

Modeling and control of Squirrel Cage Induction
Generator with Full Power Converter applied to
windmills

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You can not prevent the wind. But windmills can be built.

(Dutch quote)

The pessimist complains about the wind; the optimist expects it to change; the realist adjusts the sails.

(William Arthur Ward)

If a man knows not what harbor he seeks, any wind is the right wind.

(Lucius Annaeus Seneca)





Contents

Abstract	9
Glossary	11
Foreword	17
Aim	19
Reach	21
Introduction	23
1 System's Analysis	25
1.1 Introduction	25
1.2 Turbine	25
1.3 Gear Box	27
1.4 Squirrel Cage Induction Generator	28
1.5 Full Converter	31
1.5.1 DC Bus	32
1.5.2 Crow Bar	32
1.6 Grid connection	32
2 Generator system	35
2.1 Change of abc basis to qd0	35
2.1.1 Introduction	35
2.1.2 qd0 Transformation	36
2.2 Park Transform application on Induction machine equations	37
2.2.1 Voltage equations	37
2.2.2 Torque equations	39
2.3 State space of SCIG generator	39



3	Control system design	43
3.1	Introduction	43
3.2	High level control	43
3.2.1	Constant tip speed ratio scheme control	44
3.3	Control of the converter	45
3.3.1	Control of the electrical system of the stator	45
3.3.2	Control of the electrical system of the grid side	49
4	Modeling of the system	53
4.1	Introduction	53
4.2	Modeling of the Mechanical System	53
4.3	Modeling of SCIG	54
4.4	Modeling of Full Power Converter	54
4.4.1	Stator Side Converter	54
4.4.2	DC Link	54
4.4.3	Grid Side Converter	55
4.5	Modeling of the Grid	55
4.5.1	Grid Filter	55
4.5.2	Grid	55
5	Simulation of the model and Analysis of the results	57
5.1	Generator connected directly to the grid	57
5.2	Complete system	58
6	Conclusions	61
A	Controllability and Observability of the SCIG Linear System	65
A.1	Controllability	66
A.2	Observability	67
A.3	A lower bound on the distance to a non-controllable system	67
A.4	A lower bound on the distance to a non-observable system	68
B	Space Vector Pulse Width Modulation (SVPWM)	71
C	Introduction to current loops	73
D	Definition of magnetizing current	75
	Bibliography	78



List of Figures

1	Location of windfarms in Finland.[1]	17
1.1	Generic Nacelle of a Windmill	25
1.2	Windmill. Source <i>www.winwind.fi</i>	25
1.3	Illustration of the Area which provide us of the wind power.	27
1.4	GearBox Input & Output	27
1.5	Simplification of the GearBox	28
1.6	Induction machine's schematic illustration	28
1.7	Full converter illustration	32
1.8	Simplification of the grid connection	33
2.1	Any current vector in abc reference	35
2.2	Representation of the Clarke Transformation	36
2.3	Rotation of the reference (α, β) an angle θ	36
3.1	Representation of the control system	43
3.2	Schematic of the IMC scheme	45
3.3	Alternative structure of the IMC scheme	46
3.4	Block diagram of the system	47
3.5	Block diagram of the system of the BUS control	51
4.1	Simulink® model of the whole windmill	53
4.2	Simulink® model of the mechanical part	54
4.3	Simulink® model of the generator	54
4.4	Simulink® model of the Full Power Converter	54
4.5	Simulink® model of the Stator Side of the Back-to-Back	54
4.6	Simulink® model of the DC Link of the converter	54
4.7	Simulink® model of the grid side of the frequency converter	55
4.8	Simulink® model of the grid	55



4.9	Simulink® model of the grid filter	55
4.10	Simulink® model of the grid voltage generation.	55
5.1	Mechanical Torque produced by the Generator	57
5.2	Stator currents in qd reference	57
5.3	Evolution of $C_p(\lambda)$	58
5.4	Graphic of characteristic C_p of each turbine.	58
5.5	Graphic of the evolution of the grid filter voltage	58
5.6	Voltage E of the DC Link	58
5.7	Stator voltage in qd reference	58
5.8	Stator current in qd reference	58
5.9	Mechanical Torque produced by the Generator	58
5.10	Park Transform of the voltage of the grid	59
B.1	Electrical scheme of an inverter	71
B.2	Commutation hexagon	71



Abstract

The project aims to develop a dynamic model, of a generation system of electrical energy with a variable speed wind turbine using a squirrel cage induction generator which is connected to the grid by a back to back frequency converter, for testing purposes.

In the project has been done an analysis of the mathematical equations of the whole system, separating the mechanical system (turbine and gearbox) and the electrical (generator and converter).

We study the control design of the machine which allow us to obtain the maximum wind power extraction, and we also study the control of the converter which couple the generator with the network.

The proper functioning of the model is checked comparing the obtained results with some commonly known results.





Glossary

Symbols

\dot{m}	mass flow
v_w	wind speed
A	area where the air flow could pass through
v_{w1}	wind speed before the turbine
v_{w2}	wind speed after the turbine
P_{wind}	mechanical power extracted by the converter
P_{wind0}	mechanical power that could be converted
c_p	Power coefficient
c_i	set of values greater or equal than zero, these are known as <i>turbine's characteristic coefficients</i>
R	radius of the turbine, that means, the length of blades
c	Viscosity constant of the turbine's shaft
k	Elasticity constant of the turbine's shaft
J_m	Inertia moment of the generator
J_t	Inertia moment of the turbine
v_s^{abc}	stator winding's voltage vector
v_r^{abc}	rotor winding's voltage vector
i_s^{abc}	stator winding's current vector
i_r^{abc}	rotor winding's current vector

r_s	resistance of the stator windings
r_r	resistance of the rotor windings
L_{ss}	self-inductance of the stator windings without the winding owing the dispersion flow
L_{rr}	self-inductance of the rotor windings without the winding owing the dispersion flow
L_{sm}	coupling inductances between stator windings
L_{rm}	coupling inductances between rotor windings
L_{sr}	maximum value reached by coupling inductances between stator and rotor windings
L_{ls}	dispersion inductance of the stator windings
L_{lr}	dispersion inductance of the rotor windings
$L(\theta_r)$	Induction machine's coupling inductance matrix
P	number of poles that the Induction Machine has
E_0	DC Bus voltage at t=0
C	DC Bus capacity
i_{DCE}	Current which flows through the condenser
i_{DCg}	Current which came from grid's converter
i_{DCs}	Current which came from stator's converter
\underline{Z}	Value of the impedance
R	Value of the resistance parameter
L	Value of the inductance parameter
l	Value of the length of the wire from converter to electrical transformer
X_{ccpu}	Value of the short circuit inductance in PU reference
V_{1b}	Base voltage in the primary
V_{1n}	Nominal voltage in the primary
S_b	Base power
S_n	Nominal power



x^{qd0}	x^{abc} after apply Park Transform on it
$T(\theta)$	Park Transform
v_s^{qd0}	stator winding's voltage vector in qd0 reference
v_r^{qd0}	rotor winding's voltage vector in qd0 reference
i_s^{qd0}	stator winding's current vector in qd0 reference
i_r^{qd0}	rotor winding's current vector in qd0 reference
$C(s)$	IMC controller.
$G(s)$	Induction Generator's transfer function.
$G(s)$	Internal model of the Induction Generator's transfer function.
I	Identity or unit matrix
\underline{i}_m	Magnetizing current
$K_{c_p _{\theta_{pitch}=0}}$	Optimum Torque coefficient of the turbine



Greek Symbols

ρ	air density
ω_t	turbine's spin speed
λ	tip speed ratio
θ_{pitch}	spin blade angle
θ_m	Orientation angle of the motor's shaft
θ_t	Orientation angle of turbine
ν	Gear ratio of the multiplier
τ_m	Mechanical torque of the engine.
τ_t	Mechanical torque of the turbine
λ_s^{abc}	stator winding's concatenated flows vector
λ_r^{abc}	rotor winding's concatenated flows vector
ω_r	generator's shaft's orientation angle from electric system
Γ_r	Torque on the rotor shaft
Γ_m	Mechanical Torque of the machine
ε_{cc}	short circuit voltage
α	Controller's bandwidth in close loop



Acronyms

DFIG	Double Fed Induction Generator
SCIG	Squirrel Cage Induction Generator
IGBT	Insulated Gate Bipolar Transistor
SVPWM	Space Vector Pulse Width Modulation
PU	Per Unit
MPPT	Maximum Power Point Tracking
IMC	Internal Model Control
PI	Proportional and Integer controller
LTI	Linear Time-Invariant





Foreword

The wind power capacity in Finland is 144 MW, 119 wind turbines (May 2009). Wind power production in 2008 was about 260 GWh which is 0.3% of the Finnish electricity consumption,[1]. The target of the Finnish Government is to have 2000 MW of installed wind energy capacity by 2020,[2].

The 90% of Finns would want further investments in wind energy in September 2007,[3]. Therefore there are a lot of ongoing projects of building new windmills and the public desire of improvement, these are slow because they are too much expensive, and this deceleration of the construction of the windmills implies do not reach the wind energy installation target. This fact causes several studies to improve the existing windmills.

This project is thought for to get a dynamic simulation model which allow us to study if a squirrel cage induction generator with a full power converter connected to its stator side works better than the conventional control systems.

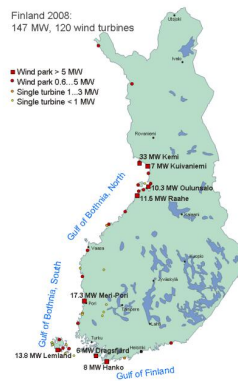


Figure 1: Location of windfarms in Finland.[1]





Aim

The target of this project is to obtain a dynamic model of the windmill, which is equipped with a squirrel cage induction generator (SCIG) and a full power converter to do the control. Moreover, this dynamic model will allow us make some simulations and understand the behavior of the generator with this kind of a control. Finally, with the results of the simulations we will be able to conclude if it is good enough and how it works.

Then, we can resume briefly that aim of the project as ” Provide a simulation model of our SCIG windmill which permit us to test it”.





Reach

This project could be applied to reduce manufacturing costs of windmills, changing the generator typology.

The most used generator is the double feed induction generator (DFIG), and we suggest to change it by squirrel cage induction generator (SCIG), owing to the robustness of SCIG, and since it has low cost with an almost null maintenance.

Thanks to this, we are able to reduce the generator expenses and the rest of the costs are the same because we are coupling the same full converter and applying the same control what is to be used in DFIG.





Introduction

The windmill, also called as the wind turbine, is a means of exploitation the kinetic energy of the wind and converting it into electrical energy using an electrical generator.

The windmills can be divided in two essential parts:

- I. **How we obtain the wind energy.** To realize this task we have to use a *turbine*. There are two options for the turbine, vertical axis and horizontal axis. We will choose for our study the horizontal turbine, because this the type which is used more often.

- II. **How we convert kinetic energy into electrical energy.** To make this, there exist two different approaches, Fixed speed and Variable Speed, where each one implies quite different configurations. We will select for our project the variable speed, because, again, is the most used method.

In this project we use a squirrel cage induction generator, despite this type of a generator, usually, is applied by the fixed speed windmills, which is directly connected to the grid, or can include a condenser between the generator and the grid to compensate the reactive power.

In our case, we will introduce a full power converter between generator and the grid, thanks to this we will use the variable speed windmill. The full power converter allow us control the generator.

It is important to remark that the squirrel cage is rather common as a motor. Therefore, the control in this way is quite well studied and it also implies a robust machine that need little, or almost null, maintenance.





Chapter 1

System's Analysis

1.1 Introduction

In this chapter we will be present the equations that describe each part of our windmill. This windmill is a variable speed turbine with squirrel cage induction generator with a vector control.

The windmill could be divided as follows:

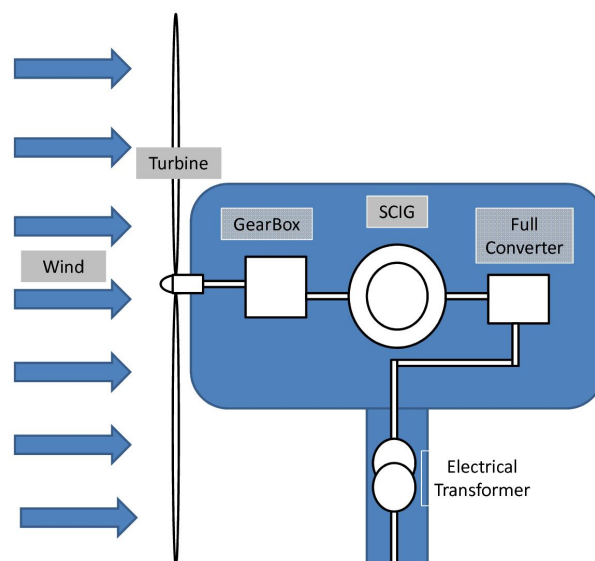


Figure 1.1: Generic Nacelle of a Windmill

1.2 Turbine

A windmill is a machine responsible for transforming the kinetic energy of the wind into the electrical energy. If the wind flow is known, the kinetic power could be expressed, as it is well known, by means the follow equation:

$$P_{wind} = \frac{1}{2} \dot{m} v_w^2 = \frac{1}{2} \rho A v_w^3 \quad (1.1)$$

where

- \dot{m} mass flow
- v_w wind speed
- ρ air density
- A area where the air flow could pass through



Figure 1.2: Windmill. Source www.winwind.fi

The power described in (1.1) is just a mathematical description about kinetic power. However, this power could not be obtained by a wind turbine. The power that could be achieved in the best situation, is 0.593 times P_{wind} . This value is called as the *Betz number*. It is a power coefficient and the turbine efficiency limit, for more details how to obtain the Betz number see [4].

The **Power coefficient** can be defined as the ratio between the mechanical power extracted by the converter and the power of the undisturbed air stream (see [5]).

$$\begin{aligned} c_p &= \frac{P_{wind}}{P_{wind0}} \\ &= \frac{\frac{1}{4} \rho A (v_{w1}^2 - v_{w2}^2) (v_{w1} + v_{w2})}{\frac{1}{2} \rho A v_{w1}^3} \\ &= \frac{1}{2} \left(1 - \left(\frac{v_{w2}}{v_{w1}} \right)^2 \right) \left(1 + \frac{v_{w2}}{v_{w1}} \right) \end{aligned} \quad (1.2)$$

with



v_{w1}	wind speed before the turbine
v_{w2}	wind speed after the turbine
ρ	air density
A	area where the air flow could pass through. as could be seen in (1.3)
P_{wind}	mechanical power extracted by the converter
P_{wind0}	mechanical power that could be converted

It should be noted that the quotient between v_{w2} and v_{w1} never can overcome $\frac{1}{3}$ because of the *Betz number*.

The kinetic power obtained by the turbine can than be defined as:

$$P_{wind} = c_p P_{wind0} \quad (1.3)$$

where c_p is defined in (1.2). However, this coefficient depends directly on each turbine, on the *tip speed ratio* λ , which is defined below in (1.5) and, just in the case that the rotor is equipped with blade pitch control, on the θ_{pitch} called *pitch*. The coefficient's value can be found on tables for some specific turbines or determined by analytic function, as follows [6].

$$c_p(\lambda, \theta_{pitch}) = c_1 \left(c_2 \frac{1}{\Lambda} - c_3 \theta_{pitch} - c_4 \theta_{pitch}^{c_5} - c_6 \right) e^{-c_7 \frac{1}{\lambda}} \quad (1.4)$$

with

$$\lambda = \frac{\omega_t R}{v_{w1}} \quad (1.5)$$

$$\frac{1}{\Lambda} = \frac{1}{\lambda + c_8 \theta_{pitch}} - \frac{c_9}{1 + \theta_{pitch}^3} \quad (1.6)$$

with

c_i	set of values greater or equal than zero, these are known as <i>turbine's characteristic coefficients</i> .
ω_t	turbine's spin speed.
R	radius of the turbine, that means, the length of blades.

