

k = 1,55 E09   *a* = 34,62586   *b* = 0,091711

For this model the value of the sum of squared errors is: 4,0639 E+15

The value just examined is greater than previously calculated., then we can say that the value of K = 1,5 E+09 may be the best.

In summary:

K	*a*	*b*	*Sum Square Error*
1,00E+10	228,24	0,086978463	4,32E+15
1,00E+11	2377,964	0,089045943	4,76584E+15
1,00E+09	22,9255	0,098131255	4,24461E+15
1,50E+09	33,46712	0,091837003	4,05E+15
1,55E+09	34,62586	0,091710662	4,06E+15

1.5.3 Chi-Square Test

f_0 and f_e are observed frequencies and expected frequencies.

H0 is the null hypothesis and H1 is the alternative hypothesis.

H0: There is a difference between the observed and expected frequencies.

H1: There is a difference between the observed and expected frequencies.

Test statistic:

$$\chi^2 = \sum \left[\frac{(f_o - f_e)^2}{f_e} \right]$$

The critical value is a value taken from a random variable chi-square with degrees of freedom (k-1), where k is the number of classes in which the random sample is grouped.

Step 1: Fix null hypothesis and alternative.

H0: There is agreement between the data.

H1: There is no agreement between the data.

In our case, the data are:

- internet subscribers in millions.
- estimated data using the logistic equation.

Our goal is to apply the Chi Square Test to verify the consistency of the actual data and those estimated in terms of square error.

Step 2: Select the level of significance α.

Let $\alpha = 0.01$

The **level of significance** of a test is usually given by a test of hypothesis testing. In the simplest case is defined as the probability of accepting or rejecting the null hypothesis. The decision in this case is done using the p-value: if the value p (p-value) is less than the significance level, then the null hypothesis is rejected. The lower the p value, the more significant is the result.

Step 3: Select the test statistic

How to use the test $\chi 2$ statistics.

Step 4: H0 is rejected if the p-value is less than α = 0.01.

We calculate the test statistic at each logistic curve identified in the first part of the analysis:

1. *Logistic curve with K=1E+10*

Sum value χ^2 ***= 50305881.73,15***

This value was obtained by applying the formula:

$$\chi^2 = \sum \left[\frac{(f_o - f_e)^2}{f_e} \right]$$

Degrees of freedom = 16-1 = 15

The p ($\chi 2$>50305881.73,15) = 0 has been calculated using the method *chi2cdf* ($\chi 2$ = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(50305881.73,15)
>> chi =
>>   0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of $\chi 2$, which is equal to 30,58 (tabulated value), and then the p-value of 0 is less than the chosen significance level (α = 0.01).

Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

2. *Logistic curve with K=1E+11*

Sum value ***χ^2 = 55494414.78,15***

Degrees of freedom = 16-1 = 15

The p (χ2>55494414.78,15) = 0 has been calculated using the method *chi2cdf* (χ2 = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(55494414.78,15)
>> chi =
>>  0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of χ2, which is equal to 30,58 (tabulated value), and then the p-value of 0 is less than the chosen significance level (α = 0.01).

Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

3. *Logistic curve with K=1E+09*

Sum value ***χ^2 = 49275555.04,15***

Degrees of freedom = 16-1 = 15

The p (χ2>49275555.04,15 = 0 has been calculated using the method *chi2cdf* (χ2 = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(49275555.04,15)
>> chi =
>>   0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of χ2, which is equal to 30,58 (tabulated value), and then the p-value of 0 is less than the chosen significance level (α = 0.01).
Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

4. *Logistic curve with K=1,5E+09*

Sum value χ^2 ***= 47200035.08,15***

Degrees of freedom = 16-1 = 15

The p (χ2>47200035.08,15) = 0 has been calculated using the method *chi2cdf* (χ2 = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(47200035.08,15)
>>  chi =
>>  0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of χ2, which is equal to 30,58 (tabulated value), and then the p-value of 0 is less than the chosen significance level (α = 0.01).
Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

5. *Logistic curve with K=1,55E+09*

Sum value χ^2 ***= 47311124.44,15***

Degrees of freedom = 16-1 = 15

The p ($\chi2$>47311124.44,15) = 0 has been calculated using the method *chi2cdf* ($\chi2$ = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(47311124.44,15)
>> chi =
>>  0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of $\chi2$, which is equal to 30,58 (tabulated value), and then the p-value of 0 is less than the chosen significance level (α = 0.01). Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

In summary:

K	$\chi2$
1,00E+10	50305881.73
1,00E+11	55494414.78
1,00E+09	49275555.04
1,50E+09	47200035.08
1,55E+09	47311124.44

The chi-square test was applied to test whether the logistic curve that best approximates the performance of the input data coincides with the one identified in the first phase of the study. According to the statistical hypothesis test, the observed data are significantly different from the actual data. However, the calculation of statistics shows that the lower value is in correspondence of *K = 1,50E+09*.

This coincides with what is assumed in the previous phase, namely that the logistic curve that best approximates the performance of the input data is described by the following equation:

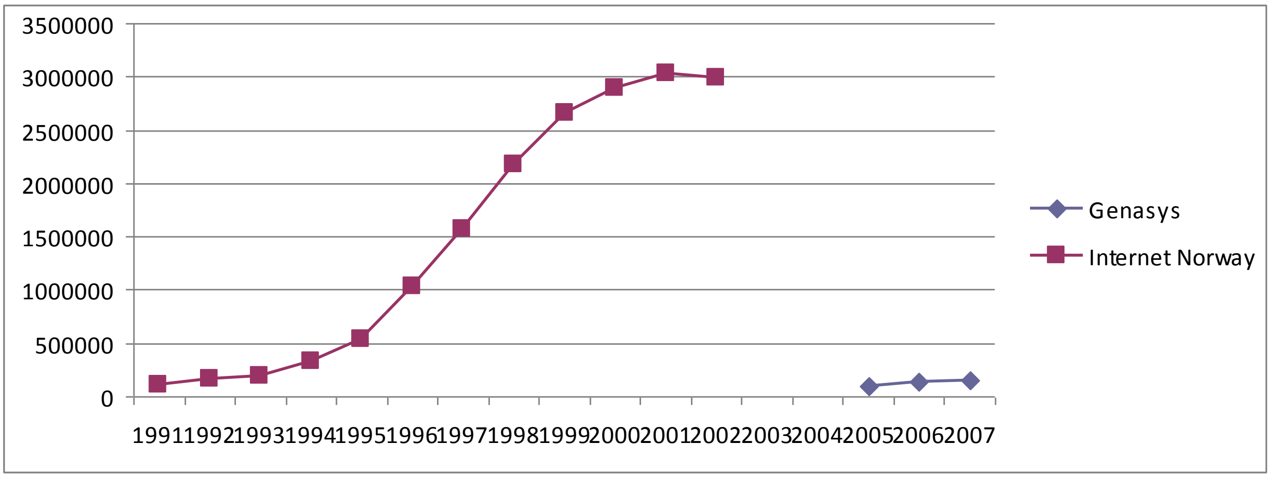
```
y = 1,50E+09 / (1 + 33,46712 *  exp (-0,091837 * t)).
```

1.6 Internet in Norway

1.6.1 Considerations on the relationship between the logistic curves of the Internet in Norway and location services

At this stage we are dedicated to research and reports of possible links that may exist between the logistic curve for Internet users in Norway and that of location services.

Comparing these curves in the same graph we can see that there is a "time shift" between the curve for the number of registered users of Internet in Norway and the curve representing the development of location services.

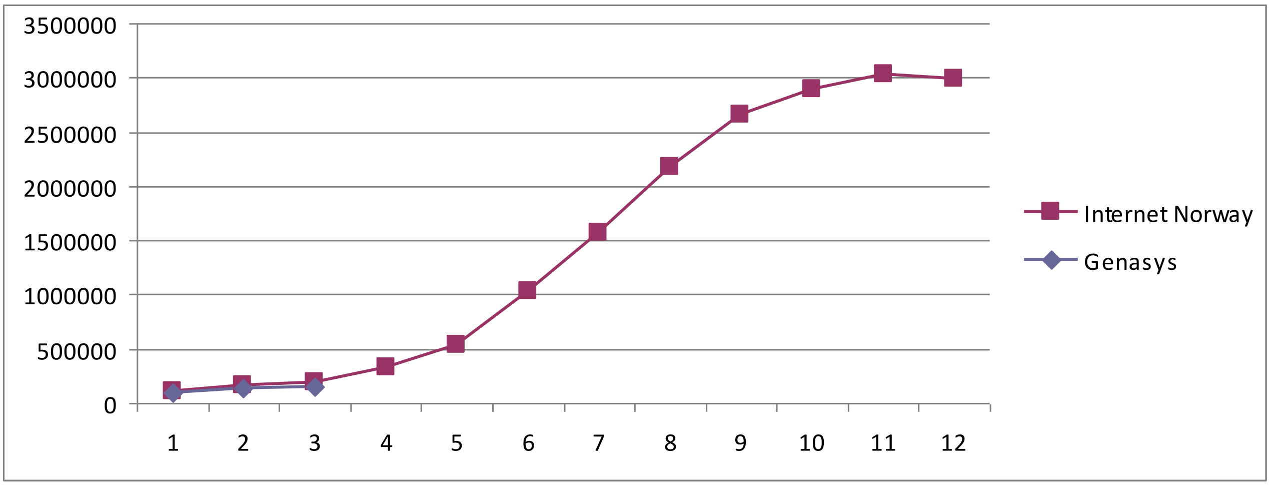


With the data at our disposal we can make a finer analysis on existing relationships.

With regard to the location services available data are those related to the first 3 years of development (2005 to 2007) in contrast to the Internet service whose data are related to a period of 12 years. Such availability may be related to the fact that location services are of recent development and distribution, as opposed to services Internet. Such availability may be related to the fact that location services are a recent development and distribution, unlike the Internet that saw its birth in the 90's and for which the information is clearer and more precise.

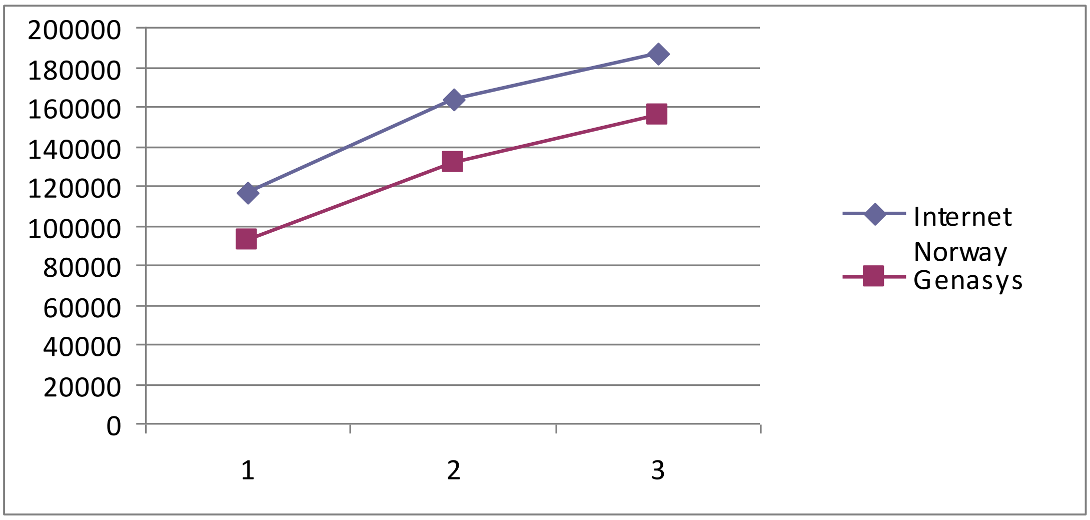
To make a more detailed analysis, we identify the similarities and differences between the two curves in their early years of development. In this way we can examine what happened during the introduction of such services.

To do this we insert the same graph the two curves in parallel.



At least in the first three years of analysis, the two curves have same trends. The two curves seem to overlap.

In reality, with particular attention to the evolution of the curves in the first three years, we can verify that the location services have a slower pace than the curve of the Internet in Norway in terms of volume. On the contrary, we can verify that the two curves have the same trend in terms of math performance.



The curve of location services, provided by the company's data Genasys presents a curve with a slope almost identical to that of the Internet in Norway. We might think that the two curves can be developed by an almost similar trend as the comparison of these first two years.

1.6.2 Applications to find the best parameters for the logistic curve of the Internet in Norway

As a first step, we want to make a fitting with the logistic model of growth through transformation of variables (in order to be reduced to a linear equation) and then by linear regression. Recall that the logistic equation

$$y = K / (1 + a * \exp(-bt))$$

becomes linear with the following transformation

$$\log(K/y - 1) = \log a - bt .$$

The data we suppose that the population limit (equilibrium) and K (remember that is the horizontal asymptote of the logistic function). Then we determine a and b by linear regression.

Since we are not aware of the extent to which value will reach the number of registered users of Internet go forward in search of that parameter. We have made the analysis on different values of K. We stopped our search when the value of the mean square error made from the model was worse than that calculated in the previous system.

To make this process we assumed different values of K, we left the value of K = 1E+10

The idea is to calculate

$$\log(K/y - 1).$$

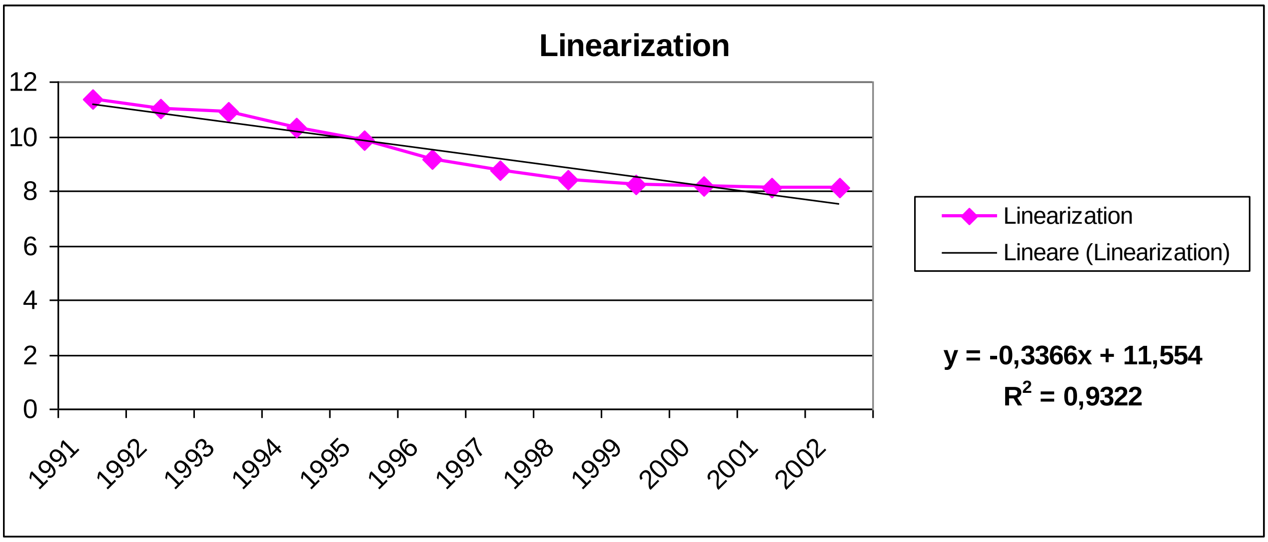
To this end we write in a cell of the Excel spreadsheet the following formula

```
= LN(1E+10/B2-1).
```

Where B2 is the first value of the function y. This calculation is done for all values of y available.

Now draw the chart with "chart" entering the x-axis the days and y axis, the column data as soon as detected.

At the same time insert the regression “trendline" together with its equation.



Then

$$\log(a) = 11{,}554 \qquad a = 104192{,}976 \qquad b = 0{,}3366$$

We find the logistic curve

$$y = 1E{+}10 \;/\; (1 + 104192{,}976 * \exp(-0{,}3366 * t))$$

Then we tried to optimize the choice of parameters using ''Solver'' (''Risolutore'') of Excel.

In another paper we have entered the data file according to the formula given by the logistic growth model

$$y = K/(1 + a * \exp(-b*t))$$

To do this, we inserted the formula in column writing for example

```
= I2 /( 1+( I3 * EXP(- I4 * A2)))
```

Note that the parameters used are those included in Excel spreadsheet cells. Box I2 is that of the parameter K, I3 is the box for the parameter a, while the I4 box refers to the parameter b.

