A NETWORK-BASED EVOLUTIONARY MODEL OF THE SPACE ENVIRONMENT

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ABSTRACT

Recently, the first treatment of the space population as a multi-layer temporal network was introduced. This model has allowed the application of network theory techniques for a holistic treatment of the space environment. In this work, we go one step further by focusing on the physical layer of the network, investigating its structure, dynamics, and stability. The interactions among resident space objects are modeled through their collision rates: each node represents a population of satellites in a given orbital class, and each link represents their time-varying collision rates with other nodes. Satellites can flow in between these classes and populations, depending on several sink and source phenomena, including explosions, collisions, natural decay due to atmospheric drag, post-mission disposal strategies, operational lifetime duration, and new launches. The underlying dynamics on the network is modeled as a stochastic contact process, in which nodes can assume two possible states: operational and non-operational. The network, therefore, presents a time-varying structure where the number of active and inactive objects, their distribution, their time-varying collision rates, and many other key variables can be studied. The conceptual simplicity and versatility of the network allow to study various topologies and dynamics, and therefore investigate interactions among space objects under different levels. In this framework, several aspects are studied: first of all, two network topologies are introduced. Then, the concept of global stability of the network is introduced and discussed, and a stochastic evolutionary network model is built to simulate the sink and source phenomena in the space environment and evolve the overall population of objects in low Earth orbit for long periods of time. Finally, we perform experiments using a lattice network structure to show how this model can be used to probabilistically study the evolution of space objects and all the key variables involved, such as the number of collisions, the fragments generated, the collision rates evolution, and many others.

1. INTRODUCTION

The space industry is one of today's most growing sectors and, as a consequence, the number of resident space objects is continuously rising, due to new investments and launches from institutional and commercial partners. This poses a threat to currently operating satellites, whose lifetime might be jeopardized by the increased risk of collision with other objects. It is therefore essential to develop the necessary techniques to model, understand and predict the current space environment and its future evolution [1]. For this reason, the space community has developed a plethora of techniques and tools to model the current environment and analyze possible evolution scenarios, accounting for sink and source mechanisms, such as explosions, collisions, new launches, decay of satellites, active removal, and de-orbiting [2]. Other studies have also numerically studied the impact of the planned launches of megaconstellations on the current space environment and their effect on its long-term stability [3]. The purpose of these studies and experiments is to gain insights into the driving factors that regulate the space environment stability and to analyze control strategies for avoiding a collisional domino effect and feedback runaway mechanism that could hinder space activities and access to space [4]. While most of these models offer a probabilistic view of the space environment, they often are computationally intensive and lack of versatility in studying the relationships among resident space objects under different orbital configurations. In this work, we propose a temporal network model of the space environment, where nodes can assume two states: either operational or non-operational, and they represent specific orbital classes (e.g. orbital shells, keplerian elements sets, etc.) and populations (e.g. rocket bodies, megaconstellations, mission related objects, etc.). The model embodies several source and sink phenomena (such as future launches, collisions, explosions, natural decay) and is able to evolve a given population of objects for long periods of time while leaving to the user the possibility to tweak and tune several hyperparameters that influence the space environment evolution (e.g. number of launches, post-mission disposal duration, collision avoidance maneuvers rate of success, etc.). We show that depending on how orbital classes are defined, the network displays different topologies, whose features and interactions with the underlying population stability and evolution can be studied using graph theoretic tools. Then, we discuss two types of network configurations and present how the study of the stability of these networked systems can be approached. Finally, we present several experiments performed with a lattice network structure and show its key features and usage.

