# The Mereological Problem of Entanglement

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9th Apr 2021

Preprint. https://philarchive.org/archive/NGETMPv6

The mereological problem of entanglement concerns the question which objects there are on the micro and macro level, when a system is in an entangled state, and which of the objects carries which properties. This paper proceeds from a recent taxonomy of the possible mereological models for entangled systems and systematically discusses which of them is compatible with the quantum mechanical evidence. It reveals that entangled quantum systems neither describe undivided wholes nor objects that stand in irreducible relations. The appropriate model assumes that the entangled property is an irreducible non-relational plural property carried collectively by the micro objects while there is no macro object.

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## 1 Introduction

The peculiar features of entangled quantum systems (Einstein, Podolsky, and Rosen 1935; Schrödinger 1935) pose at least four philosophical puzzles: a spatio-temporal problem, a problem for individuation, a causal problem, and a mereological problem. The latter is at present the least developed of the four riddles. It concerns the question which objects there are, when a system is in an entangled state, which of them stand in the parthood relation, and, if one understands the problem in a wider sense, also which of them carries which properties, especially the entangled property. This paper aims at a precise formulation and at solving this mereological puzzle of entanglement.

On the one hand, the problem does not seem unfamiliar: Due to some kind of holism that entangled quantum states exhibit (e.g. Esfeld 2001, also Bohm and Hiley 1993), it has long been conjectured that also the mereological features of such systems might be unusual. There are two widespread main ideas: One is that entangled states describe intrinsic properties of undivided wholes, while a rival conception assumes that they refer to irreducible entanglement relations between micro objects ("relational holism"; Teller 1986). What kind of holism exactly do we have and what does it imply for the involved objects? Often, in this early debate about the mereology of entanglement, the focus has been on the irreducible entangled property, whereas the implications for the objects and their mereological relations have been treated *en passant*; accordingly the mereological conclusions tend to rest on imprecise concepts, intuitive inferences and implicit assumptions.

 $<sup>^{\</sup>rm 1}$  Non-locality or backwards causation? Bell 1964, Maudlin 1994.

<sup>&</sup>lt;sup>2</sup> Non-individuals or weak discernibility? Redhead and Teller 1992, Saunders 2006.

<sup>&</sup>lt;sup>3</sup> Causally unexplained correlations or causal unfaithfulness? Näger 2016.

Only in recent years philosophers have started to apply the precise concepts, principles and positions of mereology to the quantum realm, putting the question for the objects and their mereological relations to the center of interest and on solid grounds. This includes both applying the formal theory of parts and wholes (classical extensional mereology)<sup>4</sup> as well as considering the positions of the qualitative, metaphysical debate (initiated by Inwagen 1990). It seems that in the discussion among mereologists the dominant view is that the irreducible entangled state requires the existence of a macro object (Schaffer 2010; Calosi and Tarozzi 2014; Calosi and Morganti 2016), which can either have parts or not. In contrast, Bohn (2012) and Brenner (2018) argue that entangled systems need not require the existence of a macro object since the irreducible entangled property can be carried collectively by the micro objects.

The claim of collectively carried properties makes clear that the inference from properties to the existence of objects, which is at the heart of the mereological problem of entanglement, is not straightforward. In particular, it reveals that arguments for the existence of a macro object that do not explicitly rule out collective properties might have been to hasty. On the other hand, neither Bohn nor Brenner provide evidence from the quantum mechanical formalism why to believe in their proposal.

In this paper I shall examine the quantum mechanical evidence for and against possible solutions to the mereological problem in a systematic way. I start (section 2) from a recent comprehensive survey of the mereological models that are available for entangled systems (Näger and Strobach, 2020) and proceed to approach an appropriate model by ruling out all others. I first discard certain obvious but inappropriate strategies for this task (section 3), before I present reliable arguments that are based on evidence from quantum mechanics (sections 4–6). The aim is to justify the choice of the mereological model by the best scientific theory that we have of the phenomenon.

My arguments aim to show that quantum mechanics speaks against modeling entangled systems as objects that stand in irreducible relations (sections 4) as well as against understanding them as undivided wholes (section 5); it even speaks against assuming a macro object in addition to the micro objects (section 6). My somewhat surprising result then will be that quantum theory suggests a model along the ideas of Bohn that the entangled property is an irreducible non-relational plural property carried by the micro objects while there is no macro object.<sup>5</sup>

#### 2 Possible models

#### 2.1 Systems, levels and ontology

The most simple systems that are capable of having an entangled state are so called "two-particle systems", say, a system of two electrons. There are two levels of description: On

<sup>&</sup>lt;sup>4</sup> Tarski 1929; Leonard and Goodman 1940; Simons 1987

<sup>&</sup>lt;sup>5</sup> Brenner's idea is similar, but he assumes that the plural property is relational, which turns out to be in conflict with the quantum mechanical description.

the micro level one considers each one-particle system ("micro system") in itself while on the macro level one considers the two-particle system ("macro system") as a whole. So the distinction "micro" vs. "macro" here is a relative rather than an absolute one, and especially "macro system" does *not* denote systems of perceptible size (of the magnitude of  $10^{23}$  objects). However, much of what I say about the macro system can easily be generalized to n-particle systems (with  $n \geq 2$ ), insofar these systems are correctly described by quantum mechanics.<sup>6</sup>

I take "system" to be a concept on the epistemic level which is ontologically neutral: Talk about a system does not imply that there is an object corresponding to the system. Especially, the fact that there is a micro-level description and a macro-level description does not per se imply that the micro objects or the macro object must exist. It will be a central part of our below discussion which features the descriptions must have in order to provide evidence for or against the existence of the corresponding objects.

I denote the possibly existing disjunct objects corresponding to the one-particle systems as a and b, respectively (the single electrons, the "micro objects"), and the possibly existing object corresponding to the two-particle system as c (the "macro object"). Since a and b have the same role, I consider only models according to which either both a and b exists or none.

When a and b are of the same kind (e.g. electrons), quantum mechanics requires a certain (anti-)symmetry of their states.<sup>7</sup> The micro objects of such systems have been interpreted to be either non-individuals or to only have weak identity (individuational problem of entanglement, see Footnote 2), which poses two problems: First, this seems to mingle the mereological problem at hand with the individuational problem. Second, non-individuals and weakly identical objects would not be suited to be described by classical mereology, which is based on first order predicate logic and therefore presupposes that objects can be given names. In order to avoid both problems in this paper, I just mimic quantum mechanics' strategy to deal with such states: I refer to them by usual names and require symmetry of properties, i.e. that a has a certain property if and only if b has that property.<sup>8</sup> While our pretension of being able to attach names should not be interpreted literally, the symmetrization blurs the inappropriately strong assumptions such that our results should be compatible with different solutions of the individuational problem, which then can be neglected here and tackled separately.

Note that also "particle" is meant to be an ontologically neutral concept: To say that there is an object corresponding to a one-particle system is not to say that the object is a tiny billiard ball; and to say that there is an object corresponding to a two-particle system neither implies that there are two separate objects. In order to avoid these misleading

 $<sup>^{6}</sup>$  It is an empirical question and I leave it open here whether the generalization includes systems with a size of even  $10^{23}$  particles.

<sup>&</sup>lt;sup>7</sup> Symmetry for bosons, antisymmetry for fermions.

<sup>&</sup>lt;sup>8</sup> Quantum mechanics describes macro states as composed of micro states, labels the micro states by unique indices and then requires to (anti-)symmetrize the macro state.