1.3 Gear Box

A transmission or gearbox provides speed and torque conversions from a rotating power source to another device using gear ratios.

The gearbox in a wind turbine converts the slow, high-torque rotation of the turbine into much faster rotation of the electrical generator. Usually, it contains three stages to achieve an overall gear ratio from 40:1 to over 100:1, depending on the size of the turbine. The first stage of the gearbox is usually a planetary gear, for compactness, and to distribute the enormous torque of the turbine over more teeth of the low-speed shaft.

The gearbox will be described as a *black box* that receive a low speed and transforms it to

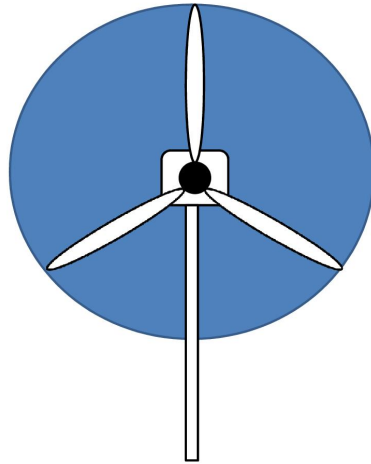


Figure 1.3: Illustration of the Area which provide us of the wind power.



Figure 1.4: GearBox Input & Output

a faster rotation, as is shown in the illustration (1.4).

There are two ways that are commonly used for the modeling of the gearbox [7]:

1. The **Lumped model**, which assumes that all the rotating masses can be treated as one concentrated mass.



2. The **Two-mass model**, which considers an equivalent system with an equivalent stiffness and damping factor on the wind turbine on the rotor side.

We will use the *Two-mass model*. Since, if we want to use the model to realize the transient analysis, Lumped model will give us wrong results. Moreover, it is necessary to remark that the inertia moment of the turbine is, almost 90% of the inertia of all the drive train (Turbine and GearBox), and it is known the high efficiency of the transmission. Then, we are able to neglect friction torque and, only, consider the inertia moment of the turbine and the inertia of the rotor's generator. Finally, we get the following equation to describe our model that could be written as (1.5).

$$\begin{Bmatrix} \ddot{\theta}_m \\ \ddot{\theta}_t \\ \dot{\theta}_m \\ \dot{\theta}_t \end{Bmatrix} = \begin{bmatrix} -\frac{\nu^2 c}{J_m} & \frac{\nu c}{J_m} & -\frac{\nu^2 k}{J_m} & \frac{\nu k}{J_m} \\ \frac{\nu c}{J_t} & -\frac{c}{J_t} & \frac{\nu k}{J_t} & -\frac{k}{J_t} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{\theta}_m \\ \dot{\theta}_t \\ \theta_m \\ \theta_t \end{Bmatrix} + \begin{bmatrix} \frac{1}{J_m} & 0 \\ 0 & \frac{1}{J_t} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \tau_m \\ \tau_t \end{Bmatrix} \quad (1.7)$$

where

- θ_m Orientation angle of the motor's shaft.
- θ_t Orientation angle of turbine.
- ν Gear ratio of the multiplier.
- c Viscosity constant of the turbine's shaft.
- k Elasticity constant of the turbine's shaft.
- J_m Inertia moment of the generator.
- J_t Inertia moment of the turbine.
- τ_m Mechanical torque of the engine.
- τ_t Mechanical torque of the turbine.

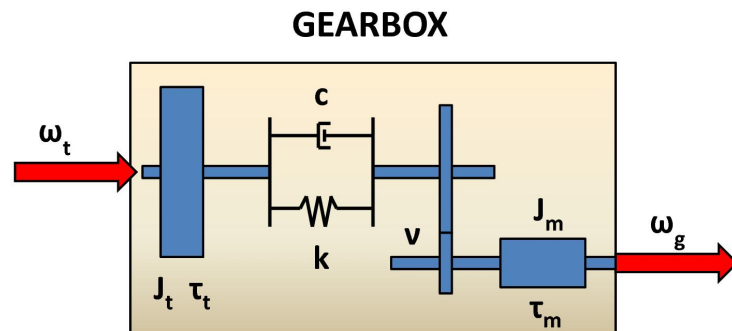


Figure 1.5: Simplification of the GearBox

1.4 Squirrel Cage Induction Generator

Three-phase induction machines have three windings in the stator and three windings more in the rotor, although, these can be real or imaginary.

As it is known, all electrical machines can be described as motor and generator as well, consequently, they can be described with the same set of equations. It is appropriate to remember that these equations govern the operation of the electrical machines. These equations are divided in two groups, *Voltage equations and Torque equations in machine variables* and other which are expressed in the axes of the reference variables.

With the goal of simplifying these equations, it is common in the technical literature, e.g. [8], to consider the following hypothesis:

- Symmetric and balanced three-phase induction machine, with a single winding rotor (Squirrel cage simple) and constant gap.
- Material is assumed to be linear, that is to say, the iron saturation is discarded.
- The iron magnetic permeability is assumed to be infinite in front of the air permeability, which means that the magnetic flux density is radial to the gap.
- All kind of losses in the iron are neglected.
- Both the stator windings as the rotor windings represent distributed windings which always generate a sinusoidal magnetic field distribution in the gap.

All hypothesis that we have explained before, using the induction motor's illustration, guide us to the following system of equations which describe the dynamic behavior of the induction machine.

$$\begin{Bmatrix} v_s^{abc} \\ v_r^{abc} \end{Bmatrix} = \begin{bmatrix} r_s^{abc} & 0 \\ 0 & r_r^{abc} \end{bmatrix} \begin{Bmatrix} i_s^{abc} \\ i_r^{abc} \end{Bmatrix} + \frac{d}{dt} \begin{Bmatrix} \lambda_s^{abc} \\ \lambda_r^{abc} \end{Bmatrix} \quad (1.8)$$

with

v_s^{abc}	stator winding's voltage vector
v_r^{abc}	rotor winding's voltage vector
i_s^{abc}	stator winding's current vector
i_r^{abc}	rotor winding's current vector
λ_s^{abc}	stator winding's concatenated flows vector
λ_r^{abc}	rotor winding's concatenated flows vector

The relationship between concatenated flows, rotor and stator's current is given by

$$\begin{Bmatrix} \lambda_s^{abc} \\ \lambda_r^{abc} \end{Bmatrix} = \begin{bmatrix} L_{ss}^{abc} & L_{sr}^{abc} \\ L_{rs}^{abc} & L_{rr}^{abc} \end{bmatrix} \begin{Bmatrix} i_s^{abc} \\ i_r^{abc} \end{Bmatrix} \quad (1.9)$$

where each term represents a 3-dimensional matrix or a three-dimensional vector. Then, the

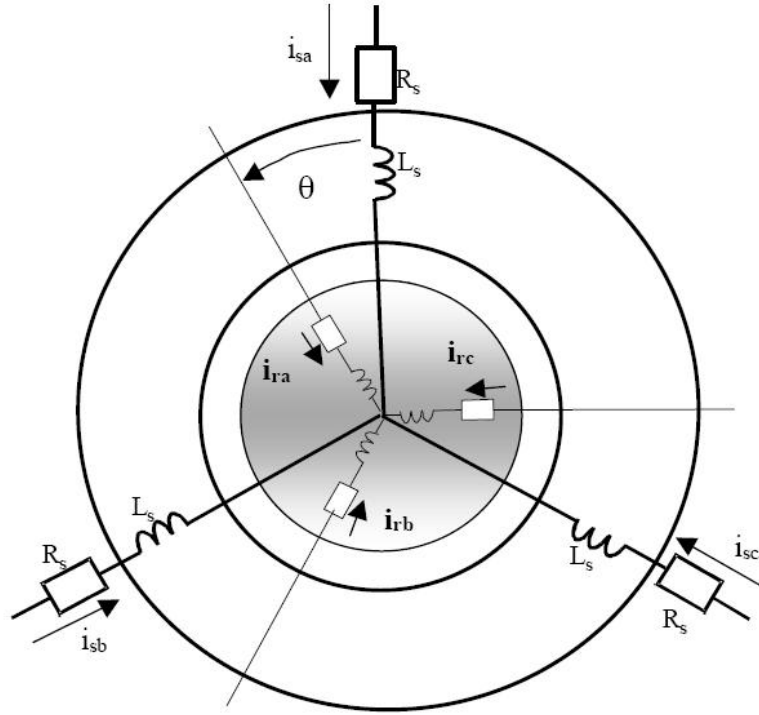


Figure 1.6: Induction machine's schematic illustration

vectors can be written as

$$v_s = \begin{pmatrix} v_{sa} \\ v_{sb} \\ v_{sc} \end{pmatrix}, v_r = \begin{pmatrix} v_{ra} \\ v_{rb} \\ v_{rc} \end{pmatrix}, i_s = \begin{pmatrix} i_{sa} \\ i_{sb} \\ i_{sc} \end{pmatrix}, i_r = \begin{pmatrix} i_{ra} \\ i_{rb} \\ i_{rc} \end{pmatrix}$$

and impedance matrices as

$$r_s^{abc} = \begin{bmatrix} r_s & 0 & 0 \\ 0 & r_s & 0 \\ 0 & 0 & r_s \end{bmatrix} \quad (1.10)$$

$$r_r^{abc} = \begin{bmatrix} r_r & 0 & 0 \\ 0 & r_r & 0 \\ 0 & 0 & r_r \end{bmatrix} \quad (1.11)$$

$$L_{ss}^{abc} = \begin{bmatrix} L_{ls} + L_{ss} & L_{sm} & L_{sm} \\ L_{sm} & L_{ls} + L_{ss} & L_{sm} \\ L_{sm} & L_{sm} & L_{ls} + L_{ss} \end{bmatrix} \quad (1.12)$$



$$L_{sr}^{abc} = \{L_{rs}^{abc}\}^t = L_{sr} \begin{bmatrix} \cos(\theta_r) & \cos(\theta_r + \frac{2\pi}{3}) & \cos(\theta_r - \frac{2\pi}{3}) \\ \cos(\theta_r - \frac{2\pi}{3}) & \cos(\theta_r) & \cos(\theta_r + \frac{2\pi}{3}) \\ \cos(\theta_r + \frac{2\pi}{3}) & \cos(\theta_r - \frac{2\pi}{3}) & \cos(\theta_r) \end{bmatrix} \quad (1.13)$$

$$L_{rr}^{abc} = \begin{bmatrix} L_{lr} + L_{rr} & L_{rm} & L_{rm} \\ L_{rm} & L_{lr} + L_{rr} & L_{rm} \\ L_{rm} & L_{rm} & L_{lr} + L_{rr} \end{bmatrix} \quad (1.14)$$

with:

- ω_r generator's shaft's orientation angle from electric system
- r_s resistance of the stator windings
- r_r resistance of the rotor windings
- L_{ss} self-inductance of the stator windings without the winding owing the dispersion flow
- L_{rr} self-inductance of the rotor windings without the winding owing the dispersion flow
- L_{sm} coupling inductances between stator windings
- L_{rm} coupling inductances between rotor windings
- L_{sr} maximum value reached by coupling inductances between stator and rotor windings
- L_{ls} dispersion inductance of the stator windings
- L_{lr} dispersion inductance of the rotor windings

The electromechanical conversion theory provides the following equation:

$$\Gamma_r = \frac{1}{2} [i]^t \frac{\delta[L(\theta_r)]}{\delta(\theta_r)} [i] \quad (1.15)$$

where

Γ_r Torque on the rotor shaft

$L(\theta_r)$ Induction machine's coupling inductance matrix. $L(\theta_r) = \begin{bmatrix} L_{ss} & L_{sr} \\ L_{rs} & L_{rr} \end{bmatrix}$

Usually the induction machines are designed with number of poles over 1. Theoretically, this could be understood as a ideal multiplier with a transmission ratio P between shaft's mechanical angle (θ_m) and electrical system's angle.

$$\Gamma_m = \frac{P}{2} [i]^t \frac{\delta[L(\theta_r)]}{\delta(\theta_r)} [i] \quad (1.16)$$

with

P number of poles for the Induction Machine.



Without the loss of generality we may suppose the number of poles is exactly one. However, we want to remark the results obtained generalize to the situation of multiple poles.

The equation (1.16) expresses the torque developed by the induction machine at any time, depending on the instantaneous currents circulating by each one of the six windings, and the separation angle between the stator winding 1 and the rotor winding 1. This equation is obtained by the electrical system energy balance.

Developing the equation (1.15), it is easily simplified due to L_{ss} and L_{rr} are not θ_r dependent. Then the derivative of this constant vanishes. So the new equation can be written as

$$\Gamma_r = \frac{1}{2} \begin{Bmatrix} i_s^{abc} \\ i_r^{abc} \end{Bmatrix}^t \begin{bmatrix} 0 & N_{sr} \\ N_{rs} & 0 \end{bmatrix} \begin{Bmatrix} i_s^{abc} \\ i_r^{abc} \end{Bmatrix} \quad (1.17)$$

where

$$N_{sr}^{abc} = \{N_{rs}^{abc}\}^t = -L_{sr} \begin{bmatrix} \sin(\theta_r) & \sin(\theta_r + \frac{2\pi}{3}) & \sin(\theta_r - \frac{2\pi}{3}) \\ \sin(\theta_r - \frac{2\pi}{3}) & \sin(\theta_r) & \sin(\theta_r + \frac{2\pi}{3}) \\ \sin(\theta_r + \frac{2\pi}{3}) & \sin(\theta_r - \frac{2\pi}{3}) & \sin(\theta_r) \end{bmatrix} \quad (1.18)$$

Also it is possible to make the same operation, finding the mechanical torque produced by the generator. This can be achieved just by multiplying by the transmission ratio P . Then, we obtain the following equation:

$$\Gamma_m = \frac{P}{2} \begin{Bmatrix} i_s^{abc} \\ i_r^{abc} \end{Bmatrix}^t \begin{bmatrix} 0 & N_{sr} \\ N_{rs} & 0 \end{bmatrix} \begin{Bmatrix} i_s^{abc} \\ i_r^{abc} \end{Bmatrix} \quad (1.19)$$

The generator which we will be the studied object is a **SCIG** (*Squirrel Cage Induction Generator*), this type of generator is known also as *Short-Circuit Induction Generator*, owing to rotor windings are then connected in short-circuit. So, the only part of the generator connected to the grid will be the stator. Because of this connection to the rotor windings, we are able to get the next simplification $v_r^{abc} = 0$. Then, the equation described before (1.17) could be written as follows:

$$\begin{Bmatrix} v_s^{abc} \\ 0^{abc} \end{Bmatrix} = \begin{bmatrix} r_s^{abc} & 0 \\ 0 & r_r^{abc} \end{bmatrix} \begin{Bmatrix} i_s^{abc} \\ i_r^{abc} \end{Bmatrix} + \frac{d}{dt} \begin{Bmatrix} \lambda_s^{abc} \\ \lambda_r^{abc} \end{Bmatrix} \quad (1.20)$$

In the next chapter, we will be describe the most common way to work with these equations and how to obtain them.

1.5 Full Converter

If the induction generator is connected directly to the grid, then a capacitor bank is connected in stand-alone systems to the stator to provide the magnetizing current for the reactive power. However, for grid connected systems the reactive power is drawn from the grid. Induction generators also are able to use back to back converters. Then, this fact let us make

several configurations. Therefore it is a good idea to couple this converter to SCIG, because this decoupling of the generator and the grid allows us to work with different frequencies on each side. This leads us to the advantage to change the usual operation typology of SCIG, because as we are able to work with different frequencies, is not necessary to be worried about the rotor speed. Then we will study a variable speed windmill.

