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Títol

Vehicle Trajectory Estimation based on Newell's Simplified Kinematic Wave Model with Heterogeneous Data

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Abstract

Trajectory estimation provides valuable information for travelers and decision makers, and can lead to an improvement in the performance of the global traffic system that maximizes the benefit of users and society. This paper presents a new and effective method to estimate individual vehicle trajectories from Newell's simplified kinematic wave model based on heterogeneous data sources. The method is initially developed and analyzed considering First In First Out (FIFO). Then, an important step forward is taken by introducing FIFO violation in Newell's model, greatly outperforming the results obtained in the initial approach. Different from existing studies, the proposed method finds a new way to obtain vehicle trajectories from Newell's model, improving the previous attempts and representing a major breakthrough with the introduction of FIFO violation. Finally, the estimation method is verified using NGSIM data and the results support its consistency.

Key Words--- Trajectory Estimation, Newell's Simplified Kinematic Wave Model, FIFO violation, Vehicle Reidentification, Heterogeneous Data, NGSIM data

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1 Introduction

Real time traffic control is essential to improve the performance of the global traffic system and to maximize the benefit of users and society. The better the information about current traffic state is, the better decisions and strategies can be made. Estimating trajectories is one of the best ways to have a complete picture of the traffic state in a road network, since it provides valuable information to deduce other traffic variables. Besides, having vehicle trajectories over a freeway link is useful to develop other important applications, such as estimating emissions, computing travel times or quantifying congestion, among others.

There are many existing studies about estimating vehicle trajectories. The methods used for this purpose vary from processing the images recorded by video cameras to more complex systems merging global-positioning-system (GPS) data with Kalman Filtering and geographic-information-system (GIS) data (Barrios and Motai, 2011). In the middle, several different approaches can be found, such as computer vision algorithms using monocular vision (Ponsa and Lopez, 2007) or other estimation strategies with GPS or radar devices. Apart from these, there are also some studies that deal with the problem from the traffic flow theory perspective. For example, the study in Coifman (2002) presented an interesting new method to solve the problem of estimating vehicle trajectories. The idea was to develop an estimation of link travel times and vehicle trajectories based on dual loop detectors, without requiring any new hardware.

However, in spite of all the progress made in trajectory estimation, there are still some limitations. First, most of the above mentioned methods are not enough accurate to be considered for future applications, while others use expensive technologies that cannot be implemented everywhere. Furthermore, many traffic flow models that could be used to estimate trajectories include an important underlying assumption that makes the results inaccurate, which is the First In First Out (FIFO) assumption. This is the case of the cell transmission model (CTM) proposed in Daganzo (1994), for example. Some discussions about the importance of the FIFO hypothesis and its influence on traffic models are included in (Jin et al., 2006) and (Jin and Li, 2007).

The aim of the present study is to develop a simple and efficient method that can be easily applied to obtain individual vehicle trajectories, dealing at the same time with the violation of the FIFO hypothesis. This method is based on Newell's simplified kinematic wave model (Newell, 1993a,b,c), which does not include the FIFO assumption among its foundations. Different from previous studies, the method presented in this paper stands on one of the simplest and most recognized traffic flow models and proposes a major breakthrough by introducing FIFO violation. In particular, we start assuming FIFO and deduce a naive but consistent algorithm to predict trajectories. Then, we go one step further, including the FIFO violation approach and deriving the correspondent estimation method. This is the most important result presented in the study and it is the point that makes the great difference with other studies, since it is a simple method that allows us to obtain more realistic results.

Once the estimation method is fully developed, we will test it using the Next Generation

Simulation (NGSIM) data (USDOT, 2008). In this stage, we need to calibrate the model's parameters, such as free flow speed, shock wave speed and jammed density, as well as the initial state (i.e. initial number of vehicles inside the segment). In most studies, these parameters are determined by fitting the fundamental diagram based on "point measurements" within a road segment, and the initial conditions are often ignored. However, in this paper we are going to use the method described in (Sun et al., 2015), where traffic parameters and states are simultaneously estimated with respect to the road segment other than a single point.

Regarding the heterogeneous data used to make the estimation, we focus on two specific data types, Lagrangian data from the vehicle reidentification sensors and Eulerian data from flow counting sensors. The vehicle reidentification system refers to a set of technologies and algorithms whereby a vehicle detected at an upstream location is matched with the same vehicle at a downstream location (Sun et al., 1999). The vehicle reidentification system can be implemented based upon different sensing technologies, such as video camera, AVI (automatic vehicle identification) tags, and loop detectors (Jeng, 2007). Various applications have been developed based on the vehicle reidentification, including travel time estimation (Coifman and Cassidy, 2002), performance evaluation (Jeng, 2007; Oh et al., 2005), and OD estimation (Oh et al., 2002). Besides, we consider a homogeneous link, since a general traffic network can be modeled as combinations of individual links with a single entry and exit. When both ends of the link are monitored by static and vehicle reidentification sensors, the traffic volume and entry/exit time of a proportion of vehicles can be collected.

The paper is organized as follows: in section 2, we review Newell's simplified kinematic wave model and the parameters estimation method described in (Sun et al., 2015). In section 3, we assume FIFO and derive the trajectory estimation method based on the Newell's simplified kinematic wave model, presenting an implementation algorithm to obtain the results. Section 4 introduces FIFO violation and explains a new approach of the previous estimation method to obtain more accurate trajectories. In section 5, we use the NGSIM data to verify our methods and give a measurement of the error in both cases. We conclude the whole study in section 6.