2. BACKGROUND

More than 34,000 objects above 10 cm in size are known to orbit Earth and around 1 million are estimated above 1 cm in size[1]. Although space debris mitigation guidelines are increasingly adopted by space actors, these trends are expected to worsen due to the growing interest in space technologies both from the private and public sectors. Previous studies have shown that the risk of collision among objects in space has been growing in the last decades and is expected to worsen in the near future [5]. These trends and the risk of a domino effect of collisions among objects can hinder future access to space. In this regard, debris evolutionary models are pivotal to improve our understanding of the space environment and its evolution. The first orbital debris model dates back to the 1970s when it was developed at NASA [6]. Since then, several other models have been developed by private partners and space agencies, such as EVOLVE 4.0 [7], LEGEND [8], DELTA [9], MEDEE [10], LUCA2 [11], IDEMN [12], etc. Usually, these models propagate the current population of objects in time and compute all the effects of sink and source mechanisms in such a population. These effects include possible collisions among objects, possible explosions, orbital decay due to atmospheric drag or other perturbative effects, future launches of objects in space, adoption of post-mission disposal strategies, adoption of collision avoidance strategies, and active debris removal. Often, when a collision or explosion event is simulated, NASA's breakup model is used to simulate the fragments number, area to mass ratio, and change in velocity: this model employs an empirically derived set of probability density functions to describe the distributions of these variables after a disruption [13]. Most of the aforementioned models compute the collision rates using algorithms that assess how long surrounding objects stay in the proximity of each object and then derive a collision rate from that. Some examples of these algorithms are Orbit-Trace [14] and CUBE [15]. The cost of the propagation of the entire population added to the computational burden of these collision rates estimation algorithms can make these models quite slow and less versatile. Reference [16] proposed a different approach, by computing collision rates through orbital debris flux and by extracting the colliding masses using the geometric average of the orbital region of interest. While this approach solves the computational complexity issues, it averages the colliding masses, therefore failing to capture all the possible collision configurations. To combine the speed of the latter model with the accuracy of the former, we build a simulator that employs a fully probabilistic approach and that is versatile to different orbital configurations. In the next section, we will detail the overall structure and features of the aforesaid model.

[1] `https://www.esa.int/Safety_Security/Space_Debris/Space_debris_by_the_numbers`, April 2021.

3. TEMPORAL NETWORK MODEL

3.1. General Form

As already mentioned, the growing rate of launches and tracked objects in space makes it cumbersome to maintain and develop a population dynamics model that keeps track of every single object. For this reason, we focused on a stochastic approach that treats each collision event as a random event where the collision time, the colliding masses, and the generated fragments number and characteristics are treated in a probabilistic manner. This type of model is able to capture the uncertainties in the problem and inform the user about the probability of certain scenarios taking place in the future. This tool can be extremely important in assessing the environment evolution and provides the user with a general model that governs the population dynamics and stability. We formulate this model as a time-varying network, where each node is a population of a certain class, and each link represents the relationship among nodes, which can be expressed in terms of in/out-flow of mass or in terms of collision probability among classes and populations. By assuming that:

- each node represents a type of population of either active on inactive objects in a given orbital class k;
- we use y to indicate the active objects in population j and x to indicate the inactive objects in the same population. Thus the active objects of population j in class k are y_{jk} and the inactive objects in the same population of the same class are x_{jk};
- mass can flow from one class to another;

the general form of such a temporal network model is the following:

$$\begin{aligned} dy_{jk} &= f_{y_{jk}}(x_{jk}, y_{jk})dt + g_{y_{jk}}(x_{jk}, y_{jk})dB \\ dx_{jk} &= f_{x_{jk}}(x_{jk}, y_{jk})dt + g_{x_{jk}}(x_{jk}, y_{jk})dB, \end{aligned} \tag{1}$$

where B denotes a Wiener process and f and g are two generic functions that describe the drift and diffusive parts of the dynamics, respectively.