As was said before, to take advantage of the variable speed operation a power electronic interface must be provided between the machine terminals and the grid. The back to back converter is a suitable option for Cage induction machine in wind Power application. The back to back is formed by a rectifier, an inverter and a condenser between both. The rectifier and the inverter, are composed by IGBT and these are controlled by Space Vector Pulse Width Modulation (SVPWM)¹

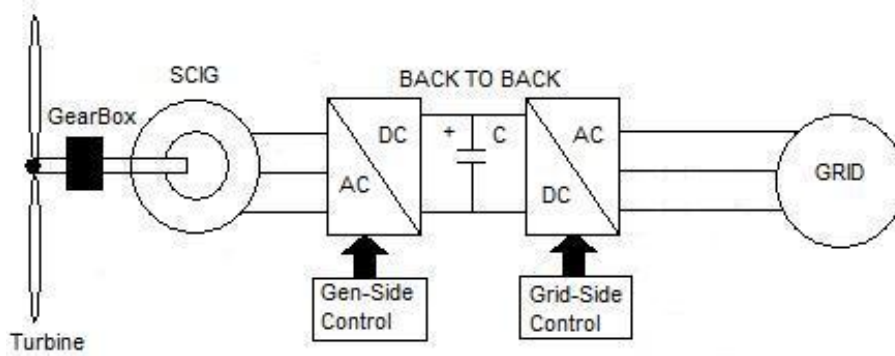


Figure 1.7: Full converter illustration

1.5.1 DC Bus

DC Bus is connected between rotor's converter and grid's converter. DC Bus' voltage V has the following expression:

$$E = E_0 + \frac{1}{C} \int_0^t i_{DCE} dt = E_0 + \frac{1}{C} \int_0^t (i_{DCg} - i_{DCs}) dt \quad (1.21)$$

- with
- E_0 DC Bus voltage at $t=0$
 - C DC Bus capacity
 - i_{DCE} Current which flows through the condenser
 - i_{DCg} Current which came from grid's converter
 - i_{DCs} Current which came from stator's converter

1.5.2 Crow Bar

The *Crow Bar* is a protection circuit used to prevent an overvoltage of a power supply unit from destroying the IGBTs and diodes of the converter.

¹The description of this technique is described in the appendix B



The Crow Bar operates, putting a short circuit or low resistance path across the voltage source, when excessive currents or voltages are detected. It means that when the power is too big and the converter is not able to absorb it all. It usually happens in voltage sags. Crow Bar will not be included in our model because it is not the aim of the project to study the behavior of the generator during a sag.

1.6 Grid connection

The grid connection will be described just until the electrical transformer, because after these the rest is part of the wind farm.

The cable which are between the converter and the electrical transformer could be called "short cable" because the length of the wire is shorter than 50 km. [9] Then, it is possible make to the simplification:

- Neglect the effect of capacity and focus the effect of their parameters in a series impedance equal to the total impedance of the line.

$$\underline{Z}_t = (R + j\omega L)l \quad (1.22)$$

where

- Z Value of the impedance.
- R Value of the resistance parameter.
- L Value of the inductance parameter.
- l Value of the length of the wire from converter to electrical transformer.

Then, the electrical circuit which connects the grid with the grid side converter can be described by the *Ohm law* ($\Delta V = Z \cdot I$). Applying this equation and the impedance of the line, the electrical circuit can be written as:

$$v_z^{abc} - v_l^{abc} = r_l i_l^{abc} + L_l \frac{d}{dt} i_l^{abc} \quad (1.23)$$

The Electrical transformer will be described with just an inductance. It can be approximated in this way in the PU (per unit) reference, and the inductance has the following equation:

$$x_{ccpu} = \varepsilon_{cc} \left(\frac{V_{1n}^2 S_b}{V_{1b} S_n} \right) \quad (1.24)$$

where

- ε_{cc} short circuit voltage.
- X_{ccpu} Value of the short circuit inductance in PU reference.
- V_{1b} Base voltage in the primary.
- V_{1n} Nominal voltage in the primary.
- S_b Base power.
- S_n Nominal power.

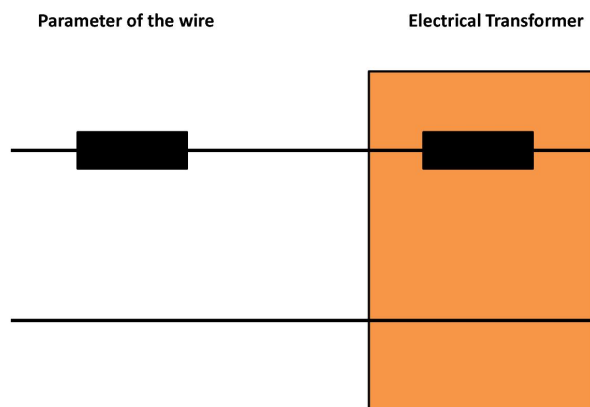


Figure 1.8: Simplification of the grid connection



Chapter 2

Generator system

On electrical system studies, it is usual to utilize mathematical transform of variables that are used to remove the time dependence of some parameters.

2.1 Change of abc basis to qd0

Quadrature–direct–zero (qd0) transformation is a mathematical transformation, used to simplify the analysis of three-phase circuits. In the case of balanced three-phase circuits, application of the qd0 transform reduces the three AC quantities to two DC quantities. Simplified calculations can, then, be carried out on these imaginary DC quantities before, performing the inverse transform to recover the actual three-phase AC results. It is often used in order to simplify the analysis of three-phase synchronous machines, or to simplify calculations for the control of three-phase inverters.

2.1.1 Introduction

The main goal of this introduction to give a brief resume of how to reach the qd0 transformation that will be used in rest of the document.

Mathematical theory

Let A , B be two square matrices, if these matrices could be describe as $T(A) = S^{-1}AS$ it is called *Similarity Transformation*, and S is known as the change of basis matrix. The change of basis matrix can be obtained by the eigenvectors of A .

Clarke Transformation

Clarke Transformation allow us to change the 3-dim system to 2-dim system (abc to $\alpha\beta$). For example, the current vector that is shown in (2.1).

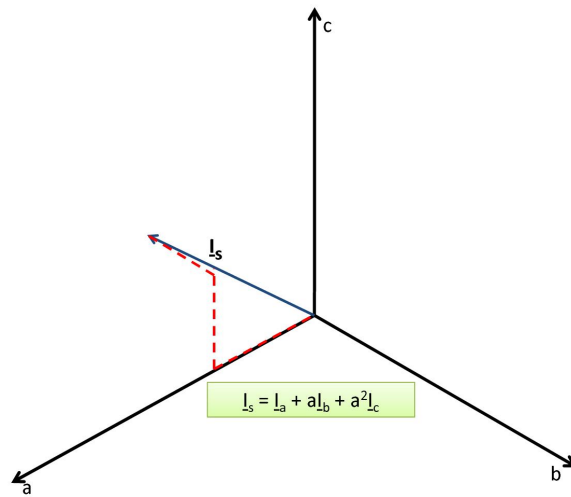


Figure 2.1: Any current vector in abc reference

$$\begin{Bmatrix} \alpha \\ \beta \end{Bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{Bmatrix} a \\ b \\ c \end{Bmatrix} \quad (2.1)$$

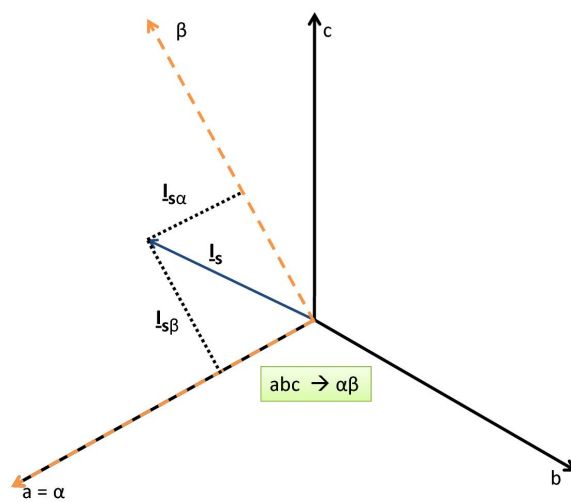


Figure 2.2: Representation of the Clarke Transformation



Park Transformation

The Park Transformation describes a rotation of an orthogonal system ((α, β) to (q, d)).

$$\begin{Bmatrix} q \\ d \end{Bmatrix} = \begin{bmatrix} -\sin(\theta_{field}) & \cos(\theta_{field}) \\ \cos(\theta_{field}) & \sin(\theta_{field}) \end{bmatrix} \begin{Bmatrix} \alpha \\ \beta \end{Bmatrix} \quad (2.2)$$

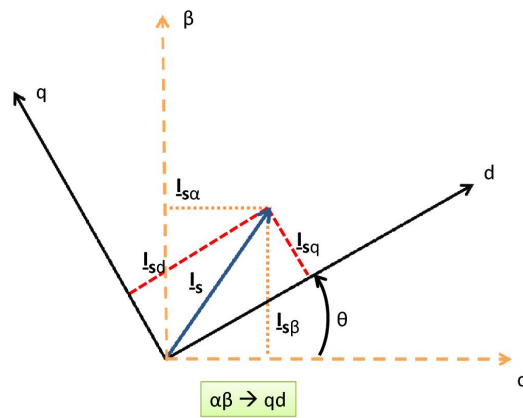


Figure 2.3: Rotation of the reference (α, β) an angle θ

2.1.2 qd0 Transformation

For the calculations, the **Park Transform**¹ be used which is proportional to Direct–quadrature–zero with a $\sqrt{3}$ ratio. Because of this, it is usual to describe the *Park Transform* as qd0 too. These equations are the ”shortcut” for the Clark and Park Transformations, instead of use both matrices with just one is enough.

The Park Transformation with an angle θ of a vector either $(x^{abc} \in \mathbb{R})$ is defined as:

$$x^{qd0} = T(\theta)x^{abc} \quad (2.3)$$

with

$$T(\theta) = \frac{2}{3} \begin{bmatrix} \cos(\theta) & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ \sin(\theta) & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad (2.4)$$

¹Throughout the document, qd0 Transformation will be called *Park Transform*, although it could be known as **Clarke-Park Transformation**

Then could be said that the vector x^{qd0} is the vector x^{abc} in $dq0$ on the angle θ reference. As a change of basis matrix $T(\theta)$ is invertible. So, it's true that:

$$x^{abc} = T^{-1}(\theta)x^{qd0} \quad (2.5)$$

with

$$T^{-1}(\theta) = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 1 \\ \cos(\theta - \frac{2\pi}{3}) & \sin(\theta - \frac{2\pi}{3}) & 1 \\ \cos(\theta + \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) & 1 \end{bmatrix} \quad (2.6)$$

Basis change qd0 to other reference in qd0 variables

From one vector x^{abc} could be obtained x^{qd0} using the Park Transformation with the θ , and from the same vector could be calculated \hat{x}^{qd0} using the Park Transformation with the $\hat{\theta}$. Then, the relationship between x^{qd0} and \hat{x}^{qd0} is:

$$x^{qd0} = P(\theta - \hat{\theta})\hat{x}^{qd0} \quad (2.7)$$

where:

$$P(\theta - \hat{\theta}) = \begin{bmatrix} \cos(\theta - \hat{\theta}) & -\sin(\theta - \hat{\theta}) & 0 \\ \sin(\theta - \hat{\theta}) & \cos(\theta - \hat{\theta}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2.8)$$

2.2 Park Transform application on Induction machine equations

To be able to apply Park Transform to Induction machine equations, it is necessary to define a matrix which includes 2 matrices of Park. The first one transforms the stator variables and the second converts the rotor variables, both into synchronous reference.

$$T(\theta, \theta - \theta_r) = \begin{bmatrix} T(\theta) & 0 \\ 0 & T(\theta - \theta_r) \end{bmatrix} \quad (2.9)$$

2.2.1 Voltage equations

From the equation (1.8) is known that:

$$v_s^{abc} = r_s^{abc}i_s^{abc} + \frac{d}{dt} (L_{ss}^{abc}i_s^{abc} + L_{sr}^{abc}i_r^{abc}) \quad (2.10a)$$

$$v_r^{abc} = r_r^{abc}i_r^{abc} + \frac{d}{dt} (L_{rs}^{abc}i_s^{abc} + L_{rr}^{abc}i_r^{abc}) \quad (2.10b)$$



Applying the Park Transform to (2.10a) and (2.10b) and defining some parameters

$$\begin{aligned} L_s &= L_{ss} - L_{sm} + L_{ls} \\ L_r &= L_{rr} - L_{rm} + L_{lr} \\ M &= \frac{3}{2}L_{sr} \end{aligned} \quad (2.11)$$

the equation is:

$$\begin{aligned} \begin{Bmatrix} v_s^{qd0} \\ v_r^{qd0} \end{Bmatrix} &= \begin{bmatrix} \dot{\theta}[Y]L_{ss}^{qd0} + r_s^{qd0} & \dot{\theta}[Y]L_{sr}^{qd0} \\ (\dot{\theta} - \theta_r)[Y]L_{rs}^{qd0} & (\dot{\theta} - \theta_r)[Y]L_{rr}^{qd0} + r_r^{qd0} \end{bmatrix} \begin{Bmatrix} i_s^{qd0} \\ i_r^{qd0} \end{Bmatrix} \\ &+ \begin{bmatrix} L_{ss}^{qd0} & L_{sr}^{qd0} \\ L_{rs}^{qd0} & L_{rr}^{qd0} \end{bmatrix} \frac{d}{dt} \begin{Bmatrix} i_s^{qd0} \\ i_r^{qd0} \end{Bmatrix} \end{aligned} \quad (2.12)$$

where

$$r_s^{qd0} = T_{qd0}(\theta)r_s^{abc}T_{qd0}^{-1}(\theta) = \begin{bmatrix} r_s & 0 & 0 \\ 0 & r_s & 0 \\ 0 & 0 & r_s \end{bmatrix} \quad (2.13a)$$

$$r_r^{qd0} = T_{qd0}(\theta - \theta_r)r_r^{abc}T_{qd0}^{-1}(\theta - \theta_r) = \begin{bmatrix} r_r & 0 & 0 \\ 0 & r_r & 0 \\ 0 & 0 & r_r \end{bmatrix} \quad (2.13b)$$

$$L_{ss}^{qd0} = T_{qd0}(\theta)L_{ss}^{abc}T_{qd0}^{-1}(\theta) = \begin{bmatrix} L_s & 0 & 0 \\ 0 & L_s & 0 \\ 0 & 0 & L_{ss} + 2L_{sm} + L_{ls} \end{bmatrix} \quad (2.13c)$$

$$L_{sr}^{qd0} = T_{qd0}(\theta)L_{sr}^{abc}T_{qd0}^{-1}(\theta - \theta_r) = \begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M \end{bmatrix} \quad (2.13d)$$

$$L_{rs}^{qd0} = T_{qd0}(\theta - \theta_r)L_{rs}^{abc}T_{qd0}^{-1}(\theta) = \begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M \end{bmatrix} \quad (2.13e)$$

$$L_{rr}^{qd0} = T_{qd0}(\theta - \theta_r)L_{rr}^{abc}T_{qd0}^{-1}(\theta - \theta_r) = \begin{bmatrix} L_r & 0 & 0 \\ 0 & L_r & 0 \\ 0 & 0 & L_{rr} + 2L_{rm} + L_{lr} \end{bmatrix} \quad (2.13f)$$

$$Y = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (2.13g)$$

By the development of this expression, we are able to get an equation of voltage where the component 0 only depends on the current 0 and its derivatives. Meanwhile, voltages q and d are depend only on the currents and their derivatives. Therefore, they could be expressed

as follows:

$$\begin{aligned} \begin{Bmatrix} v_{sq} \\ v_{sd} \\ v_{rq} \\ v_{rd} \end{Bmatrix} &= \begin{bmatrix} L_s & 0 & M & 0 \\ 0 & L_s & 0 & M \\ M & 0 & L_r & 0 \\ 0 & M & 0 & L_r \end{bmatrix} \frac{d}{dt} \begin{Bmatrix} i_{sq} \\ i_{sd} \\ i_{rq} \\ i_{rd} \end{Bmatrix} \\ &+ \begin{bmatrix} r_s & L_s \dot{\theta} & 0 & M \dot{\theta} \\ -L_s \dot{\theta} & r_s & -M \dot{\theta} & 0 \\ 0 & M(\dot{\theta} - \dot{\theta}_r) & r_r & L_r(\dot{\theta} - \dot{\theta}_r) \\ -M(\dot{\theta} - \dot{\theta}_r) & 0 & -L_r(\dot{\theta} - \dot{\theta}_r) & r_r \end{bmatrix} \begin{Bmatrix} i_{sq} \\ i_{sd} \\ i_{rq} \\ i_{rd} \end{Bmatrix} \end{aligned} \quad (2.14a)$$

$$v_{s0} = (L_{ss} + 2L_{sm} + L_{ls}) \frac{di_{s0}}{dt} + r_s i_{s0} \quad (2.14b)$$

$$v_{r0} = (L_{rr} + 2L_{rm} + L_{lr}) \frac{di_{r0}}{dt} + r_r i_{r0} \quad (2.14c)$$