<sup>&</sup>lt;sup>9</sup> "Two-particle system" rather refers to the facts that the system's state space is in a non-trivial sense

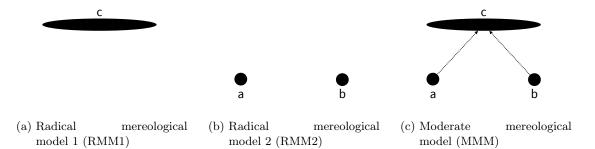


Figure 1: Possible mereological models for a quantum mechanical two-particle system. Blobs represent existing objects, dotted arrows denote the is-part-of relation.

associations, I shall mostly speak of micro and macro systems in the following rather than of one- and two-particle systems.

## 2.2 Mereological models

Any mereological model must answer two questions:

- (i) Which objects are there?
- (ii) Which of the existing objects stand in the is-part-of relation?

Consider again a two-particle quantum system. By what we have said in Section 2.1, there are three possible mereological models for such a system (Figure 1). While the models RMM1 and RMM2 are radical in the sense that only the macro object or the micro objects exist, respectively, MMM is moderate in that it assumes the existence of objects on both levels such that the following claims are true: each a and b, respectively, is a part of c; c is the mereological sum of a and b; a and b compose c.

Examining the existence of objects (and below whether instances of properties exist) it is important to distinguish the following cases (where X is some entity):

- 1. X exists and its existence does not depend on other entities (fundamental existence, strong ontological realism).
- 2. X exists and its existence depends on other entities (dependent existence, weak ontological realism).
- 3. X does not exist but there is a paraphrase for X such that "X exists" is true given the paraphrase (reductive existence; ontological reductionism).
- 4. X does not exist and there is no sensible paraphrase that would make "X exists" true (non-existence; ontological eliminativism).

composed of the state spaces of two one-particle systems and that when measuring at the system, one finds two localized objects ("particles"), which, however, is compatible with the fact that *before* measurement there was only one macro object.

Let us agree here that when a mereological model claims the existence of an object it means fundamental or dependent existence. <sup>10</sup> Especially, if an object exists only reductively, it is not noted in our mereological models. Mereological nihilism, for instance, would only assume the existence of mereological atoms in its models since according to the position non-trivial composite objects like tables do not exist in a fundamental or dependent, but only in a reductive sense: "tables exist" is true by the paraphrase "table-wise arranged atoms exist". I further emphasize that to require composite objects to exist fundamentally or dependently excludes composition as identity, <sup>11</sup> since identity clearly amounts to reductive existence. While this assumption is not shared by all mereologists, <sup>12</sup> I think it is justified by the fact that it provides the concept of composition with a substantive ontological content: To say that there is a mereological sum, then means that there is a further object that is distinct from the parts. Hence, when I talk about existence simpliciter in the following I always mean fundamental or dependent existence.

Note further that usual mereological models only indicate parthood relations but not which of the objects exist fundamentally and which dependently. Model MMM, for instance, does not tell whether a and b are fundamental or whether c is. While the natural assumption is that parts are more fundamental than the objects they compose, this thesis has been attacked in the quantum realm: Schaffer (2010) claims that entangled quantum wholes are more fundamental than their parts. In the following I understand mereological models to be neutral concerning this question, if not enhanced by further assumptions.

#### 2.3 Part-property models for entangled systems

I shall now extend the mereological models to include properties. This is a natural thing to do since the main reason why we assume the existence of objects is for them to carry properties, and it is the properties that seem to be unusual according to the quantum mechanical description.

#### Quantum states, properties, bearers

In quantum theory, properties are described by states. Similar to the concept "system" I understand "state of a system" as a concept on an epistemic level which is ontologically neutral:<sup>13</sup> It aims at describing properties but does not per se imply that in fact there is

<sup>&</sup>lt;sup>10</sup> I.e. existence inside the "ontology room" (Van Inwagen, 2014, p. 1).

<sup>&</sup>lt;sup>11</sup> While there are different varieties of the view labelled "composition as identity", I take it that they all commit to the following claim: If the xs compose y, then y is identical to the xs. (Wallace, 2011; Calosi, 2016b)

<sup>&</sup>lt;sup>12</sup> Calosi (2016a; 2016b) derives some interesting consequences of composition as identity.

<sup>&</sup>lt;sup>13</sup> Note that this is not to assume an epistemic interpretation of quantum states (which holds that the state of a system describes our state of knowledge about a system, such that changes in state are changes in our knowledge). The neutral concept of "system" I have in mind here is both compatible with realist and anti-realist readings of quantum states.

a corresponding property instance. In this sense, the relation between states and properties resembles that between predicates and properties. Whether there are properties corresponding to certain states, will be a central topic of our subsequent discussion. A "micro state" is the state of one micro system and does not refer to other systems; a "macro state", in contrast, is the state of a macro system comprising several micro systems.

In this paper we assume a realist understanding of quantum theory. One central tenet of such interpretations is

(QSR) Quantum state realism: Each fundamental quantum state of a suitable quantum model appropriately describes a property instance.

Fundamental states are those that are required to derive all other states according to the theory in a given model. Consider, for instance, a spin state of a two-particle quantum system that is in product form,

$$|\psi\rangle_{12} = |\uparrow\rangle_1|\downarrow\rangle_2 \tag{1}$$

(" $|\uparrow\rangle_1$ " means that particle 1 has spin direction up, and " $|\downarrow\rangle_2$ " ascribes spin direction down to particle 2.) Since the tensor product is the rule of composition in quantum theory, in this case the macro state  $|\psi\rangle_{12}$  is separable into the micro states  $|\uparrow\rangle_1$  and  $|\downarrow\rangle_2$ . Then, the micro states are fundamental, whereas the macro state can be derived as the tensor product of the micro states. Consequently, the realist is committed to the claim that there is a property instance described by  $|\uparrow\rangle_1$  and one described by  $|\downarrow\rangle_2$ , but she should not assume the existence of a property instance that corresponds to  $|\psi\rangle_{12}$ . The latter exists only reductively and its existence can be paraphrased by referring to the instances described by  $|\uparrow\rangle_1$  and  $|\downarrow\rangle_2$ .

Besides having product form, the states of quantum macro systems can be entangled. One of the simplest examples is the Bell singlet state, <sup>14</sup> describing again the spin direction of a two-particle system:

$$|\psi\rangle_{12} = \frac{1}{\sqrt{2}}|\uparrow\rangle_1|\downarrow\rangle_2 - \frac{1}{\sqrt{2}}|\downarrow\rangle_1|\uparrow\rangle_2 \tag{2}$$

In contrast to (1), an entangled state like the Bell singlet state neither has product form (it is an equally weighted sum of two product states), nor can it be rewritten in product form. Since, as we have just said, the tensor product is the condition of separability in quantum theory, this implies that the entangled state cannot be separated into micro states; it is a non-separable macro state. This moreover implies:

<sup>&</sup>lt;sup>14</sup> In this exposition we presuppose that quantum states are rays in Hilbert space ("ray view"). Only later we generalize our arguments to the alternative view that quantum states are statistical operators in Hilbert space ("statistical operator view").

<sup>&</sup>lt;sup>15</sup> Precisely: There is no basis relative to which the entangled state can be written as a tensor product  $|\psi\rangle_{12} \neq |\psi\rangle_{1} \otimes |\psi\rangle_{2}$  of states from its subspaces,  $|\psi_{i}\rangle \in \mathcal{H}_{i}$  and  $\mathcal{H}_{12} = \mathcal{H}_{1} \otimes \mathcal{H}_{2}$ .