By expanding the non-diffusive form, we get:

$$\begin{aligned} dy_{jk} =& \Big[IN^y_{jk} - \sum_{l,m} y_{jk}(1-s_{CAM})(C_2\tau_{2kjlm}x_{lm} + \\ & + C_1\tau_{1kjlm}y_{lm}) - OUT^y_{jk}y_{jk}\Big]dt \\ dx_{jk} =& \Big[\sum_{l,m}[x_{jk}(C_4\tau_{4kjlm}y_{lm} + C_3\tau_{3kjlm}x_{lm}) + \\ & + y_{jk}(1-s_{CAM})(C_2\tau_{2kjlm}x_{lm} + \\ & + C_1\tau_{1kjlm}y_{lm})] + IN^x_{jk} - OUT^x_{jk}x_{jk}\Big]dt, \end{aligned} \tag{2}$$

where s_{CAM} refers to the probability of successfully performing collision avoidance maneuvers (i.e., $s_{CAM} = 1$ if all collision avoidance maneuvers are performed successfully), IN and OUT refer to the source and sink terms, respectively, of the given j population of active or non-active objects, at the kth orbital class. In particular, IN includes explosions, new launches, and inflow of masses from neighboring classes; whereas OUT includes outflow of masses towards neighboring classes or towards Earth (in the case of re-entry). Furthermore, τ represents a time-varying normalized collision rate among population and classes, where τ_1 refers to the collision rate between active objects, whereas τ_2 between active and inactive ones, τ_3 between inactive ones and τ_4 between inactive and active ones. It is clear that each class and population can interact with any object of the neighboring populations and classes, in terms of collisions. The signs of the collision rates can be understood considering that the number of inactive objects grows and the number of active objects decreases as collisions increase. Furthermore, the exact definition of these terms is strictly related to the topology of the network. Finally, the C coefficients are needed to account for the number of objects produced in each collision. In the general case where a temporal network with only populations of active and inactive objects (i.e., j=1) is considered, where orbital classes are grouped according to Keplerian elements values, we would have a fully connected graph structure, as shown in Figure 1a. On the other hand, by dividing the classes into orbital shells, one would get a lattice structure as shown in Figure 1b, where only the members of the same class (i.e., orbital altitude range) can interact in terms of collisions and mass can in and out-flow in between neighboring classes. In the following section, we investigate the key features of this network.

3.2. Lattice Network

This type of network describes the case in which the orbital classes are orbital shells between 200 and 2,000 km, assuming that objects in a shell can only collide with others in the same shell. Such a model is widely studied in literature and therefore provides a baseline to compare our proposed method [16], [12]. In this case, the only collision rates that survive from Equation (2) are only those with $k = l$, since we assume that only elements of the same orbital shell can collide. Moreover, assuming to only consider active and inactive objects with no other specific population, we would get:

$$\begin{aligned} dy_k =& \big[IN_{y_k} - y_k(1 - s_{CAM})(\tau_{5kk}x_k + \tau_{6kk}y_k) \\ &- OUT_k y_k\big]dt \\ dx_k =& \big[y_k(1 - s_{CAM})(\tau_{5kk}x_k + \tau_{6kk}y_k) + \tau_{7kk}x_k x_k \\ &- OUT_k x_k + IN_{x_k}\big]dt \end{aligned} \tag{3}$$

where τ_{6kk} is the collision rate among operational satellites, τ_{7kk} among inactive, and τ_{5kk} between operational and inactive. We can express these coefficients, using a debris flux assumption, as [16]:

$$\begin{aligned} \tau_{5kk} =& \pi(R_{x_k} + R_{y_k})^2 \rho_{x_k}\rho_{y_k} v_r V_{bin,k} \\ \tau_{6kk} =& 4\pi R_{y_k}^2 \rho_{y_k} v_r \frac{(\rho_{y_k} V_{bin,k} - 1)}{2} \\ \tau_{7kk} =& 4\pi R_{x_k}^2 \rho_{x_k} v_r \frac{(\rho_{x_k} V_{bin,k} - 1)}{2} \end{aligned} \tag{4}$$