Applying the same hypothesis as in the equation (1.20), it is possible for us to write (2.14a) as follows:

$$\begin{aligned} \begin{Bmatrix} v_{sq} \\ v_{sd} \\ 0 \\ 0 \end{Bmatrix} &= \begin{bmatrix} L_s & 0 & M & 0 \\ 0 & L_s & 0 & M \\ M & 0 & L_r & 0 \\ 0 & M & 0 & L_r \end{bmatrix} \frac{d}{dt} \begin{Bmatrix} i_{sq} \\ i_{sd} \\ i_{rq} \\ i_{rd} \end{Bmatrix} \\ &+ \begin{bmatrix} r_s & L_s \dot{\theta} & 0 & M \dot{\theta} \\ -L_s \dot{\theta} & r_s & -M \dot{\theta} & 0 \\ 0 & M(\dot{\theta} - \dot{\theta}_r) & r_r & L_r(\dot{\theta} - \dot{\theta}_r) \\ -M(\dot{\theta} - \dot{\theta}_r) & 0 & -L_r(\dot{\theta} - \dot{\theta}_r) & r_r \end{bmatrix} \begin{Bmatrix} i_{sq} \\ i_{sd} \\ i_{rq} \\ i_{rd} \end{Bmatrix} \end{aligned} \quad (2.15)$$

2.2.2 Torque equations

Taking into account the equation (1.19):

$$\begin{aligned} \Gamma_m &= \frac{1}{2} \begin{Bmatrix} i_s^{abc} \\ i_r^{abc} \end{Bmatrix}^t \begin{bmatrix} 0 & N_{sr}^{abc} \\ N_{rs}^{abc} & 0 \end{bmatrix} \begin{Bmatrix} i_s^{abc} \\ i_r^{abc} \end{Bmatrix} \\ &= \frac{1}{2} \left(\{i_s^{abc}\}^t N_{sr}^{abc} i_r^{abc} + \{i_r^{abc}\}^t N_{rs}^{abc} i_s^{abc} \right) \end{aligned} \quad (2.16)$$

and applying the Park Transform on it, we will obtain the following expression:

$$\Gamma_m = \frac{3}{2} M (i_{sq} i_{rd} - i_{sd} i_{rq}) \quad (2.17)$$



2.3 State space of SCIG generator

The dynamic behavior of the machine can be studied by means of a dynamic linear system, as we saw in the equation (2.15). Although, most adequate mathematical expression to realize the system simulation is the state space, which usually is expressed as $(\dot{X} = A \cdot X + B \cdot U)$. Then, by the reordering of the equations (2.15), (2.14b), (2.14c) we are able to obtain our goal with just the few next steps:

First, it is necessary to put on the different side of the equal sign derivative variables and non-derivative.

$$\begin{bmatrix} L_s & 0 & M & 0 \\ 0 & L_s & 0 & M \\ M & 0 & L_r & 0 \\ 0 & M & 0 & L_r \end{bmatrix} \frac{d}{dt} \begin{Bmatrix} i_{sq} \\ i_{sd} \\ i_{rq} \\ i_{rd} \end{Bmatrix} = \begin{Bmatrix} v_{sq} \\ v_{sd} \\ 0 \\ 0 \end{Bmatrix} - \begin{bmatrix} r_s & L_s \dot{\theta} & 0 & M \dot{\theta} \\ -L_s \dot{\theta} & r_s & -M \dot{\theta} & 0 \\ 0 & M(\dot{\theta} - \dot{\theta}_r) & r_r & L_r(\dot{\theta} - \dot{\theta}_r) \\ -M(\dot{\theta} - \dot{\theta}_r) & 0 & -L_r(\dot{\theta} - \dot{\theta}_r) & r_r \end{bmatrix} \begin{Bmatrix} i_{sq} \\ i_{sd} \\ i_{rq} \\ i_{rd} \end{Bmatrix} \quad (2.18)$$

Second, we have to determine the inverse of the matrix which is with the derivative. It is needed because of $A^{-1} * A = I$.

$$\begin{bmatrix} L_s & 0 & M & 0 \\ 0 & L_s & 0 & M \\ M & 0 & L_r & 0 \\ 0 & M & 0 & L_r \end{bmatrix}^{-1} = \frac{1}{L_s L_r - M^2} \begin{bmatrix} L_r & 0 & -M & 0 \\ 0 & L_r & 0 & -M \\ -M & 0 & L_s & 0 \\ 0 & -M & 0 & L_s \end{bmatrix} \quad (2.19)$$

Third, to leave only the derivative variables without multiplying constants, we should multiply the inverse in both sides.

$$\begin{aligned} \frac{d}{dt} \begin{Bmatrix} i_{sq} \\ i_{sd} \\ i_{rq} \\ i_{rd} \end{Bmatrix} &= -\frac{1}{L_s L_r - M^2} \begin{bmatrix} L_r & 0 & -M & 0 \\ 0 & L_r & 0 & -M \\ -M & 0 & L_s & 0 \\ 0 & -M & 0 & L_s \end{bmatrix} \begin{bmatrix} r_s & L_s \dot{\theta} & 0 & M \dot{\theta} \\ -L_s \dot{\theta} & r_s & -M \dot{\theta} & 0 \\ 0 & M(\dot{\theta} - \dot{\theta}_r) & r_r & L_r(\dot{\theta} - \dot{\theta}_r) \\ -M(\dot{\theta} - \dot{\theta}_r) & 0 & -L_r(\dot{\theta} - \dot{\theta}_r) & r_r \end{bmatrix} \begin{Bmatrix} i_{sq} \\ i_{sd} \\ i_{rq} \\ i_{rd} \end{Bmatrix} \\ &+ \frac{1}{L_s L_r - M^2} \begin{bmatrix} L_r & 0 & -M & 0 \\ 0 & L_r & 0 & -M \\ -M & 0 & L_s & 0 \\ 0 & -M & 0 & L_s \end{bmatrix} \begin{Bmatrix} v_{sq} \\ v_{sd} \\ 0 \\ 0 \end{Bmatrix} \end{aligned} \quad (2.20)$$

Finally, we have to multiply the matrices and discard unnecessary elements of the system. Then, we get the following equation:



$$\underbrace{\frac{d}{dt} \begin{Bmatrix} i_{sq} \\ i_{sd} \\ i_{rq} \\ i_{rd} \end{Bmatrix}}_{\dot{X}} = \underbrace{-\frac{1}{L_s L_r - M^2} \begin{bmatrix} L_r r_s & M^2 \dot{\theta}_r + (L_s L_r - M^2) \dot{\theta} & -M r_r & M L_r \dot{\theta}_r \\ -M^2 \dot{\theta}_r - (L_s L_r - M^2) \dot{\theta} & L_r r_s & -M L_r \dot{\theta}_r & -M r_r \\ -M r_s & -M L_s \dot{\theta}_r & L_s r_r & (L_s L_r - M^2) \dot{\theta} - L_s L_r \dot{\theta}_r \\ M L_s \dot{\theta}_r & -M r_s & -(L_s L_r - M^2) \dot{\theta} - L_s L_r \dot{\theta}_r & L_s r_r \end{bmatrix}}_A \underbrace{\begin{Bmatrix} i_{sq} \\ i_{sd} \\ i_{rq} \\ i_{rd} \end{Bmatrix}}_X \\
 + \underbrace{\frac{1}{L_s L_r - M^2} \begin{bmatrix} L_r & 0 \\ 0 & L_r \\ -M & 0 \\ 0 & -M \end{bmatrix}}_B \underbrace{\begin{Bmatrix} v_{sq} \\ v_{sd} \\ 0 \\ 0 \end{Bmatrix}}_U$$

(2.21a)

2

$$\frac{di_{s0}}{dt} = \frac{r_s}{L_{ss} + 2L_{sm} + L_{ls}} i_{s0} - \frac{1}{L_{ss} + 2L_{sm} + L_{ls}} v_{s0} \quad (2.21b)$$

$$\frac{di_{r0}}{dt} = \frac{r_r}{L_{rr} + 2L_{rm} + L_{lr}} i_{r0} - \frac{1}{L_{rr} + 2L_{rm} + L_{lr}} v_{r0} \quad (2.21c)$$

²In the appendix A will be studied the controllability and the observability of the system.



Chapter 3

Control system design

3.1 Introduction

Control plays a very important role in modern wind energy conversion systems. In fact, the principle target of control is to enable us to obtain as much quantity of energy from the wind as is possible during certain weather conditions, and deliver it to the grid at the best possible conditions. As well as, to keep the system on a safe working area to reduce aerodynamic and mechanical loads, and to protect the electrical devices, for example, *pitch controlled* that spin the blades in his own axis to regulate the position of it, or *stall controlled* which makes us loose aerodynamical power, this type of control will have constant θ_{pitch} . Additionally, the control can be used to perform as reactive power suppliers or consumers according to the power system requirements.

The control can be divided in 2 levels, high level control system and control of the converter system.

3.2 High level control

Usually in the windmills there are distinguished two different work regions: partial load and total load.

When the windmill are working in total load, the high level control system keep the nominal rotation regime and it orients the blades to extract the nominal power.

Instead, when the windmill is working on partial load the control keep the orientation of the blades and it regulates the rotational speed to get the maximum power that is possible from the wind. There are, basically, two ways to control the system when it is in partial load: *constant tip speed ratio constant scheme* and the *scheme of maximum power point tracking (MPPT)*.

In our study we assume a partial load working zone controlled by constant tip ratio constant scheme, which is the most common used control scheme, supposing pitch angle constant ($\theta_{pitch} = 0$).

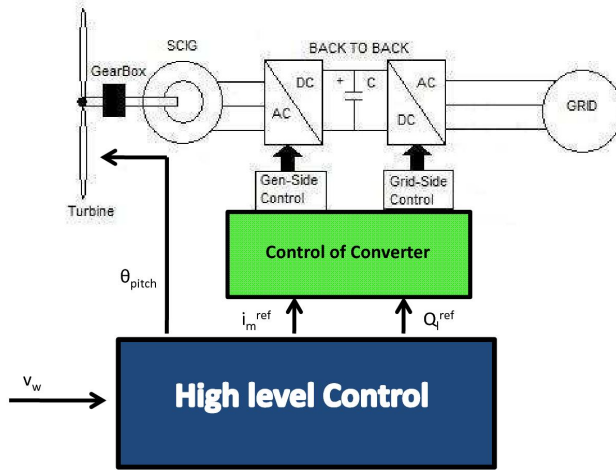


Figure 3.1: Representation of the control system

3.2.1 Constant tip speed ratio scheme control

Constant tip speed ratio control consist in reach an optimal torque curve which is a function of the rotation speed in design step of the control, and consign it to the converter so that the mechanical system is stable in the optimal point of wind power extraction.

Fundamentals of the control

Remembering the equation (1.3), it is clear that the unique variable parameter which we are able to control is c_p , because the wind speed is completely variable and uncontrollable, the area is determined by our structure and the air density is, almost, constant. Moreover, we know how this coefficient can be described (1.4) and this is a function of the **tip speed ratio** (λ), θ_{pitch} and some constant parameters which are defined depending on the windmill.

As our target is to obtain as much power as possible, always working in a security bounds and we are able to regulate c_p . Then, a good solution will be to find the optimal value of c_p , which can be calculated by deriving $c_p(\theta_{pitch}, \lambda)$ by λ . It is important to remark that the calculation are done with $\theta_{pitch} = 0$ already, because for $\theta_{pitch} \neq 0$, the system is quite hard to solve.

$$\frac{d}{d\lambda} c_1 \left(c_2 \left(\frac{1}{\lambda + c_8 \theta_{pitch}} - \frac{c_9}{1 + \theta_{pitch}^3} \right) - c_3 \theta_{pitch} - c_4 \theta_{pitch}^5 - c_6 \right) e^{-c_7 \left(\frac{1}{\lambda + c_8 \theta_{pitch}} - \frac{c_9}{1 + \theta_{pitch}^3} \right)} = 0 \quad (3.1)$$

From the equation (3.1) we can get the *tip speed ratio* for $\theta_{pitch} = 0$ ($\lambda_{opt} |_{\theta_{pitch}=0}$), which expression is:

$$\lambda_{opt} |_{\theta_{pitch}=0} = \frac{c_2 c_7}{c_2 c_7 c_9 + c_6 c_7 + c_2} \quad (3.2)$$



Replacing this expression in the equation of c_p (1.4), we get the optimal value of c_p when $\theta_{pitch} = 0$:

$$c_{p_{opt}} = \frac{c_1 c_2 e^{-\frac{c_4 c_7 \theta_{pitch}^{c_5} + c_3 c_7 \theta_{pitch} + c_6 c_7 + c_2}{c_2}}}{c_7} \quad (3.3)$$

Including both equations (3.2) and (3.3) into the expression of the power obtained by the turbine we are able to get an equation of the turbine torque what is needed to extract as much as possible for each wind speed. So, the equation of the Torque for $\theta_{pitch} = 0$ is:

$$\begin{aligned} \Gamma_{\theta_{pitch}=0} &= \frac{c_1 e^{-\frac{c_6 c_7 + c_2}{c_2}} (c_2 c_7 c_9 + c_6 c_7 + c_2)^3}{c_2^2 c_7^2} \frac{1}{2} \rho A R^3 \omega_t^2 \\ &= K_{c_p|\theta_{pitch}=0} \omega_t^2 \end{aligned} \quad (3.4)$$

where $K_{c_p|\theta_{pitch}=0}$ is called the *Optimum Torque coefficient of the turbine*.

Assuming that the control of the converter is able to force the generator to the give us the set-point torque with a bigger speed of answer (some magnitude orders) than mechanical dynamics, and neglecting the flexion of the axis, if we want to get this optimum torque, the expression of the angular acceleration of the windmill is:

$$\frac{d\omega_t}{dt} = \frac{1}{J} (\Gamma_t - \Gamma_m) = \frac{1}{2J} \rho A \left(c_p(v_w, \omega_t) \frac{v_w^3}{\omega_t} - K_{c_p|\theta_{pitch}=0} \omega_t^2 \right) \quad (3.5)$$

3.3 Control of the converter

The control of the converter system has 2 subsystems: the control of the electrical system of the stator, and the control of the electrical system of the grid side.