In particular these cells are introduced into the formula with the $ sign to make these cells remain fixed in the calculation. So we have:

```
= $I$2 /( 1+( $I$3 * EXP(-$I$4 * A2)))
```

Then calculate the square error by including in each row of column Excel spreadsheet formulas like

```
=(C2-B2)^2
```

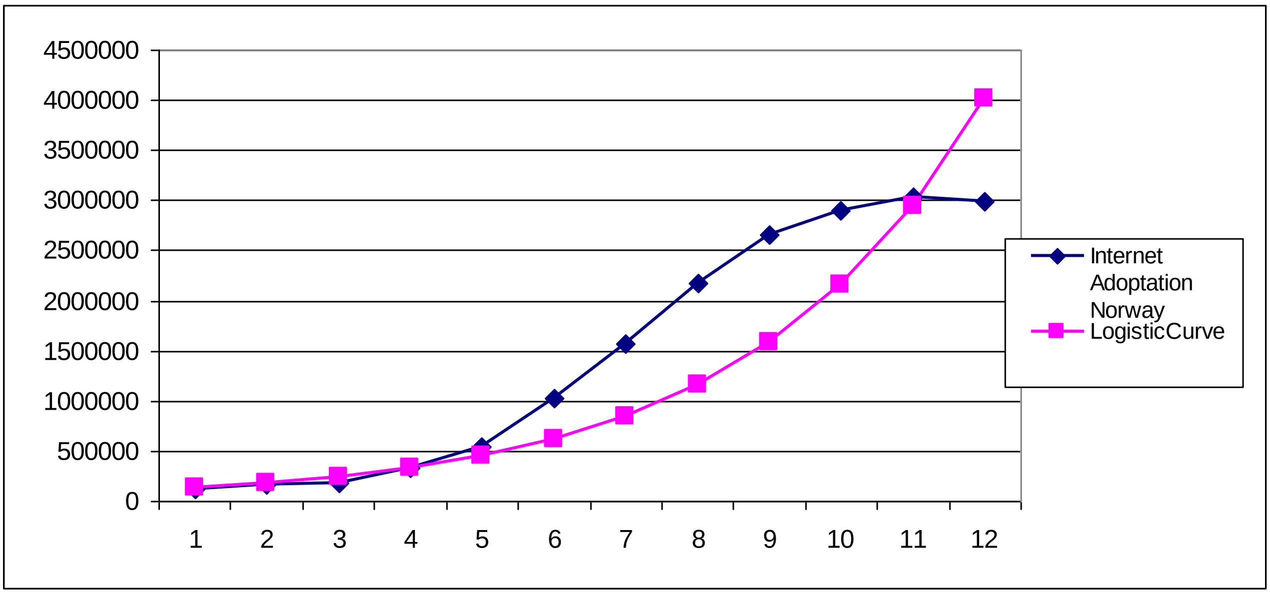
In this way we calculated the value of the mean square for the first period.

Using the "Solver" of Excel (found under the Tools menu, "Tools") optimize the parameters K, a and b by minimizing the sum of the standard deviations.

We set the "Solver" indicating that the parameters are optimized in the cells and I2-I4 (by entering the command, and then I2: I4 under “changing cells") and inserting the value of the sum of the standard deviations in cell the objective function to be minimized ("Target" cell D16), we start the "Solver".

The result of this simulation allows us to identify the best parameters a and b according to a saturation limit value set in the value K.

Finally, we can make a graph of the measured data and those calibrated.



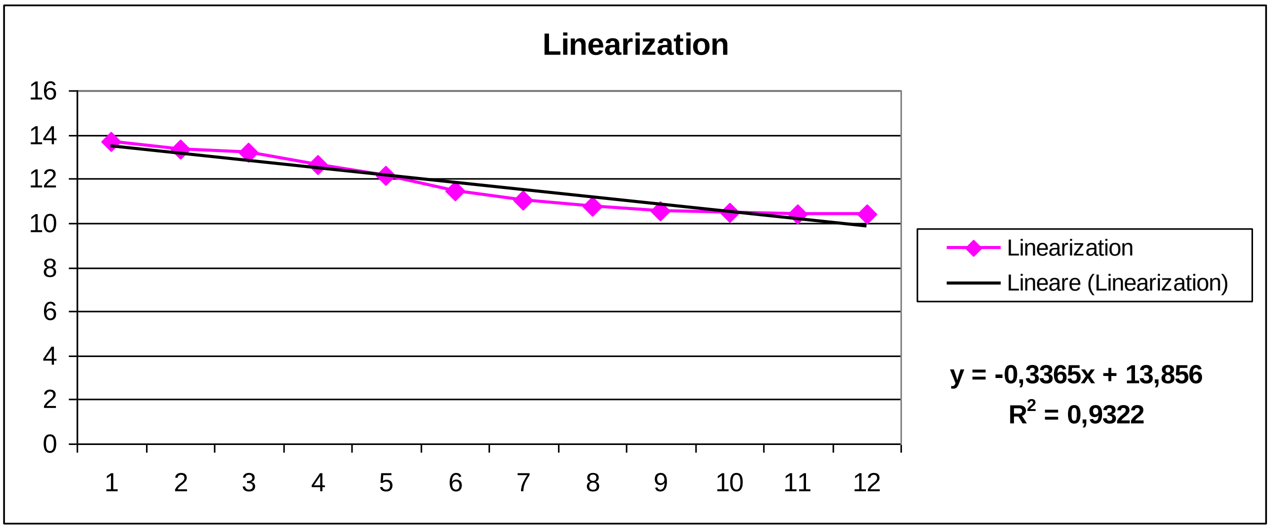
The optimized model has parameter values of:

k = 1E+10   *a* = 104192,976   *b* = 0,311305122

For this model the value of the sum of squared errors is: 4,5127E+12

This procedure was performed for different values of K.

- For $K=$ 1E+11 we have the following linearized model:



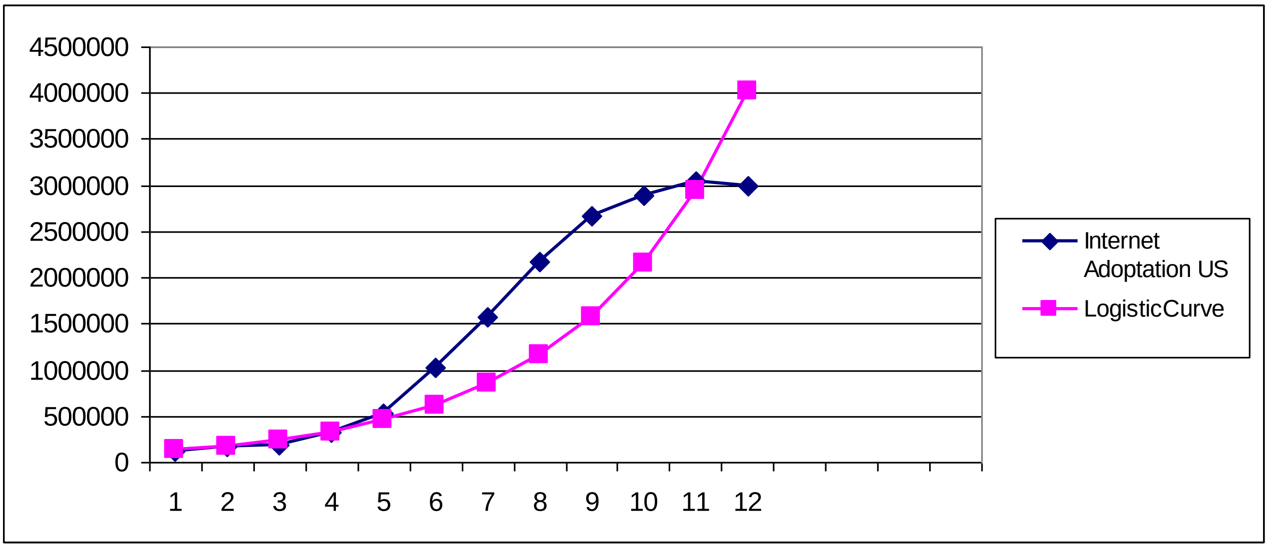
Then

log(*a*) = 13,856 *a* = 1041320,315 *b* = 0,3365

We find the logistic curve

y = 1E+11 / (1 + 1041320,315 * exp (-0,3365 * *t*))

While using the solver we have the following graph of the measured data and those calibrated.



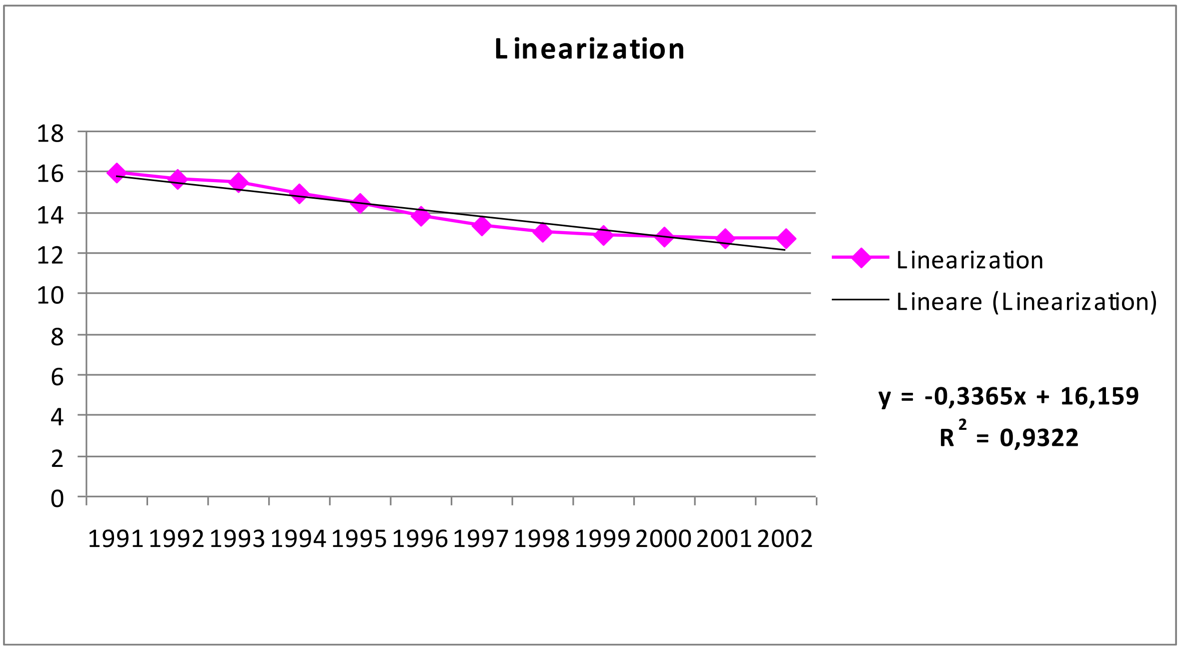
The optimized model has parameter values of:

k = 1E+11   *a* = 1041320   *b* = 0,311224

For this model the value of the sum of squared errors is: 4,51233E+12

The value of the error is less than the value calculated with the previous K, we proceed to find the optimal parameters are going to increase the value of K. Let 's see what happens.

- Per *K*= *1E+12* we have the following linearized model:



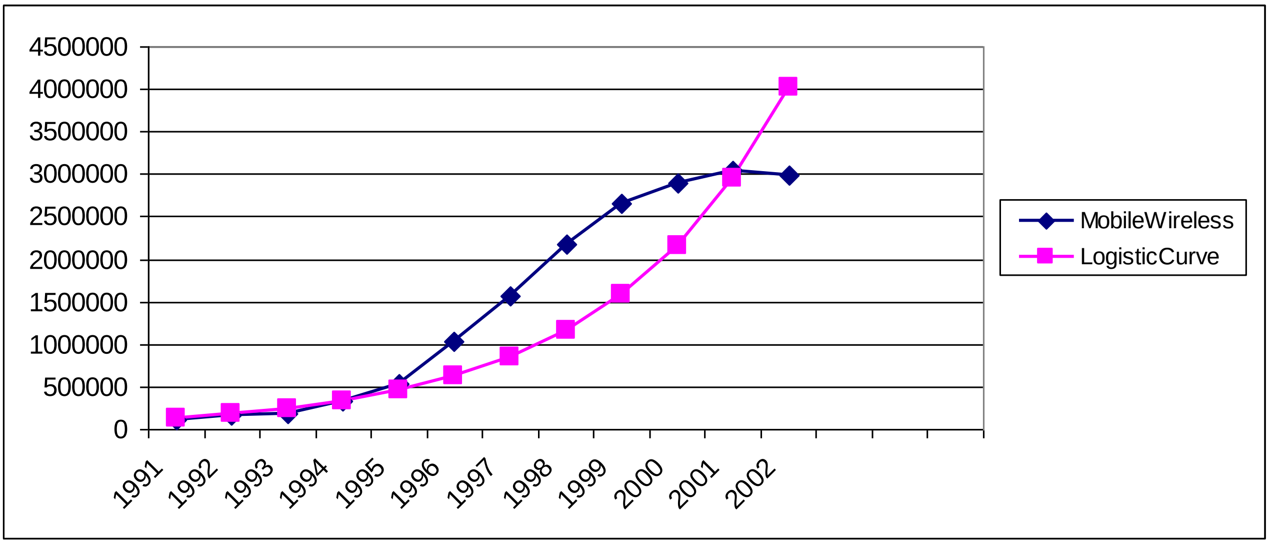
Then

log(*a*) = 16,159   *a* = 10417524,561   *b* = 0,3365

We find the logistic curve

y = 1E+12 / (1 + 10417524,561 * exp (-0,3365 * *t*))

While using the solver we have the following graph of the measured data and those calibrated.



The optimized model has parameter values of:

k = 1E+12 *a* = 9,7513 *b* = 0,000555

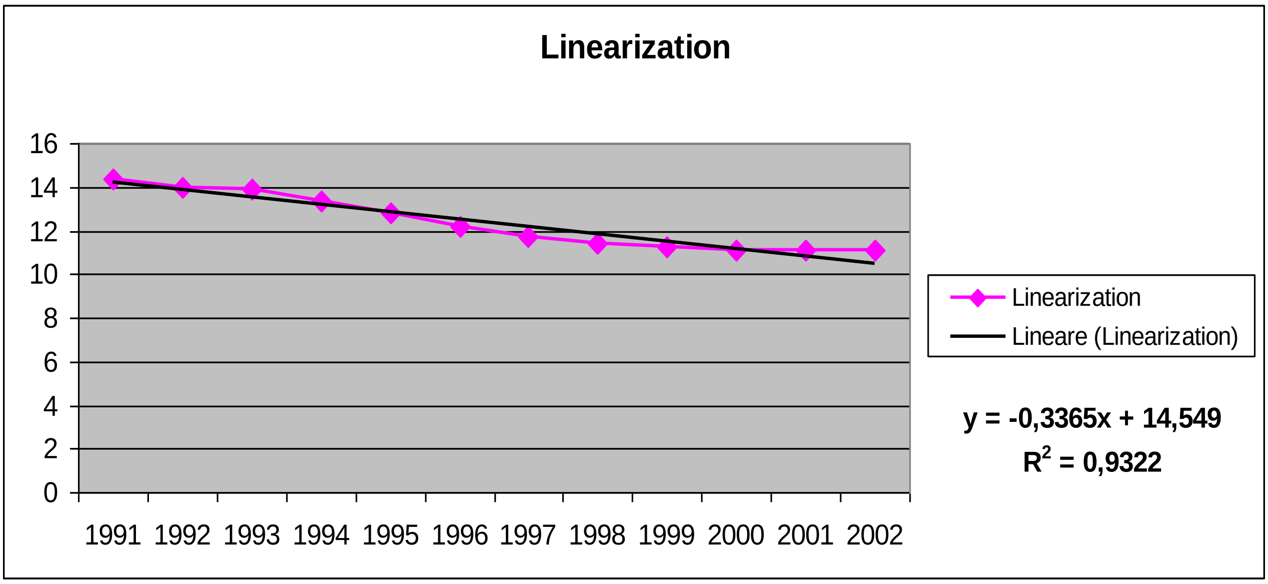
For this model the value of the sum of squared errors is: 4,51371E+12

Since the value is greater than the value just examined, calculated with K = 1E +11, proceed in the search parameters so that the value of K is taken within the Range:

1E+11 <K <1 E +12

We take a value in this interval and check if the sum of squared errors decreases.

- For *K*= *2E+11* we have the following linearized model:



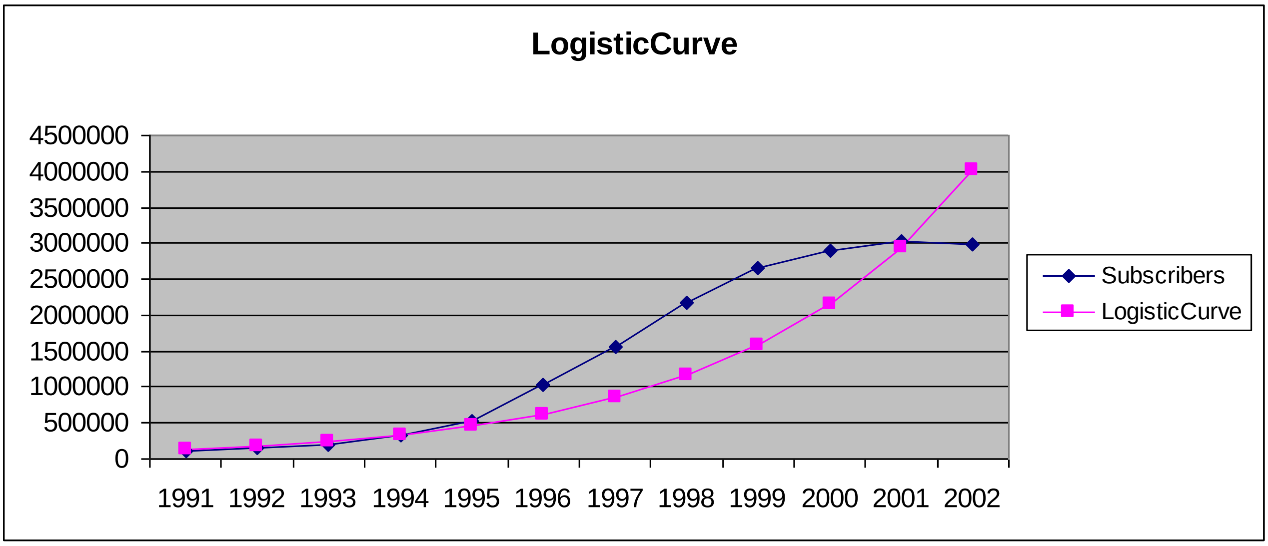
Then

`log(a) = 14,549   a = 2082334,129   b = 0,3365`

We find the logistic curve

`y = 2E+11 / (1 + 2082334,129 * exp (-0,3365* t))`

While using the solver we have the following graph of the measured data and those calibrated.



