A Probabilistic Approach to Assess the Computational Uncertainty of Ultimate Strength of Hull Girders

Shen Li[1]*, Do Kyun Kim[2]*, Simon Benson[3]*

Marine, Offshore and Subsea Technology Group, School of Engineering, Newcastle University, Newcastle upon Tyne, NE1 7RU, United Kingdom

*[1]*Corresponding email: s.li37@newcastle.ac.uk*

*[2]*Corresponding email: do.kim@newcastle.ac.uk*

*[3]*Corresponding email: simon.benson@newcastle.ac.uk*

ABSTRACT

The simplified progressive collapse method is codified in the IACS Common Structural Rules (CSR) to calculate the ultimate strength of ship hull girders in longitudinal bending. Several benchmark studies have demonstrated the uncertainty of this method, which is primarily attributed to the variation in the load-shortening curve (LSC) of local structural components adopted by different participants. Quantifying this computational uncertainty will allow the model error factor applied for the ultimate strength of hull girder in a reliability-based ship structural design to be determined. A probabilistic approach is proposed in this paper to evaluate the prediction uncertainty of ultimate strength of the hull girder caused by the critical characteristics within the LSCs. The probability distributions of critical load-shortening characteristics of stiffened panels are developed based on a dataset generated by empirical formulae and the nonlinear finite element method. An adaptable LSC formulation, with the ability to cater for specific response features of local components, is utilised in conjunction with the Monte-Carlo simulation procedure and the simplified progressive collapse method to calculate the ultimate strength of a hull girder at each sampling. The proposed method is applied to four merchant ships and four naval vessels. The computational uncertainties of the ultimate strength of the case study vessels are discussed in association with their mean values and standard deviations. The study shows that the ultimate strength of ship hull girders is subjected to different uncertainties in sagging and hogging. Whist the strength of merchant ships are primarily governed by the ultimate compressive strength of critical stiffened panels, the strength of naval vessels are also sensitive to the post-collapse response of critical members.

KEYWORDS: Probabilistic approach; uncertainty; Monte-Carlo Simulation; ultimate strength; buckling; ship hull girder; Smith method.

HIGHLIGHTS:

- A probabilistic approach for assessing the computational uncertainty of ultimate strength of hull girders;
- Application on merchant ships and naval vessels;
- The ultimate strength of hull girders are subjected to different uncertainty in sagging and hogging;
- Strength of merchants ships is dominated by the local strength of critical members;
- Strength of naval ships is affected by the complete load-shortening response of structural members.

NOMENCLATURE

β = Plate slenderness ratio

λ = Column slenderness ratio

σ_x = Average stress of stiffened panel in longitudinal direction

ε_x = Average strain of stiffened panel in longitudinal direction

σ_{xu} = Ultimate compressive strength of stiffened panel

ε_{xu} = Ultimate compressive strength of stiffened panel

$\sigma_{1.5\varepsilon_{xu}}$ = Post-collapse strength of stiffened panel at $1.5\varepsilon_{xu}$

$\sigma_{2.0\varepsilon_{xu}}$ = Post-collapse strength of stiffened panel at $2.0\varepsilon_{xu}$

$\sigma_{1.5\varepsilon_{xu}}^{CSR}$ = Post-collapse strength of stiffened panel at $1.5\varepsilon_{xu}$ predicted by standard CSR method

$\sigma_{1.5\varepsilon_{xu}}^{CSR}$ = Post-collapse strength of stiffened panel at $2.0\varepsilon_{xu}$ predicted by standard CSR method

σ_{xu}^{CSR} = Ultimate compressive strength of stiffened panel predicted by standard CSR method

ε_{xu}^{CSR} = Ultimate strain of stiffened panel predicted by standard CSR method

σ_{Yeq} = Equivalent material yield stress

ε_{Yeq} = Equivalent material yield strain

$\bar{E}_{To}$ = Normalised initial stiffness of stiffened panel

$\bar{E}_{T}$ = Normalised instantaneous stiffness of stiffened panel

k_i = Tangent stiffness of structural element

A_i = Cross sectional area of structural element

z_i = Vertical coordinate of structural element

z_G = Vertical coordinate of the neutral axis of cross section

$M_{u(sag)}$ = Ultimate strength of hull girder in sagging

$M_{u(hog)}$ = Ultimate strength of hull girder in hogging

$M_{u(sag)}^{CSR}$ = Ultimate strength of hull girder in sagging predicted by standard CSR method

$M_{u(sag)}^{CSR}$ = Ultimate strength of hull girder in hogging predicted by standard CSR method

1. INTRODUCTION

The IACS Common Structural Rules (CSR) recommends the use of the simplified progressive collapse method (Smith method), initially proposed in [1], for calculating the ultimate strength of ship hull girders in longitudinal bending [2]. The underpinning algorithms within these methods are common, relying on an evaluation of the load-shortening curves (LSC) of local stiffened panel components under in-plane loading caused by global hull girder bending. Whist the procedure of the Smith method is well-established and consistent in application, the approaches to determine the local structural members' LSC substantially differ. A benchmark study was completed by the Special Task Committee VI.2 of 14th ISSC to assess the sensitivity of ultimate hull girder strength under pure vertical bending with respect to the LSC [3]. It was concluded by the committee that an uncertainty of 10% in the ultimate compressive strength of a structural element could lead to an uncertainty of roughly 20% on the sagging ultimate bending moment. Additionally, a variation of 25% in sagging and a variation of 12% in hogging may be induced due to the post-collapse characteristics. Recently, Li et al. [4] analysed the effect of ultimate compressive strength, ultimate strain, elastic stiffness and post-collapse stiffness of structural components on hull girder strength calculations by applying the simplified progressive collapse method. They showed that the ultimate compressive strength and post-collapse stiffness have the most considerable influence on the hull girder strength calculation among four considered parameters.

Quantifying the computational uncertainty from the LSC will better determine the model uncertainty factor of the ultimate strength of hull girder in a reliability-based ship structural design. Structural reliability analysis is concerned with predicting the probability of limit state violation for an engineered structural system at any stage during its lifespan [5]. Several studies have applied reliability theory to assess the safety of marine structures [6-11]. In terms of the ultimate limit state, there are several established procedures in the literature for the stochastic modelling of ultimate strength of ship hull girders. However, the common practice in most of the ultimate limit state-based reliability analysis methods for ship hull girders only account for the variance in scantling parameters. The strength model error relating to the epistemic uncertainty is evaluated empirically or even omitted, which is likely because no credible solution is available to assist the determination of such strength model uncertainty factor.

A systematic approach to evaluate the computational uncertainty of the ultimate strength of hull girder will improve reliability analysis. This paper develops a probabilistic method to assess the uncertainty of ultimate ship hull strength prediction. Probability distributions of the critical LSC characteristics of stiffened panels are developed based on a dataset generated by empirical formulae and the nonlinear finite element method. An adaptable LSC formulation is utilised in conjunction with a Monte-Carlo Simulation procedure and a simplified progressive collapse method to predict the ultimate strength of the hull girder in each sampling. The proposed method is applied to eight case study vessels, including merchant ships and naval vessels.

2. BACKGROUND

2.1 Computational Methods for Ultimate Strength of Hull Girders

Ultimate strength is a fundamental issue in the structural design of ship structures. Different prediction methods have been developed, namely 1) Presumed stress distribution-based method; 2) Collapse strain-based method; 3)

Idealised structural unit method; 4) Nonlinear finite element method; 5) Simplified progressive collapse method (Smith method). A concise overview is given in this section to highlight the key features of these methods.

The presumed stress distribution-based method was initially proposed by Caldwell [12]. Caldwell's approach was elaborated by Paik and Mansour with refined stress distributions [13]. A modified Paik and Mansour's method was developed to account for the spread of the tensile yielding zone [14]. In this methodology, a stress distribution at the collapse state is first assumed. The ultimate moment is then evaluated by taking the first moment of the resultant stress.

The collapse strain-based method was introduced by Hughes [15]. By examining each structural segment's average strain at a given curvature, the local failure is identified. The corresponding bending moment is calculated as the product the bending stiffness and applied curvature. The calculation proceeds to an equilibrium check with the revised bending stiffness accounting for the loss of load-carrying capacity of the failed components. The procedure is performed in an incremental/iterative way and is terminated when no equilibrium can be achieved, at which the corresponding bending moment defines the ultimate strength of the hull girder.

The idealised structural unit method was initially developed by Ueda and Rashed [16]. It was intended to tackle the significant computational time in the conventional finite element method by reducing the degrees of freedom of each element. In this regard, beam-column element, plate element and stiffened panel element are developed.

The nonlinear finite element method evaluates all the buckling failure modes and their interactions. However, the computational time is substantial, especially for the hull girder analysis. Thus, it is not yet the most usual choice for the initial design of the ship structures.

The simplified progressive collapse method is a generalisation of Euler beam theory. The bending stiffness of the cross section is incrementally evaluated by accounting for the contribution of local components. This method is able to trace the complete bending moment versus increments of curvature. Various advancements to the method have been proposed including [17-19].

2.2 Quantification of Modelling Uncertainty for Reliability Engineering

It is of importance to quantify the modelling uncertainty for rational reliability analysis of an engineering system. In this section, a review is presented for the state-of-the-art research regarding the modelling uncertainty quantification. Whilst the emphasis is placed on the ultimate ship hull girder strength, a concise summary is also given for the recent works in other disciplines in a view of stimulating their marine application.

2.2.1 Uncertainty Quantification of Ultimate Strength of Hull Girders

The ultimate strength of hull girders has been subjected to extensive probabilistic studies. Guedes Soares and Teixeira indicated that the material yield stress typically has a coefficient of variance (COV) of 7% to 8% [20]. Combined with the additional uncertainty related to computation, a 15% COV was assumed for the ultimate ship hull strength. Similarly, an overall COV of 12% with a mean value of 1.1 was assumed by Parunov et al. [21-22]. Further examples may refer to [23-26], in which the uncertainty factor is assumed to follow a normal distribution with unity mean value and COV of 10% to 15%. Uncertainties in material properties, geometric dimensions and computations were expected to be covered. A more elaborate approach to account for the variability of material property and geometric dimension is the inverse application of the first-order reliability method, as employed by Teixeira and Guedes Soares [27]. For sagging, it was proposed that the mean value between the ultimate bending moment and the elastic bending moment was 1.014 and the standard deviation was 0.067. For hogging, it was proposed that the mean ratio between the ultimate bending moment and elastic bending moment was 1.195 and the standard deviation was 0.081. Alternatively, as presented by Xu et al [28], the model correction factor method may be applied where the model correction factor was found by an iterative calculation, so that the safety index estimations based on a realistic strength model and the corrected simplified model satisfies convergence criterion. In Xu's application, the NLFEM model was chosen as the baseline "realistic model". A hybrid approach was adopted by Lua and Hess where the ultimate ship hull strength prediction program ULTSTR was combined with Monte-Carlo Simulation accounting for the variability of material property and geometric dimension [29].

It is clear from the literature survey that a variety of computational methods are established to predict the ultimate strength of ship hull girders in longitudinal bending. In addition, the modelling error is primarily assessed considering the uncertainties due to geometry and material properties, whereas the computational uncertainty has not yet been thoroughly analysed. It is challenging to develop credible uncertainty factors for different computational methods. This paper will focus on the simplified progressive collapse method (Smith method), because it is the codified approach in contemporary ship structural design guidelines such as CSR.

2.2.2 Uncertainty Quantification in Other Disciplines

A number of reliability studies with consideration of the model uncertainty were available in the literature of other disciplines. Feng et al. [30] presented a time-dependent reliability-based redundancy assessment of deteriorated RC structures against progressive collapse considering corrosion effect, in which the modelling uncertainty was empirically specified. Francesco et al. [31] demonstrated a time-dependent reliability analysis of the reactor building of a nuclear power plant where the probabilistic safety margin evaluation had accounted for the epistemic uncertainty. Liu et al. [32] introduced a machine learning-based Bayesian approach to inversely quantify and reduce the uncertainties of multiphase computational fluid dynamics simulations for bubbly flows. Zugazagoitia et al. [33] compared the non-parametric, parametric and binomial approaches for uncertainty analysis in the context of nuclear engineering. An uncertainty quantification and validation framework was presented by McKeand et al. [34] to account for both aleatory and epistemic uncertainties. An integrated data and knowledge model to address the aleatory and epistemic uncertainty for oil condition monitoring was developed by Pan et al. [35].