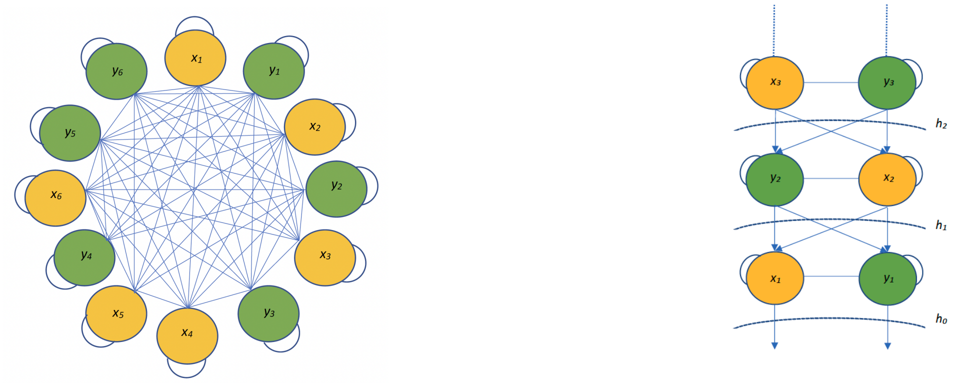
where R_{x_k} and R_{y_k} are the radii of the non-operational and operational objects, whereas $V_{bin,k}$ is the volume of the kth orbital shell, ρ_{x_k} and ρ_{y_k} are the spatial densities of inactive and active objects in the considered shell, and v_r is the relative velocity among the two colliding objects (which is assumed to be fixed at 10 km/s).

3.3. Network Model Simulation

In previous sections, the constitutive equations of the network model have been introduced and it was shown how they can be used to compute the number of inactive and active objects in each class and population. Both the number of active and inactive objects are to be considered two stochastic quantities whose value depends on an underlying (time varying) process that defines the values of the coefficients τ and the IN and OUT terms. This model captures the underlying uncertainties in the global dynamics of the space environment and thus results in a probabilistic description of its evolution. For treating this, the underlying stochastic processes are simulated at each time-step to check for increase and decrease in each population and class due to sink and source phenomena (i.e., IN and OUT), check for collisions among objects, update the collision rates (i.e., τ) and the number of active and inactive objects per population and class, and repeat these steps until the final simulation time is reached.

In short, this space debris evolutionary temporal network model leverages probability distributions to capture the underlying uncertainties in the evolution. The general description of the model is displayed in Figure 2. As it can be seen, the model features the following steps:

1. the model inputs have to be provided: these include the current population file (where orbits and physical characteristics of resident space objects are provided) and the launch file that specifies launch characteristics (for simulating future launches). Optionally, megaconstellation details can be provided. Also, the model allows to specify the operational lifetime of active satellites before they become inactive, and the post-mission disposal strategies to be implemented afterwards (e.g. 25 years lifetime before burning up during re-entry);



(a) Fully connected network: dividing classes using orbital elements.

(b) Lattice network: dividing classes using orbital shells.

Figure 1: Two types of network topology.