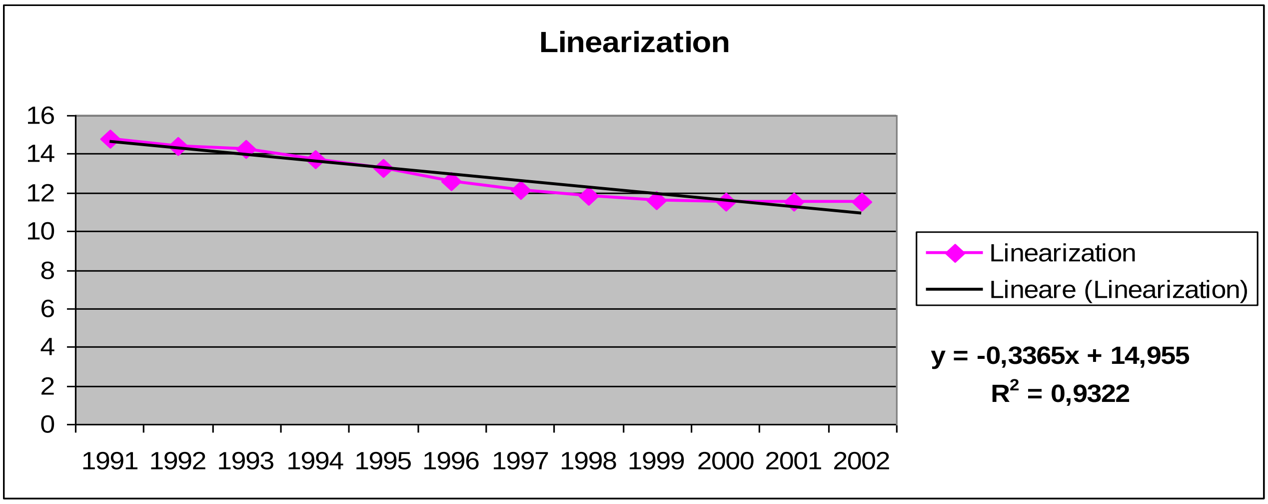
The optimized model has parameter values of:

`k = 2E+11   a = 2082334   b = 0,31121`

For this model the value of the sum of squared errors is: 4,51196E+12.

The value of the error is less than the value calculated with the previous K, we proceed to find the optimal parameters are going to increase the value of K. Let 's see what happens.

- For *K*= *3E+11* we have the following linearized model:

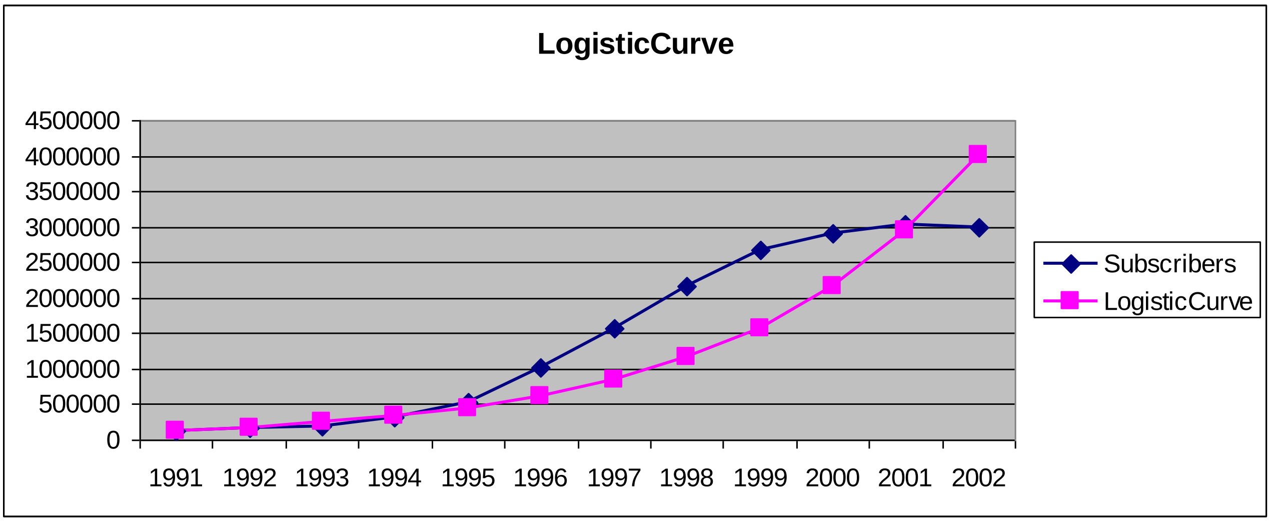


Then log(*a*) = 14,955 *a* = 3125172,376 *b* = 0,3365

We find the logistic curve

y = 3E+11 / (1 + 3125172,376 * exp (-0,3365* *t*))

While using the solver we have the following graph of the measured data and those calibrated.



The optimized model has parameter values of:

k = 3E+11 *a* = 3125172 *b* = 0,311256

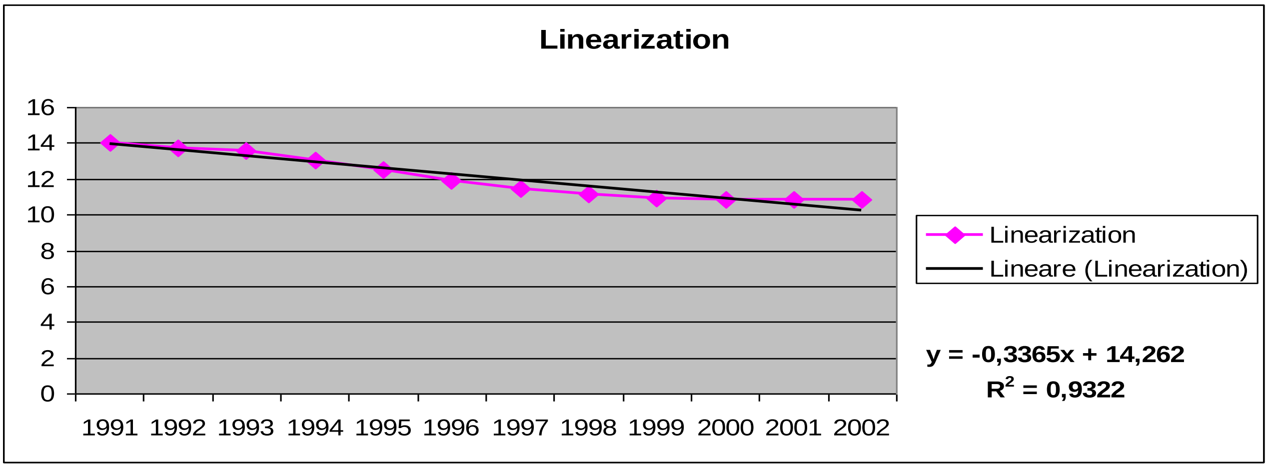
For this model the value of the sum of squared errors is: 4,51359E+12.

Since the value is greater than the value just examined, calculated with K = 2E +11, proceed in the search parameters so that the value of K is taken within the Range:

1E+11 <K <2 E +11

We take a value in this interval and check if the sum of squared errors decreases.

- For *K*= *1,50E+11* we have the following linearized model:



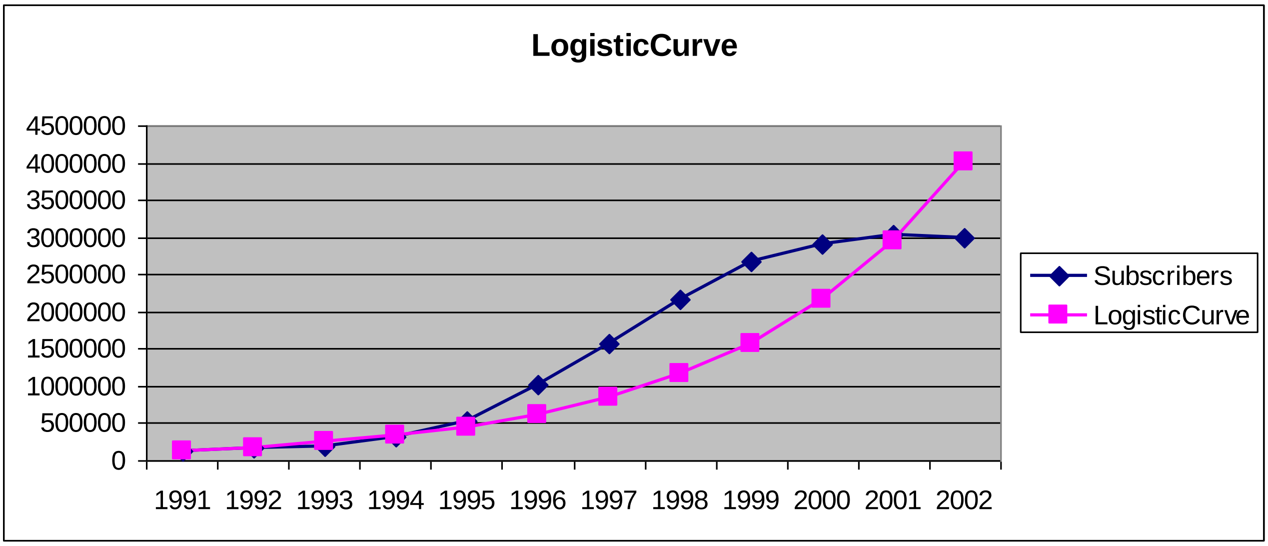
Then

log(a) = 14,262 a = 1562816,187 b = 0,3365

We find the logistic curve

$$y = 1{,}50E{+}11 / (1 + 1562816{,}187 * \exp(-0{,}3365 * t))$$

While using the solver we have the following graph of the measured data and those calibrated.



The optimized model has parameter values of:

k = 1,50E+11 a = 1562816 b = 0,31127

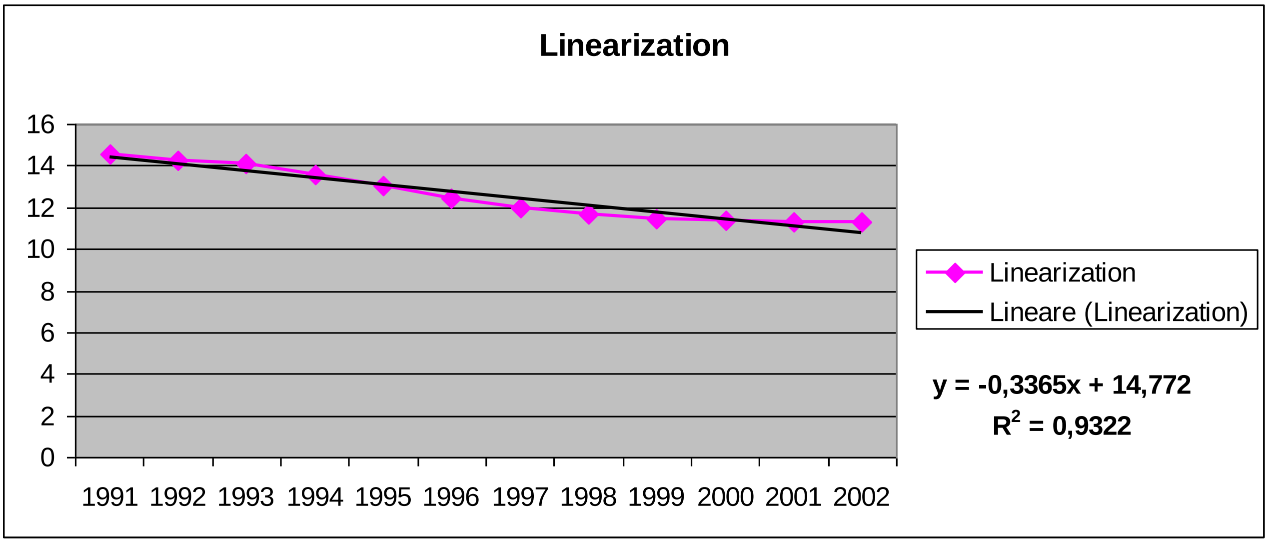
For this model the value of the sum of squared errors is: 4,51398E+12.

Since the value is greater than the value just examined, calculated with K = 2E +11, proceed in the search parameters so that the value of K is taken within the Range:

2E+11 <K <3 E +11

We take a value in this interval and check if the sum of squared errors decreases.

- For K= 2,50E+11 we have the following linearized model:



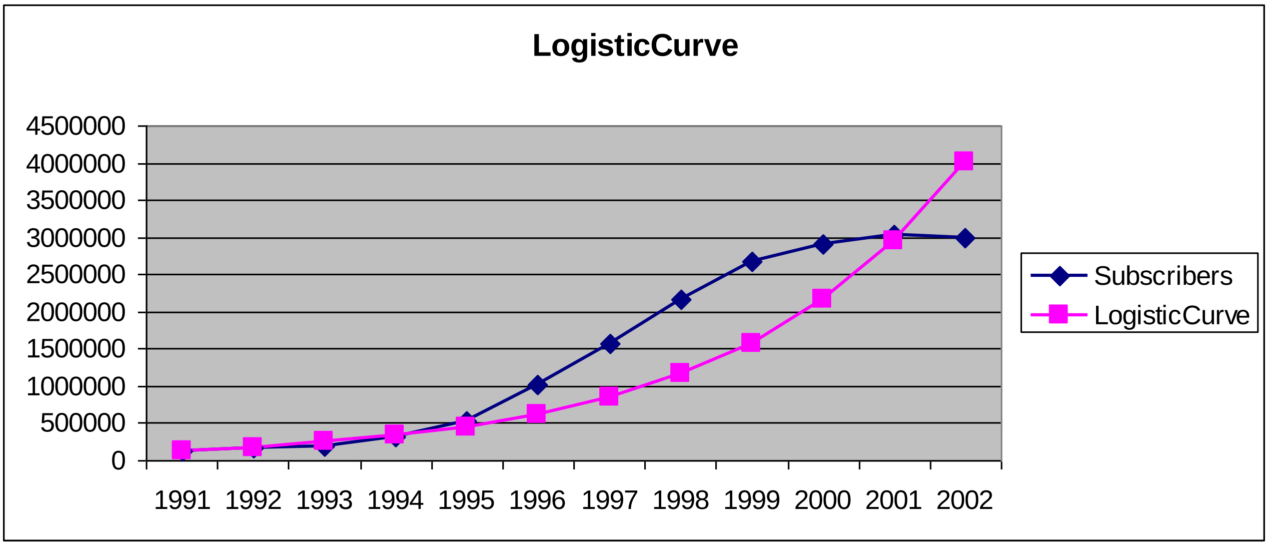
Then

log(*a*) = 14,772   *a* = 2602544,036   *b* = 0,3365

We find the logistic curve

y = 2,50E+11 / (1 + 2602544,036 * exp (-0,3365* *t*))

While using the solver we have the following graph of the measured data and those calibrated.



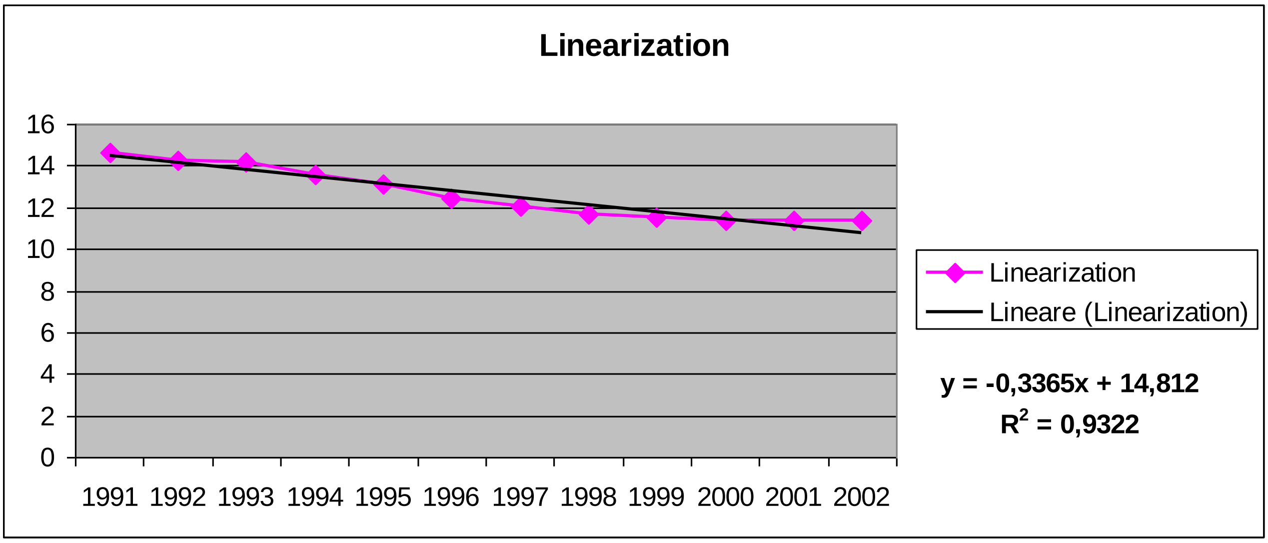
The optimized model has parameter values of:

k = 2,50E+11   *a* = 2602544   *b* = 0,311197

For this model the value of the sum of squared errors is: 4,51155E+12.