It is noticed from the literature survey in other disciplines that several recent studies were completed considering the model uncertainty quantification using dedicated methods other than empirical probabilistic models. Their applications in large-scale marine structures (e.g. ship hull girders) could be the future research objects.

3. METHODOLOGY

3.1 Overview

The proposed methodology includes four major steps. In step 1, an identification of the critical features of the LSC is conducted. Once the critical features are identified, their dataset generations are carried out in step 2. Based on the generated data, the probability distributions of the critical features of LSC are developed in step 3. Finally, the Monte-Carlo Simulation is combined with the simplified progressive collapse to sample the ship hull ultimate strength in step 4. With reference to Figure 1, the overall methodology is detailed in the following sections.

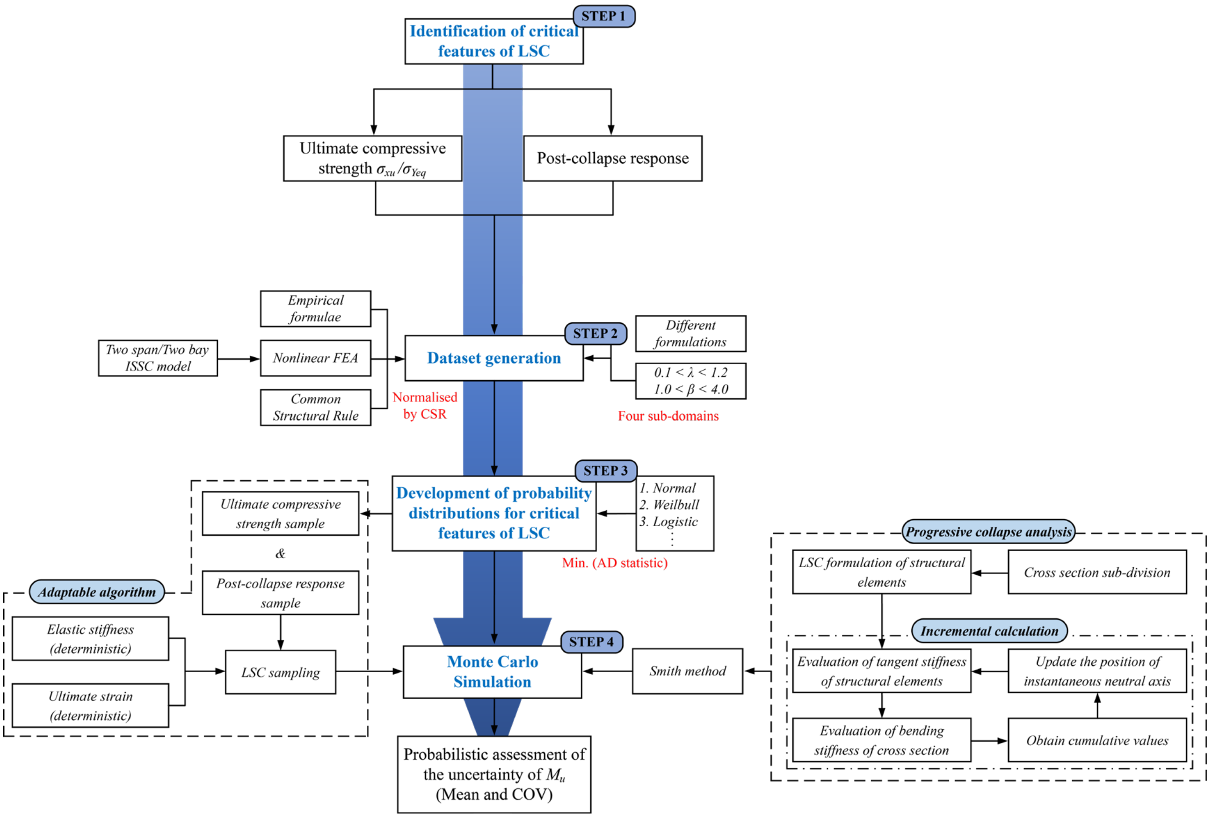

Figure 1. A flow chart illustration of the proposed methodology

3.2 Identification of the Critical Features of LSC (Step 1)

The first step is to identify the most influential features of the LSC in the ultimate ship hull strength prediction. This has been completed in a previous study by a deterministic approach [4]. With an adaptable algorithm, the effects of four LSC features, including ultimate compressive strength, ultimate strain, elastic stiffness and post-collapse stiffness, were investigated. The ultimate compressive strength is the maximum load-carrying capacity of a stiffened panel and the ultimate strain represents the corresponding average edge displacement where the collapse takes place. The elastic stiffness describes the rigidity of the panel in the initial elastic phase, while the post-collapse stiffness represents the capacity drop after the collapse takes place, which was simplified as linear in [4]. Different values of these critical LSC features were deterministically selected to generate LSCs of structural elements used in a Smith-type progressive collapse analysis. A case study on double hull cross section showed that the ultimate compressive strength and post-collapse characteristics of structural components were the two most critical response characteristics, as indicated by the sensitivity index. Based on this investigation, the present study explores their influence in a probabilistic manner.

3.3 Dataset Generation (Step 2)

The second step is to develop the dataset of critical LSC characteristics subjected to analysis, i.e. the ultimate compressive strength and post-collapse behaviour. The purpose of this dataset is to provide a collection of possible predictions that cover estimations by most of the equivalent methods available in the literature. For the ultimate compressive strength, the dataset is generated from the prediction by seven empirical formulae and the nonlinear finite element method. For the post-collapse characteristics, which is more difficult to predict by empirical formulae, the dataset is developed based on the results of the nonlinear finite element method only. For the development of the empirical formulation refer to [36-42]. A review of these empirical formulations are conducted by Kim et al. [43]. The modelling for nonlinear finite element analysis is consistent with the ISSC recommendation for boundary conditions and assumes an average-level imperfection. The present study focuses on the uncertainty due to different computational models, while the uncertainties due to imperfection and material property are out of the scope. Therefore, the average-level imperfection and nominal material property are assumed following the conventional practice for structural strength assessment of this kind. For a series of typical ship-type stiffened panels (105 panels), the computation of critical LSC characteristics of each panel is performed by all selected methods and is normalised by the standard CSR estimation. These predictions are grouped by domains to represent the prediction variance between state-of-the-art methods and the CSR approach.

For the post-collapse response, the dataset is generated only from the prediction by the nonlinear finite element method. The post-collapse response is usually characterised as a nonlinear curve and it is therefore challenging to describe its probabilistic variance. In this regard, the post-collapse response is characterised by two measurements in this paper, namely the strength at $1.5\varepsilon_{xu}$ and at $2.0\varepsilon_{xu}$ where ε_{xu} is the ultimate strain of the panel. A piecewise linear post-collapse response between these measurements is assumed. The purpose of this simplification is to better compare each sample prediction with the standard CSR method by ensuring that the principle post-collapse patterns are equivalent. An example verification is shown in Figure 2, which is typical of all results in this study, where the ultimate ship hull strength prediction with the standard CSR method and example sampling using the adaptable algorithm introduced later are closely correlated.

Different computation methods may compare well for certain structural configurations whilst deviating significantly for other configurations. This is because different computational methods were motivated by different theories and purposes. A direct probabilistic evaluation over the entire spectrum of data could omit this issue and result in a less credible probability distribution. Hence, for both ultimate compressive strength and post-collapse characteristics, the dataset is divided into four sub-domains based on slenderness ratios, i.e. sub-domain 1 ($\beta > 1.9$ & $\lambda < 0.6$), sub-domain 2 ($\beta > 1.9$ & $\lambda > 0.6$), sub-domain 3 ($\beta < 1.9$ & $\lambda < 0.6$) and sub-domain 4 ($\beta < 1.9$ & $\lambda > 0.6$). The rationale of this sub-division follows the von Karman model of plate ultimate strength together with the findings of Ozdemir et al. [44]. The former indicated that the plate behaviour could significantly change if $\beta > 1.9$, while the latter suggested that the primary failure mode of stiffened panels under compression may change from local plate buckling to beam–column buckling at the threshold of $\lambda = 0.6$. In Ozdemir's study, it was proposed that $\lambda = 1.3$ was another threshold value where the failure mode may be changed from column-type to overall collapse. However, this slenderness is out of the scope of the dataset and the application which will be presented later. Thus, it is not discussed in this paper. All of the ultimate compressive strength and post-collapse characteristics are normalised by the corresponding CSR estimation to indicate the uncertainty with respect to the CSR method as defined by Equation (1) to (3).

Normalised ultimate compressive strength = $\sigma_{xu}/\sigma_{xu}^{CSR}$ (1)

Normalised post-collapse strength at $1.5\varepsilon_{xu}$ = $\frac{\sigma_{1.5\varepsilon_{xu}}}{\sigma_{xu}} / \frac{\sigma_{1.5\varepsilon_{xu}}^{CSR}}{\sigma_{xu}^{CSR}}$ (2)

Normalised post-collapse strength at $2.0\varepsilon_{xu}$ = $\frac{\sigma_{2.0\varepsilon_{xu}}}{\sigma_{xu}} / \frac{\sigma_{2.0\varepsilon_{xu}}^{CSR}}{\sigma_{xu}^{CSR}}$ (3)

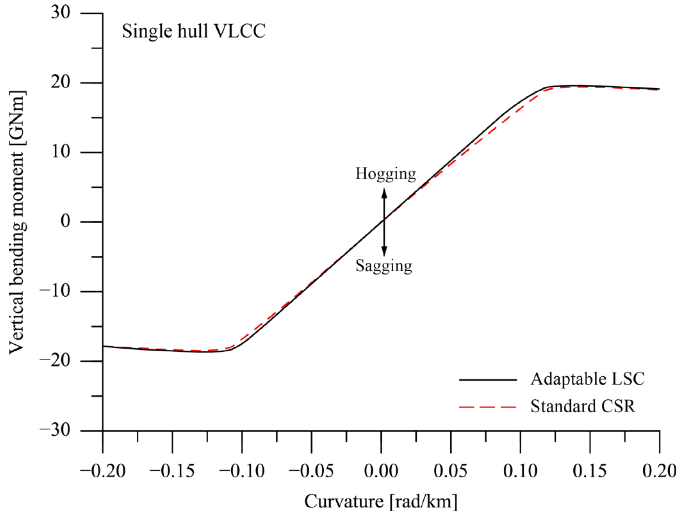

Figure 2. Comparison between standard CSR and simplified CSR with adaptable LSC

3.4 Development of the Probability Distribution of Critical Features of LSC (Step 3)

The third step is to derive the probability distribution of critical LSC characteristics based on the developed dataset. This is completed following the procedure introduced by Kim et al. [45] and further elaborated in [46]. Since there is no previous study indicating which probability model may be used for a specific LSC features, eleven candidate probability distributions are employed for fitting, including normal distribution, log-normal distribution, exponential distribution, 2-parameter exponential distribution, smallest extreme value distribution, Weibull distribution, 3-parameter Weibull distribution, largest extreme value distribution, logistic distribution, log-logistic distribution and 3-parameter log-logistic distribution. The estimation of distribution parameters is completed by the maximum likelihood method, except for normal and log-normal distributions which are based on unbiased parameter estimates. The Anderson-Darling statistics is calculated for each fitted distribution, in which the best-fit is selected justified by the minimum Anderson-Darling statistics.