Measuring at a system that is in this state yields in 50% of the cases that particle 1 is spin up and particle 2 is spin down, while in 50% of the cases particle 1 is spin down and particle 2 is spin up. This

(¬DMi) **No well-defined micro states**: The micro systems do not have a well-defined spin micro state.

And consequently:

(Irr) Irreducibility: An entangled state is not reducible to the micro states.<sup>17</sup>

Entangled states being irreducible, non-separable macro states constitutes well-known quantum holism. Since in this way, entangled states are fundamental, a realist interpretation of quantum theory needs to assume that there is a property instance described by the entangled state.

Concerning existing property instances we further require:

(EPB) **Existence of property bearers**: <sup>18</sup> For every existing property instance there is an object that carries it, or there are several objects that carry it collectively.

By this plausible principle the property instance described by the entangled quantum state needs an object that carries it. Then, two central questions emerge:

- (iii) Of which arity is the irreducible entangled property?
- (iv) Which object(s) bear the irreducible entangled property?

#### Six models

There are three main answers to these two questions, each of which comes in two variants. Näger and Strobach (2020) have argued that these six part-property models exhaust the conceivable options:

First, the property P described by the entangled quantum state might be carried by the macro object c corresponding to the two-particle system, so P is a monadic singular macro property and Pc holds. We call this scenario "monistic holism". Requiring the existence of the macro object, monistic holism is compatible with the mereological models RMM1 and MMM, so it comes in two variants: The radical variant, assuming RMM1, says that the macro object c is the only existing object, i.e. it is an undivided whole ("radical monistic holism", RMH, figure 2a) in the sense that it is a macro object that does not have parts. Since it is spatially extended and partless, it is an extended simple (cf. Simons 1987). The moderate version, assuming MMM, claims that besides c the two micro objects a and b exist as well ("moderate monistic holism", MMH, figure 2b; Schaffer (2010); Calosi and Tarozzi 2014; Calosi and Morganti 2016).

correlation of the measurement outcomes is well-known to hold also under space-like separation of the two measurements, yielding empirical evidence for famous quantum non-locality and the associated spatio-temporal problem of entanglement.

<sup>&</sup>lt;sup>17</sup> Note that the failure of micro-reduction does not rest on an incommensurability of concepts (since both levels are described by the same theory) nor on an epistemic shortcoming (since one can prove mathematically that there is no product form of the entangled state).

<sup>&</sup>lt;sup>18</sup> Cf. Calosi and Tarozzi's (2014, 70) instantiation principle.

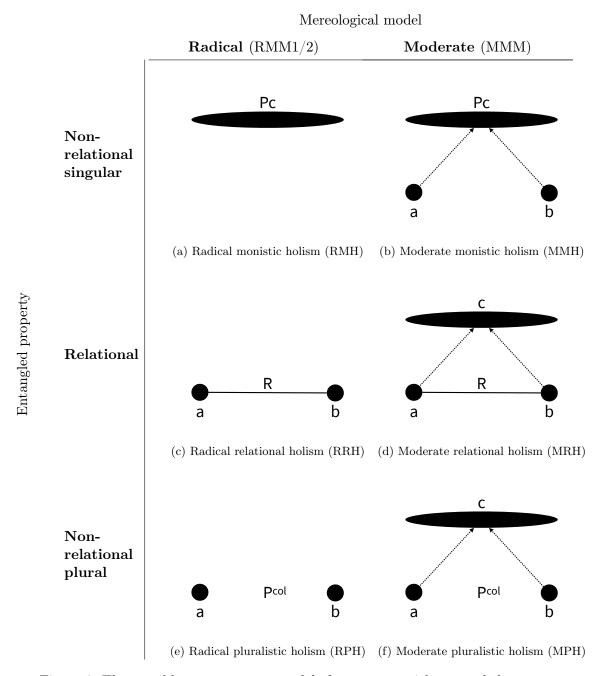


Figure 2: The possible part-property models for a two-particle entangled system

Second, the entangled state might describe a property that is carried collectively by the two particles. Since we have said above that non-separable states count as macro states this scenario claims that several micro objects collectively carry a macro property. The most straightforward way is to think of the entangled property as an irreducible relational property R holding between a and b ("relational holism"; Teller 1986, 1989; Esfeld 2004). Presupposing the existence of the micro objects, this scenario is compatible with mereological models RMM2 and MMM, yielding either the model radical relational holism (RRH, figure 2c, Brenner 2018), which assumes that only a and b exist, or the model moderate relational holism (MRH, figure 2d), which additionally assumes the existence of the macro object c.

There is a third option, introduced to the debate by Bohn (2012), that is easily overlooked and that is another form of collective attribution. The idea is that a macro property that is collectively carried by the micro objects might be a non-relational plural property  $P^{\text{col}}$ . <sup>19</sup> The idea can be explained as follows. Similar to relational properties, which are plural properties,  $P^{\text{col}}$  is plural as well in that it is jointly carried by several objects; by contrast,  $P^{\text{col}}$  is a non-relational feature of the objects taken collectively. Compare: "Five dogs surround a cat" refers to a relational plural property (since it is a statement about the spatial relations between the dogs and the cat), while "having temperature 12 °C" refers to a non-relational plural property (since temperature is proportional to the mean kinetic energy of the atoms of the system, it is carried by all atoms collectively; but temperature is not relational). <sup>20,21</sup>

While temperature reduces to the micro properties, the idea in the present proposal is that the entangled property is an *irreducible* non-relational plural property. This position, which I call "pluralistic holism", again comes in two variants, depending on which mereological model one presumes. Starting from RMM2, which assumes that only the micro objects exist, yields the model radical pluralistic holism (RPH, figure 2e, Bohn 2012), whereas presupposing MMM, which additionally involves the macro object, yields the model moderate pluralistic holism (MPH, figure 2f).

The overview of possible model reveals the following important fact:

(MaPB) Macro property bearing: A macro property can be borne either by a macro object (as a singular property) or by several micro objects collectively (as a plural property, either relational or non-relational).

This insight blocks direct inferences from the existence of an irreducible macro property to the existence of the corresponding macro object. For this reason, property holism

<sup>&</sup>lt;sup>19</sup> For the plural logic underlying collective predication see Oliver and Smiley 2016.

<sup>&</sup>lt;sup>20</sup> Generally, any mean property of micro properties seems to be a non-relational plural property.

<sup>&</sup>lt;sup>21</sup> I caution that for many cases there is a gap between predicates and properties in that a predicate being monadic plural does not imply that the denoted property is monadic plural. There are plenty of monadic plural *predicates*, like "writing a book together", "jointly carrying the sofa" or "dancing tango", that syntactically require a *single* subject (hence monadic) and the subject needs to denote *several* things (hence plural). Nevertheless, the properties referred to by these predicates typically are complex relational properties, i.e. they are not monadic plural properties.

does not per se imply object holism; the option of collective properties in some sense decouples the two questions.

Since there are good reasons that the overview of the six models is exhaustive, one of these models must be true. In this way, the taxonomy of six models gives the mereological problem of entanglement a precise from. In the following we shall investigate which of the models is the most appropriate one. The overview of six models serves as a basis for our considerations in that we shall try to find arguments that exclude certain models.