The value of the error is less than the value calculated with K=2,00E+11, we proceed to find the optimal parameters are going to increase the value of K. Let 's see what happens.

- For *K*= *2,60E+11* we have the following linearized model:



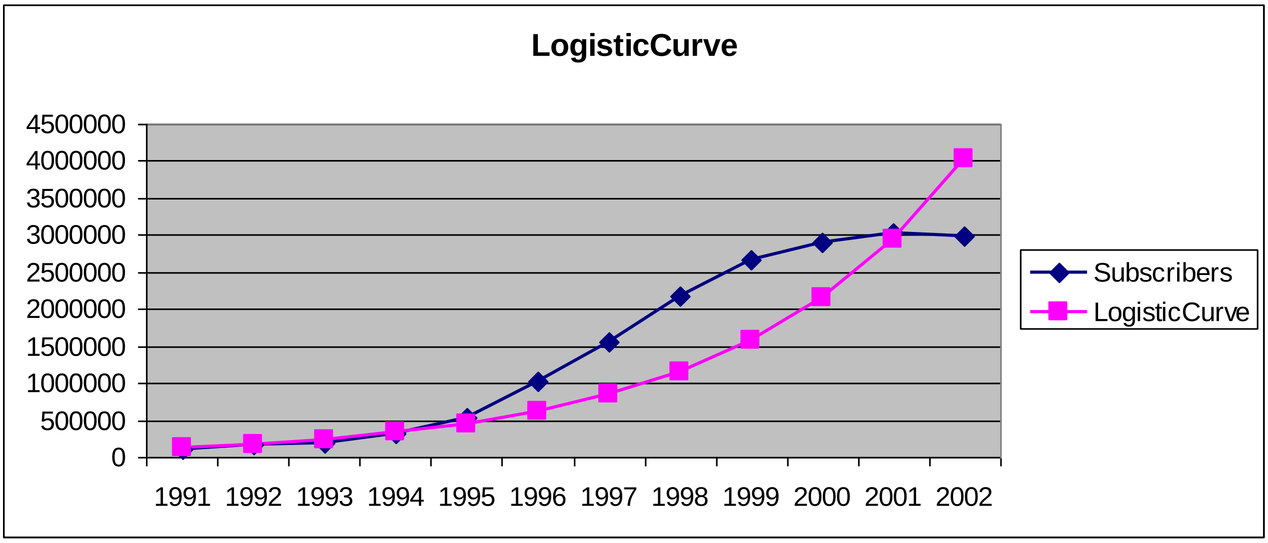
Then

log(*a*) = 14,812   *a* = 2708755,873   *b* = 0,3365

We find the logistic curve

y = 2,60E+11 / (1 + 2708755,873 * exp (-0,3365* *t*))

While using the solver we have the following graph of the measured data and those calibrated.



The optimized model has parameter values of:

k = 2,60E+11   *a* = 2708756   *b* = 0,311265

For this model the value of the sum of squared errors is: 4,51388E+12.

The value just examined is greater than previously calculated. At this moment we can say that the value of K=2,50E+11 is the best value.

In summary:

K	*a*	*b*	*Sum Square Error*
1E+10	104192,98	0,311305122	4,5127E+12
1E+11	1041320,3	0,311224029	4,51233E+12
1E+12	10417525	0,311257202	4,5137E+12
2E+11	2082334,1	0,311209524	4,51196E+12
3E+11	3125172,4	0,31125561	4,51359E+12
1,5E+11	1562816,2	0,311269558	4,51398E+12
2,5E+11	2602544	0,311196671	4,51155E+12
2,6E+11	2708755,9	0,311264575	4,51388E+12

1.6.3 Chi-Square Test

f_0 and f_e are observed frequencies and expected frequencies.

H0 is the null hypothesis and H1 is the alternative hypothesis.

H0: There is a difference between the observed and expected frequencies.

H1: There is a difference between the observed and expected frequencies.

Test statistic:

$$\chi^2 = \sum \left[\frac{(f_o - f_e)^2}{f_e} \right]$$

The critical value is a value taken from a random variable chi-square with degrees of freedom (k-1), where k is the number of classes in which the random sample is grouped.

Step 1: Fix null hypothesis and alternative.

H0: There is agreement between the data.

H1: There is no agreement between the data.

In our case, the data are:

- internet subscribers in millions.
- estimated data using the logistic equation.

Our goal is to apply the Chi Square Test to verify the consistency of the actual data and those estimated in terms of square error.

Step 2: Select the level of significance α.

Let $\alpha = 0.01$

The **level of significance** of a test is usually given by a test of hypothesis testing. In the simplest case is defined as the probability of accepting or rejecting the null hypothesis. The decision in this case is done using the p-value: if the value p (p-value) is less than the significance level, then the null hypothesis is rejected. The lower the p value, the more significant is the result.

Step 3: Select the test statistic

How to use the test $\chi 2$ statistics.

Step 4: H0 is rejected if the p-value is less than α = 0.01.

We calculate the test statistic at each logistic curve identified in the first part of the analysis:

1. *Logistic curve with K=1E+10*

Sum value $\chi^2 = 3058282{,}015$

This value was obtained by applying the formula:

$$\chi^2 = \sum \left[\frac{(f_o - f_e)^2}{f_e} \right]$$

Degrees of freedom = 12-1 = 11

The p ($\chi 2$ > 3058282,015) = 0 has been calculated using the method *chi2cdf* ($\chi 2$ = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(3058282.015,11)
>> chi =
>>   0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of $\chi 2$, which is equal to 24,73 (tabulated value), and then the p-value of 0 is less than the chosen significance level (α = 0.01).

Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

2. Logistic curve with K=1E+11

Sum value χ^2 = 3057855,693

Degrees of freedom = 12-1 = 11

The p ($\chi 2$>3057855,693) = 0 has been calculated using the method *chi2cdf* ($\chi 2$ = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(3057855.693,11)
>> chi =
>>  0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of $\chi 2$, which is equal to 24,73 (tabulated value), and then the p-value of 0 is less than the chosen significance level ($\alpha = 0.01$).
Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

3. *Logistic curve with K=1E+12*

Sum value χ^2 = 3059282,039
Degrees of freedom = 12-1 = 11

The p ($\chi 2$>3059282,039) = 0 has been calculated using the method *chi2cdf* ($\chi 2$ = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(3059282.039,11)
>> chi =
>> 0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of $\chi 2$, which is equal to 24,73 (tabulated value), and then the p-value of 0 is less than the chosen significance level ($\alpha = 0.01$).
Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

4. *Logistic curve with K=2E+11*

Sum value χ^2 = 3057476,286

Degrees of freedom = 12-1 = 11

The p (χ2>3057476.286) = 0 has been calculated using the method *chi2cdf* (χ2 = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(3057476.286,11)
>>  chi =
>>  0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of χ2, which is equal to 24,73 (tabulated value), and then the p-value of 0 is less than the chosen significance level (α = 0.01).

Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

5. *Logistic curve with K=3E+11*

Sum value χ^2 = 3059161.613

Degrees of freedom = 12-1 = 11

The p (χ2>3059161,613) = 0 has been calculated using the method *chi2cdf* (χ2 = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(3059161.613,11)
>> chi =
>> 0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of χ2, which is equal to 24,73 (tabulated value), and then the p-value of 0 is less than the chosen significance level (α = 0.01).

Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

6. Logistic curve with K=1,50E+11

Sum value χ^2 = 3059566,982

Degrees of freedom = 12-1 = 11

The p (χ2>3059566,982) = 0 has been calculated using the method *chi2cdf* (χ2 = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(3059566.982,11)
>> chi =
>>   0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of χ2, which is equal to 24,73 (tabulated value), and then the p-value of 0 is less than the chosen significance level (α = 0.01).

Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

7. Logistic curve with K=2,50E+11

Sum value χ^2 = 3057046,484

Degrees of freedom = 12-1 = 11

The p (χ2>3057046,484) = 0 has been calculated using the method *chi2cdf* (χ2 = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(3057046.484,11)
>> chi =
>>  0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of χ2, which is equal to 24,73 (tabulated value), and then the p-value of 0 is less than the chosen significance level (α = 0.01).

Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

- *Logistic curve with K=2,60E+11*

Sum value χ^2 = 3059466,696

Degrees of freedom = 12-1 = 11

The p (χ2>3059466,696) = 0 has been calculated using the method *chi2cdf* (χ2 = X, V = degrees of freedom) of Matlab, which is likely given the value of the variable (it is the cumulative distribution function, cdf).

Matlab code snippet:

```
>> chi=1-chi2cdf(3059466.696,11)
>>  chi =
>>  0
```

It rejects the null hypothesis because the value of the test statistic exceeds the critical value of $\chi2$, which is equal to 24,73 (tabulated value), and then the p-value of 0 is less than the chosen significance level ($\alpha = 0.01$).

Rejecting the null hypothesis it is concluded that there is no agreement between the estimated and actual data. This performance should be evaluated considering that it is very rare that the results obtained in specimens agree exactly with the theoretical results expected according to the rules of probability.

In summary:

K	$\chi2$
1E+10	3.058.282
1E+11	3.057.856
1E+12	3.059.282
2E+11	3.057.476
3E+11	3.059.162
1,5E+11	3.059.567
2,5E+11	3.057.046
2,6E+11	3.059.467

The chi-square test was applied to test whether the logistic curve that best approximates the performance of the input data coincides with the one identified in the first phase of the study. According to the statistical hypothesis test, the observed data are significantly different from the actual data. However, the calculation of statistics shows that the lower value is in correspondence of $K = 2{,}50E+11$.

This coincides with what is assumed in the previous phase, namely that the logistic curve that best approximates the performance of the input data is described by the following equation:

$$y = 2{,}50E+11 \;/\; (1 + 2602544 \;*\; \exp\,(-0{,}311197 * t)).$$

2 Logistic curves

In this last phase we are going to draw the curves logistical features of the Internet and Mobile systems in the world and of the Internet and mobile systems for different countries.. What we want to do is to go to compare these curves with the available data on detection systems, from this comparison trying to plot a forecast of logistics tracking systems.

2.1 Logistic Curve Internet

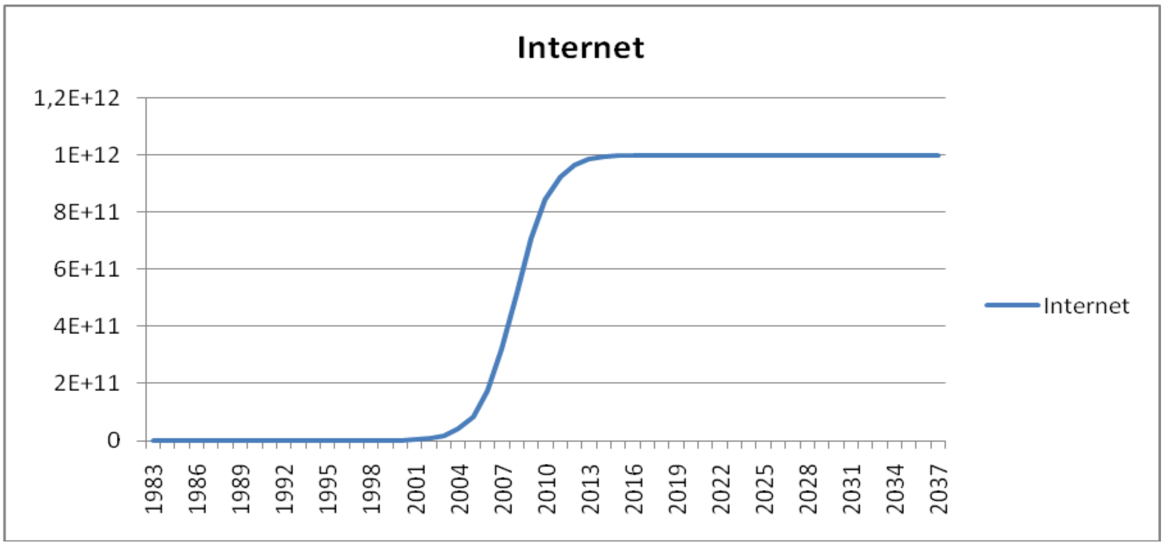
In this section, we go to graph the logistic curve of the Internet and then devote ourselves to estimates of the logistic curve detection systems.

2.1.1 Performance Internet

Through the model equation of the logistic curve that we have identified as the best for the description and the Internet, go to graphics its performance. The equation for the logistic curve is as follows: y = 1E+12 / (1 + 1401767812 * exp (-0,8125* t))

The data on the Internet are en Annex A1.

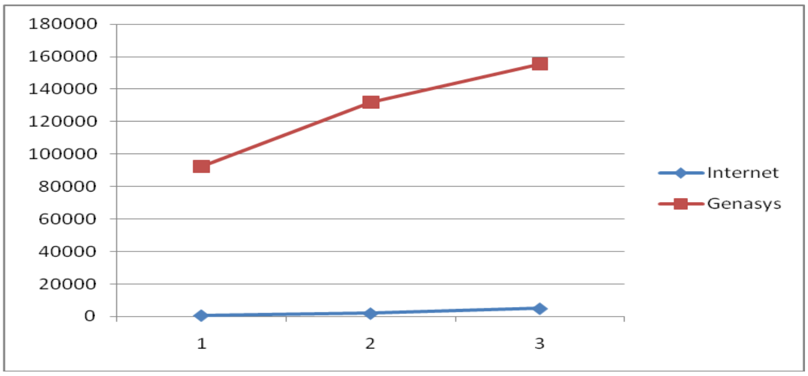
These data are plotted as follows:



2.1.2 Forecast of logistic curve of Localization Systems

At this stage, we employed the study of the evolution of location systems based on what happens on the Internet.

First, we compared the first three periods of development. We have plotted the course and then we evaluated the relationship that existed between the values. The report was made between values for the same periods.



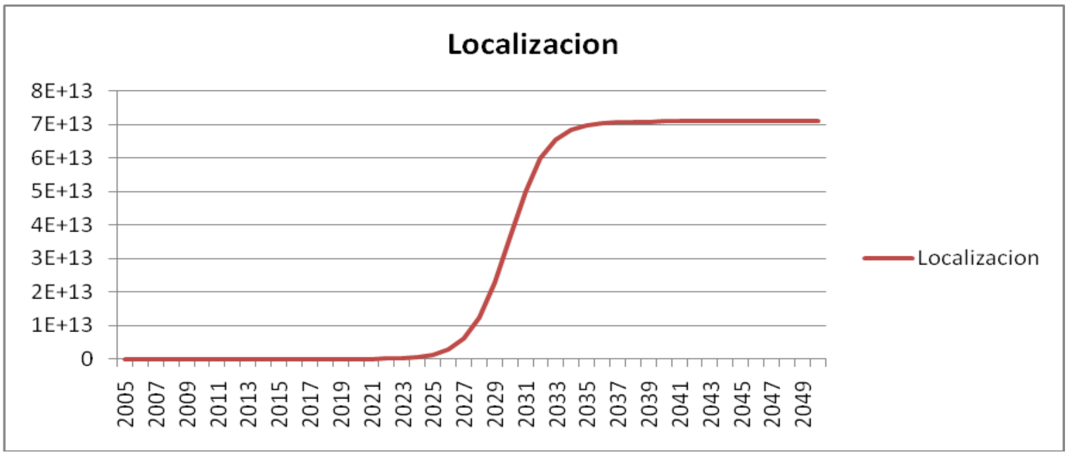
After assessing the existing relationships we have proceeded with the calculation of the average value of that relationship.

	Internet	Genasys	Coeff
1	800	92395,83	115,4948
2	2000	131927,1	65,96354
3	5000	155468,8	31,09375

70,85069	***Middle Value***

Median we went through to calculate the respective values of the logistic curve of Localization Systems.
The calculated values are en Annex A2.

These data are plotted as follows:



2.2 Logistic Curve Mobile Wireless

In this section, we go to graph the logistic curve of the Internet and then devote ourselves to estimates of the logistic curve detection systems.

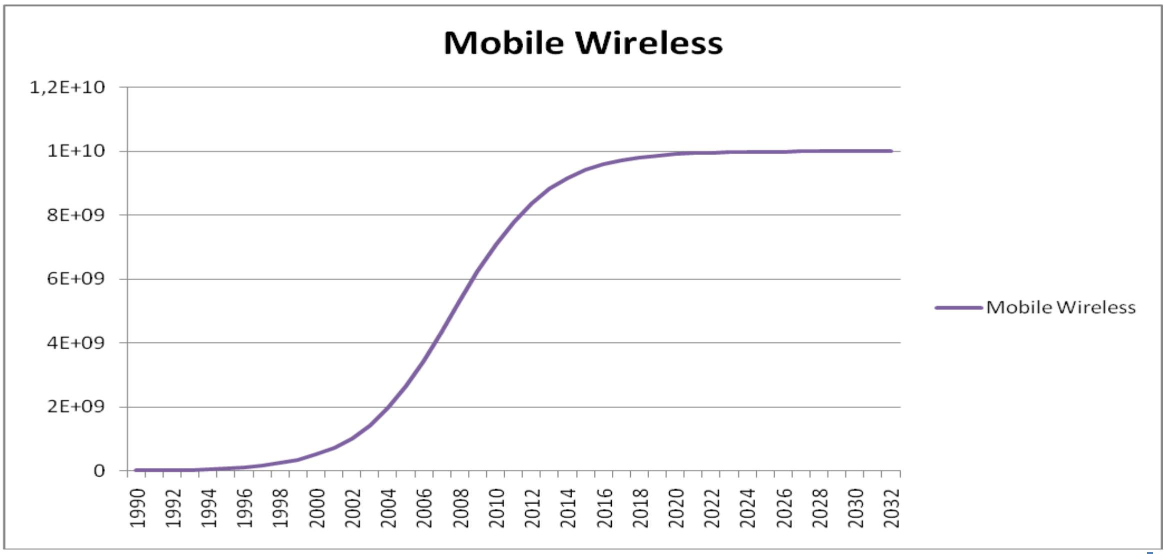
2.2.1 Performance Mobile Wireless

Through the model equation of the logistic curve that we have identified as the best description of the development of Mobile Wireless, go to graphics its performance.The equation for the logistic curve is as follows:

$$y = 1E+10 / (1 + 1234,8441 * \exp(-0,395 * t))$$

The data on the Internet are en Annex B1.