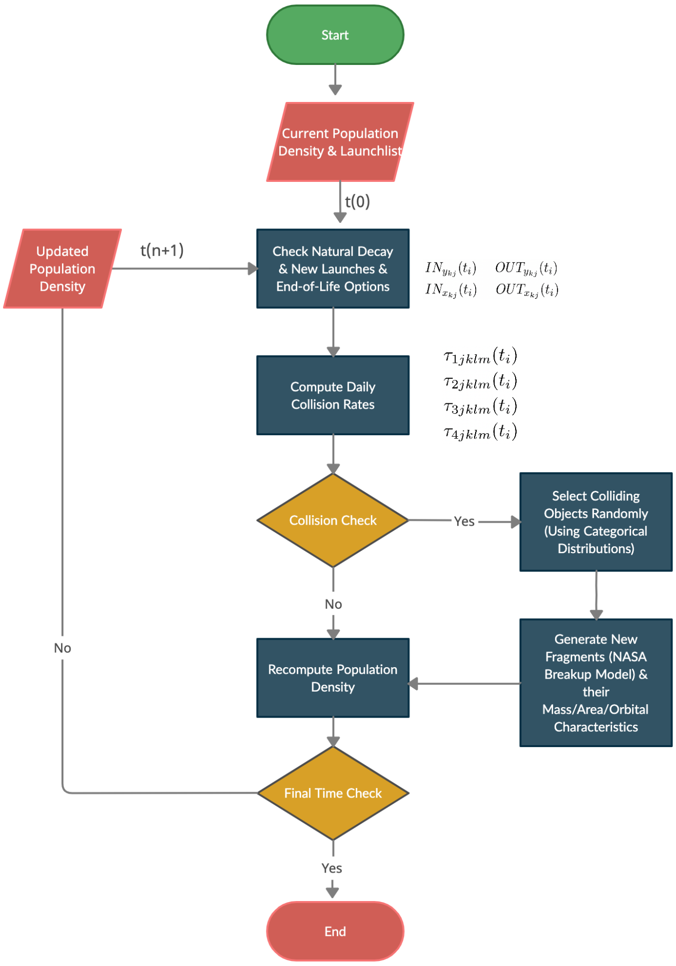


Figure 2: Space debris evolutionary model workflow

2. the time stamp is advanced, and new launches (i.e., $IN_{y_k}(t_i)$), end-of-life options (i.e., $OUT_{y_k}(t_i) \rightarrow IN_{x_k}(t_i)$) and natural decay (i.e., $OUT_{x_k}(t_i)$) are checked and updated if needed;

3. collision rates are computed (i.e., τ_{1jklm}, τ_{2jklm}, τ_{3jklm}, τ_{4jklm}). For instance, for the lattice network case, Equation (4) is used;

4. Poisson distributions are used to establish if a collision between objects has to happen. In a certain time interval Δt, whether a collision between class

k, population j and class l, population m happens is determined by sampling a Poisson distribution. In particular, the probability of having a collision at each time interval Δt can be written as:

$$Pr(collision) = \tau e^{-\tau}$$

where τ is the collision rate in the considered time interval. This is computed for all the populations and classes (e.g. τ_{1kjlm}, τ_{2kjlm}, τ_{3kjlm}, τ_{4kjlm} are used as rates);

5. in case a collision check returns a positive answer, categorical distributions are used to establish the target and chaser masses involved in the collision for the considered population and orbital classes. The colliding masses are then removed from the relevant populations and classes;

6. NASA standard breakup model is then used to simulate the number of fragments generated (which will be used to update the number of inactive objects in the considered region), their area, mass, and their change in velocity: this model leverages mixtures of normal distributions to generate the area to mass ratio of the produced fragments, which in turn are dependant on the colliding masses. More details about this model can be found in literature [13];

7. then, all the contributions of the sink and source terms described above are used to assess the increment/decrement in the number of active and inactive objects in each population and class (which means that dy_{kj} and dx_{kj} are computed). Finally, the number of inactive and active objects per class and population (i.e., $y_{kj}(t_{i+1})$ and $x_{kj}(t_{i+1})$) and the population density characteristics are updated. These latter include the densities of the considered orbital regions used for computing the collision rates, as well as the area and mass characteristics of each region (useful for the collision rates computation, for the selection of the colliding masses, and for computing the characteristics of the fragments if a collision happens);

8. time is advanced and step 2 is repeated until the final time has been reached.

As discussed, the model captures the underlying problem uncertainties by means of probability distributions, in particular, Poisson, categorical and mixture distributions are used for this purpose. This makes this evolutionary model stochastic since every run of the model will output a different result, and multiple simulations can be performed to analyze possible future scenarios of the space debris population. We discuss some experiments in Section 4.