## 3 Inconclusive strategies

#### 3.1 Main mereological positions

One could try to rule certain models out by bringing in more mereological assumptions, e.g. to assume one of the main mereological positions (universalism, nihilism, moderate compositionalism). This strategy restricts the models to some extent, but it does not yield a unique result for any of the positions: Universalism, holding that any number of objects have a sum, excludes RRH and RPH; nihilism, which forbids non-trivial composition, is incompatible with the moderate positions; and moderate compositionalism, assuming that composition occurs under certain circumstances, excludes RRH and RPH, if entangled systems fulfill the circumstances (e.g. because composition is among the sufficient conditions for composition), or excludes the moderate models, if entangled systems do not fulfill the conditions.<sup>22</sup>

We shall, however, not make use of any of these restrictions in the following, because that would require to have independent reasons for the presumed position. The idea of this paper rather is to look at the evidence that the quantum mechanical formalism provides for restricting the models.

#### 3.2 Variable vs fixed arities

Schaffer (2010; cf. also Calosi and Tarozzi 2014) have argued that monism, assuming intrinsic monadic macro properties, has the advantage over relationalism that it can ascribe a spin macro property, say "spin 0", to systems with any number of components, while relationalism needs to assume a distinct spin property for each number of components. Brenner (2018) has replied that relationalists do not need to build their positions on n-place relations, which require a fixed number of objects that bear it, but can assume multigrade relations, which can be carried by a variable number of objects. Likewise, Bohn (2012, p. 219) has remarked that non-relational plural properties typically can be had by a variable number of objects as well.<sup>23</sup> So against first appearances there does

<sup>&</sup>lt;sup>22</sup> Interestingly, RMH is the position that all three positions could agree on, which, however, is not to say that it is the most appropriate one.

<sup>&</sup>lt;sup>23</sup> Though his example "being classmates" rather seems to be a relational property; but his claim is clearly true for more paradigmatic examples of non-relational plural properties like average quantities.

not seem to be a crucial difference among the models concerning this question: One can understand either of them as involving a property with a variable number of components.

On the other hand, presupposing realism concerning fundamental quantum states, the most basic properties we are concerned with here are not properties like "having macro spin 0" but rather entangled quantum states like the singlet state (2). Such states, however, typically involve a fixed number of numeric labels, 24 two in the case of the singlet state, which in some sense indicate the number of involved micro objects: in the case of relationalism and pluralism the number of labels indicates the number of actual micro objects that carry the entangled property; in the case of MMH the number of labels correspond to the number of actual micro objects that, however, do not carry the entangled spin property (the macro object does); and in the case of RMH, according to which no micro objects exist when entanglement holds, the number of labels is best understood as the number of micro objects the system can decompose into at measurement. In any case, the number of labels is crucial for the entangled state since it makes a notable difference whether one has an entangled state with e.g. two labels or eight labels, even if both imply the same total spin. For this reason I think that it seems more appropriate to assume that the quantum states we are concerned with here (from non-relativistic quantum mechanics) require a fixed numbers of objects.

This again is compatible with all three basic positions: The monist needs to assume that her monadic singular properties are internally structured in a way that corresponds to the fixed number of labels, the relationalist just assumes well-known n-place relations, and the pluralist considers plural properties that require a fixed number of objects.

Of course, in relativistic quantum physics we also have the case that the particle number can change depending on the reference frame (Unruh effect), and in quantum field theory we even have states that do not fix a certain number of particles relative to the same frame. In such cases, the fact that all models can allow for properties with variable numbers, might help (cf. Bohn, 2012, p. 219).

In sum, the idea to make a case for the one or the other model on the basis of the different arities of the properties does not seem to be conclusive.

#### 3.3 Light-weight interpretation of quantum states

#### Ray view vs statistical operator view

There are two basic views of what quantum states are: states of quantum systems are either represented by rays in Hilbert space ("ray view") or by statistical operators ("statistical operator view"). So far we have discussed entanglement on the basis of the ray view: The entangled state noted in equation (2) is a vector in Hilbert space, determining a ray in Hilbert space. Concerning the most central features of entangled

<sup>&</sup>lt;sup>24</sup> In quantum field theory there are also states that do not fix a certain number of particles.

states, however, the two views agree (Maudlin 1998): Entangled states are well-defined and they are not reducible to the micro states (Irr).

The two views differ, however, concerning the status of the micro states, and consequently concerning the justification of (Irr). It is only according to the ray view that the micro states are not well-defined: The failure to form a product of micro states implies that the entangled state is not a ray in the Hilbert spaces of the micro systems and, consequently, by the ray view, the micro states are not well-defined (¬DMi). Hence, (Irr) holds because there are no micro states from which the macro state could be derived.

The statistical operator view, in contrast, does assign a statistical operator to each micro system, <sup>25</sup> so there are well-defined micro states. Here, (Irr) holds because one cannot derive the entangled macro state from the micro states: Each micro state only allows to predict the statistics of experiments with the corresponding micro system, but it is silent about the correlations between the micro system at hand and the respective other micro system(s); only the macro state allows to predict these correlations.

In sum, the ray view says that the entangled macro state is well-defined, but the micro states are not, while the statistical operator view holds that both the entangled macro and the micro states are well-defined.

#### Intuitive inference?

There might be some intuitive appeal in inferring that the ray view favors a model according to which only the macro object exists and carries the entangled quantum state (RMH), while the statistical operator view requires the corresponding moderate variant (MMH), which additionally assumes the existence of the micro objects. If this procedure were reliable, one would have a fast-track to deciding the question of the appropriate model, based solely on a light-weight interpretation of the quantum mechanical formalism. The only question then would be to decide between RV and SOV.

The naturally seeming inference, however, must involve hidden assumptions, as one infers from the well-definiteness of states to the existence of objects. A straightforward (though probably not the only) way to formulate these assumptions is the following principle:

(SBP) **Strong bridge principle**: If and only if the state of a quantum system on level l is well-defined, the corresponding object on level l exists.

Reference to the level here secures that if and only if the state of a *macro* system is well-defined, the corresponding *macro* object exists; and if and only if the state of a *micro* system is well-defined, the corresponding *micro* object exists. While so far we have only considered spin states of quantum systems, the state of a quantum system here means the *complete* state of such systems, i.e. the spin state as well as the position state. It does not mean the state-independent properties such as mass, charge, total spin (more on these in section 5). The principle then says that if and only if both position

<sup>&</sup>lt;sup>25</sup> The statistical operator for the micro systems is the respective reduced operator.

and spin states are well-defined, a corresponding object exists. So in case the spin state of a system fails to be well-defined, the corresponding object does not exist.

Isn't this principle natural? Maybe. More importantly, however, it is a heavy-weight and complex metaphysical principle whose status is not immediately clear; it needs justification. The straightforward way to infer from a light-weight interpretation of the formalism to the appropriate mereological model, is blocked. If the inference is to be justified, we cannot avoid deep metaphysical considerations. So let us now consider whether the principle is in fact justified.

#### 3.4 The strong bridge principle

Is the strong bridge principle justified? I think it is not. Let me shortly illustrate this for the direction of inference from the states to the objects.

Suppose we have a macro system in a product state, i.e. a non-fundamental state. The state is well-defined, so by the (SBP) the corresponding macro object exists. But why assume that? We have made clear above that realism is only committed to assume that fundamental states describe property instances because property described by non-fundamental states can be reduced to the property instances described by the fundamental states. Assuming property instances that are reducible to more fundamental ones would contradict Ockham's principle. So why assume l-level objects if there are no l-level properties?

One might then be inclined to restrict the principle to fundamental states:

(SBP') **Strong bridge principle**': If and only if the state of a quantum system on level l is well-defined and fundamental, the corresponding object on level l exists.