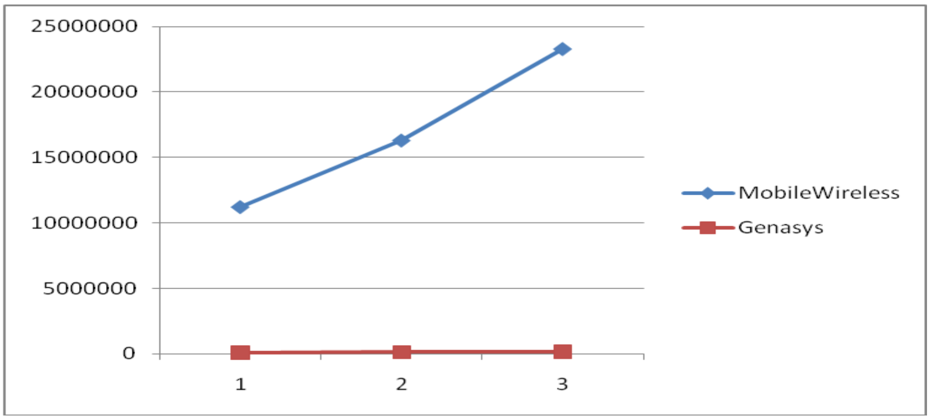
These data are plotted as follows:



2.2.2 Forecast of logistic curve of Localization Systems

At this stage, we employed the study of the evolution of location systems based on what's happening in Mobile Wireless.

First, we compared the first three periods of development. We have plotted the course and then we evaluated the relationship that existed between the values. The report was made between values for the same periods.



After assessing the existing relationships we have proceeded with the calculation of the average value of that relationship.

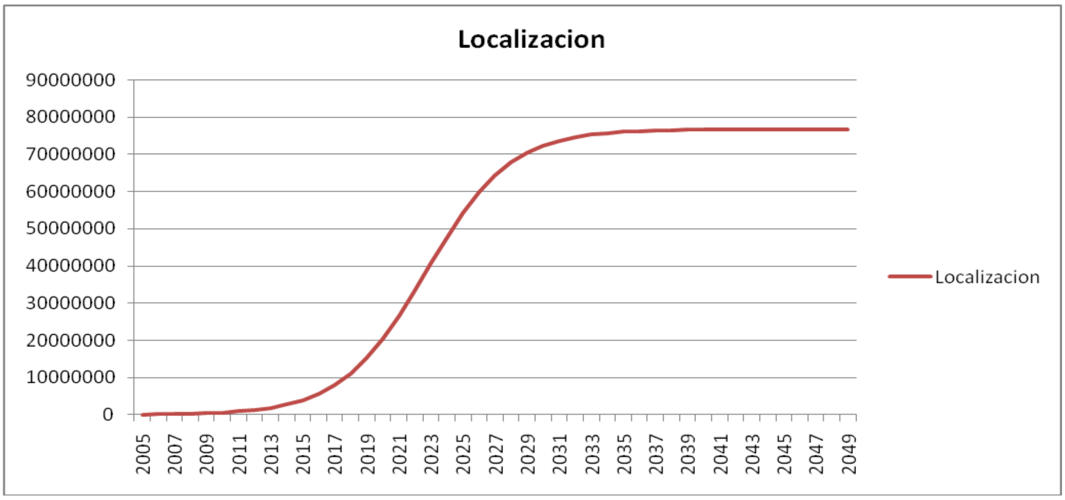
	MobileW	Genasys	Coeff
1	11210000	92395,83	0,008242
2	16280000	131927,1	0,008104
3	23250000	155468,8	0,006687

0,007678	***Middle Value***

Median we went through to calculate the respective values of the logistic curve of Localization Systems.

The calculated values are en Annex B2.

These data are plotted as follows:



2.3 Logistic Curve Internet in Saudi Arabia

In this section, we go to graph the logistic curve of the Internet in Saudi Arabia and then devote ourselves to estimates of the logistic curve detection systems.

2.3.1 Performance Internet

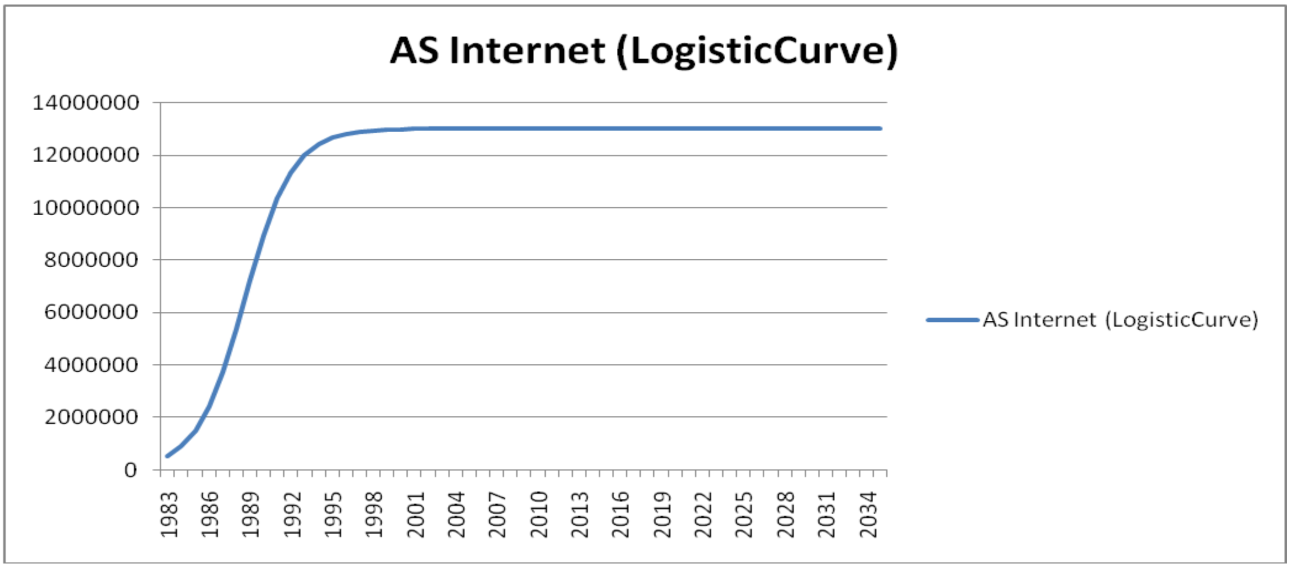
Through the model equation of the logistic curve that we have identified as the best for the description and the Internet, go to graphics its performance.

The equation for the logistic curve is as follows:

$$y = 13006411 / (1 + 42{,}13563 * \exp(-0{,}566015 * t))$$

The data on the Internet are those set in Annex C1.

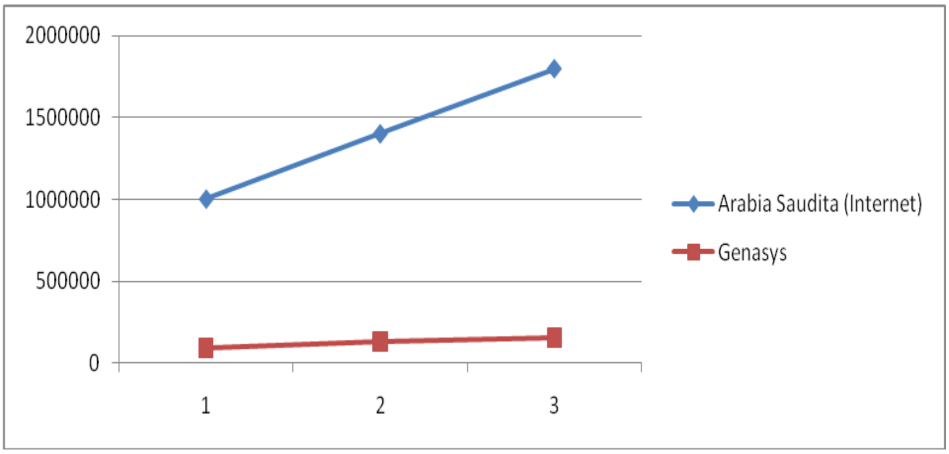
These data are plotted as follows:



2.3.2 Forecast of logistic curve of Localization Systems

At this stage, we employed the study of the evolution of location systems based on what happens on the Internet.

First, we compared the first three periods of development. We have plotted the course and then we evaluated the relationship that existed between the values. The report was made between values for the same periods.



After assessing the existing relationships we have proceeded with the calculation of the average value of that relationship.

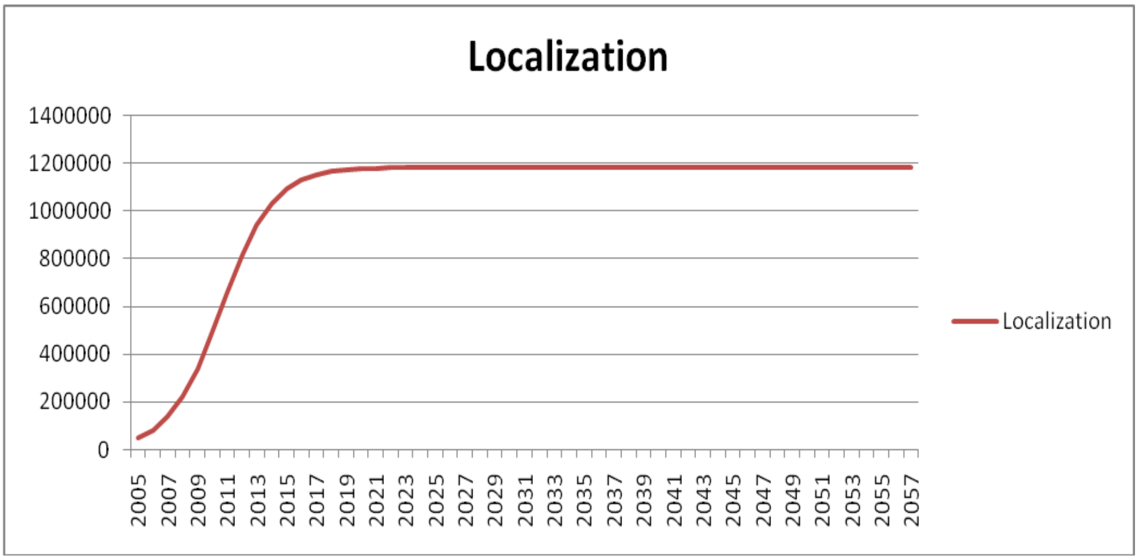
	Internet	Genasys	Coeff
1	800	92395,83	115,4948
2	2000	131927,1	65,96354
3	5000	155468,8	31,09375

70,85069 ***Middle Value***

Median we went through to calculate the respective values of the logistic curve of Localization Systems.

The calculated values are those set out in Annex C2.

These data are plotted as follows:



2.4 Logistic Curve Mobile in Saudita Arabia

In this section, we go to graph the logistic curve of the Mobile in Saudi Arabia and then devote ourselves to estimates of the logistic curve detection systems.

2.4.1 Performance Mobile Wireless

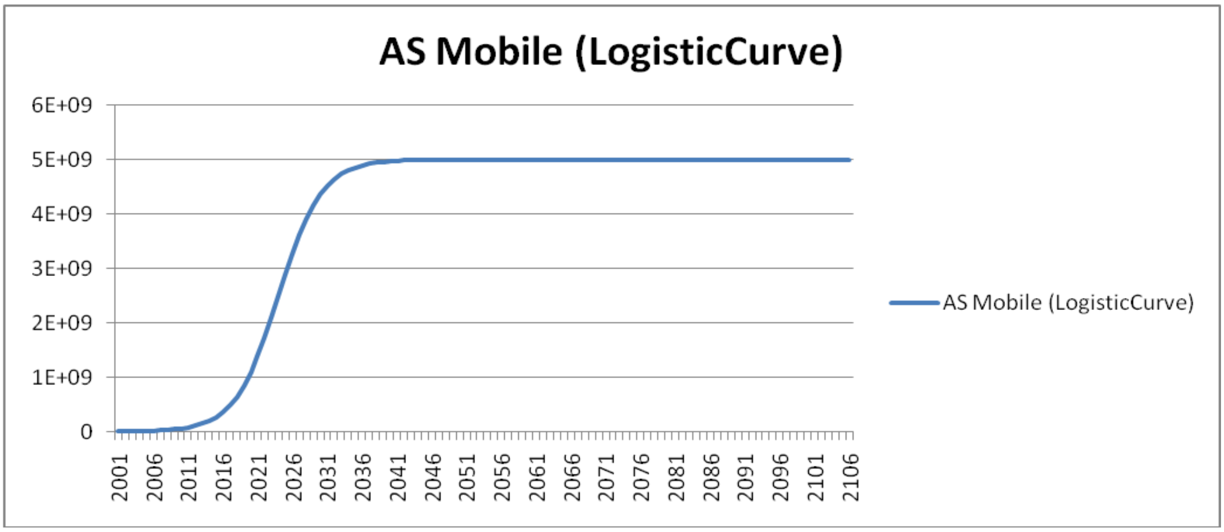
Through the model equation of the logistic curve that we have identified as the best description of the development of Mobile Wireless, go to graphics its performance.

The equation for the logistic curve is as follows:

$$y = 5E{+}09 \,/\, (1 + 2137{,}9 \ast \exp(-0{,}3194 \ast t))$$

The data on the Internet are those set in Annex D1.

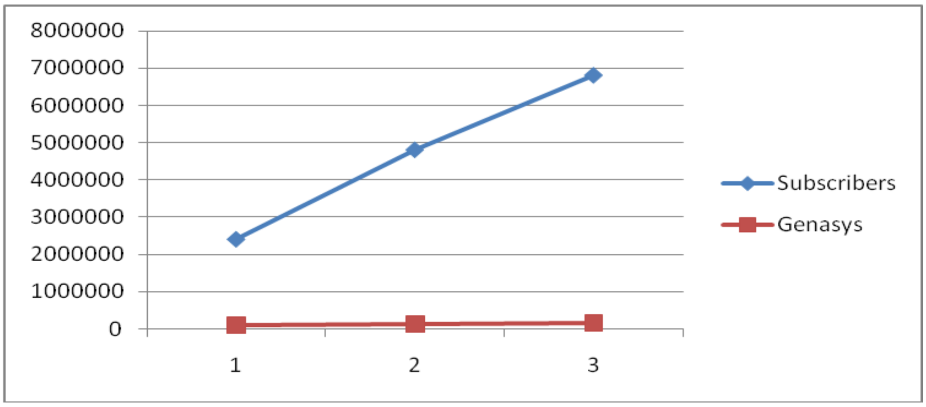
These data are plotted as follows:



2.4.2 Forecast of logistic curve of Localization Systems

At this stage, we employed the study of the evolution of location systems based on what's happening in Mobile Wireless.

First, we compared the first three periods of development. We have plotted the course and then we evaluated the relationship that existed between the values. The report was made between values for the same periods.



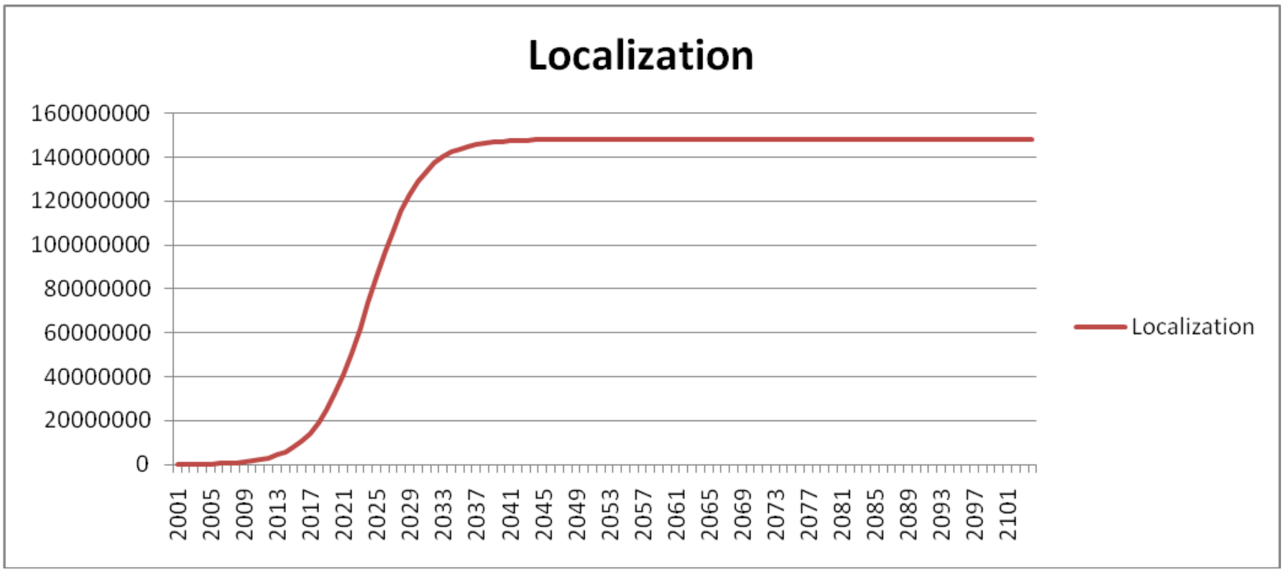
After assessing the existing relationships we have proceeded with the calculation of the average value of that relationship.