3.3.1 Control of the electrical system of the stator

The control of the electrical system of the stator can be realized through different current loops.¹

To make the control of Torque and induction generator's speed, will be used by the IMC (*Internal Model Control*) because it is very insensitive to changes of parameters, and can easily tune controllers [10].

This control strategy allows us to use a control structure which encapsulates the process system, as is described later. Then, as Induction Machine is in synchronism reference has a transfer function which is a first order system, as it will be shown in 3.3.1, the control structure that we get is quite similar to PI regulators. The main advantages are:

- Setpoint values are constant in the synchronism reference. We are able to get a **zero static error** if we choose this error, and will be neglecting any noise or perturbation,

¹In the appendix C are shown some of them.

as could be seen on (3.3).

- The control parameters are obtained based on known parameters of the machine, and of a parameter (the bandwidth desired closed loop control).

Brief description of IMC

The control strategy based on IMC was developed for the chemical engineering applications [11]. The Internal Model Control (IMC) concept relies on the Internal Model Principle which says that the control can be achieved only if the control system includes some representation of the process to be controlled.

The illustration (3.2) show the IMC scheme.

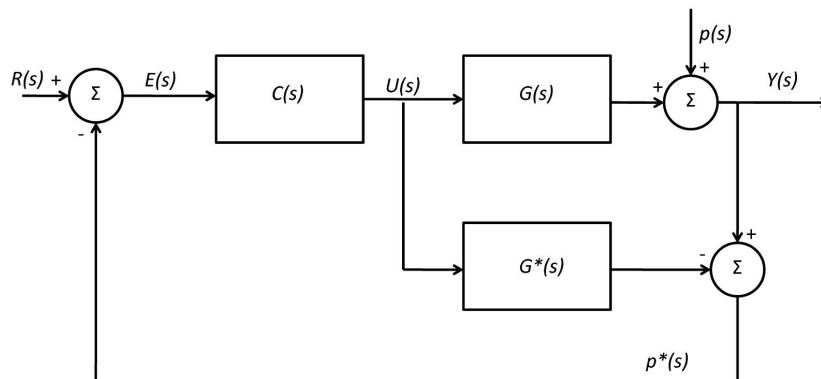


Figure 3.2: Schematic of the IMC scheme

where

$C(s)$ IMC controller.

$G(s)$ Induction Generator's transfer function.

$G^*(s)$ Internal model of the Induction Generator's transfer function.

As can be deduced from the 3.2, if $G^*(s) = G(s)$ (it means that the approximation of the Induction generator model is perfect), then doesn't exist feedback and the transfer function's matrix close loop is: $G_c(s) = G(s)C(s)$. So, the system will be stable, only, if both transfer function are stable.

For that, it seems to be a good option to choose the transfer function as $C(s) = G^{*-1}(s)$, because the rise time will be instantaneous. But, this gives us a lot of disadvantages, for example, detune of the parameter of the model or too high variables which, maybe, are not possible to apply.



To improve robustness, it is possible for us to include a modification of the model. The idea is to define the transfer function matrix equal that system transfer function in series with a low-pass filter. Then the controller's function matrix becomes:

$$C(s) = G^{*-1}L(s); L(s) = \frac{\alpha}{s + \alpha}I \quad (3.6)$$

where

I Identity or unit matrix.

α Controller's bandwidth in close loop.

In the first order systems, the rise time, is related to the bandwidth by the following equation:
 $t_r = \ln\left(\frac{9}{\alpha}\right)$

Current's Control Loop of the SCIG

Known the IMC propose, it is needed to apply it on *current's control loop of the induction generator*. To do this, we need the transfer function of the control loop, equation which could be reached directly from Induction generator's state equation on synchronism reference (2.15). If we suppose current constant for all the time, then it is correct to neglect the terms which are affected by magnetizing current, due to it will be, just, a little disturbance that could be corrected easily with a integer. The transfer function becomes:

$$\begin{Bmatrix} v_{sq} \\ v_{sd} \end{Bmatrix} = \begin{bmatrix} r_s & L_s\dot{\theta} \\ -L_s\dot{\theta} & r_s \end{bmatrix} \begin{Bmatrix} i_{sq} \\ i_{sd} \end{Bmatrix} + \begin{bmatrix} L_s & 0 \\ 0 & L_s \end{bmatrix} \frac{d}{dt} \begin{Bmatrix} i_{sq} \\ i_{sd} \end{Bmatrix} \quad (3.7)$$

Applying Laplace transform to (3.7) we obtain

$$\begin{Bmatrix} v_{sq} \\ v_{sd} \end{Bmatrix} = \begin{bmatrix} r_s + L_s s & L_s \dot{\theta} \\ -L_s \dot{\theta} & r_s + L_s s \end{bmatrix} \begin{Bmatrix} i_{sq} \\ i_{sd} \end{Bmatrix} \Rightarrow G^{*-1}(s) = \begin{bmatrix} r_s + L_s s & L_s \dot{\theta} \\ -L_s \dot{\theta} & r_s + L_s s \end{bmatrix} \quad (3.8)$$

To reach controller's transfer function, we apply the equation (3.6).

$$C(s) = G^{*-1}(s)L(s) = \frac{\alpha}{s + \alpha} \begin{bmatrix} r_s + L_s s & L_s \dot{\theta} \\ -L_s \dot{\theta} & r_s + L_s s \end{bmatrix} \quad (3.9)$$

It can be observed from the scheme, that the transfer function $F(s)$ is the transfer function resulting of the control loop which contains $C(s)$ and G^{*-1} .

$$F(s) = [1 - C(s) \cdot G^{*-1}]^{-1} \cdot C(s) \Rightarrow F(s) = \frac{\alpha}{s} \begin{bmatrix} r_s + L_s s & L_s \dot{\theta} \\ -L_s \dot{\theta} & r_s + L_s s \end{bmatrix} \quad (3.10)$$

The system is a control (F), a plant (G) and a unity feedback, and can be represented as it is shown in the figure 3.4.

Then, the transfer function of the whole system can be obtained simplifying the diagram. The final transfer function is L , introduced in 3.6, which is a first order system with an

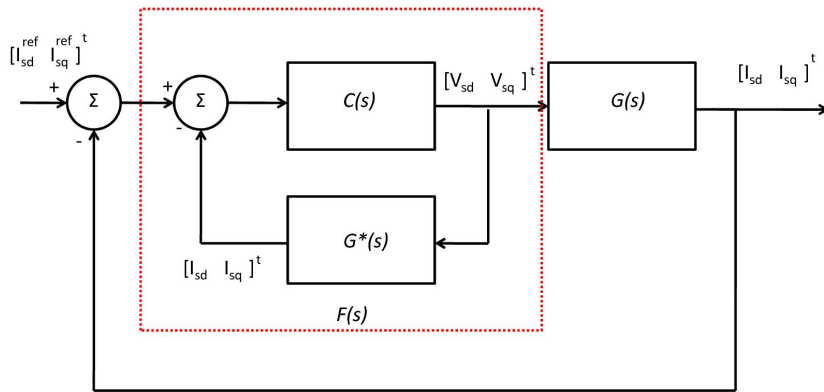


Figure 3.3: Alternative structure of the IMC scheme

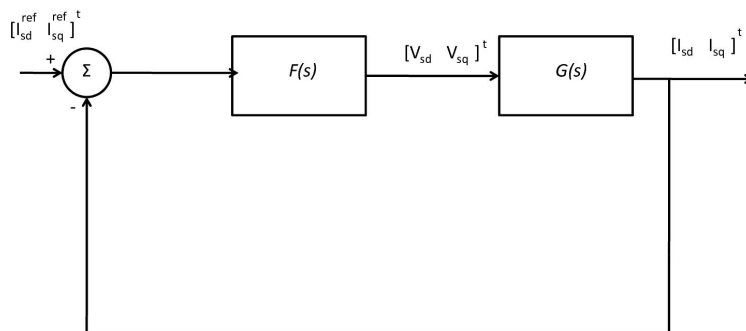


Figure 3.4: Block diagram of the system

unitary gain and time constant α^{-1} .



$$CS(s) = [I + F(s) \cdot G(s)]^{-1} \cdot F(s) \cdot G(s) = \begin{bmatrix} \frac{\alpha}{s+\alpha} & 0 \\ 0 & \frac{\alpha}{s+\alpha} \end{bmatrix}^2 \quad (3.11)$$

How to define the currents of reference

To be able to realize the control of the converter of the rotor's side, we need to define the reference currents, as is shown in the following control equation

$$\begin{Bmatrix} \dot{i}_{sq} \\ \dot{i}_{sd} \end{Bmatrix} = \begin{bmatrix} \frac{\alpha}{s+\alpha} & 0 \\ 0 & \frac{\alpha}{s+\alpha} \end{bmatrix} \begin{Bmatrix} \dot{i}_{sq}^{ref} \\ \dot{i}_{sd}^{ref} \end{Bmatrix} \quad (3.12)$$

To find these references, first of all, we should make a change of variables which is frequently used in the study of Induction machines in steady state. This new variable is called the *magnetizing current*³ which is the image of the magnetic field in the gap, in our case, with a magnetic field seen from the rotor. It is described for the following relationship:

$$\dot{i}_m = \dot{i}_s + \frac{L_r}{M} \dot{i}_r \quad (3.13)$$

Applying this change of variable to the equation system of SCIG (2.15) whose reference and orientation is synchronized by the flux of the rotor, we obtain:

$$\begin{Bmatrix} v_{sd} \\ v_{sq} \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} L_s - \frac{M^2}{L_r} & 0 & \frac{M^2}{L_r} & 0 \\ 0 & L_s - \frac{M^2}{L_r} & 0 & \frac{M^2}{L_r} \\ 0 & 0 & M & 0 \\ 0 & 0 & 0 & M \end{bmatrix} \frac{d}{dt} \begin{Bmatrix} i_{sd} \\ i_{sq} \\ i_{md} \\ i_{mq} \end{Bmatrix} + \begin{bmatrix} r_s & -\left(L_s - \frac{M^2}{L_r}\right)\dot{\theta} & 0 & -\frac{M^2}{L_r}\dot{\theta} \\ \left(L_s - \frac{M^2}{L_r}\right)\dot{\theta} & r_s & \frac{M^2}{L_r}\dot{\theta} & 0 \\ -\left(\frac{r_r}{L_r}M\right) & 0 & r_r \frac{M}{L_r} & -M\dot{\theta}_r \\ 0 & -\left(\frac{r_r}{L_r}M\right) & -M\dot{\theta}_r & r_r \frac{M}{L_r} \end{bmatrix} \begin{Bmatrix} i_{sd} \\ i_{sq} \\ i_{md} \\ i_{mq} \end{Bmatrix} \quad (3.14)$$

Assuming that there is a current control voltage inverter (CCVSI), we can dismiss the equations which are referred to stator because we do not need to calculate the voltage because of the CCVSI control will do. Furthermore, there are infinite synchronism references, if we choose the reference which its direct component is oriented by the maximum of the magnetic field in the gap (seen from rotor), then the quadratic component of the magnetizing current will be zero ($i_{mq} = 0$).

So, finally the equation (3.14) can be written as:

$$\begin{cases} 0 = -r_r \frac{M}{L_r} \dot{i}_{sd} + \frac{r_r M}{L_r} \dot{i}_{md} + M \frac{d}{dt} \dot{i}_{md} \\ 0 = -r_r \frac{M}{L_r} \dot{i}_{sq} + M \dot{\theta}_r \dot{i}_{md} \end{cases} \quad (3.15)$$

²The inverse of a two-dimensions matrix is easily calculated. From a matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, its inverse is defined as $A^{-1} = \frac{1}{\det\{A\}} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$

³In the appendix D is explained how it is defined.

Applying the same changes and assumptions to the torque equation (2.17), it becomes:

$$\Gamma = \frac{M^2}{L_r} \cdot i_{md} \cdot i_{sq} \quad (3.16)$$

Supposing an steady state of the system and the system in a save zone of work, we are able to neglect the derivative of the magnetizing current due to it is, almost, null.

Then, we are able to find our desired references as:

$$i_{sq}^{ref} = \frac{L_r}{r_r} \dot{\theta}_r i_{md}^{ref} \quad (3.17a)$$

$$i_{sd}^{ref} = i_{md}^{ref} \quad (3.17b)$$

Can be seen that our control system, only, has one degree of freedom, which is i_{md}^{ref} . Replacing the value of i_{sq}^{ref} (3.17a) in the equation of the torque (3.16), we can find a equation which give us i_{md}^{ref} from the desired torque (Γ^{ref}) what will be defined by the wind speed because we are able to include, also, the hypothesis of the *tip speed ratio constant* which offer us a constant relationship between wind speed and turbine rotational speed, and assuming the perfect conversion of turbine rotational speed and rotor rotational speed ⁴.

$$i_{md}^{ref} = \sqrt{\frac{r_r \Gamma^{ref}}{M^2 \dot{\theta}_r}} = \sqrt{\frac{r_r \frac{1}{2} C_{p_{opt}} \rho A R^2 v_w}{M^2 \lambda_{opt} |_{\theta_{pitch}=0} \cdot \nu}} \quad (3.18)$$

3.3.2 Control of the electrical system of the grid side

The control of the electrical system of the grid is configured to control the DC Bus voltage and the reactive power which is consumed or supplied by the converter of the grid side.

The control of the this side of the converter can be defined by the same way that for the others machines which are using the same converter back to back. This is possible because the converter decouple both sides, so the grid side is a single element by itself.[12]

Current's Control Loop of the grid

Applying Park transformation to the equations of the circuit of the grid (1.23) in v_z reference, the equations are:

$$\begin{Bmatrix} v_{zq} \\ 0 \end{Bmatrix} - \begin{Bmatrix} v_{lq} \\ v_{ld} \end{Bmatrix} = \begin{bmatrix} r_l & -L_l \dot{\theta}_e \\ L_l \dot{\theta}_e & r_l \end{bmatrix} \begin{Bmatrix} i_{lq} \\ i_{ld} \end{Bmatrix} + \begin{bmatrix} L_l & 0 \\ 0 & L_l \end{bmatrix} \frac{d}{dt} \begin{Bmatrix} i_{lq} \\ i_{ld} \end{Bmatrix} \quad (3.19)$$

where

$\dot{\theta}_e$ It is the frequency of the grid ($2\pi f$)

The evolution of this differential equations system depends on the frequency of the grid. Applying a *feed-forward* ⁵to the system, we delete the dependence of the dynamics with the

⁴ $\omega_t \cdot \nu = \omega_r = \dot{\theta}_r$

⁵Method used to keep constant the state of the system. The system answer to the perturbations in a known - defined way.



frequency of the grid. We define the feed-forward as:

$$\begin{Bmatrix} v_{lq} \\ v_{ld} \end{Bmatrix} = \begin{Bmatrix} \widehat{v}_{lq} + v_{zq} - L_l \dot{\theta}_e i_{ld} \\ -\widehat{v}_{ld} - L_l \dot{\theta}_e i_{lq} \end{Bmatrix} \quad (3.20)$$

Moreover, after the incorporation of the feed-forward, the system becomes decoupled between q and d. With the following system:

$$\begin{Bmatrix} \widehat{v}_{lq} \\ \widehat{v}_{ld} \end{Bmatrix} = \begin{bmatrix} r_l & 0 \\ 0 & r_l \end{bmatrix} \begin{Bmatrix} i_{lq} \\ i_{ld} \end{Bmatrix} + \begin{bmatrix} L_l & 0 \\ 0 & L_l \end{bmatrix} \frac{d}{dt} \begin{Bmatrix} i_{lq} \\ i_{ld} \end{Bmatrix} \quad (3.21)$$

Performing the Laplace Transform to our system (3.21), we obtain the following transfer function:

$$\begin{Bmatrix} v_{lq} \\ v_{ld} \end{Bmatrix} = \begin{bmatrix} r_s + L_s s & 0 \\ 0 & r_s + L_s s \end{bmatrix} \begin{Bmatrix} i_{lq} \\ i_{ld} \end{Bmatrix} \Rightarrow S^{*-1}(s) = \begin{bmatrix} r_s + L_s s & 0 \\ 0 & r_s + L_s s \end{bmatrix} \quad (3.22)$$

To make the control of this system we will use again the **IMC method**, and the whole system is:

$$\begin{Bmatrix} i_{lq} \\ i_{ld} \end{Bmatrix} = \begin{bmatrix} \frac{\alpha}{s+\alpha} & 0 \\ 0 & \frac{\alpha}{s+\alpha} \end{bmatrix} \begin{Bmatrix} i_{lq}^{ref} \\ i_{ld}^{ref} \end{Bmatrix} \quad (3.23)$$

How to define the currents of reference

The control loops of Bus voltage and Reactive power are responsible of generate the references for the current loops.