3.4. Stability Condition

The general study of the stability of a networked system described by a set of stochastic differential equations can be carried out using Lyapunov theory. By assuming the generic network form expressed in Equation (1), the stability study involves the study of a Lyapunov function. In particular, defining the following operator:

$$L = \frac{\partial}{\partial t} + \sum_{i=1}^{d} f_i \frac{\partial}{\partial x_i} + \frac{1}{2} \sum_{i=1}^{d} [gg^T]_{ij} \frac{\partial}{\partial x_i \partial x_j}. \tag{5}$$

If a positive-definite Lyapunov function $V(x,t)$ exists such that:

$$LV \leq 0 \qquad \forall x, t \in [t_0, \infty), \tag{6}$$

then the trivial (i.e., equilibrium) solution is stable. Whereas if $LV < 0$, then the trivial solution is said asymptotically stable. These stochastic stability concepts are a direct generalization of Lyapunov's direct method for ordinary differential equations [17]. Moreover, more general concepts exist in the context of the study of the stability of stochastic differential equations, such as exponential stability and stability in probability [18]. The study of the stability for random processes is far from trivial, and the thorough analysis of these concepts goes beyond the scope of this work.

For showing an application of the stability analysis, we limit ourselves to Equation (2) and to the case in which IN and OUT terms are constant. This resembles a known structure studied in multigroup epidemic models [19]. In this case, by grouping the multi-indexes jk into one single index k, for ease of notation, we can define the following Lyapunov function:

$$V(y_k, x_k) = \sum_k \bigg(y_k - y_k^* - y_k^* \log(\frac{y_k}{y_k^*}) + x_k - x_k^* - x_k^* \log(\frac{x_k}{x_k^*}) \bigg),$$

where x_k^* and y_k^* refer to the equilibrium solutions of Equation (2). Then:

$$
\begin{aligned}
LV = \sum_k \Bigg(& y_k^* OUT_k \big[2 - \frac{y_k}{y_k^*} - \frac{y_k^*}{y_k}\big] \\
& + \sum_j \Bigg[\tau_{2kj} \big[y_k^* x_j^* - \frac{y_k^*}{y_k} y_k^* x_j^* + y_k^* x_j \\
& - \frac{x_j y_k x_k^*}{x_k} - \frac{y_k^* x_j^* x_k}{x_k^*} + y_k^* x_j^*\big] \\
& + \tau_{1kj} \big[y_k^* y_j^* - \frac{y_k^*}{y_k} y_k^* y_j^* + y_k^* y_j \\
& - \frac{y_k y_j x_k^*}{x_k} - \frac{y_k^* y_j^* x_k}{x_k^*} + y_k^* y_j^*\big] \\
& + \tau_{3kj} \big[x_k x_j - x_k^* x_j - x_j^* x_k + x_j^* x_k^*\big] \\
& + \tau_{4kj} \big[x_k y_j - x_k^* x_j - x_j^* x_k + y_j^* x_k^*\big] \Bigg] \Bigg).
\end{aligned}
$$

Redefining:

$$
\begin{aligned}
\overline{\tau}_{2kj} &= \tau_{2kj} x_j^* y_k^* \\
\overline{\tau}_{1kj} &= \tau_{1kj} y_j^* y_k^* \\
\overline{\tau}_{3kj} &= \tau_{3kj} x_j^* x_k^* \\
\overline{\tau}_{4kj} &= \tau_{4kj} y_j^* x_k^*.
\end{aligned}
$$

We get:

$$
\begin{aligned}
LV = \textstyle\sum_k & \Bigg[y_k^* OUT_k \big[2 - \frac{y_k}{y_k^*} - \frac{y_k^*}{y_k}\big] \\
+ \textstyle\sum_j & \Bigg(\overline{\tau}_{2kj} (2 + \frac{x_j}{x_j^*} - \frac{y_k^*}{y_k} - \frac{x_j y_k x_k^*}{x_k x_j^* y_k^*} - \frac{x_k}{x_k^*}) \\
+ \overline{\tau}_{1kj} & (2 + \frac{y_j}{y_j^*} - \frac{y_k^*}{y_k} - \frac{y_k y_j x_k^*}{x_k y_k^* y_j^*} - \frac{x_k}{x_k^*}) \\
+ \overline{\tau}_{3kj} & (1 - \frac{x_k}{x_k^*} - \frac{x_j}{x_j *} + \frac{x_k x_j}{x_k^* x_j^*}) \\
+ \overline{\tau}_{4kj} & (1 - \frac{x_k}{x_k^*} - \frac{y_j}{y_j^*} + \frac{x_k y_j}{x_k^* y_j^*}) \Bigg) \Bigg].
\end{aligned} \tag{7}
$$