	Arabia Saudita (Mobile)	Genasys	Coeff
1	2400000	92395,83	0,038498
2	4800000	131927,1	0,027485
3	6800000	155468,8	0,022863

0,029615 ***Middle Value***

Median we went through to calculate the respective values of the logistic curve of Localization Systems. The calculated values are those set out in Annex D2.

These data are plotted as follows:



2.5 Logistic Curve Internet in Us

In this section, we go to graph the logistic curve of the Internet in Us and then devote ourselves to estimates of the logistic curve detection systems.

2.5.1 Performance Internet

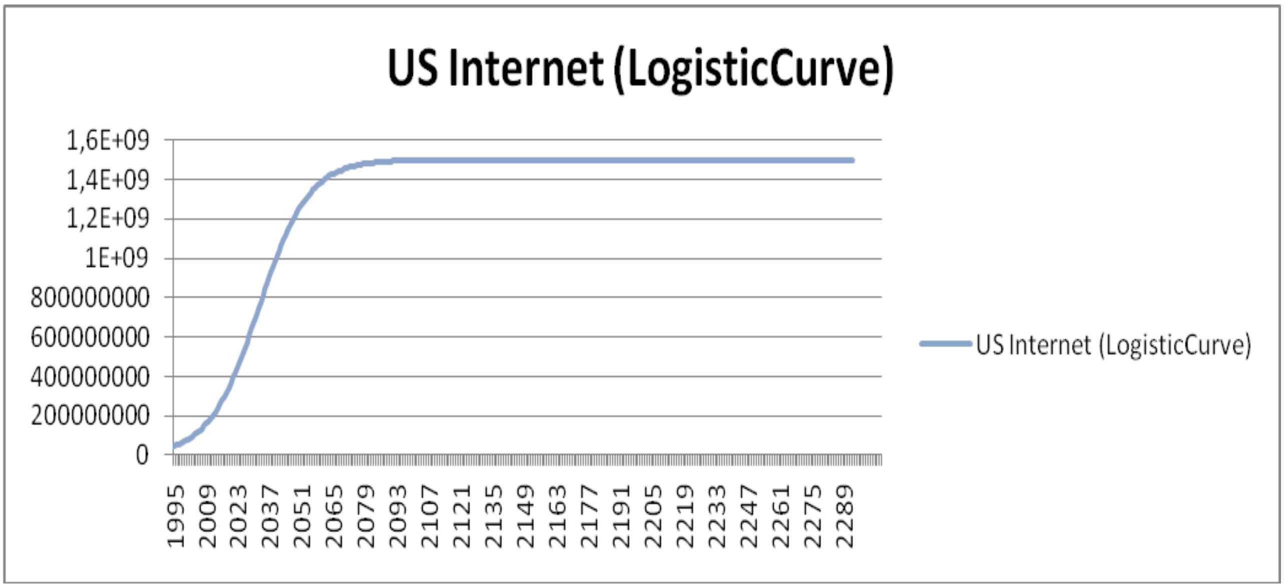
Through the model equation of the logistic curve that we have identified as the best for the description and the Internet, go to graphics its performance.

The equation for the logistic curve is as follows:

$$y = 1{,}5\ E{+}09 \,/\, (1 + 33{,}4671 * \exp(-0{,}09184 * t))$$

The calculated values are those set out in Annex E1.

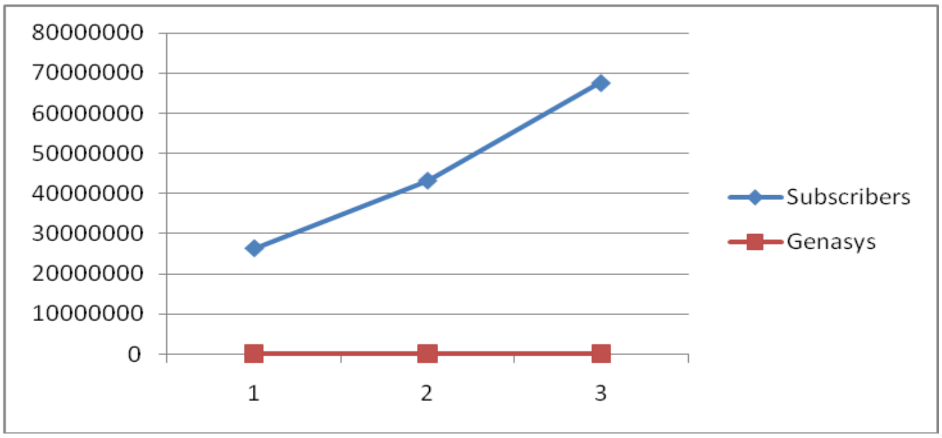
These data are plotted as follows:



2.5.2 Forecast of logistic curve of Localization Systems

At this stage, we employed the study of the evolution of location systems based on what happens on the Internet.

First, we compared the first three periods of development. We have plotted the course and then we evaluated the relationship that existed between the values. The report was made between values for the same periods.



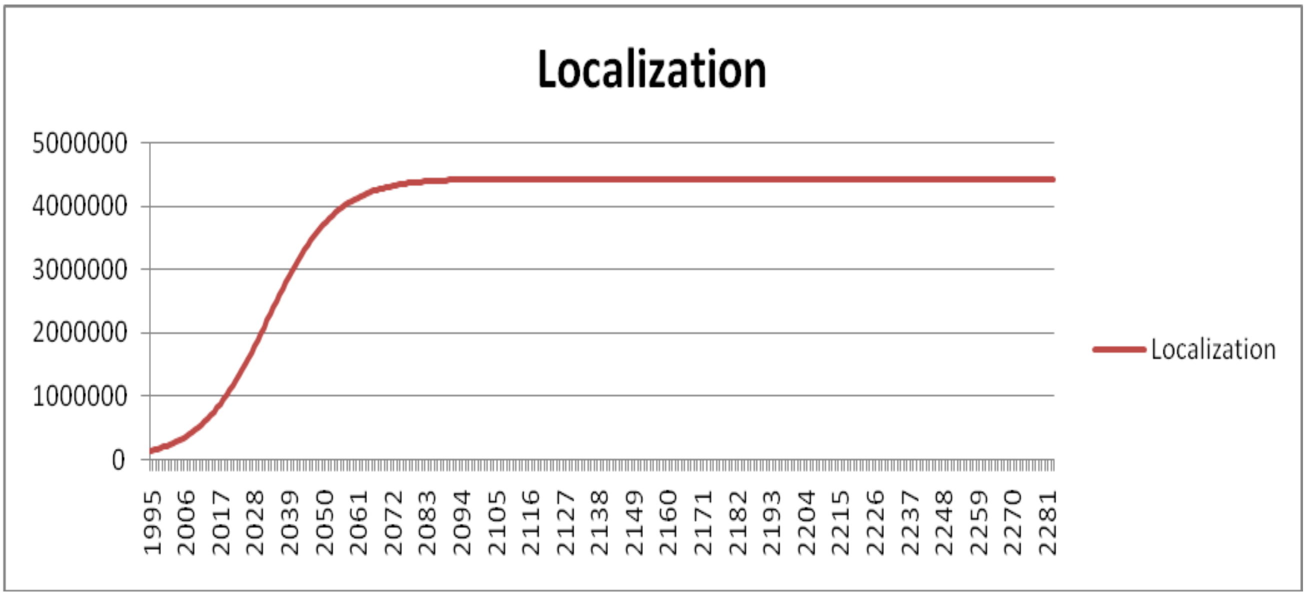
After assessing the existing relationships we have proceeded with the calculation of the average value of that relationship.

	US (Internet)	Genasys	Coeff
1	26288888,92	92395,83	0,003515
2	43188888,94	131927,1	0,003055
3	67600000,08	155468,8	0,0023

0,002956	***Middle Value***

Median we went through to calculate the respective values of the logistic curve of Localization Systems.The calculated values are those set out in Annex E2.

These data are plotted as follows:



2.6 Logistic Curve Internet in Norway

In this section, we go to graph the logistic curve of the Internet in Norway and then devote ourselves to estimates of the logistic curve detection systems.

2.6.1 Performance Internet

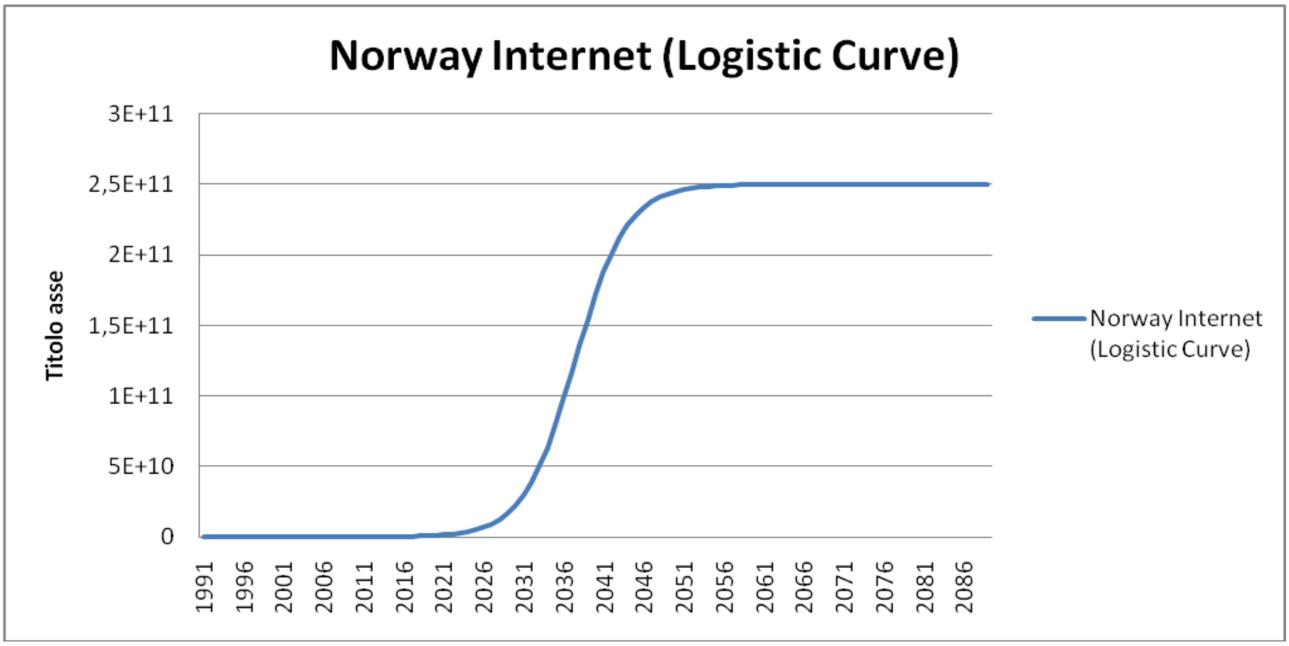
Through the model equation of the logistic curve that we have identified as the best for the description and the Internet, go to graphics its performance.

The equation for the logistic curve is as follows:

$$y = 2,5E+11 / (1 + 2602544,04 * \exp(-0,31119667 * t))$$

The data on the Internet are those set in Annex F1

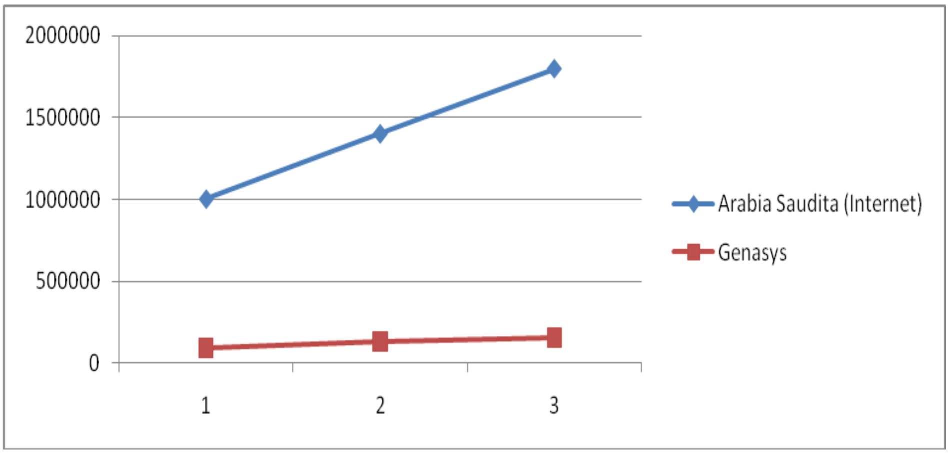
These data are plotted as follows:



2.6.2 Forecast of logistic curve of Localization Systems

At this stage, we employed the study of the evolution of location systems based on what happens on the Internet.

First, we compared the first three periods of development. We have plotted the course and then we evaluated the relationship that existed between the values. The report was made between values for the same periods.



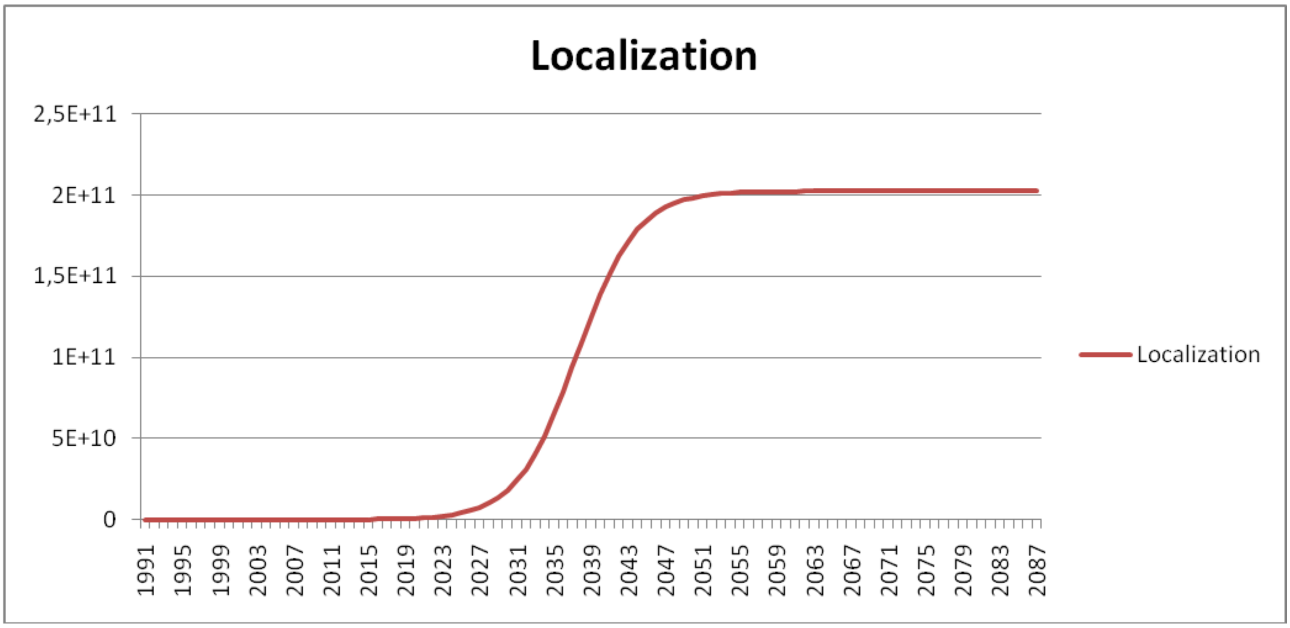
After assessing the existing relationships we have proceeded with the calculation of the average value of that relationship.

	Norway(Internet)	Genasys	Coeff
1	116851,275	92395,83	0,790713
2	163591,785	131927,1	0,806441
3	186962,04	155468,8	0,831552

0,809569	***Middle Value***

Median we went through to calculate the respective values of the logistic curve of Localization Systems. The calculated values are those set out in Annex F2.

These data are plotted as follows:


Localization
2,5E+11
2E+11
1,5E+11
1E+11
5E+10
0
1991
1995
1999
2003
2007
2011
2015
2019
2023
2027
2031
2035
2039
2043
2047
2051
2055
2059
2063
2067
2071
2075
2079
2083
2087
Localization

3 Quantitative and qualitative analysis of the logistic curve identified

At this stage we are going to make qualitative and quantitative analysis of the logistic curve that we have identified. Such analysis will be accomplished by comparing some characteristic parameters of mathematical curves.

3.1 Comparison of the logistic curve of Internet and the logistic curve of location systems

To highlight more easily and effectively the differences between the two curves, we input the data on them in a table.

The curves that we compare are that of the Internet and the logistic curve detection systems. This curve is the result of the data held company Genasys and as identified for the Internet.

	Internet	***Localization System***
Year Beginning	1983	2005
Year End	2035	2057
Year Saturation	2026	2048
Starting year Rise Time	2005	2027
Ending year Rise Time	2011	2033
Rise Time	6	6
Quantity of volume in Rise Time	8E+11	5,66806 E+13
Quantity of volume in the first during	99999998365	7,08507 E+12
Length of introduction time	22	22
Volume reached in the first during	99999999972	7,08507 E+12
Volume reached at the time of saturation	1 E+12	7,08E+13
Value Chi-Square	4245245095	-

3.2 Comparison of the logistic curve of Mobile Wireless and the logistic curve of location systems

To highlight more easily and effectively the differences between the two curves, we input the data on them in a table.