By this principle, there is only one level of objects, namely the one that correspond to the level of fundamental states. In the case of entangled systems this would be the macro object, and the micro objects would not exist, not even if one presumes the SOV.

The crucial problem, however, is that from the fact that there is an l-level property, even if the property is fundamental, it does not follow that there is an l-level object. As l-level properties can be carried collectively by l-1-level objects (MaPB), one cannot infer from the existence of a property instance on level l to the existence of an object on level l without further information.

In sum, deriving mereological consequences on the basis of the (SBP) (or the (SBP')) is not justified. A straightforward, "intuitive" interpretation of the RV as well as of the SOV fails. We now need to turn to arguments that are based on more appropriate principles. Rather than formulating one principle that rules out all but one model, we shall collect different small pieces of evidence from the quantum mechanical formalism that yields several separate arguments against different models.

# 4 Against a relational entangled property

## 4.1 Non-relative relational properties

Relational holism (models RRH and MRH) claims that the entangled property is an irreducible relation. According to this view, the singlet state (2) can be understood relationally in the following sense: Object a has opposite spin to object b. The relation being irreducible means that there are no micro properties to which the relation reduces as, for instance, "Alice being 5 cm taller than Mary" reduces to "Mary being 1.68 m tall" and "Alice being 1.73 m tall". We have just the relation and the micro properties are either not defined at all (RV) or they are defined but do not allow to derive the relation (SOV).

The features of systems in the singlet state provide no evidence against (and rather some evidence for) the case that it describes a relational property. We here first need to be explicit about these features in order to understand their failure for other entangled states, which then provide evidence against a relational property. Discussing the features of entangled states one can either argue on the theoretical level (regarding the structure of the states and their dispositions for yielding certain measurement results) or on on the empirical level (discussing the statistics resulting from measurements at such states). Here we take the latter route and present the results of spin measurements at a system in the singlet state:

- $|\uparrow\rangle_1$  and  $|\downarrow\rangle_2^{26}$  in  $50\%^{27}$  of the cases,
- $|\downarrow\rangle_1$  and  $|\uparrow\rangle_2$  in 50% of the cases,
- and this statistics holds for any measurement direction.

These results fulfill four general features that are necessary conditions for an irreducible relational spin macro property before measurement:

- (i) There is a rotational symmetry of the measurement results. ("Having opposite spin to" does not prefer any direction.)
- (ii) The micro properties (after measurement) vary from run to run. ("Having opposite spin to" does not determine which spin direction a particle in itself has; a well-defined spin direction can only be determined at measurement when the entangled state collapses.)
- (iii) The different possible outcomes are statistically equally distributed. ("Having opposite spin to" does not prefer any of the possible outcomes.)
- (iv) In all measurement runs the measured micro properties either (a) have opposite spin or (b) they have equal spin. ("Having opposite spin to" requires that the spins are opposite, while "having equal spin to" requires that the spins agree.)

<sup>&</sup>lt;sup>26</sup> Read: object 1 has spin up and object 2 has spin down.

<sup>&</sup>lt;sup>27</sup> The probability that a certain combination of micro properties emerges at measurement is given by the square of the factor that appears in the corresponding term in the entangled state (before measurement), e.g.  $(1/\sqrt{2})^2 = 50\%$  for the result  $|\uparrow\rangle_1$  and  $|\downarrow\rangle_2$ .

I should mention that the conditions summarized here hold for spin properties understood as *non-relative* relational properties (the relative notion is a generalization which we shall explain in the subsequent Section 4.2).

While the antisymmetric singlet state fulfills (i)-(iii) and (iva), its symmetric counterpart

$$|\psi'\rangle_{12} = \frac{1}{\sqrt{2}}|\uparrow\rangle_1|\uparrow\rangle_2 + \frac{1}{\sqrt{2}}|\downarrow\rangle_1|\downarrow\rangle_2,\tag{3}$$

has the following similar but different measurement statistics,

- $|\uparrow\rangle_1$  and  $|\uparrow\rangle_2$  in 50% of the runs,
- $|\downarrow\rangle_1$  and  $|\downarrow\rangle_2$  in 50% of the runs,
- and this statistics holds for any measurement direction.

and therefore fulfills (i)–(iii) and (ivb), providing some evidence that it describes the property "having the same spin direction as".

## 4.2 Relative relational properties and superpositions thereof

The singlet state (and its counterpart), however, are special cases of entanglement in many ways, and when one considers less special cases the idea of a relational property quickly becomes doubtful. Consider, for instance, the entangled state

$$|\psi''\rangle_{12} = \frac{1}{\sqrt{2}}|\uparrow\rangle_1|\downarrow\rangle_2 + \frac{1}{\sqrt{2}}|\downarrow\rangle_1|\uparrow\rangle_2 \tag{4}$$

State (4) looks similar to the singlet state, and the statistics is similar but with a crucial difference:

- $|\uparrow\rangle_1$  and  $|\downarrow\rangle_2$  in 50% of the cases,
- $|\downarrow\rangle_1$  and  $|\uparrow\rangle_2$  in 50% of the cases,
- ullet and this statistics only holds for one measurement direction and *changes for dif*ferent directions to:<sup>28</sup>
  - $-\mid\uparrow\rangle_1$  and  $\mid\downarrow\rangle_2$  in a fraction  $p_1$  of the runs,
  - $-\mid\downarrow\rangle_1$  and  $\mid\uparrow\rangle_2$  in a fraction  $p_1$  of the runs.
  - $-\mid\uparrow\rangle_1$  and  $\mid\uparrow\rangle_2$  in a fraction  $\frac{1}{2}-p_1$  of the runs,
  - $-\mid\downarrow\rangle_1$  and  $|\downarrow\rangle_2$  in a fraction  $\frac{1}{2}-p_1$  of the runs,

The state in the rotated basis has the form  $|\psi''\rangle_{12} = \sqrt{\frac{1}{2} - p_1|\uparrow\rangle_1|\uparrow\rangle_2 + \sqrt{p_1}|\uparrow\rangle_1|\downarrow\rangle_2 + \sqrt{p_1}|\downarrow\rangle_1|\uparrow\rangle_2 - \sqrt{\frac{1}{2} - p_1}|\downarrow\rangle_1|\downarrow\rangle_2,$  and  $p_1$  is a function of the rotation angle.

where  $0 \le p_1 \le \frac{1}{2}$ . Hence, here two of the above mentioned necessary conditions for a relational property are violated, namely that there is a rotational symmetry (i) and that all possible outcomes are either opposite or equal spins (iv). While this seems to speak against a relational property, I shall now demonstrate how one can push the concept of a relational property such that even this case might be covered. (Note that I do not defend the following proposal, I just want to show here how far one can stretch the concept of a relation in order to subsequently make clear where the conflict really lies.)

In requiring the condition of rotational symmetry we have implicitly assumed, as might seem natural, that the relational property does not prefer any direction of space, such as for relations of spatial distance between two objects. The rotational symmetry, however, does not hold generally for relations, it is, for instance, not true of relational properties that hold relative to a spatial direction. An example might be the relation "is larger in extension": Alice might be larger in extension than Mary in vertical direction, but Mary might be larger in extension than Alice in horizontal direction. If one accepts that the macro spin direction is a relative relational property, the spin property could change in dependence of the measurement direction, violating rotational symmetry. Hence, the claim that the macro spin direction is a relative relational property does not imply condition (i) but only requires conditions (ii)—(iv).