The reference i_{ld}^{ref} is easy to define because reactive power expression of the grid side is simple in our reference:

$$Q_z = -\frac{3}{2} v_{zq} i_{ld} \quad (3.24)$$

then, the reference can be found from the referenced reactive power.

$$i_{ld}^{ref} = -\frac{2Q_z^{ref}}{3v_{zq}} \quad (3.25)$$

Otherwise, first of all, the goal of the Bus voltage control is obtain i_{lq}^{ref} holding constant the DC Link voltage (E). Moreover, to define the Bus voltage control it is necessary make some assumptions:

1. The grid side converter has an efficiency of 100% and it makes perfect the conversion between DC /AC.
2. Bus condenser capacity is large that implies a slow voltage evolution, the condenser voltage is almost constant and we can neglect the grid side inductance losses.

3. The current loop is fast enough to consider that we have an ideal current source.

Making a balance of power through converter of the grid and taking account of the first hypothesis that we had defined before, we get the following expression:

$$\begin{aligned} P_{DCI} = P_{lq} &\Rightarrow v_{DCI} \cdot i_{DCI} = \frac{3}{2} (v_{ld} \cdot i_{ld} + v_{lq} \cdot i_{lq}) \\ &\Rightarrow v_{DCI} \cdot i_{DCI} = \frac{3}{2} (v_{lq} \cdot i_{lq}) \end{aligned} \quad (3.26)$$

Then, we obtain an equation for i_{DCI} current:

$$i_{DCI} = \frac{3}{2} \frac{v_{lq}}{E} i_{lq} \quad (3.27)$$

Using the second assumption which explain us $E \simeq \text{constant}$ and neglecting voltage drop in the inductance of the grid side ($v_{lq} \simeq v_{zq}$). Then, equation becomes:

$$i_{DCI} = \frac{3}{2} \frac{v_{zq}}{E} i_{lq} \quad (3.28)$$

where, now, v_{zq} and E are values always constants.

Making a little review of our equations and of our target, we can say that: We have a DC Link between 2 converters, the DC Link follow the equation (1.21): $E = \frac{1}{C \cdot s} (i_{DCI} - i_{DCs})$ ⁶ and we have obtained an expression that relates i_{DCI} and i_{lq} (3.28).

Finally, the resultant system is linear and, then, can be easily designed a controller what be able to give us a current reference i_{lq}^{ref} which hold $E \text{ constant}$ (steady error null) for an current input (i_{DCs}) with step shape and reply speed slow enough to be agree with the third assumption.

Then, the block diagram of the system described before is:

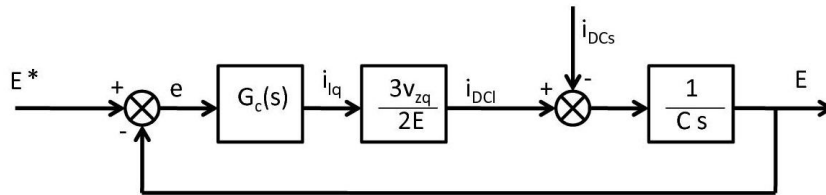


Figure 3.5: Block diagram of the system of the BUS control

The error expression of this system can be expressed as:

$$e(s) = \frac{s}{s + \frac{3v_{zq}}{2EC} G_c(s)} E^*(s) + \frac{\frac{1}{C}}{s + \frac{3v_{zq}}{2EC} G_c(s)} i_{DCs}(s) \quad (3.29)$$

Due to we want to assure a null error, we must choose a controller which help us to reach it.

⁶Is the equation (1.21), in the Laplace variables



To calculate the steady error of the system we use the *Final Value Theorem*⁷. Assuming the inputs E^* and i_{DCs} as a constant, we can define the controller to reach our desire. As is quite easy to see for the input E^* is only needed a proportional controlled, but for i_{DCs} is necessary a PI controller. Then we choose the more restrictive of both, *PI controller*. To determine the values of the parameters of the controller ($K_p + \frac{K_i}{s}$), we introduce this definition to our system.

$$e(s) = \frac{\frac{1}{C}}{s + \frac{3v_{zq}}{2EC} \left(K_p + \frac{K_i}{s} \right)} = \frac{\frac{s}{C}}{s^2 + \frac{3v_{zq}K_p}{2EC}s + \frac{3v_{zq}K_i}{2EC}} \quad (3.30a)$$

$$\frac{\frac{s}{C}}{s^2 + 2\zeta\omega_n s + \omega_n^2} \Rightarrow \frac{\frac{s}{C}}{s^2 + \frac{3v_{zq}K_p}{2EC}s + \frac{3v_{zq}K_i}{2EC}} \quad (3.30b)$$

$$\begin{aligned} \omega_n^2 = \frac{3v_{zq}K_i}{2EC} &\Rightarrow K_i = \frac{2EC}{3v_{zq}}\omega_n^2 \\ 2\zeta\omega_n = \frac{3v_{zq}K_p}{2EC} &\Rightarrow K_p = \frac{4EC}{3v_{zq}}\zeta\omega_n \end{aligned} \quad (3.30c)$$

We can check if our PI controller works:

$$\lim_{s \rightarrow 0} s \cdot e(s) = \frac{\frac{s^2}{C}}{s^2 + \frac{3v_{zq}K_p}{2EC}s + \frac{3v_{zq}K_i}{2EC}} \Rightarrow \frac{1}{C} \overset{8}{\rightarrow} 0 \overset{9}{\rightarrow} 0 \quad (3.31)$$

⁷ $x(t \rightarrow \infty) = \lim_{s \rightarrow 0} s C(s)$

⁸Because there is the same maximum potence on numerator and denominator.

⁹Applying the second assumption of the large capacity of the Bus Link condenser.





Chapter 4

Modeling of the system

4.1 Introduction

The equations developed in previous chapters have been implemented in Simulink®¹ to simulate the system response for a given situation. In this chapter could be seen the details of the system implementation.

The main idea of the model is keep the same basic structure of the system, at least, the most important groups: Mechanical system (Turbine and GearBox), Generator, Converter with his control and Grid.

The complete model is implemented as is shown in the figure (4.1).

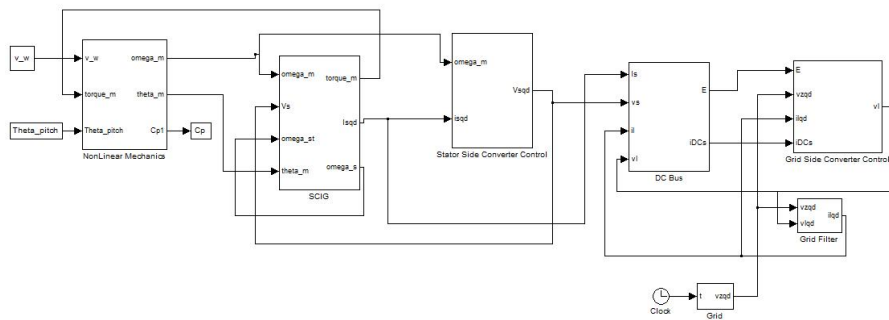


Figure 4.1: Simulink® model of the whole windmill

4.2 Modeling of the Mechanical System

The model of the mechanical system includes both Turbine system and Gear Box. To determine this model have been necessary some simplifications of the original equations obtained before, which are:

¹Simulink®, (Registered trademark of MathWorks) is an environment for multidomain simulation and Model-Based Design for dynamic and embedded systems.

- The Gear Box model is replaced by only the transmission ratio. We can make this simplification because we are mainly interested in the electrical behavior of the system.
- The dynamics of the windmill is described in the model as $\frac{d\omega_t}{dt} = \frac{1}{J} (\Gamma_t - \Gamma_m)$, where J will be only the Inertia of the turbine because is some orders of magnitude bigger than the Inertia of the generator, as can be seen in [13].²

The implemented mechanical system (4.2) requires three inputs: wind speed (v_w), pitch angle (θ_{pitch}) and Generator Torque (Γ_m). And give us as output: C_p value, mechanical angle (θ_m) and mechanical speed (ω_m or $\dot{\theta}_m$).

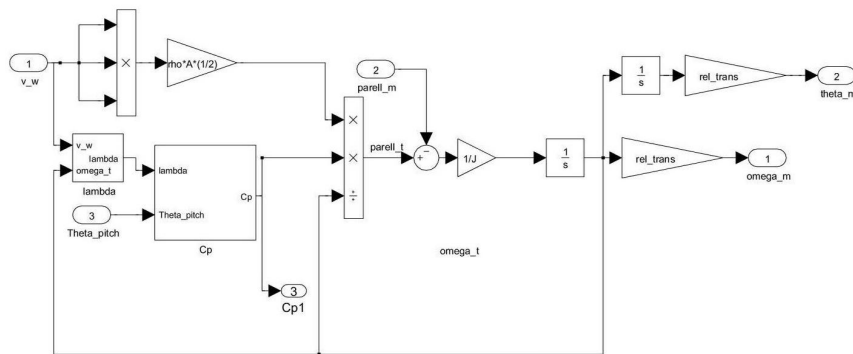


Figure 4.2: Simulink[®] model of the mechanical part

4.3 Modeling of SCIG

The model of the generator (figure 4.3) have been defined using the equations of the generator obtained before. In this box, it is calculated the electrical torque of the generator, and the active and reactive power of the stator.

4.4 Modeling of Full Power Converter

The model of the full power converter (figure 4.4) have been divided in 3 parts (Generator side, DC Link and Network side), as is indeed the converter (rectifier , dc bus and inverter).

²Typical Inertias Range: $J_{turb}[3 \cdot 10^6 - 9 \cdot 10^6]$ and $J_{gen}[65 - 130] [kgm^2]$

4.4.2 DC Link

The block of the DC Link (figure 4.6) is defined directly by the equation (1.21).

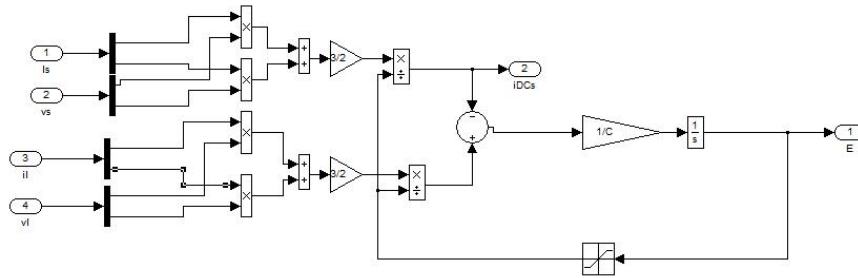


Figure 4.6: Simulink[®] model of the DC Link of the converter

4.4.3 Grid Side Converter

The block of the grid converter (figure 4.7) includes the DC Link voltage control and the reactive power control delivered to the grid.

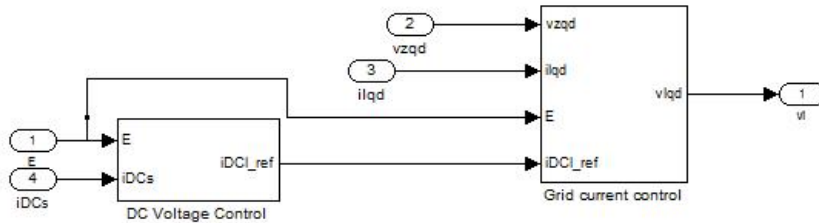


Figure 4.7: Simulink[®] model of the grid side of the frequency converter

4.5 Modeling of the Grid

The model of the grid (figure 4.8) have been divided in 2 parts (Grid Filter and Grid).

4.5.1 Grid Filter

The block of grid filter (figure 4.9) represents the little voltage sag which is produced by the impedances of the wire, as is defined in the equation (1.23).

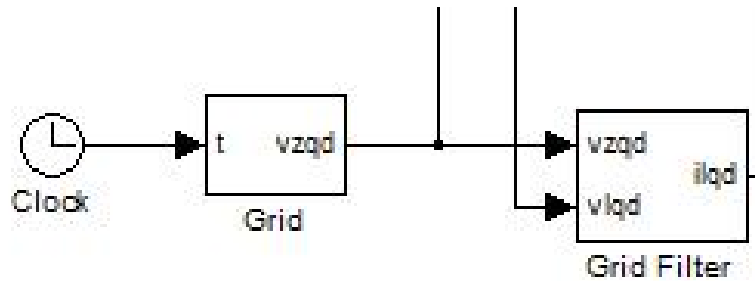


Figure 4.8: Simulink® model of the grid

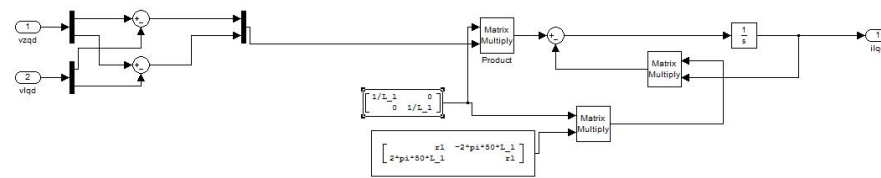


Figure 4.9: Simulink® model of the grid filter

4.5.2 Grid

The block of grid (figure 4.10) represents all the grid, which could be defined as an ideal voltage source. In this block can be found a Park transformation box to change the abc voltage to qd.

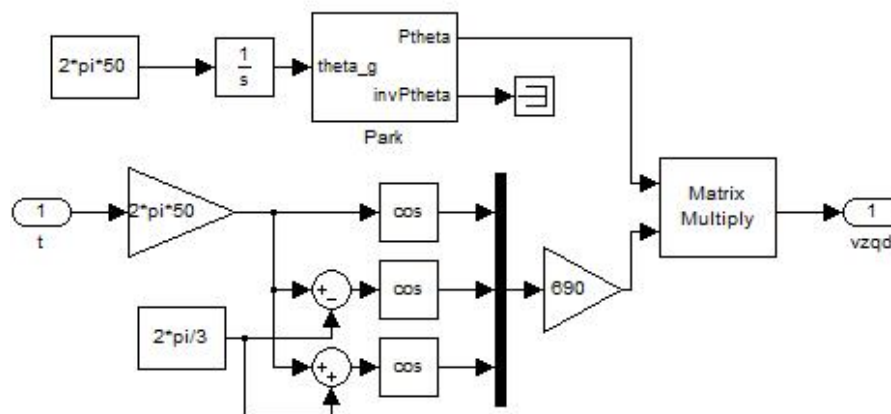


Figure 4.10: Simulink® model of the grid voltage generation.