The curves that we compare are that of the logistic curve Wireless and Mobile Systems Location This curve is the result of the data held company Genasys and as identified for Mobile.

	Mobile Wireless	***Localization System***
Year Beginning	1990	2005
Year End	2037	2053
Year Saturation	2028	2044
Starting year Rise Time	2002	2017
Ending year Rise Time	2014	2029
Rise Time	12	12
Quantity of volume in Rise Time	7999887262	61423134,4
Quantity of volume in the first during	1017035487	7808798,47
Length of introduction time	12	12
Volume reached in the first during	1028875298	7899704,535
Volume reached at the time of saturation	1,00E+10	7,68E+07
Value Chi-Square	327637768.1	-

At this stage we are going to make qualitative and quantitative analysis of the logistic curve that we have identified. Such analysis will be accomplished by comparing some characteristic parameters of mathematical curves.

3.3 Comparison of the logistic curve of the Internet in Saudi Arabia and the logistic curve of location systems

To highlight more easily and effectively the differences between the two curves, we input the data on them in a table.

The curves that we compare are that of the Internet in Saubi Arabia and the logistic curve of location systems. This curve is the result of the data held company Genasys and as identified for the Internet.

	(AS)Internet	*Localization System*
Year Beginning	1983	2005
Year End	2035	2057
Year Saturation	2007	2029
Starting year Rise Time	1985	2007
Ending year Rise Time	1993	2015
Rise Time	9	9
Quantity of volume in Rise Time	1,04E+07	946870,1802
Quantity of volume in the first during	7,79E+05	70870,757
Length of introduction time	3	3
Volume reached in the first during	1,3E+06	118358,773
Volume reached at the time of saturation	1,3E+07	1183587,73
Value Chi-Square	1035873,31	-

3.4 Comparison of the logistic curve of the Mobile in Saudi Arabia and the logistic curve of location systems

To highlight more easily and effectively the differences between the two curves, we input the data on them in a table.

The curves that we compare are that of the logistic curve Wireless and Mobile Systems Location This curve is the result of the data held company Genasys and as identified for Mobile.

	(AS)Mobile	*Localization System*
Year Beginning	2001	2005
Year End	2106	2108
Year Saturation	2056	2060
Starting year Rise Time	2017	2021
Ending year Rise Time	2031	2035
Rise Time	15	15
Quantity of volume in Rise Time	4E+09	118461499,2
Quantity of volume in the first during	4,97E+08	14712422,61
Length of introduction time	17	17
Volume reached in the first during	5E+08	14807687,4
Volume reached at the time of saturation	5E+09	148076874
Value Chi-Square	6443895,289	-

3.5 Comparison of the logistic curve of the Internet in US and the logistic curve of location systems

To highlight more easily and effectively the differences between the two curves, we input the data on them in a table.

The curves that we compare are that of the logistic curve Wireless and Mobile Systems Location This curve is the result of the data held company Genasys and as identified for Mobile.

	(US) Internet	*Localization System*
Year Beginning	1995	2005
Year End	2279	2293
Year Saturation	2141	2151
Starting year Rise Time	2007	2017
Ending year Rise Time	2057	2067
Rise Time	51	51
Quantity of volume in Rise Time	1,2E+09	3547648,522

Quantity of volume in the first during	1,02E+08	302812,5976
Length of introduction time	13	13
Volume reached in the first during	1,5E+08	443456,0652
Volume reached at the time of saturation	1.5E+09	4434560,652
Value Chi-Square	47200035,08	-

3.6 Comparison of the logistic curve of the Internet in Norway and the logistic curve of Location Systems

To highlight more easily and effectively the differences between the two curves, we input the data on them in a table.

The curves that we compare are that of the logistic curve Wireless and Mobile Systems Location This curve is the result of the data held company Genasys and as identified for Mobile.

	(Norway) Internet	***Localization System***
Year Beginning	1991	2005
Year End	2089	2101
Year Saturation	2070	2084
Starting year Rise Time	2030	2044
Ending year Rise Time	2045	2059
Rise Time	16	16
Quantity of volume in Rise Time	2E+11	1,61914E+11
Quantity of volume in the first during	2,5E+10	20239109295
Length of introduction time	40	40
Volume reached in the first during	2,5E+10	20239215451
Volume reached at the time of saturation	2,5E+11	2,02392E+11
Value Chi-Square	3.057.046	-

4 Annex

Annex A1

Below, the values of the logistic curve of the Internet in the world.

		Internet			Internet
1	1983	1607,687674	29	2011	9,2431E+11
2	1984	3623,092712	30	2012	9,64938E+11
3	1985	8165,019225	31	2013	9,84132E+11
4	1986	18400,72663	32	2014	9,92896E+11
5	1987	41467,96559	33	2015	9,96835E+11
6	1988	93452,40298	34	2016	9,98593E+11
7	1989	210604,7627	35	2017	9,99375E+11
8	1990	474619,7867	36	2018	9,99723E+11
9	1991	1069604,811	37	2019	9,99877E+11
10	1992	2410463,343	38	2020	9,99945E+11
11	1993	5432215,435	39	2021	9,99976E+11
12	1994	12241983,62	40	2022	9,99989E+11
13	1995	27588170,11	41	2023	9,99995E+11
14	1996	62170683,45	42	2024	9,99998E+11
15	1997	140097234,1	43	2025	9,99999E+11
16	1998	315668364,8	44	2026	1E+12
17	1999	711110350,8	45	2027	1E+12
18	2000	1601134393	46	2028	1E+12
19	2001	3601096129	47	2029	1E+12
20	2002	8078976317	48	2030	1E+12
21	2003	18024273085	49	2031	1E+12
22	2004	39722056593	50	2032	1E+12
23	2005	85271651970	51	2033	1E+12
24	2006	1,7361E+11	52	2034	1E+12
25	2007	3,21318E+11	53	2035	1E+12
26	2008	5,16196E+11	54	2036	1E+12
27	2009	7,06271E+11	55	2037	1E+12
28	2010	8,44207E+11			

Annex A2 - Below, the values of the logistic curve of the localization systems, estimated by the logistic curve of the Internet in the world.

Date	*Localizacion*
2005	113905,781
2006	256698,6186
2007	578497,2459
2008	1303704,178
2009	2938033,975
2010	6621167,233
2011	14921492,76
2012	33627139,38
2013	75782238,87
2014	170782991,1
2015	384876211,8
2016	867352986,1
2017	1954640888
2018	4404835820
2019	9925985706
2020	22365321455
2021	50382659020
2022	1,13441E+11
2023	2,5514E+11
2024	5,72401E+11
2025	1,27703E+12
2026	2,81434E+12
2027	6,04156E+12
2028	1,23004E+13
2029	2,27656E+13
2030	3,65729E+13
2031	5,00398E+13
2032	5,98127E+13

Date	*Localizacion*
2033	6,5488E+13
2034	6,83665E+13
2035	6,97264E+13
2036	7,03474E+13
2037	7,06265E+13
2038	7,0751E+13
2039	7,08064E+13
2040	7,0831E+13
2041	7,0842E+13
2042	7,08468E+13
2043	7,0849E+13
2044	7,08499E+13
2045	7,08504E+13
2046	7,08505E+13
2047	7,08506E+13
2048	7,08507E+13
2049	7,08507E+13
2050	7,08507E+13
2051	7,08507E+13
2052	7,08507E+13
2053	7,08507E+13
2054	7,08507E+13
2055	7,08507E+13
2056	7,08507E+13
2057	7,08507E+13
2058	7,08507E+13

Annex B1- Below, the values of the logistic curve of the Mobile Wireless in the World.

Period	Date	*Mobile Wireless*
1	1990	11839810,48
2	1991	17321190,28
3	1992	25333802,96
4	1993	37039219,04
5	1994	54123744,46
6	1995	79026086,22
7	1996	115253227,8
8	1997	167806716,5
9	1998	243732812,8
10	1999	352779940,9
11	2000	508074503,7
12	2001	726581480,6
13	2002	1028875298
14	2003	1437443207
15	2004	1972579902
16	2005	2645389713
17	2006	3449082115
18	2007	4352444652
19	2008	5300952882
20	2009	6228208834
21	2010	7073505151
22	2011	7796387910
23	2012	8381564439

Period	Date	*Mobile Wireless*
24	2013	8834574353
25	2014	9173290890
26	2015	9420026312
27	2016	9596362563
28	2017	9,72E+09
29	2018	9,81E+09
30	2019	9,87E+09
31	2020	9,91E+09
32	2021	9,94E+09
33	2022	9,96E+09
34	2023	9,97E+09
35	2024	9,98E+09
36	2025	9,99E+09
37	2026	9,99E+09
38	2027	9,99E+09
39	2028	1,00E+10
40	2029	1,00E+10
41	2030	1,00E+10
42	2031	1,00E+10
43	2032	1,00E+10
44	2033	1,00E+10
45	2034	1,00E+10

Annex B2 - Below, the values of the logistic curve of the localization systems, estimated by the logistic curve of the Mobile Wireless in the world.

Date	*Localizacion*
2005	90906,06485
2006	132992,099
2007	194512,9391
2008	284387,1238
2009	415562,11
2010	606762,29
2011	884914,2832
2012	1288419,969
2013	1871380,537
2014	2708644,386
2015	3900996,039
2016	5578692,608
2017	7899704,535
2018	11036688,94
2019	15145468,49
2020	20311302,21
2021	26482052,48
2022	33418070,04
2023	40700716,23
2024	47820187,43
2025	54310372,55
2026	5,99E+07
2027	6,44E+07

Date	*Localizacion*
2028	6,78E+07
2029	7,04E+07
2030	7,23E+07
2031	7,37E+07
2032	7,46E+07
2033	7,53E+07
2034	7,58E+07
2035	7,61E+07
2036	7,63E+07
2037	7,65E+07
2038	7,66E+07
2039	7,66E+07
2040	7,67E+07
2041	7,67E+07
2042	7,67E+07
2043	7,67E+07
2044	7,68E+07
2045	7,68E+07
2046	7,68E+07
2047	7,68E+07
2048	7,68E+07
2049	7,68E+07

Annex C1 - Below, the values of the logistic curve of the Internet in Saudi Arabia.

	AS Internet (LogisticCurve)		
1983	521844,4276	2026	13006411,25
1984	891851,0647	2027	13006411,25
1985	1492835,691	2028	13006411,25
1986	2417970,13	2029	13006411,25
1987	3730656,567	2030	13006411,25
1988	5393014,518	2031	13006411,25
1989	7219573,522	2032	13006411,25
1990	8938462,506	2033	13006411,25
1991	10335658,47	2034	13006411,25
1992	11342309,36	2035	13006411,25
1993	12006252,82		
1994	12419014,76		
1995	12666257,39		
1996	12811069,37		
1997	12894774,65		
1998	12942789,79		
1999	12970211,49		
2000	12985832,87		
2001	12994719,19		
2002	12999770,11		
2003	13002639,69		
2004	13004269,55		
2005	13005195,14		
2006	13005720,74		
2007	13006019,18		
2008	13006188,64		
2009	13006284,86		
2010	13006339,49		
2011	13006370,51		
2012	13006388,12		
2013	13006398,12		
2014	13006403,8		
2015	13006407,02		
2016	13006408,85		
2017	13006409,89		
2018	13006410,48		
2019	13006410,81		
2020	13006411		
2021	13006411,11		
2022	13006411,17		
2023	13006411,21		
2024	13006411,23		
2025	13006411,24		

Annex C2 - Below, the values of the logistic curve curve detection systems estimated by the localization systems identified the logistic curve of the Internet in Saudi Arabia.

	Localization		
2005	47488,01548	2047	1183587,724
2006	81158,74182	2048	1183587,724
2007	135848,5416	2049	1183587,725
2008	220036,0814	2050	1183587,725
2009	339490,9813	2051	1183587,725
2010	490766,1045	2052	1183587,725
2011	656983,5779	2053	1183587,725
2012	813403,0439	2054	1183587,725
2013	940548,3391	2055	1183587,725
2014	1032153,902	2056	1183587,725
2015	1092572,977	2057	1183587,725
2016	1130134,45		
2017	1152633,612		
2018	1165811,549		
2019	1173428,757		
2020	1177798,151		
2021	1180293,535		
2022	1181715,085		
2023	1182523,743		
2024	1182983,379		
2025	1183244,511		
2026	1183392,83		
2027	1183477,059		
2028	1183524,888		
2029	1183552,047		
2030	1183567,467		
2031	1183576,223		
2032	1183581,194		
2033	1183584,017		
2034	1183585,62		
2035	1183586,53		
2036	1183587,047		
2037	1183587,34		
2038	1183587,506		
2039	1183587,601		
2040	1183587,655		
2041	1183587,685		
2042	1183587,703		
2043	1183587,712		
2044	1183587,718		
2045	1183587,721		
2046	1183587,723		

Annex D1 - Below, the values of the logistic curve of the Mobile in Saudi Arabia.

	AS Mobile (LogisticCurve)		
2001	3216734,316	2044	4991588079
2002	4426109,288	2045	4993885190
2003	6089611,089	2046	4995555569
2004	8377272,298	2047	4996769947
2005	11522347,69	2048	4997652669
2006	15844429,3	2049	4998294240
2007	21780663,48	2050	4998760500
2008	29927597,85	2051	4999099334
2009	41096690,59	2052	4999345555
2010	56386839,94	2053	4999524471
2011	77277085,49	2054	4999654478
2012	105741225	2055	4999748943
2013	144382301,8	2056	4999817583
2014	196576857,5	2057	4999867457
2015	266603876	2058	4999903695
2016	359708682,7	2059	4999930026
2017	482016977,9	2060	4999949157
2018	640175198,5	2061	4999963058
2019	840572910,2	2062	4999973158
2020	1088046828	2063	4999980497
2021	1384134454	2064	4999985830
2022	1725260894	2065	4999989704
2023	2101595512	2066	4999992519
2024	2497415940	2067	4999994564
2025	2893365963	2068	4999996051
2026	3270064317	2069	4999997130
2027	3611723226	2070	4999997915
2028	3908429457	2071	4999998485
2029	4156532031	2072	4999998899
2030	4357513330	2073	4999999200
2031	4516179239	2074	4999999419
2032	4638908650	2075	4999999578
2033	4732350696	2076	4999999693
2034	4802640860	2077	4999999777
2035	4855036857	2078	4999999838
2036	4893829986	2079	4999999882
2037	4922407720	2080	4999999914
2038	4943382192	2081	4999999938
2039	4958734449	2082	4999999955
2040	4969949156	2083	4999999967
2041	4978129493	2084	4999999976
2042	4984090135	2085	4999999983
2043	4988430025	2086	4999999987

2087	4999999991
2088	4999999993
2089	4999999995
2090	4999999996
2091	4999999997
2092	4999999998
2093	4999999999
2094	4999999999
2095	4999999999
2096	4999999999
2097	5000000000
2098	5000000000
2099	5000000000
2100	5000000000
2101	5000000000
2102	5000000000
2103	5000000000
2104	5000000000
2105	5000000000
2106	5000000000

Annex D2 - Below, the values of the logistic curve detection systems estimated by the localization systems identified the logistic curve of the Mobile in Saudi Arabia.

2005	95264,79239	2048	147827751,8
2006	131080,8854	2049	147895781,6
2007	180346,1148	2050	147945250,5
2008	248096,0589	2051	147981214,8
2009	341238,6453	2052	148007356,9
2010	469238,712	2053	148026357,3
2011	645042,5123	2054	148040165,7
2012	886317,027	2055	148050200,4
2013	1217093,895	2056	148057492,3
2014	1669917,398	2057	148062791
2015	2288589,85	2058	148066641,2
2016	3131566,01	2059	148069438,8
2017	4275935,983	2060	148071471,6
2018	5821697,311	2061	148072948,7
2019	7895573,71	2062	148074021,9
2020	10652907,45	2063	148074801,7
2021	14275113,46	2064	148075368,3
2022	18959028,44	2065	148075779,9
2023	24893881,78	2066	148076079,1
2024	32222914,61	2067	148076296,4
2025	40991660,62	2068	148076454,3
2026	51094248,01	2069	148076569,1
2027	62239538,76	2070	148076652,4
2028	73961909,07	2071	148076713
2029	85688117,43	2072	148076757
2030	96844180,36	2073	148076789
2031	106962537	2074	148076812,2
2032	115749603,2	2075	148076829,1
2033	123097253,9	2076	148076841,4
2034	129049390,5	2077	148076850,3
2035	133748340,8	2078	148076856,8
2036	137383018,3	2079	148076861,5
2037	140150339,5	2080	148076864,9
2038	142232009,1	2081	148076867,4
2039	143783736,2	2082	148076869,2
2040	144932609,2	2083	148076870,5
2041	145778949,5	2084	148076871,4
2042	146400116,4	2085	148076872,1
2043	146854779,2	2086	148076872,6
2044	147186907	2087	148076873
2045	147429170,7	2088	148076873,3
2046	147605697,4	2089	148076873,5
2047	147734224,8	2090	148076873,6

2091	148076873,7
2092	148076873,8
2093	148076873,8
2094	148076873,9
2095	148076873,9
2096	148076873,9
2097	148076873,9
2098	148076873,9
2099	148076874
2100	148076874
2101	148076874
2102	148076874
2103	148076874
2104	148076874
2105	148076874
2106	148076874
2107	148076874
2108	148076874

Annex E1- Below, the values of the logistic curve of the Internet in Us.