3.5 Combined Monte-Carlo Simulation (Step 4)

The fourth step is to perform Monte-Carlo simulation combined with the simplified progressive collapse method. The ultimate compressive strength and post-collapse strength are sampled based on their probability distributions. Following the Monte-Carlo simulation procedure, a random number between zero to one is first generated from a uniform distribution. This random number is taken as the probability of a sample, by which the corresponding value of the LSC characteristics is evaluated via the inverse cumulative density function. These sample values are then input into the adaptable algorithm for deriving the complete LSC as illustrated by Figure 3 and given by Equation (4) to (8). Note that all the material properties and geometric dimensions are taken as mean values. Meanwhile, the normalised elastic stiffness is assumed as unity and the ultimate strain is computed by the formula provided in [47]. Although there should be a correlation between the ultimate compressive strength and the post-collapse strength of stiffened panels, they are assumed as statistically independent in this paper. This correlation may be revealed through a comprehensive finite element analysis. However, in many analytical methods, such as those reviewed in section 1, this correlation may not be properly defined. This is because predicting the post-collapse behaviour is particularly challenging in an analytical way and thus it is sometimes evaluated empirically without considering any correlation with the ultimate compressive strength. Hence, in this manuscript, they are considered as statistically independent. To consider the dependence between variables in future research, the insights developed in [48-50] may be followed.

$$\frac{\sigma_x}{\sigma_{Yeq}} = \bar{E}_{To}\frac{\varepsilon_x}{\varepsilon_{Yeq}} \qquad \text{for } \frac{\varepsilon_x}{\varepsilon_{Yeq}} \leq \frac{\varepsilon_{xe}}{\varepsilon_{Yeq}} \tag{4}$$

$$\frac{\sigma_x}{\sigma_{Yeq}} = \frac{\sigma_{xu}}{\sigma_{Yeq}} - R + Rcos[-tan^{-1}(\bar{E}_T)] \qquad \text{for } \frac{\varepsilon_{xe}}{\varepsilon_{Yeq}} < \frac{\varepsilon_x}{\varepsilon_{Yeq}} < \frac{\varepsilon_{xu}}{\varepsilon_{Yeq}} \tag{5}$$

$$\frac{\sigma_x}{\sigma_{Yeq}} = \frac{\sigma_{xu}}{\sigma_{Yeq}} + \bar{E}_{p1}\left(\frac{\varepsilon_x}{\varepsilon_{Yeq}} - \frac{\varepsilon_{xu}}{\varepsilon_{Yeq}}\right) \qquad \text{for } \frac{\varepsilon_{xu}}{\varepsilon_{Yeq}} \leq \frac{\varepsilon_x}{\varepsilon_{Yeq}} < \frac{1.5\varepsilon_{xu}}{\varepsilon_{Yeq}} \tag{6}$$

$$\frac{\sigma_x}{\sigma_{Yeq}} = \frac{\sigma_{1.5\varepsilon_{xu}}}{\sigma_{Yeq}} + \bar{E}_{p2}\left(\frac{\varepsilon_x}{\varepsilon_{Yeq}} - \frac{1.5\varepsilon_{xu}}{\varepsilon_{Yeq}}\right) \qquad \text{for } \frac{\varepsilon_x}{\varepsilon_{Yeq}} \geq \frac{1.5\varepsilon_{xu}}{\varepsilon_{Yeq}} \tag{7}$$

$$\frac{\varepsilon_{xu}}{\varepsilon_{Yeq}} \geq \frac{\sigma_{xu}}{\sigma_{Yeq}}/\bar{E}_{To} \tag{8}$$

where

$$R = \frac{cos[tan^{-1}(\bar{E}_{To})]\left(\bar{E}_T\frac{\varepsilon_{xu}}{\varepsilon_{Yeq}} - \frac{\sigma_{xu}}{\sigma_{Yeq}}\right)}{1 - cos[tan^{-1}(\bar{E}_{To})]}$$

$$\frac{\varepsilon_{xe}}{\varepsilon_{Yeq}} = \frac{\varepsilon_{xu}}{\varepsilon_{Yeq}} + Rsin[-tan^{-1}(\bar{E}_{To})]$$

$\bar{E}_{To}$ = Normalised initial stiffness

$\bar{E}_T$ = Normalised instantaneous stiffness

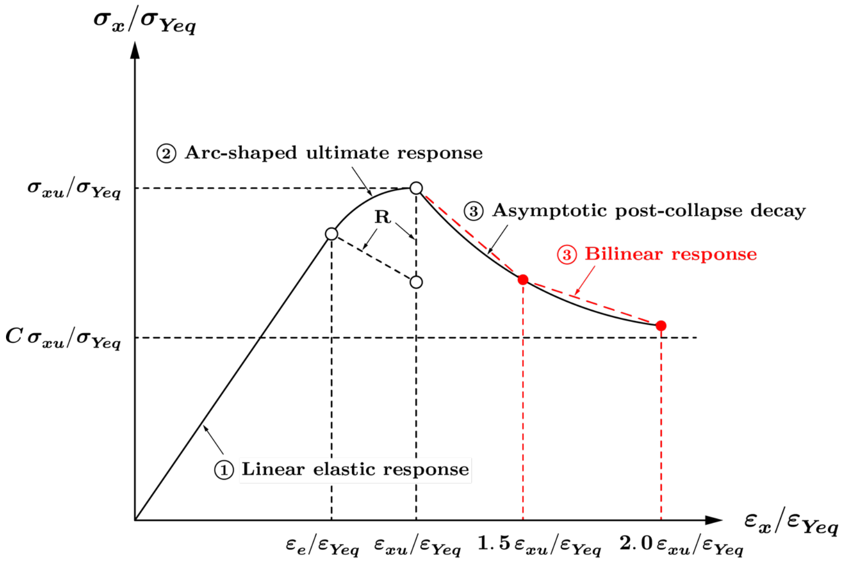

Figure 3. Schematic illustration of the adaptable algorithm for predicting LSC of structural component

Once the LSC sampling is completed, the ultimate strength of hull girder is calculated following the standard Smith method procedure. The ship hull girder cross section is sub-divided into structural elements, which are assigned with sampled LSC to characterise their responses under in-plane compression and tension. The analysis procedure of the Smith-type progressive collapse method may be summarised as follows:

1. The ship hull cross section is sub-divided into structural elements.
2. A load-shortening curve (LSC) characterising the response under monotonic in-plane load is assigned to each element.
3. Evaluate the tangent stiffness k_i of each element at present strain using the load-shortening curve.
4. Calculate the position of instantaneous neutral axis z_G using Equation (9).
5. Evaluate the vertical bending stiffness of the cross section with respect to the instantaneous neutral axis using Equation (10).
6. Calculate the strain increment of each element using Equation (11).
7. Calculate the vertical bending moment increment ΔM_V using Equation (12).
8. Obtain the cumulative bending moment, curvature and elemental strain.
9. Return to step 3.

In this paper, 2000 samples are conducted for each case study ship. To reduce the computational effort of Monte-Carlo Simulation, there are several recent studies discussing the efficient sampling methods [51-53]. Future research can be performed to apply these sampling techniques. The ultimate ship hull strength is normalised by the standard CSR computation to demonstrate the uncertainty with respect to CSR, i.e. Equation (13) and (14).

$z_G = (\sum_{i=1}^{n} z_i k_i A_i)/(\sum_{i=1}^{n} k_i A_i)$ (9)

$D_{VV} = \sum_{i=1}^{n} k_i A_i (z_i - z_G)^2$ (10)

$\Delta\varepsilon_i = (z_i - z_G)\Delta\chi_V$ (11)

$\Delta M_V = D_{VV}\Delta\chi_V$ (12)

Normalised ultimate sagging strength = $M_{u(sag)}/M_{u(sag)}^{CSR}$ (13)

Normalised ultimate hogging strength = $M_{u(hog)}/M_{u(hog)}^{CSR}$ (14)

4. PROBABILITY DISTRIBUTION OF LOAD-SHORTENING CHARACTERISTICS VARIANCE

The generated datasets are shown from Figure 4 to 6. The mean values of ultimate compressive strength variance in Domain 3 are close to unity, whereas the mean values in Domain 2 and Domain 4 are smaller than the standard CSR prediction by about 10% and the variance in Domain 1 is larger than the standard CSR. In terms of the post-collapse characteristics, the strength at $1.5\varepsilon_{xu}$ variance in Domain 3 is about 20% lower than the standard CSR, while the other domains are relatively close to CSR. Besides, the mean value of the strength at $2.0\varepsilon_{xu}$ all deviate from the standard CSR prediction by less than 10%. In general, the variance in ultimate compressive strength and strength at $1.5\varepsilon_{xu}$ are significantly larger than that of strength at $2.0\varepsilon_{xu}$. Additionally, these uncertainties appear to be dependent on the slenderness of the stiffened panel. For instance, for lower plate slenderness (Domain 3 & 4), the COV of ultimate compressive strength is around 10%, whereas the COVs of stiffened panels with higher plate slenderness are 0.1895 (Domain 1) and 0.2616 (Domain 2). Conversely, a larger uncertainty of the post-collapse response (strength at $1.5\varepsilon_{xu}$) is found on Domain 3 and Domain 4.

The domain sub-division in this paper is based on the difference in the primary buckling failure modes. Alternatively, the overall spectrum may be divided into three sub sets, namely stocky ($\lambda < 0.5$), intermediate ($0.5 < \lambda < 1.5$) and slender ($\lambda > 1.5$). The stocky panels generally corresponds to the Domain 1 and Domain 3 and the intermediate panels corresponds to Domain 2 and Domain 4, whereas the slender panels are out of the scope. Thus, the insights discussed above may be interpreted using the two different sub-domain divisions.

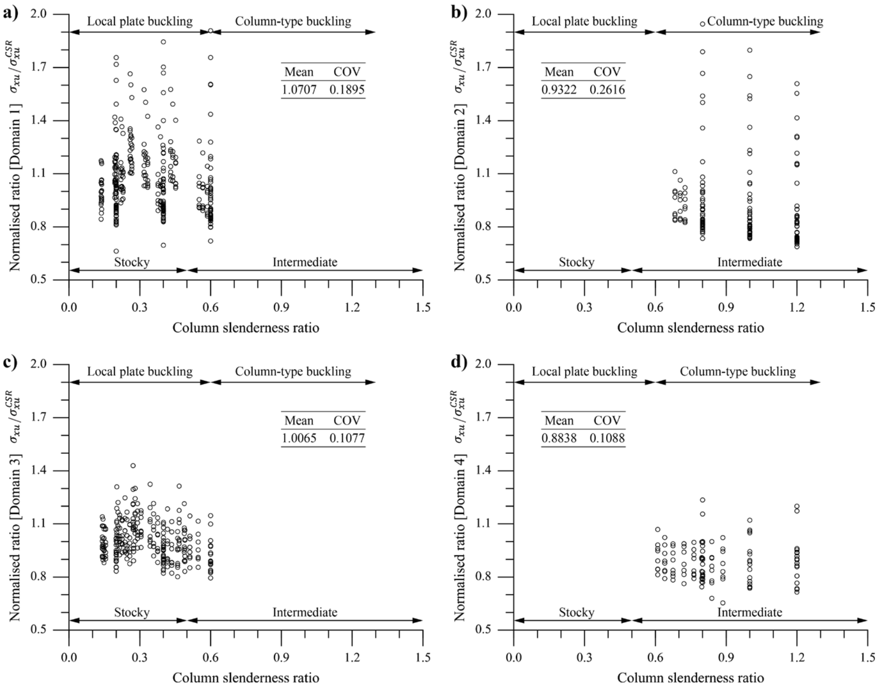

Figure 4. Dataset of the ultimate compressive strength variance of structural component for the four sub-domains: a) Domain 1; b) Domain 2; c) Domain 3; d) Domain 4

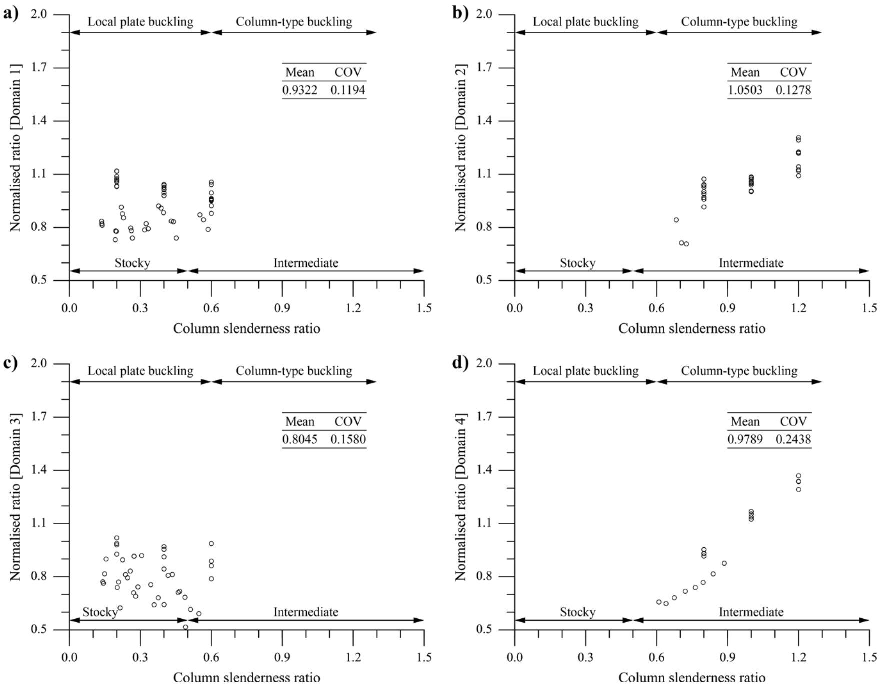

Figure 5. Dataset of the post-collapse strength variance (strength at $1.5\varepsilon_{xu}$) of structural component for the four sub-domains: a) Domain 1; b) Domain 2; c) Domain 3; d) Domain 4