The relativized relational property, however, is still in tension with the measurement statistics. For relativizing the relational spin property would not allow for arbitrary changes under change of measurement direction: Since spin values are quantized and can only be up or down, a relation between them can only be "having opposite spin" or "having equal spin". So at any measurement direction the possible measurement results may either be a distribution of opposite spins  $(|\uparrow\rangle_1|\downarrow\rangle_2$  or  $|\downarrow\rangle_1|\downarrow\rangle_2$  with some probability each) or of equal spins  $(|\uparrow\rangle_1|\uparrow\rangle_2$  or  $|\downarrow\rangle_1|\downarrow\rangle_2$  with some probability each), but no mixture of opposite and equal terms. The latter, however, is the case for the state above for most measurement directions.

In order to allow for such a mixture of opposite and equal terms one would have to further generalize the concept: One would need to allow for a relational property that is a *superposition* (with possibly different weights) of relations that hold relative to a spatial direction: "being in a superposition of 'having opposite spin' with probabilistic weight  $2p_1$  and of 'having equal spin' with probabilistic weight  $1-2p_1$  relative to a certain direction  $u_1$ ". This further generalization of the concept, if a superposition of properties makes any sense, would only require conditions (ii) and (iii) and hence be compatible with the state (4) and its statistics. So in principle one can make states of form (4) compatible with a generalized concept of a relational state. (Recall that I do not defend this proposal.)

#### 4.3 The failure of relational properties

A relational conception, however, definitely fails for the state

$$|\psi'''\rangle_{12} = \sqrt{p_1}|\uparrow\rangle_1|\uparrow\rangle_2 + \sqrt{1-p_1}|\downarrow\rangle_1|\downarrow\rangle_2,\tag{5}$$

where  $p_1$  is a number between 0 and 1 (representing the probability that  $|\uparrow\rangle_1$  and  $|\uparrow\rangle_2$  is measured, see Fn. 27).

While for  $p = \frac{1}{2}$  the state (5) reduces to the state (3), for any value  $p \neq \frac{1}{2}$ , say p = 0.2, we get a different state that yields the following measurement statistics:

- $|\uparrow\rangle_1$  and  $|\uparrow\rangle_2$  in 20% of the runs,
- $|\downarrow\rangle_1$  and  $|\downarrow\rangle_2$  in 80% of the runs,
- and this statistics only holds for one measurement direction and changes for different directions.

This statistics again violates the rotational symmetry (i), but that point, as we have explained, can be healed, so we leave it aside here. What is more important is that according to the statistics the different possible outcomes are statistically unequally distributed, violating (iii). Why would the relational property "having equal spin as" give more statistical weight to one of the two possible outcomes than to the other? I cannot see any plausible reason. If there is a truly irreducible relation, then it should not prefer one combination of intrinsic micro properties over the other. Hence, such entangled properties cannot be interpreted as relations.

If these more general ones cannot, one should not interpret the other, more specific ones to be relations. They differ from the former just in that their weights are equally distributed, but that symmetry can be explained by a gradual difference; there is no reason to suppose that there is a qualitative difference here.

The upshot is that the idea to understand an entangled quantum state as describing a relational property fails. Since the entangled state is a superposition the relational interpretation would be a non-literal one anyway, providing, however, a prima facie plausible and uniting understanding. But the more complex cases of entangled states have shown that this way is not open. Both models of relational holism are not appropriate. What remains is to understand the entangled state more literally as a superposition of combinations of different intrinsic states, which is what monistic holism and pluralistic holism assume.

# 5 For the existence of the micro objects

Besides the quantum state, which can vary over time and is described by the wave function (including the spin component of the wave function that we have considered so far), each quantum micro system is characterized by state-independent variables, which are characteristic for each type of system and most centrally comprise mass and charge.

Every type of quantum system has a specific fixed set of values for these two variables, <sup>29</sup> e.g. each electron has the same certain mass and the same certain charge.

I shall now show that the state-independent variables provide an argument for the existence of the micro objects, and this argument comes in two steps. I first argue that mass and charge of the *micro* systems play a fundamental role in the quantum mechanical description of the *macro* systems that cannot be reduced to other variables, especially not to macro variables (sections ??–5.2). I then argue that the state-independent variables describe fundamental properties requiring that the corresponding micro objects exist (section 5.3).

## 5.1 The micro charges are fundamental properties

On the one hand, it is well-known that in quantum mechanics (like in classical theories) the macro charge just is the sum of the micro charges  $(Q = \sum_i q_i)$ , which suggests a reduction of the macro to the micro charges. That presupposes, however, that it makes sense to speak of micro charges, and precisely the answer to that question is not clear in entangled systems. If, for instance, entangled systems were undivided wholes, assuming micro charges in order to show a reduction to the micro level, would be question begging. So we need to consider the quantum mechanical description of an entangled macro system and the roles that micro and macro charges play in that description.

Being coupling constant of the electromagnetic field, the role of micro and macro charges in macro systems can be evaluated by examining the dynamical equation of macro systems with charged components. As an example consider a macro system describing two electrons. The dynamical equation of the system (in the non-relativistic case) is the following Schrödinger equation,

$$i\hbar \frac{\partial}{\partial t} \Psi(\boldsymbol{x}_1, \boldsymbol{x}_2, t) = \left( -\frac{\hbar^2}{2m_1} \nabla_1^2 - \frac{\hbar^2}{2m_2} \nabla_2^2 + V(\boldsymbol{x}_1, \boldsymbol{x}_2, t) \right) \Psi(\boldsymbol{x}_1, \boldsymbol{x}_2, t), \tag{6}$$

where  $x_1$  and  $x_2$  are the spatial coordinates of electron 1 and 2, respectively,  $\Psi$  is the wave function of the macro system,  $m_1$  and  $m_2$  are the masses of electron 1 and 2, respectively, and V is the potential describing the interactions.

Each electron is negatively charged ( $q_1$  and  $q_2$ , respectively) and therefore they interact with each other, which is described by the potential V. The most simple case that the electrons are in a stationary state and interact with each other in a static way, is described by the Coulomb potential

<sup>&</sup>lt;sup>29</sup> Spin is only state-independent for fundamental particles. Macro spin depends on the macro state and does not supervene on the micro states. It is then a matter of controversy whether composite systems that agree in their components (and hence in mass and charge) but differ in macro spin are of the same kind or constitute different kinds. Some examples suggest that they belong to the same kind but differ in state (e.g. different electronic states of an atom), other suggest that they are different kinds of systems (e.g. nucleons and Δ-baryons).

$$V(x_1, x_2) = \frac{q_1 q_2}{4\pi\epsilon_0 |\mathbf{x}_1 - \mathbf{x}_2|}. (7)$$

Non-stationary and relativistic cases are more complicated, but they all agree in the general fact that the micro charges  $q_1$  and  $q_2$  play a central role in the description of the macro system, while the macro charge does not appear at all in the description, not even by its reducing paraphrase  $q_1 + q_2$ . Clearly, the micro charges are relevant variables in the dynamics of the macro system, but the macro charges are not.

Then, the conclusion seems inevitable that even in entangled systems it is the micro charges which are fundamental according to the quantum mechanical description.

#### 5.2 The micro masses are fundamental properties

Similarly, macro mass is known to be the sum of the micro masses  $(M = \sum_i m_i)$ , and the crucial question again is whether micro masses are defined in entangled systems. The system of two electrons described by equation (6) illustrates that the inertial masses  $m_1$  and  $m_2$  of the micro systems play a fundamental role, while the macro mass does not appear in any sense.