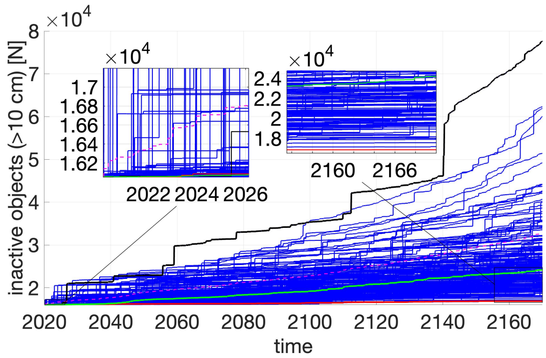
Due to the fact that x_k and y_k are always positive and $-a \leq -(1 + \log(a))$ (since $a - 1 - \log a \geq 0$, if $a > 0$) and applying logarithm's properties, one can derive the following stability condition:

$$
\begin{aligned}
LV & \leq \textstyle\sum_k \left[y_k^* OUT_k (2 - \frac{y_k}{y_k^*} - \frac{y_k^*}{y_k}) \right] \\
& = - \textstyle\sum_k OUT_k \frac{(y_k - y_k^*)^2}{y_k} \leq 0\,.
\end{aligned} \tag{8}
$$

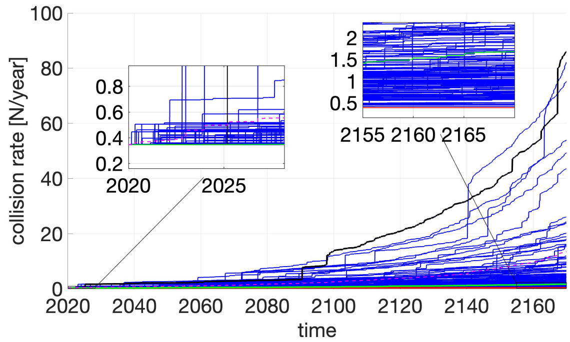
Therefore, with the above-mentioned assumptions, the equilibrium solution is always stable, when it exists.

4. EXPERIMENTS

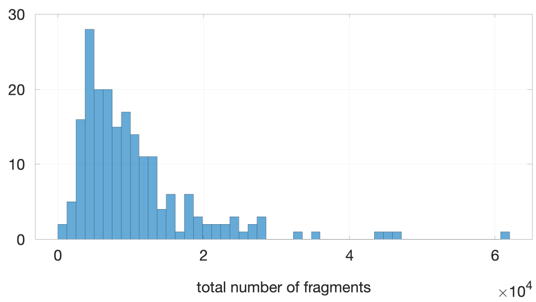
For the experiments, we use the current population (as of January 2021) and 2010-2018 MASTER launch data to simulate future launches. Moreover, we do not consider megaconstellations and we study the lattice network structure. Operational satellites are assumed to become non-operational after 8 years from their launch, and to be disposed with 100% success rate after 25 years they have become non-operational. It is assumed that collision avoidance maneuvers are executed with 100% success rate on operational satellites (i.e., $s_{CAM} = 1$), which means that only collisions among inactive objects can happen. For every simulation run, we keep track of all the variables involved, including but not limited to the number of operational and non-operational objects, their characteristics, the number of collisions, the number of generated fragments, etc. Furthermore, in these simulations, only objects above 10 cm in size have been considered. The considered population spans from 200 to 2000 km, with each orbital shell, defined every 20 km. Therefore, the lattice network comprises 90 orbital shells and 180 nodes, with a total of 360 links that describe collision rates among nodes. In Figure 3, we show several key parameters monitored during 200 simulations for 150 years each. As can be seen, we monitored the number of active objects, inactive objects, and the collision rate (expressed as the number of collisions per year) as a function of the years. In green, we highlight the median value of the variables, while in red and black the lower and upper bounds, respectively, and with the dashed line the standard deviation. The number of active objects grows from 2907 to 15352 in 150 years, and its behavior is the same across all the simulations, due to the fact that we assume that collision avoidance maneuvers are always performed successfully, and therefore collisions with active objects are not possible. The collision rates are computed by summing, at each time step, the collision rates of every orbital class. Due to this, the model allows to investigate the growth of every class collision rate, therefore informing the user on which are the most critical nodes in the network. Moreover, we also inspected the total number of fragments and collisions in each run and reported their histograms. On average, after 150 years, the simulated space environment scenarios end up reaching around 1.5 times the current inactive objects population and three times their current collision rate (which is currently around 0.347). These results appear in agreement with previous results [12], [5]. However, there are rare events, which show that the population of inactive objects can grow as much as 4 times its current value and the collision rate can reach up to 80 collisions per year, with more than 2000 collisions happening in the next 150 years. These very rare scenarios represent cases in which the collisions trigger a domino effect that causes full development of a Kessler syndrome.