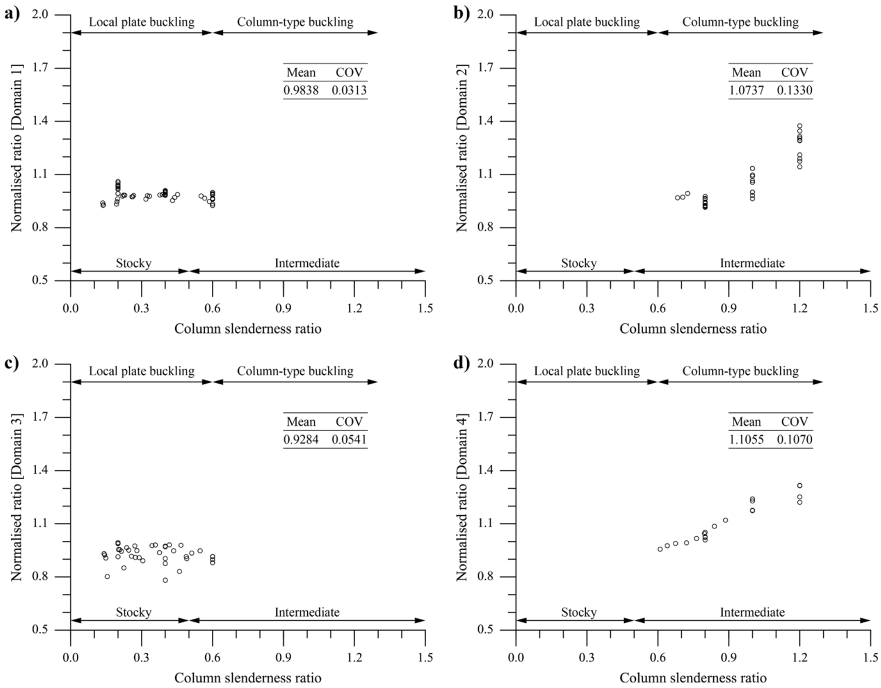

Figure 6. Dataset of the post-collapse strength variance (strength at $2.0\varepsilon_{xu}$) of structural component for the four sub-domains: a) Domain 1; b) Domain 2; c) Domain 3; d) Domain 4

The Anderson-Darling (AD) statistics of each candidate probability distribution for the ultimate compressive strength and post-collapse characteristics are given in Table 1, Table 2 and Table 3 where the AD statistics of the best-fit probability distribution (minimum AD statistics) is highlighted. Meanwhile, Figure 7 to 9 illustrate the probability density with best-fitted distribution function of each load-shortening characteristics variance. A 3-parameter log-logistic distribution is selected for the ultimate compressive strength variance in Domain 1 and Domain 2, while a 3-parameter Weibull and a largest extreme value distributions are chosen for Domain 3 and Domain 4 respectively. Besides, the smallest value distribution and logistic distribution are fitted for the strength at $1.5\varepsilon_{xu}$ variance in Domain 1 and Domain 2 respectively, whist the Domain 3 is fitted with the normal distribution and Domain 4 is fitted with the log-normal distribution. For the strength at $2.0\varepsilon_{xu}$, the fitted distributions are log-logistic, 2-parameter exponential, smallest extreme value and 3-parameter Weibull for Domain 1 to Domain 4 respectively. With regards to bin-width, there are several methods, i.e., Anderson-Darling (A-D) test, Akaike information criterion, Chi-squared test, Cramer-von Mises criterion, Hosmer-Lemeshow test, Kolmogorov-Smirnov test, Shapiro-Wilk test, and others, to select the best interval or bin width as Kim et al. [46] summarised. It is known that the bin-width of the histogram (= interval) may influence the probability distribution and fitting results, and multiple bin-width based fitting is also studied from recent research. In this study the A-D test is adopted.

Table 1. Anderson-Darling statistics of the candidate probability distribution of ultimate compressive strength variance of structural components

	Domain 1	Domain 2	Domain 3	Domain 4
Normal	8.258	13.185	1.464	0.940
Log–Normal	3.494	8.263	0.594	0.491
Exponential	104.567	47.486	103.045	42.654
2 para–exponential	45.916	3.567	29.432	15.613
Smallest extreme value	23.967	20.217	8.932	4.361
Weibull	14.352	14.514	5.543	2.687
3 para–Weibull	4.977	2.700	**0.179**	0.495
Largest extreme value	1.077	5.394	0.508	**0.400**
Logistic	4.327	8.515	1.296	0.813
Log–Logistic	2.144	5.484	0.833	0.611
3–Loglogistic	**0.852**	**0.419**	0.633	0.522

Table 2. Anderson-Darling statistics of the candidate probability distribution the post-collapse characteristics variance of structural components (strength at $1.5\varepsilon_{xu}$)

	Domain 1	Domain 2	Domain 3	Domain 4
Normal	1.051	0.794	**0.261**	0.435
Log–Normal	1.181	1.100	0.328	**0.371**
Exponential	19.629	12.170	13.406	5.624
2 para–exponential	4.322	6.498	5.706	0.722
Smallest extreme value	**0.870**	1.068	0.519	0.663
Weibull	0.910	0.855	0.347	0.482
3 para–Weibull	1.077	0.805	0.277	0.517
Largest extreme value	1.398	1.919	0.540	0.413
Logistic	1.134	**0.526**	0.337	0.461
Log–Logistic	1.237	0.607	0.356	0.408
3–Loglogistic	1.134	0.526	0.337	0.416

Table 3. Anderson-Darling statistics of the candidate probability distribution of the post-collapse characteristics variance of structural components (strength at $2.0\varepsilon_{xu}$)

	Domain 1	Domain 2	Domain 3	Domain 4
Normal	0.566	1.554	0.784	0.891
Log–Normal	0.543	1.342	0.968	0.824
Exponential	23.757	12.025	16.961	7.808
2 para–exponential	5.898	**0.384**	8.547	0.574
Smallest extreme value	1.730	2.021	**0.339**	1.059
Weibull	1.559	1.762	0.349	0.988
3 para–Weibull	0.644	0.481	0.339	**0.450**
Largest extreme value	1.083	1.196	2.273	0.779
Logistic	0.404	1.438	0.530	0.900
Log–Logistic	**0.397**	1.276	0.603	0.844
3–Loglogistic	0.403	0.550	0.531	0.463

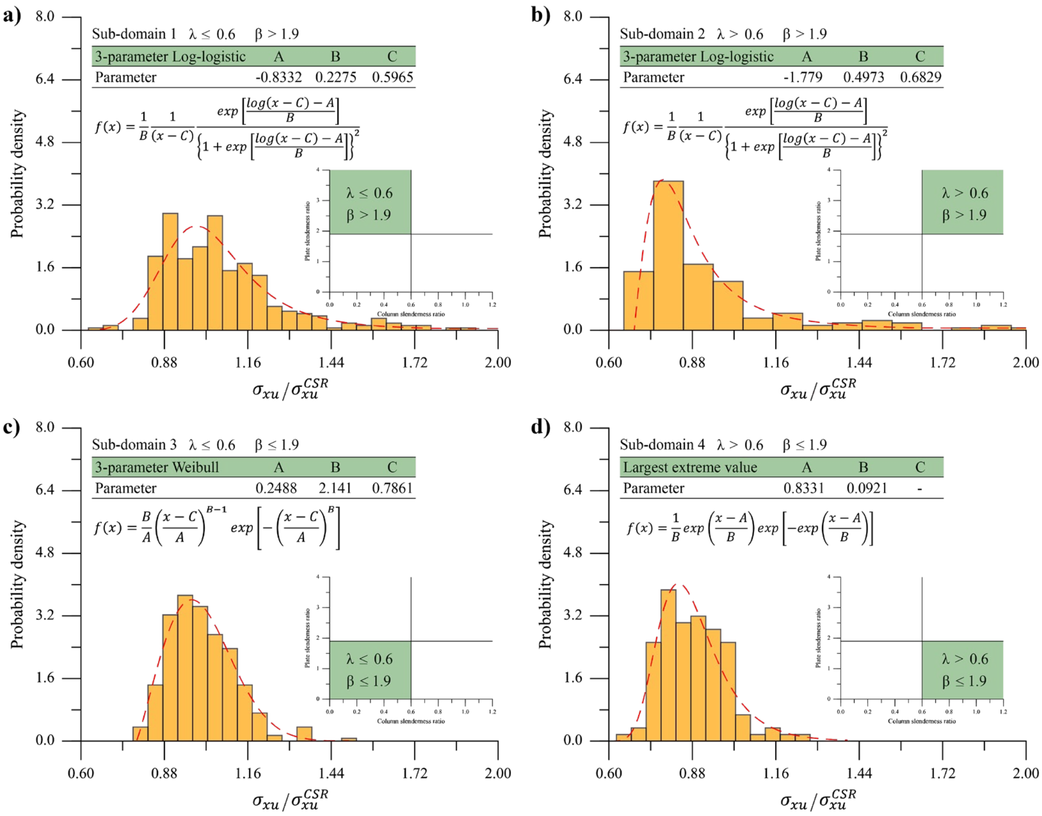

Figure 7. The probability distribution of the ultimate compressive strength variance of structural components

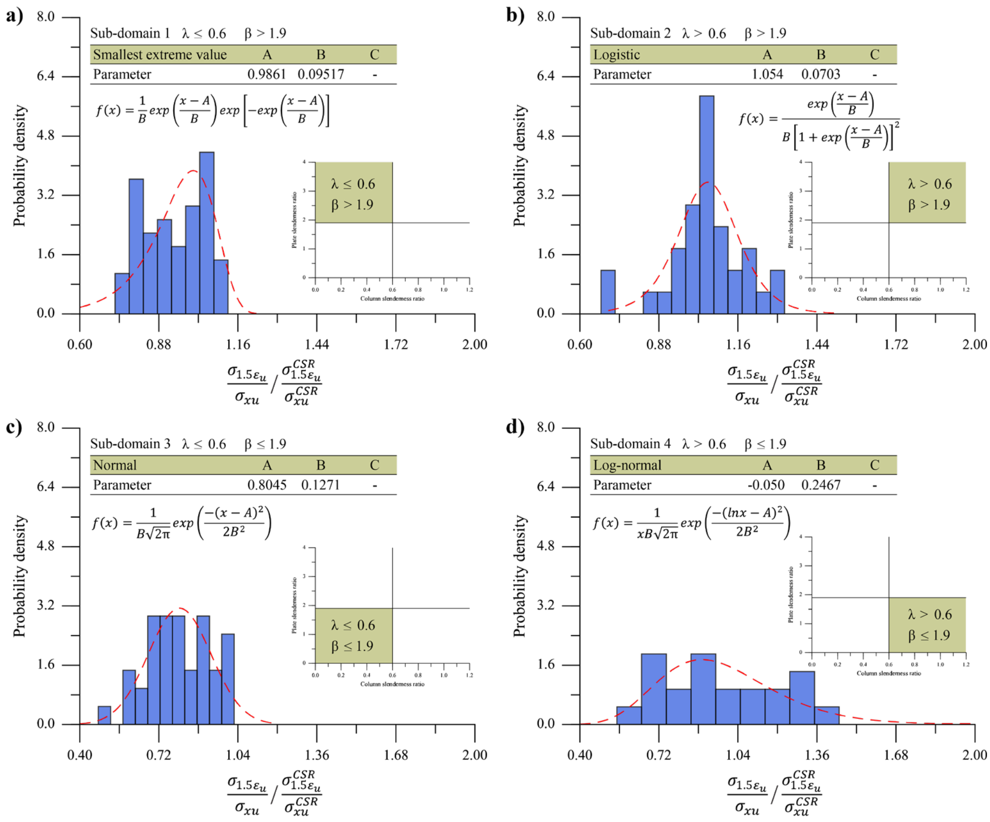

Figure 8. The probability distribution of post-collapse characteristics variance of structural components (strength at $1.5\varepsilon_{xu}$)