Assuming classical gravitational theory, the same is true for gravitational masses that can be introduced by extending (6) to the Schrödinger-Newton equation.<sup>30</sup> Then, also the micro masses describe fundamental variables.

#### 5.3 The micro objects exist and carry micro mass and charge

The upshot of these considerations is that the state-independent variables micro mass and micro charge are fundamental according to the quantum mechanical formalism. By realism (QSR) we can then infer that each of these variables on the micro level describes a micro property instance, and due to (EPB) each of these micro property instances requires an object that carries it.

Which object carries the micro property instances mass and charge? On the one hand, it is clear that the most natural scenario is to assume that they are carried by the corresponding micro objects. On the other hand, we have seen above that the existence of an *l*-level property instance does not necessarily imply the existence of an *l*-level object, since macro properties can be carried by micro objects collectively. So far, however, we have not said anything about the reverse case, which is relevant here: Can micro properties be carried by macro objects?

Imagine the case that only a (partless, undivided) macro object exists (RMH). Could such an object carry micro properties like masses  $m_1$  and  $m_2$ ? I think it is pretty clear that it cannot. If it carried the micro property mass  $m_1$  like any other macro property

<sup>&</sup>lt;sup>30</sup> It is well-known that, currently, there is no generally accepted relativistic quantum theory with gravitation.

that it carries,  $m_1$  would have to have the same status, i.e. be a macro property. So it needs to carry  $m_1$  in a different way than its macro properties. And what would be the appropriate way for the macro object to carry  $m_1$  as a micro property? It would be to carry  $m_1$  by only a part of the macro object, which contradicts the assumption of the macro object being partless.

To illustrate the case: a white object can only have red dots on its surface, if it has parts; if it did not have parts it would have to appear homogenous over its whole surface. (It is hard to imagine that one could not make a red dot on it without affecting the other parts of its surface but that is because we are used to the fact that extended objects have parts.) Or more generally: an object can only be structured, i.e. instantiate different properties of the same kind, if it has parts.<sup>31</sup> The required parts may depend ontologically on the whole (or vice versa), but I emphasize that it does not help if the parts only existed reductively, for then they would not be able to carry the micro property instance that exists non-reductively.

If these considerations are correct, we have found an asymmetry between micro and macro properties. While we have seen above that a macro property can be borne either by a macro object or by several micro objects collectively (MaPB), we here find:

(MiPB) **Micro property bearing**: A micro property cannot be borne by a macro object, but only by a micro object.

Consequently, the micro property instances of mass and charge can only be carried by the corresponding micro objects, and since any quantum macro system, including entangled systems, involves these properties, for any quantum system the micro objects exist and carry the micro properties mass and charge.

This might seem to stand in a certain tension with the holistic character of entangled systems, but our models reveal that it is in fact consistent with most of the models; only model RMH is ruled out. The latter might seem surprising since to regard entangled systems as describing undivided wholes has been a favourite model for many. The lesson here is that quantum holism is not blunt. It seems much more subtle and differentiated than many seem to have assumed.

# 6 Against the existence of the macro object

#### 6.1 The argument from duplication (of macro mass and macro charge)

This argument concerns the relation between the micro and macro level of the reducible state-independent properties mass and charge. We here present the argument for the property mass (and the argument for charge runs mutatis mutandis). We have already said that the property macro mass clearly reduces to the sum of the micro masses. Furthermore, we have said that the micro objects exist and carry the state-independent

<sup>&</sup>lt;sup>31</sup> Spinoza would probably object to this point: the partless substance, God, can have restricted modes.

micro properties (section 5). So one can easily paraphrase talk about "macro mass" as in fact meaning "the sum of the micro masses". Then, there is no need to assume that instances of macro mass exist (and neither that instances of macro charge exist), and the principle of parsimony requires that one should not assume their existence.

This leads to problems for models that assume the existence of the macro object. Recall that by exist we mean non-reductive existence, so especially an existing macro object cannot be identical to the micro objects. Then, to claim the existence of a macro object is to claim the existence of a full, distinct material object, and consequently that macro object needs to carry all properties that a material object carries, especially the state-independent properties mass and charge. Hence, to claim the existence of the macro object is to claim the existence of instances of the state-independent macro properties and these properties are distinct from the micro properties. Then it would be the case that, e.g., both the micro masses and the macro mass exist, although the latter is reducible to the former.

This yields two problems. First, there are now two candidates for the referent of "macro mass" and it is unclear to which entity the term "macro mass" refers: to the fundamental property of the macro object or to the property that supervenes on the micro masses? And if to the one, what about the other? Inconsistencies lurk. Second, having a fundamental property macro mass although macro mass supervenes on the micro masses violates the principle of parsimony.

# 6.2 The argument from causal irrelevance (of macro mass and macro charge)

We have just argued that assuming the existence of the macro object implies the existence of the properties macro mass and macro charge. Besides being in tension with supervening macro mass and charge, their existence comes with a further problem: Macro mass and macro charge are not causally relevant according to the quantum mechanical formalism; any reasonable inductive metaphysics, however, is committed to the principle that one should only assume properties that are causally relevant. Hence, since macro mass and charge are not causally relevant, one should not assume the existence of the macro object.

We should be clear about the causal irrelevance of the properties, and demonstrate explicitly that they play no causal role. There are two senses in which they might be effective: either as acting downwards on the micro properties or as acting on other macro properties. I shall examine these questions for the case of the property macro charge.

Starting with possible downwards causal roles of macro charge, we consider again the system of two electrons interacting in a static way (eq. equation (6) and equation (7)). We have seen that the micro charges  $q_1$  and  $q_2$  prominently figure in the dynamical equation via the Coulomb potential. Nowhere, however, do we find a macro charge Q or the sum of the micro charges  $q_1 + q_2$  (which equals the macro charge). The same is also true for more complex electromagnetic potentials which always involve the micro

charges and never the macro charge in the most detailed description.<sup>32</sup> So the macro charge does not play any causal role when the macro system is left to itself, not to speak of downwards causation.

What about the causal relevance of macro charge in situations in which the macro systems interacts with other macro systems? We consider the most simple case that the system with two electrons is subject to an external static electric potential due to an external charge  $q_3$ , e.g. due to a Helium nucleus. Does the property macro charge play any role in such situations? The quantum mechanical way of describing such situations is to add the external potential to the internal potential, so the total potential reads:

$$V(x_1, x_2, x_3) = \frac{1}{4\pi\epsilon_0} \left( \frac{q_1 q_2}{|x_1 - x_2|} + \frac{q_1 q_3}{|x_1 - x_3|} + \frac{q_2 q_3}{|x_2 - x_3|} \right).$$
 (8)

One can see that also in this case there is no macro charge involved. The relevant new terms in the potential (the latter two summands) each contains a product of a micro charge with the external charge, suggesting that each micro charge interacts with the external charge.

In sum, there is no doubt: It is the micro charges that are causally relevant while the macro charge does not play any relevant causal role. A similar claim is true of macro mass.

So we have found two arguments for the case that the macro object does not exist; this excludes the moderate models.

# 7 The proposed model

We have finally arrived at a unique result. In order to narrow down the possible models for the mereology of entangled quantum systems (figure 1), I have presented four main arguments. The argument against a relational entangled property excludes both models of relational holism (RRH, MRH); the argument for the existence of the micro objects rules out radical monistic holism (RMH), i.e. undivided wholes; and the two arguments against the existence of the macro object (given that the micro objects exist) speaks against any of the moderate models (MMH, MRH, MPH). In sum, my considerations rule out five of the six models, yielding the conclusion that only the model radical pluralistic holism (RPH) withstands the objections.