Chapter 5

Simulation of the model and Analysis of the results

To realize the simulation of the model have been defined some parameters. The mechanical parameters (Inertia, transmission ratio, radius and c_i [constants of the turbine power curves]) have been determined as a mean of the range of values that can be found in [13]. The electrical parameters (resistances, inductances and rated power) have been used as in [14].

We do two different simulations: Generator connected directly to the grid and Complete system.

5.1 Generator connected directly to the grid

As we explain before, this typology of connection is the most common in SCIG generators (Constant-speed wind Turbine). So, by one hand, this simulation show us how work this type of turbine and, by other hand, allow us know if the model of the generator is working correctly.

We will study how is working the generator, and to determine this we will plot the evolution of the Torque (figure 5.1)and, also, the evolution of the currents (figure 5.2).

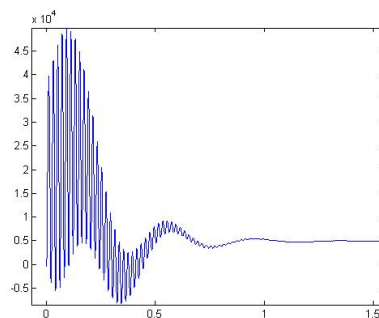


Figure 5.1: Mechanical Torque produced by the Generator

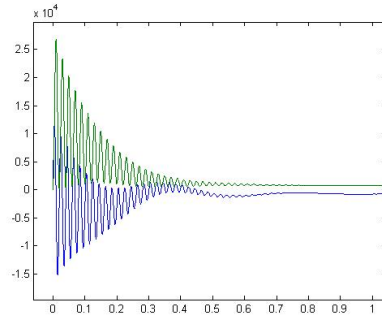


Figure 5.2: Stator currents in qd reference

As can be seen in the figures (5.1) and (5.2), the Torque and the currents reach a steady state, which implies that the generator model is correct. Can be detected a transitory which produce a large spike in the q stator current ($\cong 3000$ A).

Is necessary look how is working the mechanical system. With this purpose is plotted how change the power coefficient (C_p) as function of *tip speed ratio* (figure 5.3).

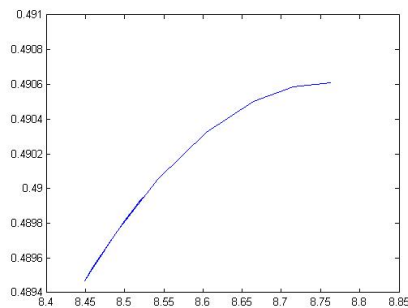


Figure 5.3: Evolution of $C_p(\lambda)$

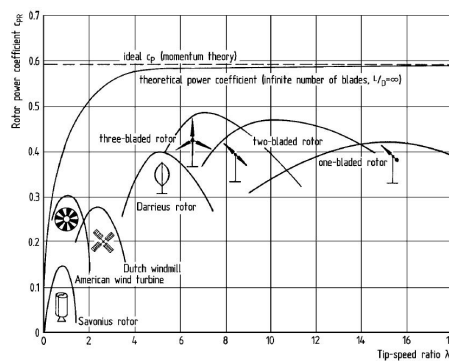


Figure 5.4: Graphic of characteristic C_p of each turbine.



Comparing the figures (5.3) and (5.4¹), we are able to say that the system is working correctly due to the values are inside the range of the three blades values that are shown. After verifying the results obtained, we can say that the generator and the mechanical system are working properly.

5.2 Complete system

In this section is checked the validity of our object of study, a SCIG connected to the grid through a back to back converter.

Using the information obtained in the previous section, we know that the model of the generator and the mechanical system are correct. Then, we need to test the model of the converter.

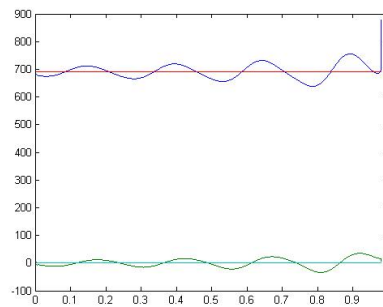


Figure 5.5: Graphic of the evolution of the grid filter voltage

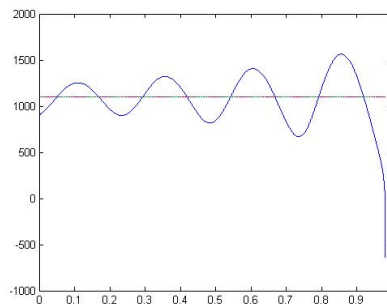


Figure 5.6: Voltage E of the DC Link

In the graphics (5.5) and (5.6) can be seen as the system tries to achieve the values referenced. Although they do not reach the values, we can interpret as positive this attempt because it means that, probably, if the system does not became saturated they will arrive to a steady state.

¹Graphic coming from [5]

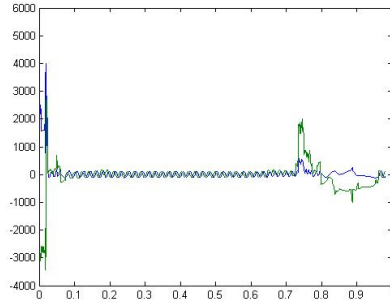


Figure 5.7: Stator voltage in qd reference

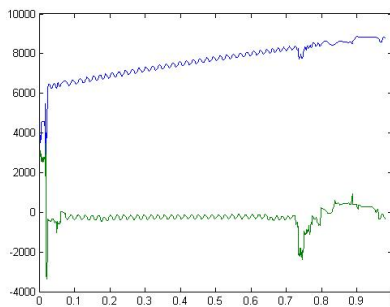


Figure 5.8: Stator current in qd reference

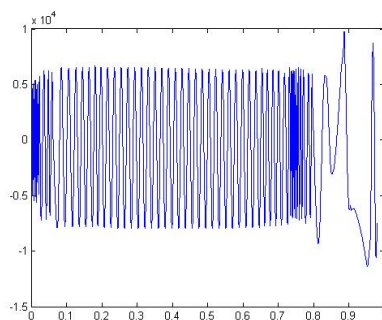


Figure 5.9: Mechanical Torque produced by the Generator

By one hand, in the graphic (5.7) can be seen how the stator voltage, after the transient, become little but never constant. By the other hand, in the graphic (5.8) can be seen the stator current which is always growing up, in principle, following the reference current offered by the control. The oscillating torque occurs with some upper and lower limits (5.9), which means no existing control on the generator.

Then, in the light of these data, can be determined that the stator side converter model is not working as we want. It can occur because of several factors: the current references are not calculated correctly, the transient affect directly to the system which can not reach



the steady state, there are too many simplifications (system without crowbar or filter in the stator side) or the chosen parameters are wrong, guiding the system to other solution (infinite or minus infinite).

Park

This point is included to show (figure 5.10) how the Park Transform acts on sine waves. As can be seen in the graphic, the Park Transform change a 3-dimension time dependent vector to a 2-dimension constant. Moreover, if the angle of definition correct, one of the components could be null.

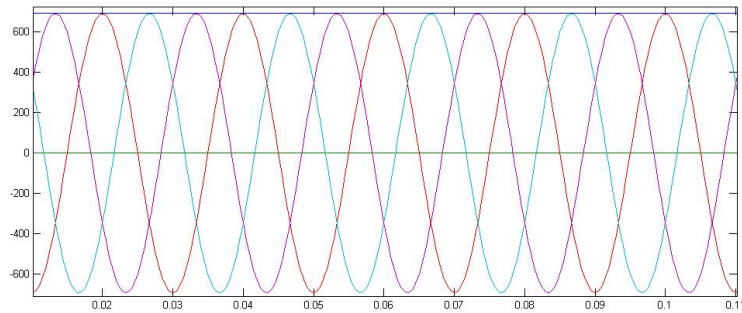


Figure 5.10: Park Transform of the voltage of the grid





Chapter 6

Conclusions

In this project, have been described deeply the equations formulation of the dynamic model of a squirrel cage induction generator connected to the grid by a back to back converter. Have been studied, also, the control strategy to extract the maximum power of the wind, and the control strategy to regulate the voltage and current which will be delivered to the grid in good conditions.

Analyzing the obtained results can be concluded that the model of the SCIG generator, the model of the mechanical system, the model of the DC Link and the model of the grid side converter are correct. But, in the model of the stator side converter there are something wrong. However, the mistake is bounded. Then is recommended make more simulations, and a deep study of the model to fix it.

For further work, are proposed follow different lines of study. Is proposed realize the study of how the system behavior change when are replaced the simplifications by the complete system. Can be also interesting include in the model the grid and the transformer, and analyze how react the grid when we connect a Farm of windmills. Is suggested study the behavior of the model when occurs a sag. Finally, can be necessary realize a prototype of our model.





Acknowledgements

I would like to acknowledge the guidance and suggestions offered by Dr. Keijo Ruotsalainen, who has helped me understand many things during the project. And thank to the University of Oulu who gave me the option of realize this project.

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Finally, I would like to thank to my family and my girlfriend who, always, gave me the strength to go ahead in each moment of my life.





Appendix A

Controllability and Observability of the SCIG Linear System

It is well known that an objective in control theory, is to be able to transfer the movement from one point in the state space to another manipulating to chosen control variables and to know which points in the state space can be reached from a given point, this leads to two concepts *controllability* and *reachability*. Another point of interest is the concept related to determining the state of a system from the knowledge of the input-output data leading to the concept of *observability*. In 1960 The engineer and mathematician R.E. Kalman [15], laid down the foundation of modern control theory. In this appendix we analyze the controllability and observability system properties [16], which allow us to be sure of we are able to control, and know our system. We have the following system:

$$\dot{X} = A \cdot X + B \cdot U \quad (\text{A.1a})$$

$$Y = C \cdot X + D \cdot U \quad (\text{A.1b})$$

Where we assume A, B, C and D like a constant. This assumption allow us define the system as **LTI** (*Linear Time-Invariant*).

In our generator we could get this system including the equation of the SCIG (2.20) and the equation of the flows (1.9) in qd reference. Then our system is:



$$\underbrace{\frac{d}{dt} \begin{Bmatrix} i_{sq} \\ i_{sd} \\ i_{rq} \\ i_{rd} \end{Bmatrix}}_{\dot{X}} = \underbrace{\frac{1}{L_s L_r - M^2} \begin{bmatrix} L_r r_s & M^2 \dot{\theta}_r + (L_s L_r - M^2) \dot{\theta} & -M r_r & M L_r \dot{\theta}_r \\ -M^2 \dot{\theta}_r - (L_s L_r - M^2) \dot{\theta} & L_r r_s & -M L_r \dot{\theta}_r & -M r_r \\ -M r_s & -M L_s \dot{\theta}_r & L_s r_r & (L_s L_r - M^2) \dot{\theta} - L_s L_r \dot{\theta}_r \\ M L_s \dot{\theta}_r & -M r_s & -(L_s L_r - M^2) \dot{\theta} - L_s L_r \dot{\theta}_r & L_s r_r \end{bmatrix}}_A \underbrace{\begin{Bmatrix} i_{sq} \\ i_{sd} \\ i_{rq} \\ i_{rd} \end{Bmatrix}}_X$$

$$+ \underbrace{\frac{1}{L_s L_r - M^2} \begin{bmatrix} L_r & 0 & -M & 0 \\ 0 & L_r & 0 & -M \\ -M & 0 & L_s & 0 \\ 0 & -M & 0 & L_s \end{bmatrix}}_B \underbrace{\begin{Bmatrix} v_{sq} \\ v_{sd} \\ 0 \\ 0 \end{Bmatrix}}_U$$

(A.2a)

$$\underbrace{\begin{Bmatrix} \lambda_{sq} \\ \lambda_{sd} \\ \lambda_{rq} \\ \lambda_{rd} \end{Bmatrix}}_Y = \underbrace{\begin{bmatrix} L_s & 0 & M & 0 \\ 0 & L_s & 0 & M \\ M & 0 & L_r & 0 \\ 0 & M & 0 & L_r \end{bmatrix}}_C \underbrace{\begin{Bmatrix} i_{sq} \\ i_{sd} \\ i_{rq} \\ i_{rd} \end{Bmatrix}}_X \quad \text{1}$$

(A.2b)

A.1 Controllability

The concept of controllability denotes the ability to move a system around in its entire configuration space using only certain admissible manipulations. The system can be defined as controllable if, for all initial times and all initial states, there exists some input function that drives the state vector to any final state at some finite time.

For continuous time invariant linear systems the concept of controllability is equivalent the reachability, as a consequence we can study controllability computing a rank of a certain matrix (reachability matrix) that in this case it is known as *controllability matrix*, which is given by:

$$R = [B \quad A \cdot B \quad A^2 \cdot B \quad \dots \quad A^{n-1} \cdot B] \quad (A.3)$$

If the rank of the matrix is maximum, then the system is controllable.

As can be realized easily, our system is controllable because B has a maximum rank.

$$\underbrace{\frac{1}{L_s L_r - M^2} \begin{bmatrix} L_r & 0 & -M & 0 \\ 0 & L_r & 0 & -M \\ -M & 0 & L_s & 0 \\ 0 & -M & 0 & L_s \end{bmatrix}}_B \quad (A.4)$$

And it is quite easy to calculate its determinant, which is different than zero.

¹D has not included because is a null matrix.



$$\det(B) = |B| = \frac{L_r^2 L_s^2 - 2L_s L_r M^2 + M^4}{L_s L_r - M^2} = L_s L_r - M^2 \neq 0 \quad (\text{A.5})$$

So, as well known, $\det(B) \neq 0 \Rightarrow$ maximum rank of B \Rightarrow maximum rank of R
We are able to say that our system is **controllable**.

A.2 Observability

A system is *observable* if, for any possible sequence of state and control vectors, the current state can be determined in finite time using only the outputs. This means that from the outputs of the system it is possible to determine the behavior of the system. If a system is not observable, the current values of some of its states can not be determined through output sensors.

The notion of observability is dual reachability, that is to say a system $\left. \begin{array}{l} \dot{X} = AX + BU, \\ Y = CX + DU \end{array} \right\}$
is observable if and only if the dual system $\left. \begin{array}{l} \dot{X} = A^t X + C^t U \\ Y = B^t X + D^t U \end{array} \right\}$ is reachable.

Taking into account the equivalence into reachability and observability we can compute the observability by means the rank of a certain matrix called *observability matrix* given by:

$$O = \begin{bmatrix} C \\ C \cdot A \\ C \cdot A^2 \\ \vdots \\ C \cdot A^{n-1} \end{bmatrix} \quad (\text{A.6})$$

If the rank of the matrix is maximum, then the system is observable.

As can be realized easily, our system is observable because C has a maximum rank.

$$\underbrace{\begin{bmatrix} L_s & 0 & M & 0 \\ 0 & L_s & 0 & M \\ M & 0 & L_r & 0 \\ 0 & M & 0 & L_r \end{bmatrix}}_C \quad (\text{A.7})$$

And it is quite easy to calculate its determinant, which is different than zero.

$$\det(C) = |C| = L_r^2 L_s^2 - 2L_s L_r M^2 + M^4 \neq 0 \quad (\text{A.8})$$

So, as well known, $\det(C) \neq 0 \Rightarrow$ maximum rank of C \Rightarrow maximum rank of O
We are able to say that our system is **observable**.

A.3 A lower bound on the distance to a non-controllable system

Given a system $\left. \begin{array}{l} \dot{X} = A \cdot X + B \cdot U, \\ Y = C \cdot X + D \cdot U \end{array} \right\}$ we are interested in to obtain a lower bound for the distance between a controllable system and the nearest non-controllable one, this bound is given in terms of the singular values of a certain matrix associated to the system.