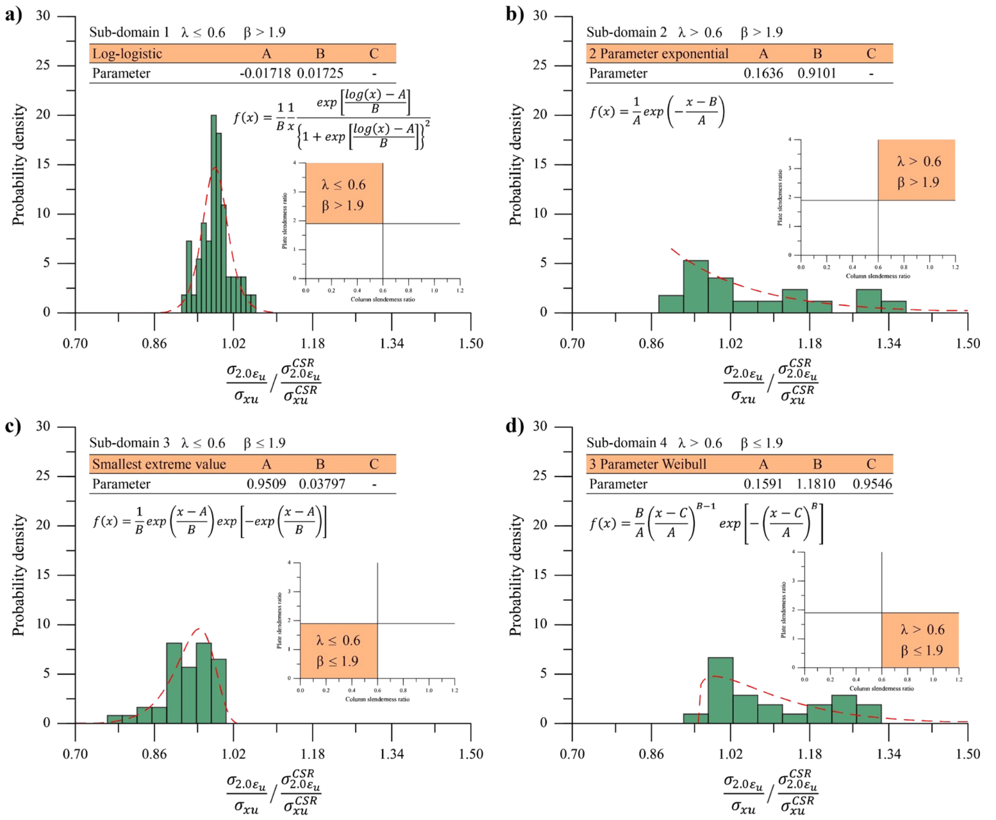

Figure 9. The probability distribution of post-collapse characteristics variance of structural components (strength at $2.0\varepsilon_{xu}$)

5. APPLICATION

5.1 Case Study Ship

The computational uncertainty of ultimate ship hull strength of eight case study vessels is analysed by the proposed methodology, including single-hull VLCC, double-hull VLCC, bulk carrier, container ship, naval frigate and three naval destroyers (Figure 10). The combinations of plate slenderness (β) and column slenderness (λ) of the case study cross sections are illustrated in Figure 11. Most of structural components correspond to Domain 1 ($\beta > 1.9\ \&\ \lambda < 0.6$) and Domain 3 ($\beta < 1.9\ \&\ \lambda < 0.6$). This aligns with the general ship hull structural design recommendation that the column slenderness ratio of the primary deck and bottom panels should generally be less than 0.45 and should never exceed to 0.55. The structural components above the elastic neutral axis, which undergo compression in sagging are generally more slender than those below the elastic neutral axis, which withstand compression in hogging. Additionally, the panels of both single hull and double hull VLCCs are stockier than the other ship types.

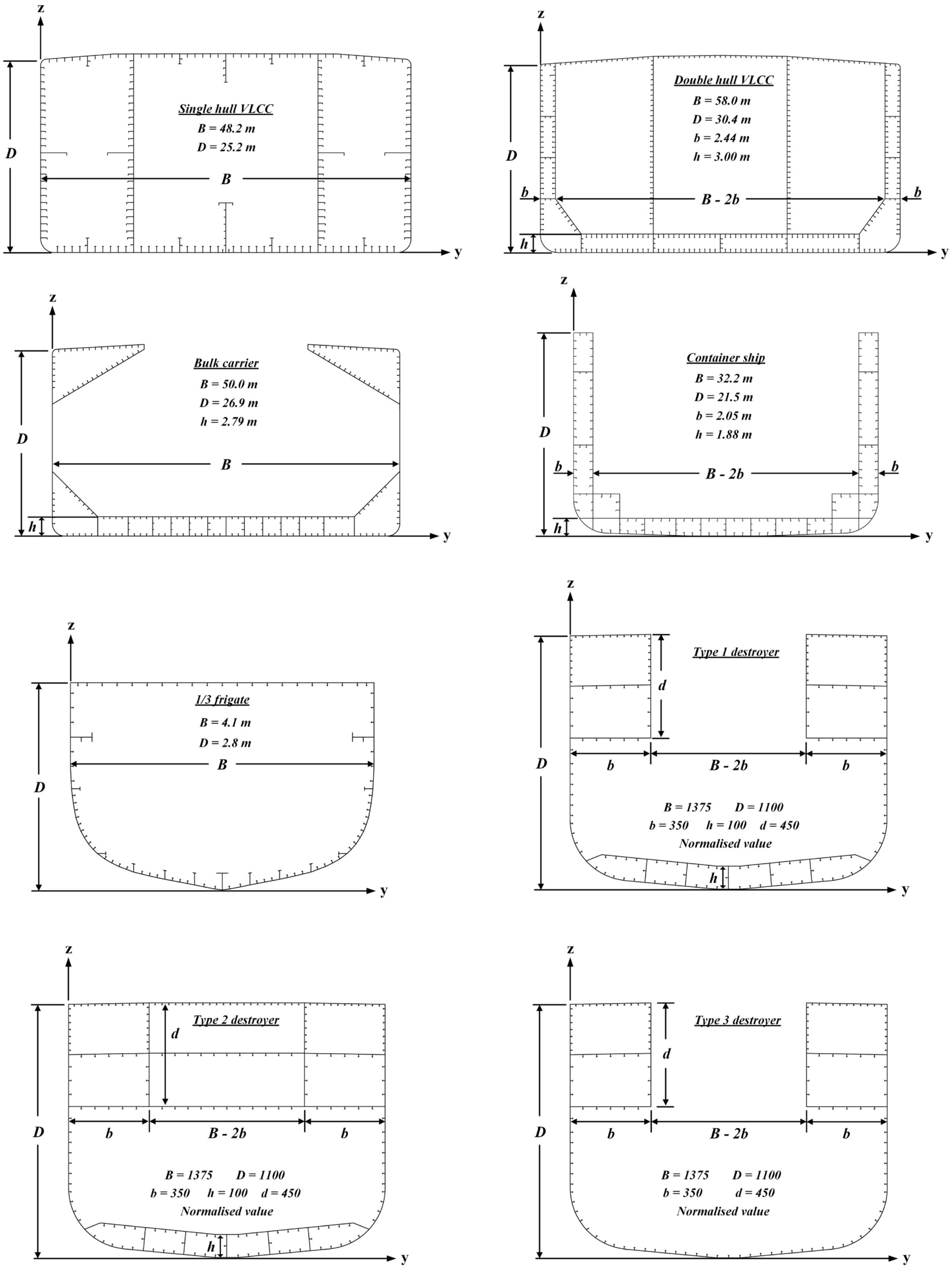

Figure 10. Cross sections of the case study models

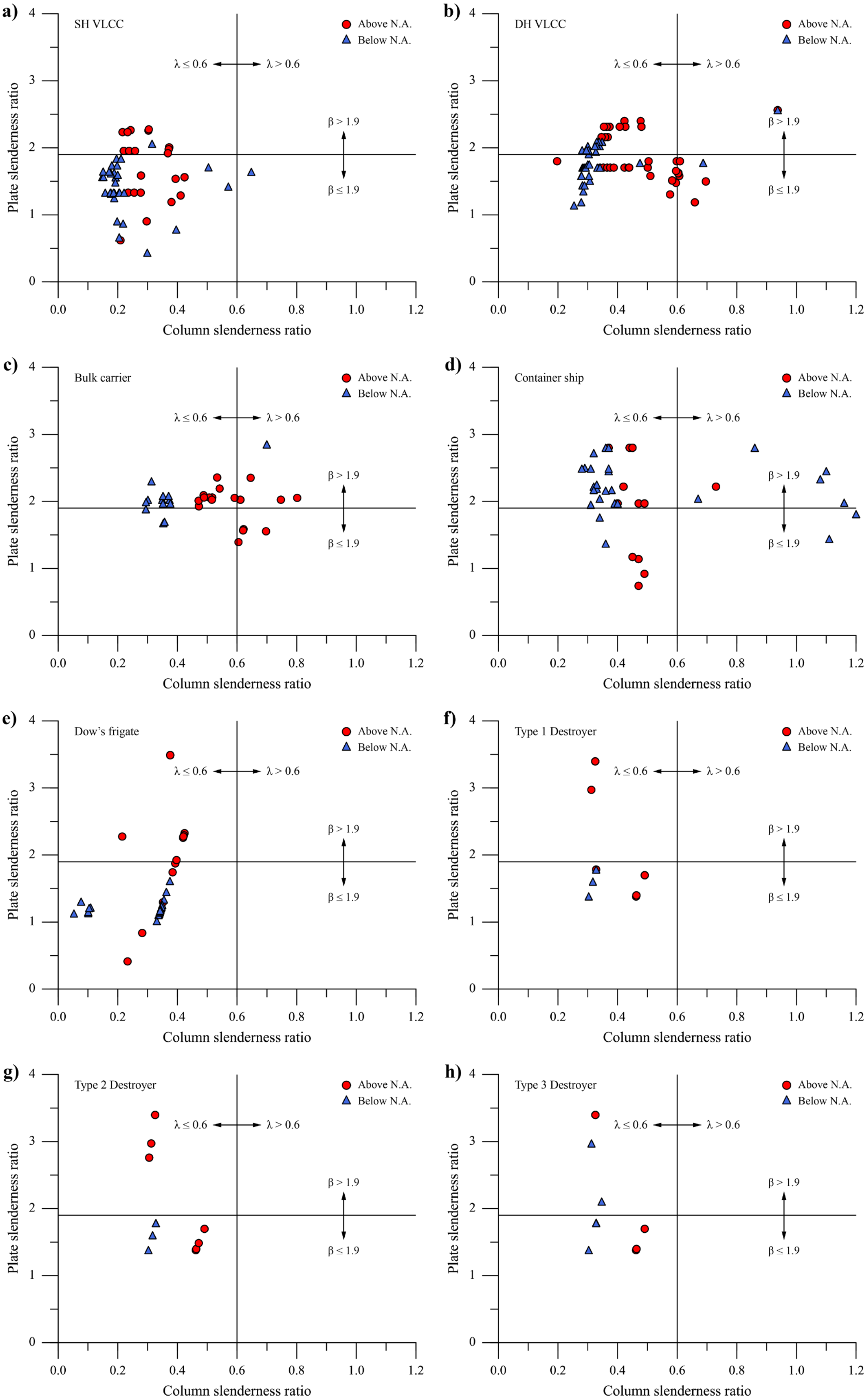

Figure 11. Plate slenderness and column slenderness combination of the case study ships

5.2 Computational Uncertainty of Ultimate Ship Hull Strength

By applying the proposed methodology, the computational uncertainty of ultimate ship hull strength of eight case study ships under vertical bending is evaluated. The two LSC features are analysed independently where a fixed post-collapse behaviour is assumed when analysing the impact of the ultimate compressive strength of stiffened panels and vice versa. This allows for the investigation of their respective effects on the hull girder strength. The histograms of normalised ultimate bending strength with respect to the standard CSR calculation are shown from Figure 12 to Figure 19. From the evaluation of computational uncertainty of ultimate ship hull strength, the following insights are developed:

- The mean ultimate strength predicted for all case study ships in hogging condition closely matches the CSR baseline, whereas for sagging the mean is about 95-97% of the CSR baseline.
- Except for the container ship, the lower prediction of the merchant ships in sagging is primarily attributed to the variance of the ultimate compressive strength of structural segments. The lower value of the naval ships in sagging is mainly due to the variance of the post-collapse characteristics of structural segments.
- A considerable variance is propagated from the variability of the ultimate compressive strength of structural components with a standard deviation ranging from 0.02 to 0.09 amongst the 8 case study vessels.
- The uncertainty of hull girder strength due to the variance in post-collapse characteristics of structural components is relatively less significant than that due to the ultimate compressive strength, with a standard deviation ranging from 0.01 to 0.03 amongst the 8 case study vessels.
- The computed uncertainty of ultimate ship hull strength substantially differs between sagging and hogging. This may be partly due to the larger deviation in the LSC characteristics of deck panels, which are under compression in sagging, compared with the bottom panels which are under compression in hogging. As an example, the progressive collapse mechanism of the double hull VLCC is shown in Figure 20. The collapse in sagging is governed by the elastoplastic buckling of the compressed panels and no tensile yielding collapse takes place. Due to the failure of the buckled panels, the instantaneous neutral axis would translate relatively significantly toward the ship bottom. As a consequence, more panels are subjected to compressive failure, which is therefore accompanied with a larger variance.
- The difference in the collapse mechanism under sagging and hogging may be another contributing factor. The collapse in hogging involves the tensile yielding collapse of the deck panels. The instantaneous neutral axis is still close to the elastic position even at the ultimate limit state. Thus, relatively fewer panels are subjected to compressive failure, which is therefore associated with a smaller uncertainty. An exception is the collapse of Dow’s frigate. As shown in Figure 21, the sagging failure of Dow’s frigate also involves the tensile yielding failure of the bottom panels. Hence, the variance of its ultimate sagging strength is smaller than other ship types due to the same reason as indicated above.
- The ultimate strength of merchant ships is insensitive to the variance in post-collapse characteristics of structural members. Conversely, a relatively considerable uncertainty is induced on the ultimate strength of naval vessels. This is attributed to the higher slenderness panels found in naval scantlings which increases the influence of the local component post-collapse behaviour to the global hull girder strength. As illustrated in Figure 22, the critical structural components of naval vessels under compression have

far exceeded their own ultimate limit states before the global hull girder collapse takes place. By contrast, for stockier merchant vessel scantlings, the global ship hull girder collapse would occur soon after the failure of critical structural members under compression. Thus, it may be concluded that the post-collapse load shedding characteristics is crucial to the structural strength of naval ships.

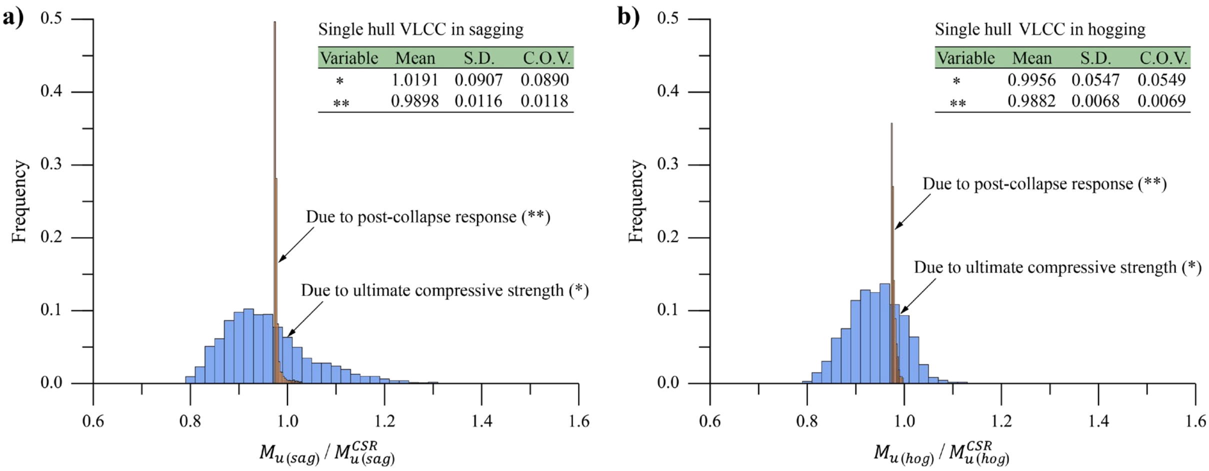

Figure 12. Histogram of the ultimate bending strength variance of single hull VLCC

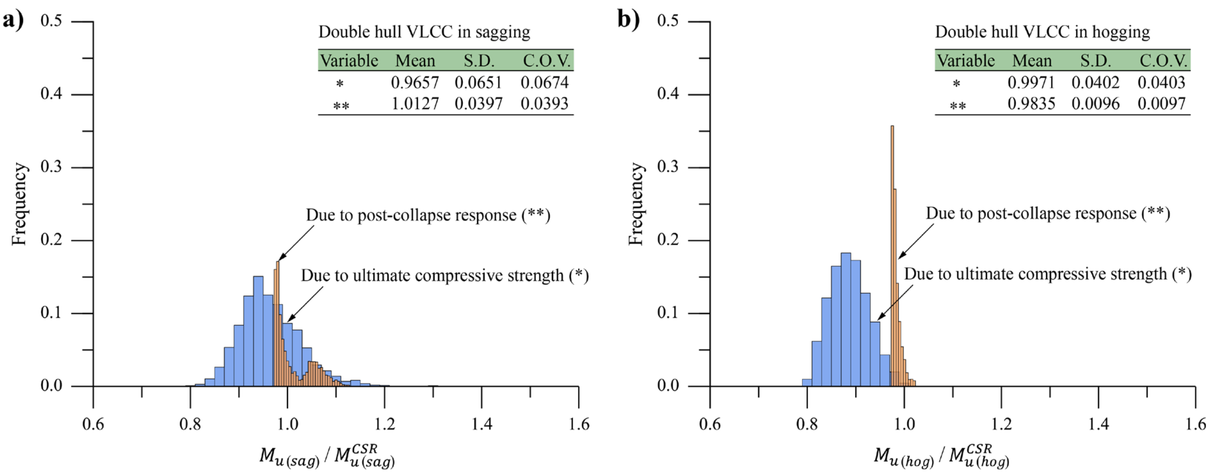

Figure 13. Histogram of the ultimate bending strength variance of double hull VLCC

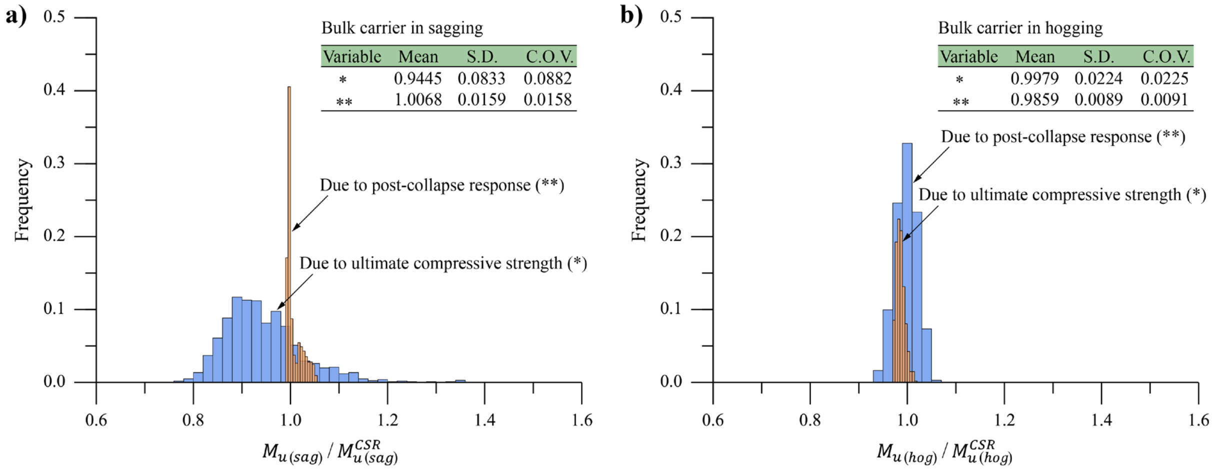

Figure 14. Histogram of the ultimate bending strength variance of bulk carrier

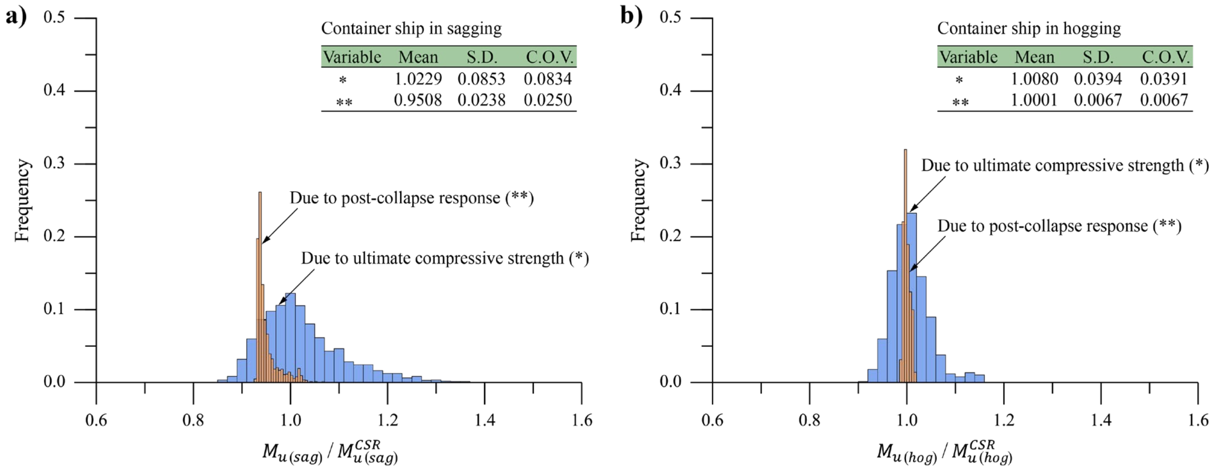

Figure 15. Histogram of the ultimate bending strength variance of container ship

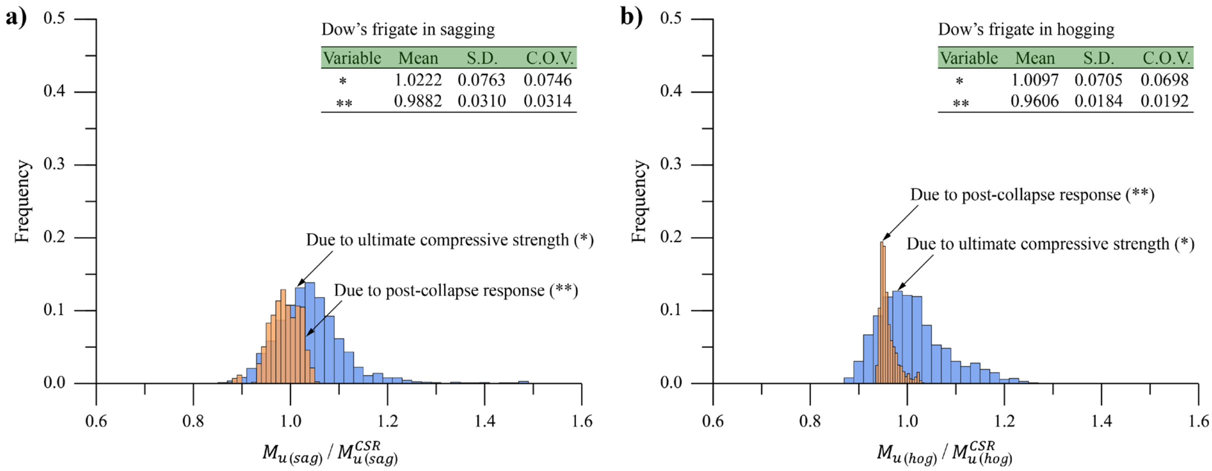

Figure 16. Histogram of the ultimate bending strength variance of Dow's frigate

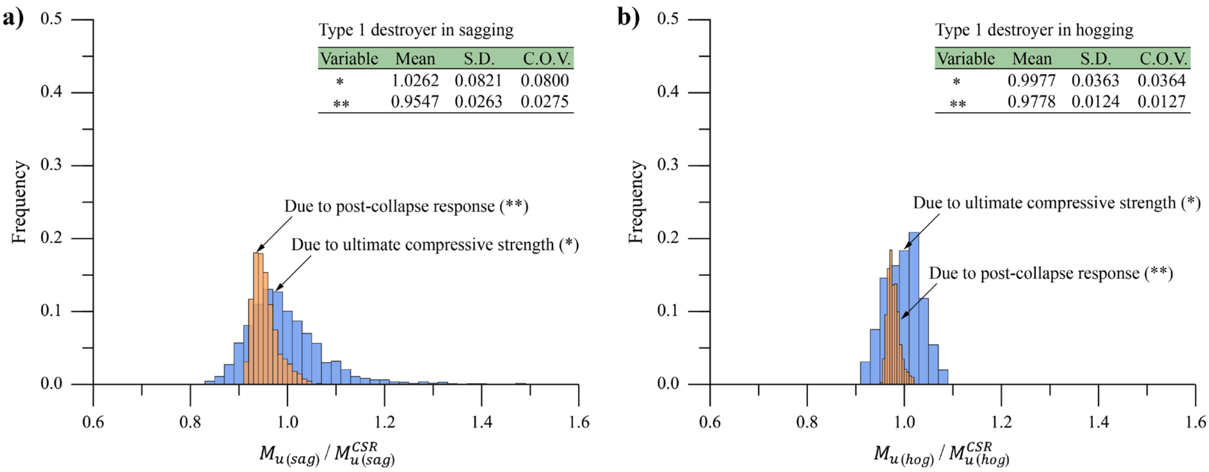

Figure 17. Histogram of the ultimate bending strength variance of Type 1 destroyer