While Bohn's and Brenner's proposals of plural properties have shown that one cannot infer without further arguments from the existence of an irreducible l-level property to

<sup>&</sup>lt;sup>32</sup> This is not to say, however, that there are no systems that can appropriately be described by a macro charge. For instance, when point charges are symmetrically arranged on the surface of a sphere, the macro system can be described as a point charge that equals the sum of the micro charges in the center of the sphere. In such cases, however, the macro charge still is the result of a summation of the micro charges and provides a more convenient summarizing description.

the existence of the corresponding object (since the l-level property might be carried collectively by the l-1-level objects), here I have provided explicit arguments against such an inference in the case of entangled systems. More precisely, it is Bohn's proposal of regarding the entangled property as a non-relational plural property that my arguments based on the quantum mechanical evidence support.

By the results of the foregoing investigation the more detailed picture of the proposed model RPH is as follows: There is no macro object. The micro objects exist and carry the state-independent micro properties mass and charge. Macro mass and macro charge supervene on these micro masses and micro charges, respectively; in this way there is no duplication of the state independent macro properties. The non-relational macro spin property is carried collectively by the micro objects. It is described by the entangled quantum state and does not supervene on the micro level.

One might object that the micro objects, if they exist, need to carry the complete set of properties that material objects usually carry: mass, charge and spin; and since their spin is not well-defined, they fail to conform to the condition. While I agree with the condition, I would not say that the micro objects fail to have a spin property: It is true that they do not have a separable singular spin property, but they collectively carry a non-separable macro spin property, which determines (the probabilities of) their individual spin behaviour.

This consideration suggests the following picture: Each quantum object only has one fundamental spin property that determines its spin behaviour: either it carries a well-defined micro spin property (and the macro spin state is described by a supervening product state) or, jointly with other quantum objects, it collectively carries a fundamental macro spin property (and its micro spin property is either not well-defined or is reducible to that macro property, depending on whether one assumes RV oder SOV).

In this way, since only one level (both for objects and properties) is fundamental one avoids duplication of properties or causal conflicts between them.

Recall that the micro objects in the models and a forteriori in the proposed model RPH are either non-individuals or objects with weak identity. It has not been the task to decide between the two variants here (which is the individuational problem of entanglement), but I have taken care to symmetrize the models (as quantum mechanics does) in order to stay as neutral as possible concerning this further question.

To say that there is no macro object according to the proposed model RPH, of course means that the macro object does not exist fundamentally or dependently, which is compatible with the fact that the macro object exists reductively. "The macro object has an entangled spin property" can be paraphrased as "the micro objects collectively carry an entangled spin macro property". "Macro object" then in fact refers to the micro objects and "singular macro property" in fact to a plural macro property.

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## 8 Discussion

(1) It might appear surprising that radical pluralistic holism comes out as the survivor of the selection process, since quantum entanglement has often been associated with holism and seems to underline the importance of the macro system. The arguments above, however, have differentiated between holistic properties and holistic objects, and while quantum mechanics clearly involves the former, the arguments show that it likely does not involve the latter.

Furthermore, the idea of a non-relational plural property might seem unusual. The reason seems to be that plural logic and its entailments have only started to become as widespread as they deserve to be. There is nothing wrong with collective properties, in fact, as I have argued with my example of temperature and others, we have ever been using them in our descriptions.

It is remarkable that exactly the proposal of a non-relational plural property allows to deal with the balancing act dictated by the arguments: On the one hand, there are good arguments against the existence of the macro object, so the entangled property must be carried by the micro objects collectively; on the other hand, the entangled property may not be a relation; hence the entangled property should be a non-relational plural property as RPH postulates.

(2) Since my arguments lead to a denial of composition in entangled quantum systems, I emphasize that they do not establish that mereological nihilism holds, since nihilism requires the absence of (non-trivial) composition for *all* kinds of systems. In this sense, I have not even attempted at answering the special composition question; my aim just was to find the appropriate mereological model for entangled quantum systems.

Nevertheless, it might be true that my result, if correct, has the potential to shake the trust in theories that assume composition, since entanglement has been regarded by many as one of the most secure cases of composition: If the non-separable states of entangled systems do not imply composition, what else should? However, in fact my result is also compatible with moderate compositionalism, which assumes that composition occurs in circumstances other than entanglement (say, due to life, Inwagen 1990).

I may add that my result is incompatible with universalism. To be fair, the principles which I have used in order to derive consequences from quantum mechanics, are designed to establish a parsimonious and as far as possible reductive metaphysics, thus principles that a universalist probably would not accept anyway.

(3) It is not only that my scope is much more narrow than the special composition question; in this paper I have neither attempted to examine the conditions under which composition occurs. Rather, I have considered possible models and have discussed which of them is in accordance with the empirical evidence and which is not. This procedure might appropriately be called "mereological modelling". The fact that entanglement is not a condition under which composition occurs can only be read off the final result of the modelling.

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(4) Trying to answer a metaphysical question by pointing to empirical evidence from one of our best scientific theory, this paper stands in the tradition of what has historically been called "inductive metaphysics" (Scholz, 2018). Inductive metaphysics is now practiced by many contemporary metaphysicians who base arguments as much as possible on scientific evidence rather than on intuitions. On hte other hand, inductive metaphysics contrasts with naturalistic metaphysics by assuming that metaphysics is a separate discipline from fundamental physics with its own methods and questions (Engelhard et al., 2021). Especially, in this paper we have seen that the physical formalism does not by itself provide the metaphysical results; rather they need to be derived by suitable principles and arguments.

- (5) While the taxonomy of six mereological models is general in that it spans the space of possible models for a system with an irreducible property, my arguments for RPH are not. My arguments rely on the specific description that the quantum mechanical formalism provides for entangled systems, and cannot be generalized to other systems.
- (6) Still, my arguments might have implications for entangled quantum systems beyond the specific mereological aim of this paper. Especially the argument in section 4 provides an objection to *all* interpretations of quantum theory that assume entangled quantum states to describe *relational* properties.

One position that seems to be challenged by this argument is ontic structural realism, which claims that the fundamental elements of reality are structure, i.e. relational properties. Entangled quantum systems understood relationally are typically cited as a major evidence for the view that relations rather than objects are fundamental entities (Esfeld, 2004; Ladyman et al., 2007; French, 2014). If, in contrast, irreducible entangled quantum states cannot be interpreted relationally, how would OSR describe them?

(7) Finally, it is a remarkable fact that the quantum mechanical description provides enough evidence to rule out all but one model, especially since the arguments have been relatively simple and the assumed principles relatively weak. It is not often the case in metaphysics, where underdetermination is the rule, that one so fortunately receives support from an empirically confirmed theory.

The clear result suggests that it is not correct to claim that scientific theories are mereologically neutral in the sense that they do not favor any mereological model. At least when one assumes a realistic reading of the theory jointly with some relatively weak metaphysical principles like the (EPB), surprisingly far-reaching consequences can be drawn. Neither does it seem true in the light of the result that mereology has no empirical content: the arguments in this paper demonstrate that certain empirical facts speak against certain mereological models — which is exactly what empirical content means.

# Acknowledgements

For valuable comments to this paper I am grateful to Ulrich Krohs, Oliver Scholz, Ansgar Seide, and Michael te Vrugt.

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