	US Internet (LogisticCurve)		
1995	47572965,62	2038	944317208,3
1996	51990225,21	2039	976044535,1
1997	56801597,27	2040	1006906269
1998	62039154,86	2041	1036812849
1999	67736989,65	2042	1065687946
2000	73931223,06	2043	1093468773
2001	80659990,81	2044	1120106058
2002	87963394,84	2045	1145563720
2003	95883416,22	2046	1169818284
2004	104463782,2	2047	1192858084
2005	113749780,1	2048	1214682302
2006	123788009,2	2049	1235299897
2007	134626065,2	2050	1254728464
2008	146312146,1	2051	1272993061
2009	158894575,5	2052	1290125052
2010	172421235,4	2053	1306160984
2011	186938904,4	2054	1321141523
2012	202492498,9	2055	1335110469
2013	219124218,6	2056	1348113858
2014	236872599,5	2057	1360199161
2015	255771484,3	2058	1371414578
2016	275848924,7	2059	1381808429
2017	297126034	2060	1391428634
2018	319615818,3	2061	1400322288
2019	343322017,9	2062	1408535304
2020	368237997,6	2063	1416112143
2021	394345729,9	2064	1423095594
2022	421614918,6	2065	1429526621
2023	450002311,9	2066	1435444258
2024	479451254,3	2067	1440885541
2025	509891520,2	2068	1445885477
2026	541239469,8	2069	1450477051
2027	573398551,4	2070	1454691242
2028	606260166,8	2071	1458557070
2029	639704896,4	2072	1462101651
2030	673604065,2	2073	1465350269
2031	707821613,7	2074	1468326450
2032	742216218,9	2075	1471052048
2033	776643598,9	2076	1473547325
2034	810958921,3	2077	1475831047
2035	845019232,1	2078	1477920563
2036	878685817	2079	1479831900
2037	911826414,7	2080	1481579842

2081	1483178014	2128	1499772937
2082	1484638963	2129	1499792858
2083	1485974232	2130	1499811031
2084	1487194433	2131	1499827611
2085	1488309315	2132	1499842736
2086	1489327829	2133	1499856534
2087	1490258189	2134	1499869121
2088	1491107929	2135	1499880604
2089	1491883952	2136	1499891080
2090	1492592587	2137	1499900637
2091	1493239630	2138	1499909355
2092	1493830387	2139	1499917308
2093	1494369715	2140	1499924564
2094	1494862058	2141	1499931183
2095	1495311484	2142	1499937221
2096	1495721710	2143	1499942729
2097	1496096136	2144	1499947754
2098	1496437872	2145	1499952339
2099	1496749758	2146	1499956521
2100	1497034390	2147	1499960336
2101	1497294142	2148	1499963816
2102	1497531180	2149	1499966991
2103	1497747484	2150	1499969887
2104	1497944863	2151	1499972530
2105	1498124968	2152	1499974940
2106	1498289307	2153	1499977139
2107	1498439258	2154	1499979145
2108	1498576077	2155	1499980975
2109	1498700912	2156	1499982644
2110	1498814812	2157	1499984167
2111	1498918733	2158	1499985556
2112	1499013548	2159	1499986824
2113	1499100053	2160	1499987980
2114	1499178977	2161	1499989035
2115	1499250982	2162	1499989997
2116	1499316676	2163	1499990874
2117	1499376610	2164	1499991675
2118	1499431289	2165	1499992406
2119	1499481174	2166	1499993072
2120	1499526685	2167	1499993680
2121	1499568205	2168	1499994234
2122	1499606083	2169	1499994740
2123	1499640639	2170	1499995202
2124	1499672165	2171	1499995623
2125	1499700926	2172	1499996007
2126	1499727163	2173	1499996357

2127	1499751100	2174	1499996677
2175	1499996969	2222	1499999960
2176	1499997235	2223	1499999963
2177	1499997477	2224	1499999966
2178	1499997699	2225	1499999969
2179	1499997901	2226	1499999972
2180	1499998085	2227	1499999974
2181	1499998253	2228	1499999977
2182	1499998406	2229	1499999979
2183	1499998546	2230	1499999981
2184	1499998674	2231	1499999982
2185	1499998790	2232	1499999984
2186	1499998896	2233	1499999985
2187	1499998993	2234	1499999987
2188	1499999081	2235	1499999988
2189	1499999162	2236	1499999989
2190	1499999235	2237	1499999990
2191	1499999303	2238	1499999991
2192	1499999364	2239	1499999992
2193	1499999420	2240	1499999992
2194	1499999471	2241	1499999993
2195	1499999517	2242	1499999994
2196	1499999559	2243	1499999994
2197	1499999598	2244	1499999995
2198	1499999633	2245	1499999995
2199	1499999665	2246	1499999996
2200	1499999695	2247	1499999996
2201	1499999722	2248	1499999996
2202	1499999746	2249	1499999997
2203	1499999768	2250	1499999997
2204	1499999789	2251	1499999997
2205	1499999807	2252	1499999997
2206	1499999824	2253	1499999998
2207	1499999840	2254	1499999998
2208	1499999854	2255	1499999998
2209	1499999866	2256	1499999998
2210	1499999878	2257	1499999998
2211	1499999889	2258	1499999999
2212	1499999899	2259	1499999999
2213	1499999908	2260	1499999999
2214	1499999916	2261	1499999999
2215	1499999923	2262	1499999999
2216	1499999930	2263	1499999999
2217	1499999936	2264	1499999999
2218	1499999942	2265	1499999999
2219	1499999947	2266	1499999999

2220	1499999951	2267	1499999999
2221	1499999956	2268	1499999999
2269	1499999999		
2270	1500000000		
2271	1500000000		
2272	1500000000		
2274	1500000000		
2275	1500000000		
2276	1500000000		
2277	1500000000		
2278	1500000000		
2279	1500000000		
2280	1500000000		
2281	1500000000		
2282	1500000000		
2283	1500000000		
2284	1500000000		
2285	1500000000		
2286	1500000000		
2287	1500000000		
2288	1500000000		
2289	1500000000		
2290	1500000000		
2291	1500000000		
2292	1500000000		
2293	1500000000		

Annex E2 - Below, the values of the logistic curve detection systems estimated by the localization systems identified the logistic curve of the Internet in Us.

2005	140643,4677		
2006	153702,538	2047	2695699,694
2007	167926,7522	2048	2791754,624
2008	183410,9301	2049	2885552,46
2009	200255,8593	2050	2976791,28
2010	218568,3285	2051	3065206,309
2011	238461,081	2052	3150571,889
2012	260052,6731	2053	3232702,397
2013	283467,2165	2054	3311452,167
2014	308833,9855	2055	3386714,532
2015	336286,8659	2056	3458420,089
2016	365963,6233	2057	3526534,348
2017	398004,9677	2058	3591054,894
2018	432553,3906	2059	3652008,213
2019	469751,7549	2060	3709446,317
2020	509741,6174	2061	3763443,292
2021	552661,2731	2062	3814091,861
2022	598643,512	2063	3861500,07
2023	647813,0919	2064	3905788,142
2024	700283,9395	2065	3947085,567
2025	756156,107	2066	3985528,445
2026	815512,5251	2067	4021257,119
2027	878415,6129	2068	4054414,085
2028	944903,8212	2069	4085142,192
2029	1014988,208	2070	4113583,115
2030	1088649,157	2071	4139876,078
2031	1165833,372	2072	4164156,825
2032	1246451,286	2073	4186556,792
2033	1330375,031	2074	4207202,483
2034	1417437,111	2075	4226215,004
2035	1507429,915	2076	4243709,751
2036	1600106,171	2077	4259796,215
2037	1695180,436	2078	4274577,897
2038	1792331,654	2079	4288152,306
2039	1891206,775	2080	4300611,03
2040	1991425,389	2081	4312039,861
2041	2092585,251	2082	4322518,968
2042	2194268,56	2083	4332123,097
2043	2296048,763	2084	4340921,801
2044	2397497,682	2085	4348979,685
2045	2498192,692	2086	4356356,658
2046	2597723,7	2087	4363108,192

2088	4369285,585	2135	4433676,477
2089	4374936,211	2136	4433754,046
2090	4380103,78	2137	4433824,811
2091	4384828,574	2138	4433889,368
2092	4389147,685	2139	4433948,263
2093	4393095,239	2140	4434001,991
2094	4396702,609	2141	4434051,006
2095	4399998,617	2142	4434095,72
2096	4403009,726	2143	4434136,513
2097	4405760,219	2144	4434173,726
2098	4408272,366	2145	4434207,675
2099	4410566,581	2146	4434238,645
2100	4412661,572	2147	4434266,898
2101	4414574,472	2148	4434292,672
2102	4416320,97	2149	4434316,185
2103	4417915,425	2150	4434337,635
2104	4419370,977	2151	4434357,203
2105	4420699,647	2152	4434375,054
2106	4421912,428	2153	4434391,339
2107	4423019,373	2154	4434406,195
2108	4424029,671	2155	4434419,748
2109	4424951,722	2156	4434432,111
2110	4425793,202	2157	4434443,39
2111	4426561,124	2158	4434453,679
2112	4427261,897	2159	4434463,065
2113	4427901,373	2160	4434471,628
2114	4428484,899	2161	4434479,44
2115	4429017,356	2162	4434486,566
2116	4429503,205	2163	4434493,066
2117	4429946,515	2164	4434498,997
2118	4430351,003	2165	4434504,407
2119	4430720,064	2166	4434509,342
2120	4431056,794	2167	4434513,844
2121	4431364,023	2168	4434517,951
2122	4431644,331	2169	4434521,698
2123	4431900,073	2170	4434525,116
2124	4432133,401	2171	4434528,234
2125	4432346,277	2172	4434531,079
2126	4432540,491	2173	4434533,674
2127	4432717,679	2174	4434536,041
2128	4432879,331	2175	4434538,201
2129	4433026,81	2176	4434540,171
2130	4433161,356	2177	4434541,968
2131	4433284,104	2178	4434543,607
2132	4433396,087	2179	4434545,103
2133	4433498,248	2180	4434546,468

2134	4433591,45	2181	4434547,712
2182	4434548,848	2229	4434560,495
2183	4434549,883	2230	4434560,509
2184	4434550,828	2231	4434560,521
2185	4434551,69	2232	4434560,533
2186	4434552,477	2233	4434560,543
2187	4434553,194	2234	4434560,553
2188	4434553,849	2235	4434560,562
2189	4434554,446	2236	4434560,57
2190	4434554,99	2237	4434560,577
2191	4434555,487	2238	4434560,584
2192	4434555,94	2239	4434560,59
2193	4434556,354	2240	4434560,595
2194	4434556,731	2241	4434560,6
2195	4434557,075	2242	4434560,605
2196	4434557,389	2243	4434560,609
2197	4434557,675	2244	4434560,613
2198	4434557,937	2245	4434560,616
2199	4434558,175	2246	4434560,619
2200	4434558,392	2247	4434560,622
2201	4434558,591	2248	4434560,625
2202	4434558,772	2249	4434560,627
2203	4434558,937	2250	4434560,63
2204	4434559,087	2251	4434560,632
2205	4434559,225	2252	4434560,633
2206	4434559,35	2253	4434560,635
2207	4434559,464	2254	4434560,637
2208	4434559,568	2255	4434560,638
2209	4434559,664	2256	4434560,639
2210	4434559,75	2257	4434560,64
2211	4434559,829	2258	4434560,642
2212	4434559,902	2259	4434560,643
2213	4434559,968	2260	4434560,643
2214	4434560,028	2261	4434560,644
2215	4434560,083	2262	4434560,645
2216	4434560,133	2263	4434560,646
2217	4434560,178	2264	4434560,646
2218	4434560,22	2265	4434560,647
2219	4434560,258	2266	4434560,647
2220	4434560,292	2267	4434560,648
2221	4434560,324	2268	4434560,648
2222	4434560,353	2269	4434560,649
2223	4434560,379	2270	4434560,649
2224	4434560,403	2271	4434560,649
2225	4434560,425	2272	4434560,649
2226	4434560,445	2273	4434560,65

2227	4434560,463	2274	4434560,65
2228	4434560,48	2275	4434560,65
2276	4434560,65		
2277	4434560,651		
2278	4434560,651		
2279	4434560,651		
2280	4434560,651		
2281	4434560,651		
2282	4434560,651		
2283	4434560,651		
2284	4434560,652		
2285	4434560,652		
2286	4434560,652		
2287	4434560,652		
2288	4434560,652		
2289	4434560,652		
2290	4434560,652		
2291	4434560,652		
2292	4434560,652		
2293	4434560,652		

Annex F1 - Below, the values of the logistic curve of the Internet in Norway.

	Norw Internet (LogisticCurve)		
1991	131127,1709	2034	63407427484
1992	178996,115	2035	79219869108
1993	244339,9544	2036	96927218396
1994	333538,0079	2037	1,15906E+11
1995	455298,3916	2038	1,35316E+11
1996	621508,1447	2039	1,54238E+11
1997	848393,685	2040	1,71841E+11
1998	1158104,865	2041	1,8752E+11
1999	1580877,247	2042	2,0095E+11
2000	2157983,617	2043	2,12078E+11
2001	2945762,788	2044	2,21045E+11
2002	4021118,767	2045	2,2811E+11
2003	5489026,753	2046	2,3358E+11
2004	7492777,97	2047	2,37756E+11
2005	10227962,07	2048	2,40911E+11
2006	13961549,48	2049	2,43277E+11
2007	19057931,55	2050	2,45039E+11
2008	26014451,53	2051	2,46346E+11
2009	35509877,47	2052	2,47313E+11
2010	48470516,99	2053	2,48026E+11
2011	66160367,89	2054	2,48551E+11
2012	90303992,9	2055	2,48937E+11
2013	123253904,6	2056	2,4922E+11
2014	168218411	2057	2,49428E+11
2015	229571439,5	2058	2,49581E+11
2016	313273234,7	2059	2,49693E+11
2017	427440496,2	2060	2,49775E+11
2018	583116965,2	2061	2,49835E+11
2019	795311085,5	2062	2,49879E+11
2020	1084386153	2063	2,49911E+11
2021	1477909425	2064	2,49935E+11
2022	2013087163	2065	2,49952E+11
2023	2739925849	2066	2,49965E+11
2024	3725252323	2067	2,49974E+11
2025	5057671869	2068	2,49981E+11
2026	6853398868	2069	2,49986E+11
2027	9262588878	2070	2,4999E+11
2028	12475233940	2071	2,49993E+11
2029	16724743720	2072	2,49995E+11
2030	22285971926	2073	2,49996E+11
2031	29462837648	2074	2,49997E+11
2032	38559542859	2075	2,49998E+11
2033	49830265171	2076	2,49998E+11

2077	2,49999E+11
2078	2,49999E+11
2079	2,49999E+11
2080	2,5E+11
2081	2,5E+11
2082	2,5E+11
2083	2,5E+11
2084	2,5E+11
2085	2,5E+11
2086	2,5E+11
2087	2,5E+11
2088	2,5E+11
2089	2,5E+11

Annex F2- Below, the values of the logistic curve detection systems estimated by the localization systems identified the logistic curve of the Internet in Norway.

	Localization		
2005	106156,464	2047	40341027068
2006	144909,6668	2048	51332673821
2007	197809,9992	2049	64133932923
2008	270021,9587	2050	78469250114
2009	368595,3642	2051	93834134178
2010	503153,5916	2052	1,09548E+11
2011	686833,042	2053	1,24867E+11
2012	937565,545	2054	1,39118E+11
2013	1279828,867	2055	1,5181E+11
2014	1747036,168	2056	1,62683E+11
2015	2384797,591	2057	1,71692E+11
2016	3255372,222	2058	1,78951E+11
2017	4443744,701	2059	1,84671E+11
2018	6065919,133	2060	1,89099E+11
2019	8280238,794	2061	1,9248E+11
2020	11302834,6	2062	1,95034E+11
2021	15428706,43	2063	1,96949E+11
2022	21060487,83	2064	1,98376E+11
2023	28747688,24	2065	1,99434E+11
2024	39240217,39	2066	2,00217E+11
2025	53561368,43	2067	2,00794E+11
2026	73107293,52	2068	2,01219E+11
2027	99782513,38	2069	2,01531E+11
2028	136184374,1	2070	2,01761E+11
2029	185853870,6	2071	2,01929E+11
2030	253616231	2072	2,02053E+11
2031	346042481,8	2073	2,02144E+11
2032	472073291,1	2074	2,0221E+11
2033	643859026,6	2075	2,02259E+11
2034	877885176,4	2076	2,02294E+11
2035	1196469332	2077	2,02321E+11
2036	1629732522	2078	2,0234E+11
2037	2218158431	2079	2,02354E+11
2038	3015847985	2080	2,02364E+11
2039	4094533253	2081	2,02372E+11
2040	5548297772	2082	2,02377E+11
2041	7498702794	2083	2,02381E+11
2042	10099559943	2084	2,02384E+11
2043	13539830398	2085	2,02386E+11
2044	18042027142	2086	2,02388E+11
2045	23852193581	2087	2,02389E+11
2046	31216602136	2088	2,0239E+11

2089	2,0239E+11
2090	2,02391E+11
2091	2,02391E+11
2092	2,02392E+11
2093	2,02392E+11
2089	2,0239E+11
2090	2,02391E+11
2091	2,02391E+11
2092	2,02392E+11
2093	2,02392E+11
2094	2,02392E+11
2095	2,02392E+11
2096	2,02392E+11
2097	2,02392E+11
2098	2,02392E+11
2099	2,02392E+11
2100	2,02392E+11
2101	2,02392E+11

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