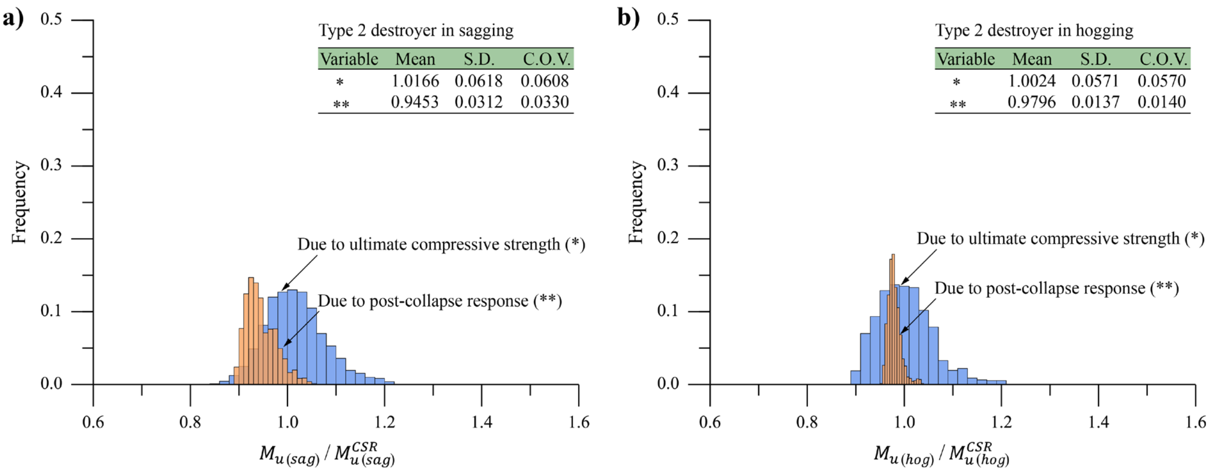

Figure 18. Histogram of the ultimate bending strength variance of Type 2 destroyer

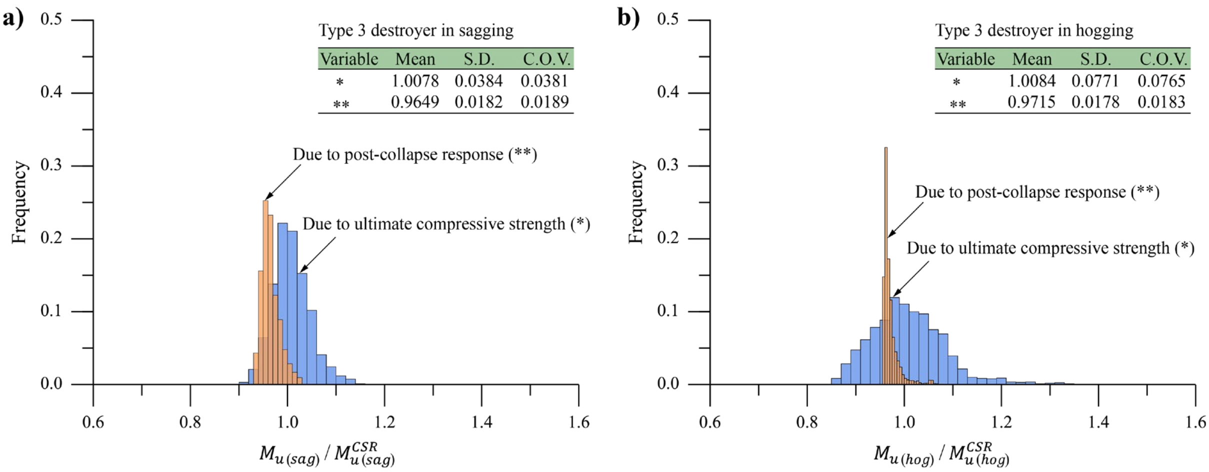

Figure 19. Histogram of the ultimate bending strength variance of Type 3 destroyer

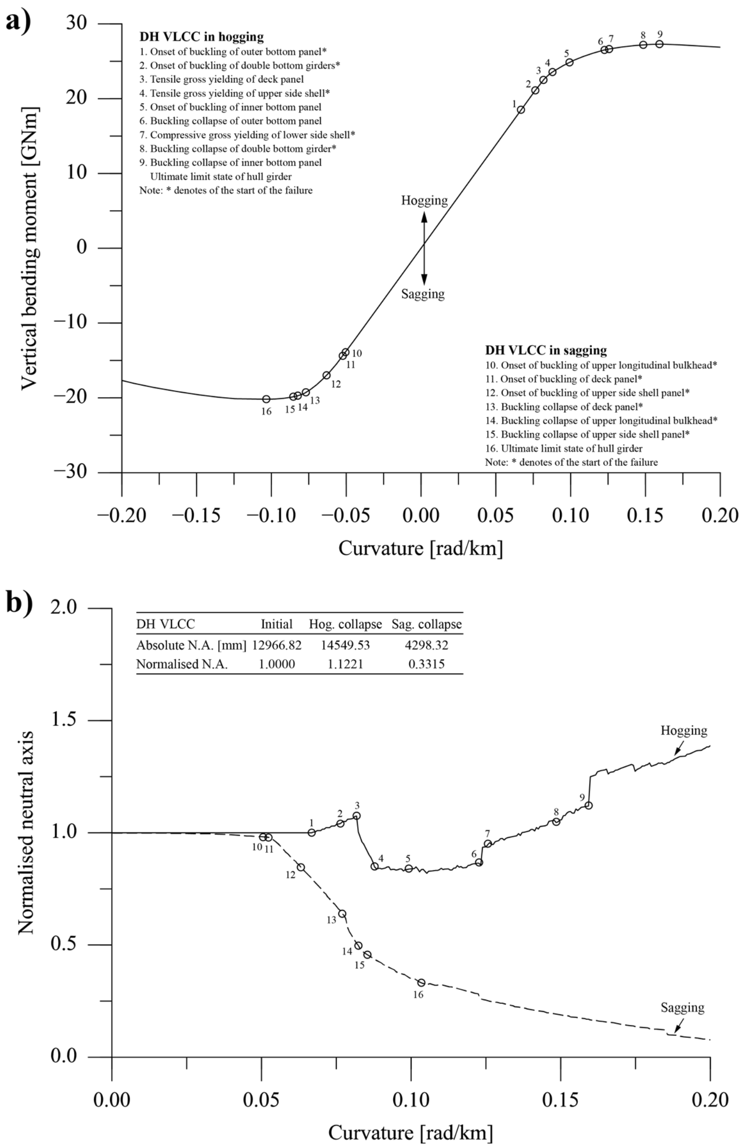

Figure 20. Example collapse mechanism of ship hull girder under vertical bending (Double hull VLCC)

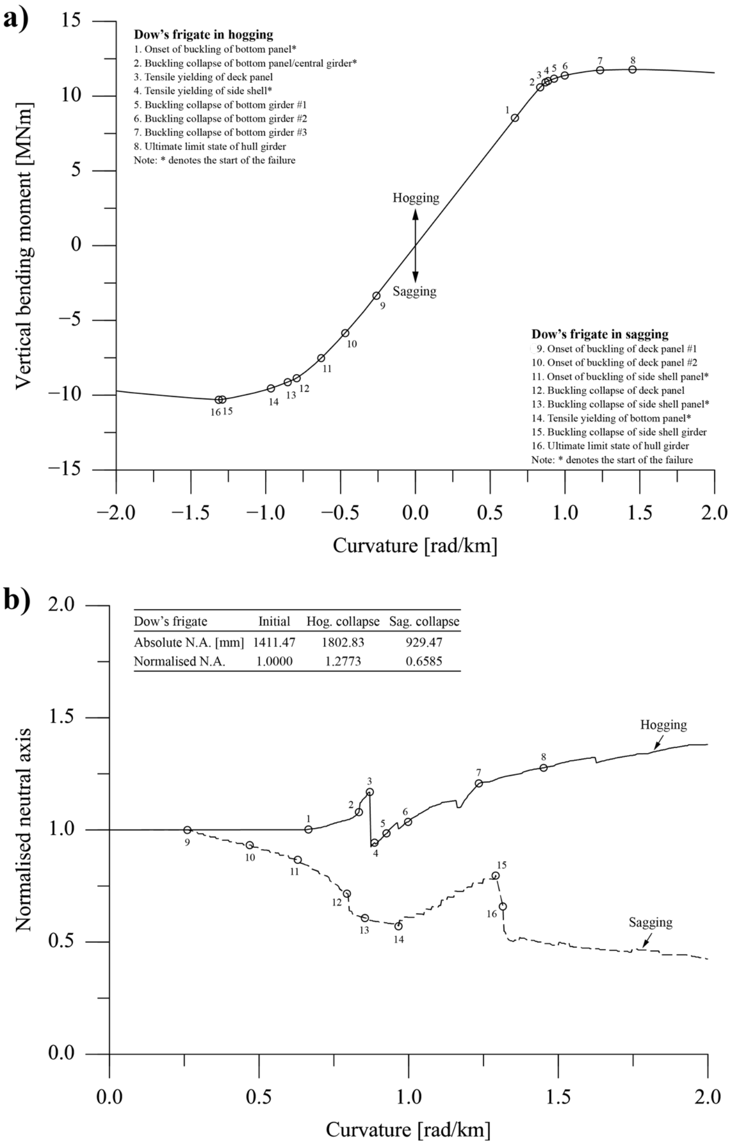

Figure 21. Example collapse mechanism of ship hull girder under vertical bending (Dow's frigate)

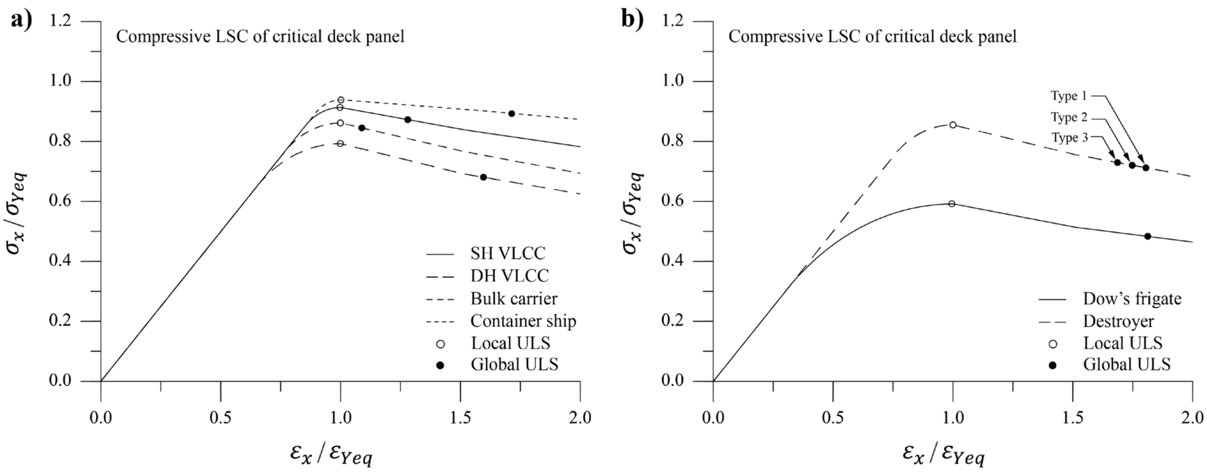

Figure 22. The relationship between the local structural component ULS and global ship hull girder ULS

6. CONVERGENCE STUDY

The probability distributions of the load-shortening charactertics presented in Figure 7 to Figure 8 utilise a dataset of 105 stiffened panels. To study the convergence of the probabilistic characteristics, an extended dataset is utilised to develop an alternative probability distribution model for the post-collapse response. A case study is presented to elucudiate its influence on the uncertainty of the ulitmate sagging strength of Dow's frigate, which is sensitive to the post-collapse response.