The problem to study controllable systems being those whose behavior does not change when applying small perturbations, in the qualitative theory of dynamical systems has been widely studied by many authors in control theory and under different points of view, (see [17] for example).

Our goal is to obtain a bound for the value of the radius of a ball which is neighborhood of a controllable system, containing only systems which are also controllable.

In our particular setup the matrix B has full rank, so if we perturb a system and the new matrix $B' = B + \varepsilon B$ has full rank, the the perturbed system is also controllable because of

$$\text{rank} \begin{bmatrix} B' & A' \cdot B' & \dots & A'^{n-1} \cdot B' \end{bmatrix} = \text{rank} B' = 4$$

So, if the perturbed system is non controllable necessarily $\text{rank} B' < 4$. Then, now the question is how far is a system such that $\text{rank} B' < 4$.

The distance we will deal with is that deduced from the Frobenius norm. We recall that given a matrix $K = (k_{ij}) \in M_{n \times m}(\mathbb{C})$, its Frobenius norm is defined as $\|K\| = \sqrt{\sum_{ij} k_{ij}^2}$.

The Eckart-Young and Minkowski theorem (see [18] for example), states that the smallest perturbation in the Frobenius norm that reduces the rank of a matrix K with $\text{rank} K = r$ from r to $r - 1$ is $\sigma_r(K)$, the smallest non-zero singular value of K .

Computing the singular values of matrix B we have

$$B^t B = \frac{1}{(L_s L_r - M^2)^2} \begin{bmatrix} L_r^2 + M^2 & 0 & -M(L_r + L_s) & 0 \\ 0 & L_r^2 + M^2 & 0 & -M(L_r + L_s) \\ -M(L_r + L_s) & 0 & M^2 + L_s^2 & 0 \\ 0 & -M(L_r + L_s) & 0 & M^2 + L_s^2 \end{bmatrix}$$

The characteristic polynomial of this matrix is

$$p(t) = \left(t^2 - \frac{L_r^2 + L_s^2 + 2M^2}{(L_s L_r - M^2)^2} t + \frac{1}{(L_r L_s - M^2)^2} \right)^2 = ((t - t_1)(t - t_2))^2$$

$$\text{with } t_1 = \frac{L_r^2 + L_s^2 + 2M^2 + \sqrt{(L_r^2 - L_s^2)^2 + 4M^2(L_r + L_s)^2}}{2(L_s L_r - M^2)^2} \geq t_2 = \frac{L_r^2 + L_s^2 + 2M^2 - \sqrt{(L_r^2 - L_s^2)^2 + 4M^2(L_r + L_s)^2}}{2(L_s L_r - M^2)^2}.$$

Consequently, the smallest non zero singular value is $\sigma_2 = \sqrt{t_2}$ and if $\|\delta B\| < \sigma_2$, the perturbed system is controllable.

In fact we can refine this bound computing the smallest non-zero singular value of the controllability matrix

$$\begin{bmatrix} B & A \cdot B & A^2 \cdot B & A^3 \cdot B \end{bmatrix}$$

getting more margin of safety.

A.4 A lower bound on the distance to a non-observable system

Analogously, given a system $\left. \begin{array}{l} \dot{X} = A \cdot X + B \cdot U, \\ Y = C \cdot X + D \cdot U \end{array} \right\}$ we are interested in to obtain a lower bound for the distance between a observable system and the nearest non-observable one, that



is to say we try to obtain a bound for the value of the radius of a ball which is neighborhood of a observable system, containing only systems which are also observable.

Computing the singular values of matrix B we have

$$C^t C = \begin{bmatrix} L_s^2 + M^2 & 0 & M(L_r + L_s) & 0 \\ 0 & L_s^2 + M^2 & 0 & M(L_r + L_s) \\ M(L_r + L_s) & 0 & M^2 + L_r^2 & 0 \\ 0 & M(L_r + L_s) & 0 & M^2 + L_r^2 \end{bmatrix}$$

The characteristic polynomial of this matrix is

$$p(t) = (t^2 - (L_r^2 + L_s^2 + 2M^2)t + (L_r L_s - M^2)^2)^2 = ((t - t_1)(t - t_2))^2$$

$$\text{with } t_1 = \frac{L_r^2 + L_s^2 + 2M^2 + \sqrt{(L_r^2 - L_s^2)^2 + 4M^2(L_r + L_s)^2}}{2} \geq t_2 = \frac{L_r^2 + L_s^2 + 2M^2 - \sqrt{(L_r^2 - L_s^2)^2 + 4M^2(L_r + L_s)^2}}{2}.$$

Consequently, the smallest non zero singular value is $\sigma_2 = \sqrt{t_2}$ and if $\|\delta B\| < \sigma_2$, the perturbed system is controllable.

In fact we can refine this bound computing the smallest non-zero singular value of the controllability matrix

$$\begin{bmatrix} C \\ C \cdot A \\ C \cdot A^2 \\ C \cdot A^3 \end{bmatrix}$$

Obviously, taking the minimum between the controllability bound and the observability bound we obtain a bound for both characters the controllability and observability.





Appendix B

Space Vector Pulse Width Modulation (SVPWM)

An three-phase inverter fed with voltage, represented in the figure (B.1), is composed by DC Bus, also called (*DC Link*), with 2 condensers of high capacity and 3 switch pairs ¹ which are linked to each phase of the grid between each pair. The inverter use a binary reference, which describe in what position are the switches.

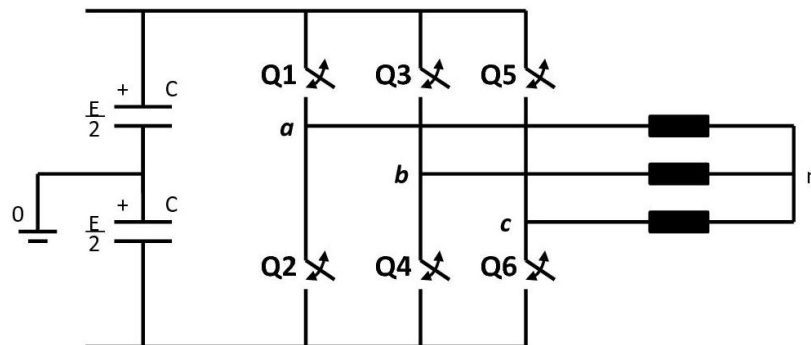


Figure B.1: Electrical scheme of an inverter

As is shown in the figure (B.1) the inverter has 6 switches, which can be only in two different positions (as we told before binary reference): blocking state or going-on state. Otherwise, both switches of the same branch had 2 freedom degrees as they have to be in opposite states. Then, the inverter are able to generate 8 (2^3 , 3 different branches which have 2 freedom degrees) different three-phasic voltages in its output.

If the 8 different voltages vectors are represented in a steady Park reference with $\theta=0$, we obtain 6 vectors which are separated 60° between each other, and two vectors with null longitude. So, six of these vectors represent the vertices of an hexagon and the other two just the center, as it is shown in the figure (B.2) and described in the table (B).

¹The switches are, in fact, IGBT with a protection diode



State	IGBT activated	Vector	v_{an}	v_{bn}	v_{cn}	v_{qn}	v_{dn}	v_{0n}
0	$Q_4Q_6Q_2$	(0 0 0)	0	0	0	0	0	$\frac{-\sqrt{2}}{2} E$
1	$Q_1Q_6Q_2$	(1 0 0)	$\frac{2}{3} E$	$-\frac{1}{3} E$	$-\frac{1}{3} E$	$\frac{2}{3} E$	0	$\frac{-\sqrt{2}}{6} E$
2	$Q_1Q_3Q_2$	(1 1 0)	$\frac{1}{3} E$	$\frac{1}{3} E$	$-\frac{2}{3} E$	$\frac{1}{3} E$	$-\frac{\sqrt{3}}{3} E$	$\frac{\sqrt{2}}{6} E$
3	$Q_4Q_3Q_2$	(0 1 0)	$-\frac{1}{3} E$	$\frac{2}{3} E$	$-\frac{1}{3} E$	$-\frac{1}{3} E$	$-\frac{\sqrt{3}}{3} E$	$\frac{-\sqrt{2}}{6} E$
4	$Q_4Q_3Q_5$	(0 1 1)	$-\frac{2}{3} E$	$\frac{1}{3} E$	$\frac{1}{3} E$	$-\frac{2}{3} E$	0	$\frac{\sqrt{2}}{6} E$
5	$Q_4Q_6Q_5$	(0 0 1)	$-\frac{1}{3} E$	$-\frac{1}{3} E$	$\frac{2}{3} E$	$-\frac{1}{3} E$	$\frac{\sqrt{3}}{3} E$	$\frac{-\sqrt{2}}{6} E$
6	$Q_1Q_6Q_5$	(1 0 1)	$\frac{1}{3} E$	$-\frac{2}{3} E$	$\frac{1}{3} E$	$\frac{1}{3} E$	$\frac{\sqrt{3}}{3} E$	$\frac{\sqrt{2}}{6} E$
7	$Q_1Q_3Q_5$	(1 1 1)	0	0	0	0	0	$\frac{\sqrt{2}}{2} E$

Table B.1: Table of commutation states of the inverter

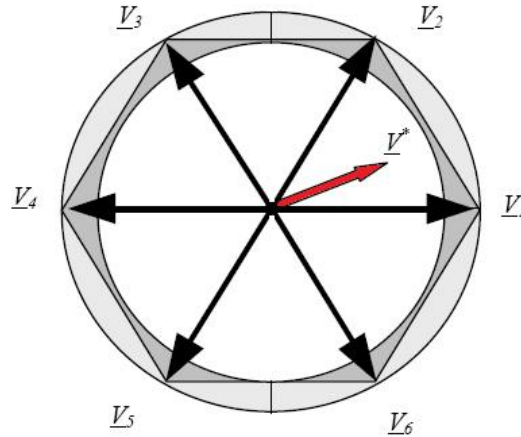


Figure B.2: Commutation hexagon

The SVPWM method to generate any three-phase voltage, consists of alternating different commutation configurations of the switches which corresponding to the two vertices of the hexagon where there is the desired voltage and one of the zero states in time periods calculated so that resultant voltage signal has a continuous component equal to the desired voltage and some high frequency components which are easy to filter.

Known a desired voltage vector v_{qd}^* , the periods $[t_i, t_{i+1}, t_0]$ in per unit, when the states should be activated, are as follow:

$$\begin{Bmatrix} t_i \\ t_{i+1} \end{Bmatrix} = \frac{1}{v_{iq}v_{(i+1)d} - v_{id}v_{(i+1)q}} \begin{bmatrix} v_{(i+1)d} & -v_{(i+1)q} \\ -v_{id} & v_{iq} \end{bmatrix} \begin{Bmatrix} v_q^* \\ v_d^* \end{Bmatrix} \quad (\text{B.1a})$$

$$t_0 = 1 - (t_i + t_{i+1}) \quad (\text{B.1b})$$



Appendix C

Introduction to current loops

This appendix is a brief introduction to few techniques which allow the control by current of a voltage inverter. [19]

The most common techniques are:

- **Hysteresis control**, also called *Bang Bang control*. It was the most useful in analogical control time, as it had a fast control (high bandwidth). In contrast, it presents a commutation frequency variable, and large current ripple.
- **PI Regulator in stationary reference**. The main idea of this regulator was to implement a PI loop for each current of the triphasic inverter. But in this way, working independent each regulator, the system is not able to reach the voltage zero state, and the error in one current have influence in the other currents. So, its behavior is not optimal.
- **PI Regulator in synchronism reference**. This controller is based in two PI regulators in synchronism reference (moreover, the reference is oriented by the maximum of the magnetic field of the rotor. The problems of this control are that it depends hardly on the estimation of the parameters of the machine, and is not able to control a over-modulation.
- **Predictive regulators**. It is based on the analysis of the trajectories that will be followed by current vector.
- **IMC method**. Our selection





Appendix D

Definition of magnetizing current

In the study of induction machine in steady state is customary to define a new current variable called *magnetizing current* \underline{i}_m , to be a representation of the magnetic field in the gap.

Otherwise, the target of the bigger part of the control strategies is make, independently, the control of the torque and the control of the induction machine, though in the equations with the stator and rotor current variables the flux does not appear as a explicit variable (but it is the sum of the flux generated by stator and the flux generated by rotor).

The flux can be defined in three different ways [20].

- Magnetic field seen from stator: $L_s \underline{i}_m = L_s \underline{i}_s + M \underline{i}_r$
- Magnetic field seen from rotor: $M \underline{i}_m = M \underline{i}_s + L_r \underline{i}_r$
- Magnetic field in the gap: $M \underline{i}_m = M \underline{i}_s + M \underline{i}_r$

To apply this change of variables there are a change of variable generic matrix, which will be function of a and b . This matrix is defined as follow:

$$\begin{Bmatrix} i_{sd} \\ i_{sq} \\ i_{md} \\ i_{mq} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ a & 0 & b & 0 \\ 0 & a & 0 & b \end{bmatrix} \begin{Bmatrix} i_{sd} \\ i_{sq} \\ i_{rd} \\ i_{rq} \end{Bmatrix} \Rightarrow \begin{Bmatrix} i_{sd} \\ i_{sq} \\ i_{rd} \\ i_{rq} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{-a}{b} & 0 & \frac{1}{b} & 0 \\ 0 & \frac{-a}{b} & 0 & \frac{1}{b} \end{bmatrix} \begin{Bmatrix} i_{sd} \\ i_{sq} \\ i_{md} \\ i_{mq} \end{Bmatrix} \quad (\text{D.1})$$

Depending on which definition of magnetizing current we want to use, we a and b will get the following values:

- Magnetic field seen from stator: $a = 1$ and $b = \frac{M}{L_s}$
- Magnetic field seen from rotor: $a = 1$ and $b = \frac{L_r}{M}$

- Magnetic field in the gap: $a = 1$ and $b = 1$

Finally, applying this change of variables in our system, choosing magnetic field seen from rotor, we obtain:

$$\begin{aligned} \begin{Bmatrix} v_{sd} \\ v_{sq} \\ 0 \\ 0 \end{Bmatrix} &= \begin{bmatrix} L_s - \frac{M^2}{L_r} & 0 & \frac{M^2}{L_r} & 0 \\ 0 & L_s - \frac{M^2}{L_r} & 0 & \frac{M^2}{L_r} \\ 0 & 0 & M & 0 \\ 0 & 0 & 0 & M \end{bmatrix} \frac{d}{dt} \begin{Bmatrix} i_{sd} \\ i_{sq} \\ i_{md} \\ i_{mq} \end{Bmatrix} \\ &+ \begin{bmatrix} r_s & -\left(L_s - \frac{M^2}{L_r}\right)\dot{\theta} & 0 & -\frac{M^2}{L_r}\dot{\theta} \\ \left(L_s - \frac{M^2}{L_r}\right)\dot{\theta} & r_s & \frac{M^2}{L_r}\dot{\theta} & 0 \\ -\left(\frac{r_r}{L_r}M\right) & 0 & r_r\frac{M}{L_r} & -M\dot{\theta}_r \\ 0 & -\left(\frac{r_r}{L_r}M\right) & -M\dot{\theta}_r & r_r\frac{M}{L_r} \end{bmatrix} \begin{Bmatrix} i_{sd} \\ i_{sq} \\ i_{md} \\ i_{mq} \end{Bmatrix} \end{aligned} \quad (\text{D.2a})$$

$$\Gamma = \frac{M^2}{L_r} \cdot i_{md} \cdot i_{sq} \quad (\text{D.2b})$$



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