MATHEMATICS AND THE LAWS OF NATURE

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The fact that the behaviour of physical objects can be described mathematically may at first sight seem remarkable; at any rate many writers have remarked on it, sometimes adding how fortunate we are to live in a world which obeys simple mathematical laws, or even referring to God as the Supreme Mathematician. The equation:

$$s = \frac{1}{2}gt^2 \qquad (1)$$

expressing a relationship between the distance traversed by a freely-falling body near the surface of the earth and the time elapsed since its release from rest, is a familiar example of a mathematical law, the general form of which was discovered by Galileo. It is certainly simple, simple enough to be taught to beginning students of physics, generations of whom might have been grateful if falling bodies had behaved in some way which eluded mathematical description.

A moment's consideration shows, however, that if equation (1) is meant to refer to falling bodies it cannot be a *mathematical* statement at all. Translated into English, it would read: "The distance fallen is equal to half the product of the acceleration and the square of the time." But *is equal to, half, product, square,* represent mathematical operations, whereas *distance, acceleration, time,* are properties of the physical world; and mathematical operations performed on physical properties result in meaningless nonsense. What does the square of a time look like? What happens when this curious hybrid is multiplied by an acceleration? And is it likely that this compound, when halved, will turn out to be a distance?

The fallacy here is easy to point out: the equation is not directly concerned with the physical properties, but with numbers which somehow represent them. The statement in its mathematical form, then, is not about falling bodies, but about numbers; it asserts a functional relationship between numbers generated by physical means. This generation we usually call *measurement,* and what it does is to translate physical relationships into mathematical language. Once translated, however, the physical significance of the terms can only obstruct calculation, and is best forgotten. Bertrand Russell once went as far as to say that mathematics is “the subject in which we never know what we are talking about, nor whether what we are saying is true,” and although he confessed that when he wrote this he was trying to be “as romantic as possible” about the subject, still the remark makes a valid point—the empirical state of affairs is irrelevant to mathematics. Nevertheless it was the behaviour of the falling body which *generated* the numbers functionally related by the equation; perhaps this fact still seems to point to mathematical structure in the world, or something of that sort.

Mathematics, however, is the study of all possible functions, and any two sets of numbers may be regarded as functionally related—after all what is needed for the specification of a function is just two sets of numbers, the elements of which are in unique one-to-one correspondence. In a trivial sense, therefore, the dream of the reluctant physics students is shattered: *no* phenomenon which submits to measurement at all can elude mathematical formulation; as long as there are two quantities involved —say time and the location of a particle— a function is being generated. Such a function is not, it is true, useful for prediction, since one cannot know until after the event which value of time corresponds to what values of the spatial coordinates, but it does constitute a mathematical description. Still there would be protest if it were maintained that this is what we meant by a “law of nature,” because the essence of a law of nature is that it is repeatedly exemplified. The so- called “uniformity of nature" is indeed remarkable. but once given this as an empirical fact the possibility of mathematical description of recurrent events follows in the same way as before. Consider a process in which some quantity, say p, varies with the time. If we adopt the convention that the beginning of the process always occurs at t=0, we get a function by taking simultaneous readings of t and of p and writing them opposite one another in two columns. The next time it happens, other things being equal, we may use this function to predict values of p, and this depends only on the fact that nature repeats itself, not on any mathematical simplicity.

This, you may feel, still evades the problem: it is not so much that *some* function can be found relating t and p, but that when found it should have a simple form, say:

$$p = kt^2. \qquad (2)$$

Here two points must be made. First, how many of the physicist’s equations actually have such a very simple form? The closer we get to nature, and the more detail we uncover, the more complicated the situation seems to become, and the fewer cases there seem to be of exact and at the same time simple laws. The *fundamental* laws, it may be said, like the one discovered by Galileo, are genuinely simple. But consider this question: Why did Galileo discover the law of falling bodies and not a law relating, say, the average number of leaves on a tree to the annual rainfall? The question is one of principle, not of history; the answer obviously is that the simple laws, if there are any, will be discovered first, but this is no argument for simplicity in nature generally. Among all the mathematical relationships which can be generated by making measurements it would be surprising if there were not *some* simple ones, and if there had been none we would have had no science.

Nor, for that matter, would we have had any mathematics, for the second point is that the basic mathematical functions themselves originate in natural relationships. Equation (2) contains a "cube," and this comes from the Greek word for a die, a familiar object long before there were any mathematicians. It is true that mathematics has come a long way from its empirical beginnings, and has generalized its subject-matter to all ordered relationships, possible as well as actual, but it bears the marks of its ancestry; the fact remains that our mathematics is a product of the world in which we find ourselves, and if we lived in a different world we would have a different mathematics. Saying: "How fortunate that the world is mathematical! because that enables us to describe it simply," is rather like saying: "How fortunate that the base of our number system is ten! because that enables us to multiply by simply adding zero."

These rather loose remarks may be summarized as follows:

1. Mathematics is the study of all possible orders; it is not surprising that the order of the world is among them.
2. There are many functions describing natural relationships; it is not surprising that some of them are simple.
3. The mathematical language that we speak originated in the description of natural relationships, simple enough to be grasped by primitive man; it is not surprising that what is derived from nature should be applicable to it.