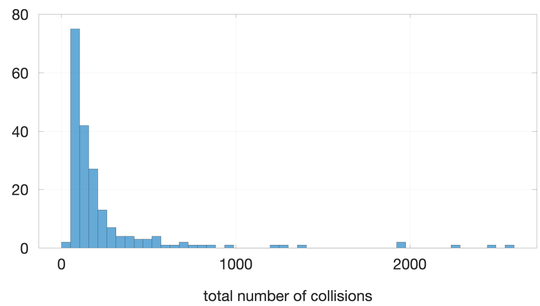
(a) Inactive objects count as a function of years.



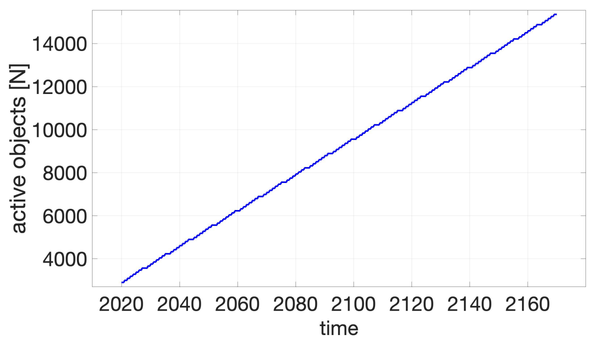
(b) Overall collision rates expressed as number of collisions per year.



(c) Total number of fragments histogram.



(d) Total number of collisions histogram.



(e) Number of active objects as a function of time.

Figure 3: Key parameters monitored during the simulations.

5. CONCLUSIONS

In this work, a temporal network model that probabilistically describes the space environment evolution over long periods of time has been introduced. This model demonstrated to retain the flexibility of previous stochastic models while permitting more versatility and conceptual simplicity. Indeed, by changing the definition of orbital classes and other network parameters, the user can investigate different network topologies, which can lead to the study of nontrivial relationships among orbital classes. Moreover, this type of approach broadens the set of tools that can be used for analyzing the space environment evolution, allowing the use of network theory tools for investigating the structure, dynamics, and stability of the network, and the presence of weak or strong links and nodes that can play a fundamental role in the space debris environment evolution. This analysis can be fundamental to understand the future evolution of collision risk in space, for any given orbital region, and therefore to inform operators on the risk profile of a given mission during its lifetime. Furthermore, such a model can also be used as a framework to investigate different mitigation strategies, such as post-mission disposal duration, collision avoidance success rate, and active debris removal, needed for stabilizing the space environment over long periods of time. In this work, we have presented a preliminary treatment of this network model, applied to a lattice structure in which orbital classes correspond to orbital shells. We then discussed several experiments, where we showcased the use of the network in investigating different possible future space environment scenarios.

In the future, we plan to study more topologies and to analyze the role of different control strategies for stabilizing the space environment evolution and reduce the risk of a Kessler syndrome.

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