The extended dataset is developed based on the original data and the added data, as shown in Figure 23. The added data contains 45 stiffened panels from sub-domain 1 ($\beta > 1.9$ & $\lambda < 0.6$), because the primary deck panel of Dow's frigate is of the same feature. The mean value and the COV of the strength at $1.5\varepsilon_{xu}$ change from 0.9322 to 0.9134 and 0.1194 to 0.1097 repsectively. The mean value and the COV of the strength at $2.0\varepsilon_{xu}$ change from 0.9838 to 0.9712 and 0.0313 to 0.0330 repsectively. As illustrated further in Figure 24, there is no significant flutuation in the mean value and the COV of the dataset in both the original and the extended sizes.

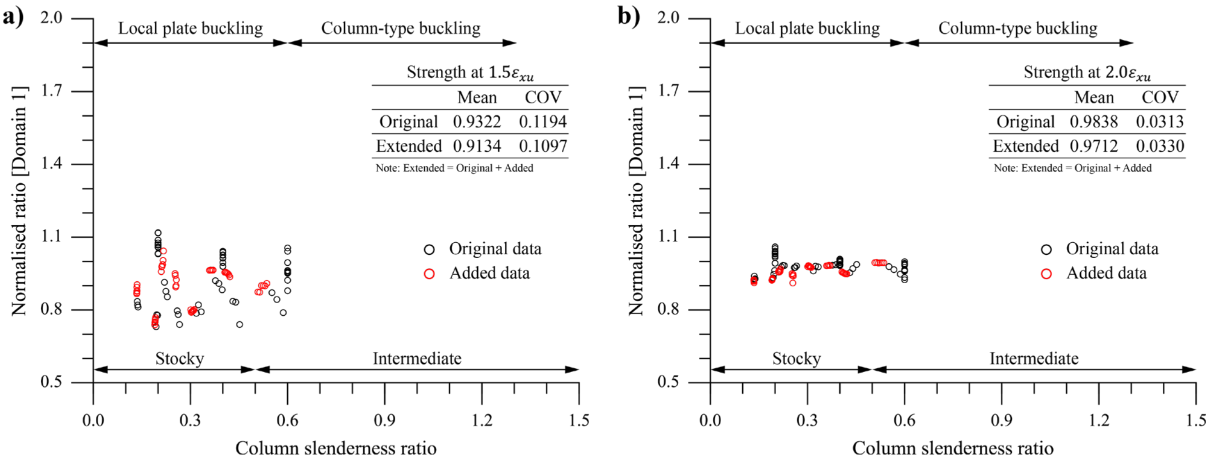

Figure 23. Extended dataset of the post-collapse strength

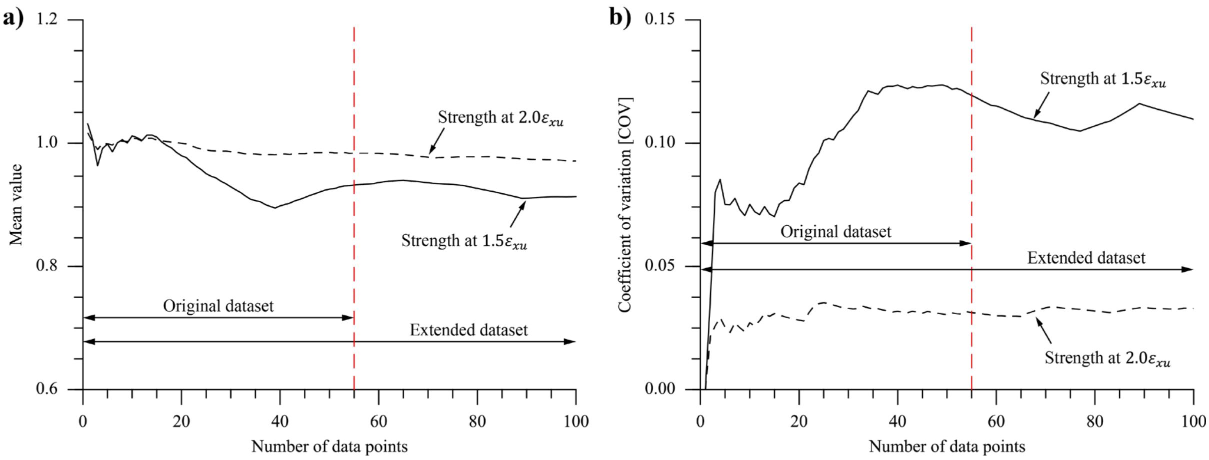

Figure 24. Variation of the mean and COV of the post-collapse strength with the number of data points

Following the same procedure described in section 3.4, the new probability distributions of the post-collapse response are formulated, as shown in Figure 25. For each post-collapse characteristic (strength at $1.5\varepsilon_{xu}$ and $2.0\varepsilon_{xu}$), two new probability models are developed. The first model is based on the original fitted function but the parameters are revised with the extended data, which is denoted as "Extended + Original function". The second model is fitted with different functions as selected based on the minimum AD-statistics cretirion, which is denoted as "Extended + Updated function". The uncertainty propagated to the ultimate hull girder strength of Dow's frigate under sagging is analysed in accordance with the procedure given in section 3.5 with the newly developed probability distribution models. The mean prediction and variance are shown in Figure 26 and summarised in Table 4. It can be seen that the mean and variance between the original model and the new model based on extended data but original function are close. Regarding the comparison between the original model and the new model based on extended data and updated function, the discrepancy is slightly larger with a smaller mean ultimate strength and an increase in the coefficient of variation.

The above comparison highlights the effect of an alternative dataset. It is concluded that the ultimate hull girder strength, at least in the case of naval vessels, is sensitive to the post-collapse respones of the local stiffened panels, whilst further research would better quntify the uncertainty.

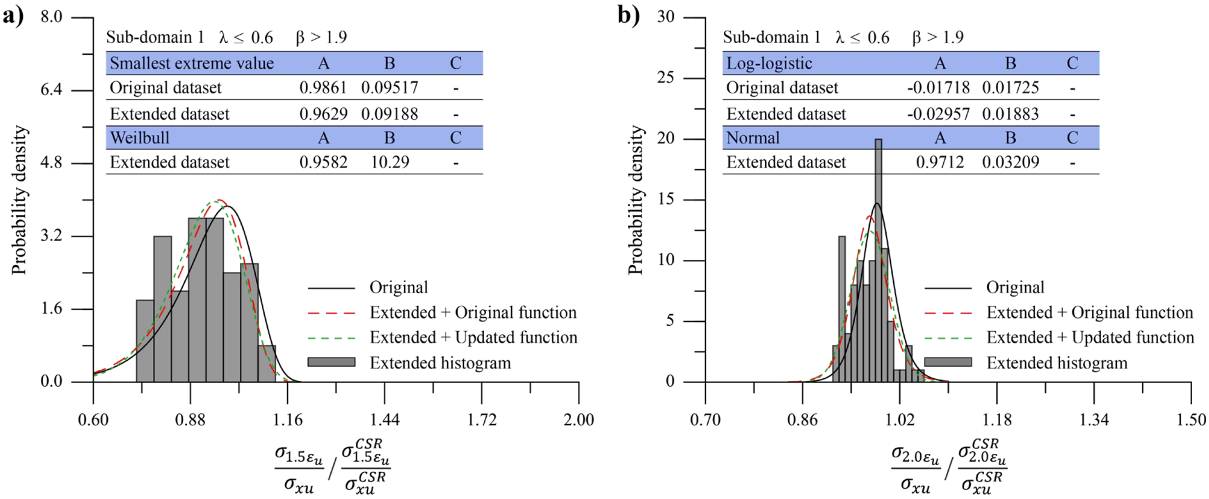

Figure 25. Comparison of the probability density functions between original dataset and extended dataset of post-collapse strength

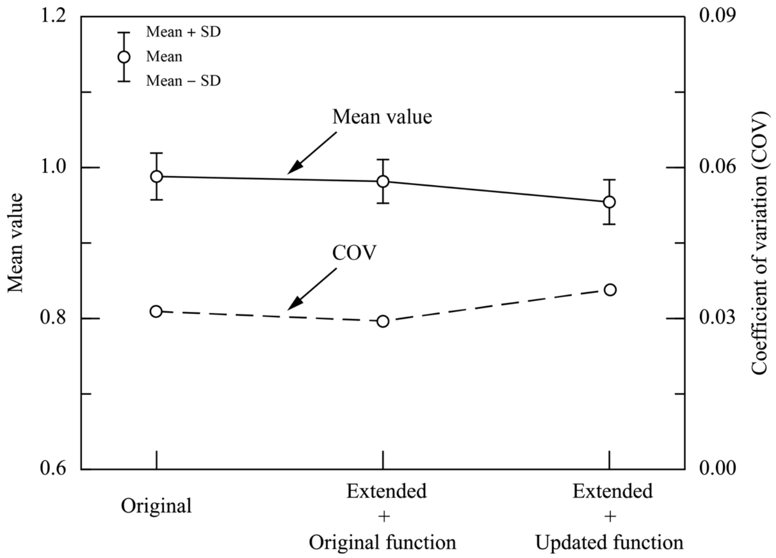

Figure 26. Comparison of the uncertainty in the ultimate sagging strength of Dow's frigate due to post-collapse response

Table 4. Summary of the mean and variance of the ultimate sagging strength of Dow's frigate due to post-collapse response

Probabilistic models	Mean	S.D.	C.O.V.
Original	0.9882	0.0310	0.0314
Extended + Original function	0.9817	0.0289	0.0295
Extended + Updated function	0.9544	0.0341	0.0357

7. CONCLUSIONS

A probabilistic approach is proposed to assess the computational uncertainty of ultimate ship hull strength caused by the variance in the compressive load-shortening characteristics of structural components. Probability distributions are developed for two critical load-shortening characteristics: ultimate compressive strength and post-collapse behaviour. The simplified progressive collapse method (Smith method) is adopted for computing the ultimate ship hull strength. The load-shortening curves of structural components are formulated using an adaptable algorithm with probabilistic sampling of the critical characteristics by Monte-Carlo Simulation. Case studies are conducted on four merchant ships and four naval vessels. From this study, the following conclusions are drawn:

- A more considerable computational uncertainty is induced on the ultimate ship hull strength under sagging compared with the hogging strength. This is because the deck panels are generally more slender than the bottom panels. These slender panels are associated with larger computational uncertainty, as indicated by the developed dataset. In addition, the difference in collapse mechanism between sagging and hogging would also contribute to the uncertainty. In the former case, elastoplastic buckling of the deck panel is dominating failure mode within the cross section, whereas tensile yielding collapse of the deck panels is found under hogging.

- The mean ultimate strength predicted for all case study ships in hogging closely match the CSR baseline, whereas for sagging the mean is about 95-97% of the CSR baseline.
- The variance in ultimate compressive strength propagates a larger uncertainty to ship hull girder strength in comparison to the variance in post-collapse characteristics.
- The strength uncertainty is ship-type dependent. The strength of merchant ships are primarily governed by the ultimate compressive strength of critical stiffened panels, while the strength of naval ships are also sensitive to the post-collapse response of critical members due to the difference in the relationship between local collapse and global collapse.

The proposed methodology could be used as a systematic approach to evaluate the computational uncertainty of ship hull girder strength, which may be further implemented in the ultimate limit state-based reliability analysis. This can be achieved by incorporating the entire procedure with a reliability analysis framework for examining the limit state equation. Alternatively, the computed statistics of hull girder ultimate strength (mean value and COV) can be directly used to formulate the strength model uncertainty factor.

Future research may be required to enhance the probabilistic approach to assess the computational uncertainties of the ultimate strength of hull girders. For instance, the computational uncertainty accounting for the combined effects of the variance in ultimate compressive and post-collapse characteristics should be evaluated. Furthermore, the analysis can be extended to the combined loading scenario, such as combined vertical and horizontal bending.

In addition, future work may be focused on an improved development of the probability distribution of critical load-shortening characteristics. A more extensive dataset would improve the probability distribution and lead to a more refined probabilistic analysis. In addition, the statistical correlation between critical LSC features should be explored and accounted for in the overall procedure.

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