2 Review of Newell's model

The trajectory estimation method developed in the present study is based on one of the most recognized traffic models, the Newell's simplified kinematic wave model. This model is a simplification under special conditions of the Lighthill-Whitham-Richard (LWR) kinematic wave model (Lighthill and Whitham, 1955), which uses a partial differential equation to describe the evolution of traffic based on density, flow rate and speed

$$\frac{\partial k(x,t)}{\partial t} + \frac{\partial Q(x,t,k)}{\partial x} = 0. \tag{1}$$

In Newell's study (Newell, 1993a), he pointed out that the kinematic wave model could be streamlined with a triangular fundamental diagram using cumulative flow $n(x, t)$ as a state variable, leading to a simpler formulation of the problem. This formulation will be introduced after listing some useful notation in Table 1

Table 1: Table of Notations

$F(t)$	The **observed** cumulative count at the upstream from 0 to t
$G(t)$	The **observed** cumulative count at the downstream from 0 to t
n_0	The initial number of vehicles within the segment
$n(x, t)$	The cumulative count at location x from 0 to t
V	Free-flow speed
W	Shock-wave speed
K	Jammed density
$X_i(t)$	Location of vehicle i at time t
Δt	Time step size
l	Length of the road segment
r_i	Entry time of vehicle i
s_i	Exit time of vehicle i

Let's consider an homogeneous road segment of length l from $x = 0$ to $x = l$. The traffic conditions on the road segment can be perfectly described using the cumulative flows, $n(x, t)$. The flow rate is

$$q(x, t) = \frac{\partial n(x, t)}{\partial t}, \tag{2}$$

and the density is

$$k(x, t) = -\frac{\partial n(x, t)}{\partial x}. \tag{3}$$

According to the definition of $G(t)$ in Table 1, we have

$$G(t) = n(l, t). \tag{4}$$

The observed cumulative count at upstream, $F(t)$, is different from $n(0, t)$ due to the presence of an initial number of vehicles within the segment. The following adjustment is needed:

$$F(t) + n_0 = n(0, t), \tag{5}$$

where $F(0) = G(0) = 0$. The upstream detector does not count the vehicles entered the segment before $t = 0$. For this reason, the initial number of vehicles, n_0, is not available from $F(\cdot)$.

In Newell's simplified kinematic wave model (Newell, 1993a), we have

$$n(x,t) = \min\left\{F(t - \frac{x}{V}) + n_0, G(t - \frac{l-x}{W}) + K(l-x)\right\} \tag{6}$$

This equation means that the cumulative flow is either determined by the upstream conditions (first term on the equation) or the downstream conditions (second term on the equation). From the information propagation point of view, the traffic state within a homogeneous link is determined by upstream and downstream combined. According to (Daganzo, 2005), Newell's model is the variational formulation of the LWR model with a triangular fundamental diagram (Munjal et al., 1971; Daganzo, 1977; Newell, 1993a): $q = \phi(k) \equiv \min\{Vk, W(K-k)\}$. This diagram is represented in figure 1. In the original Newell's model, the road is assumed to be initially empty. However, from the Hopf-Lax formula of the Hamilton-Jacobi equation for the LWR model, $n_t - \phi(-n_x) = 0$, the application domain of Newell's model can be extended for any initial conditions without a transonic rarefaction wave (Evans, 2010).

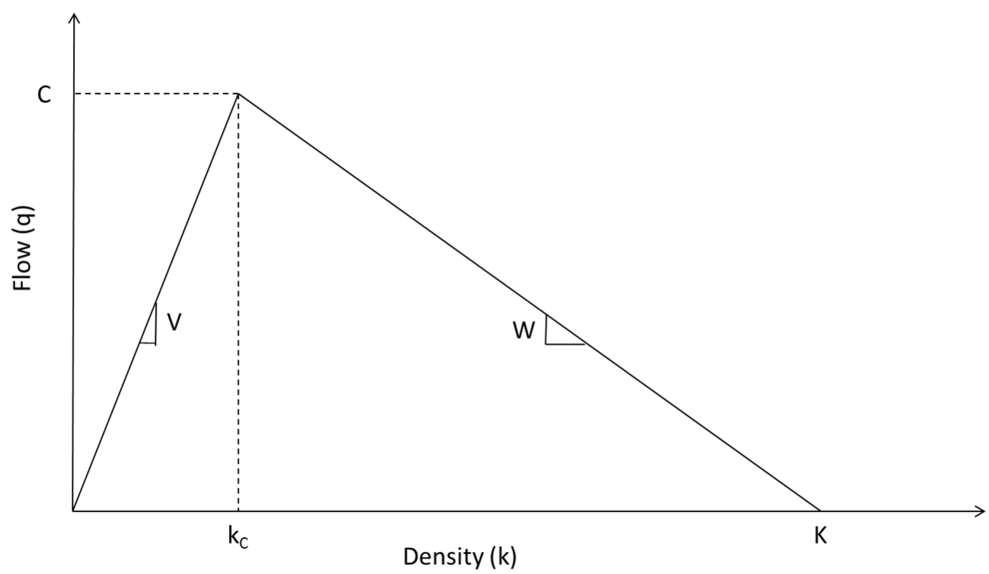


Figure 1: Triangular fundamental diagram Flow-Density

3 An estimation method without overtaking

3.1 Motivation

The partial trajectories (i.e. entry/exit time) of a number of vehicles can be obtained through vehicle reidentification. That is, for vehicle i, we have $X_i(r_i) = 0$ and $X_i(s_i) = l$ at some discrete time points. Assuming First In First Out (FIFO), the trajectory of the i-th re-identified vehicle establishes a contour line of the function $n(x, t)$. Figure 2 gives us a visual idea of this cumulative flow function and its contour lines.

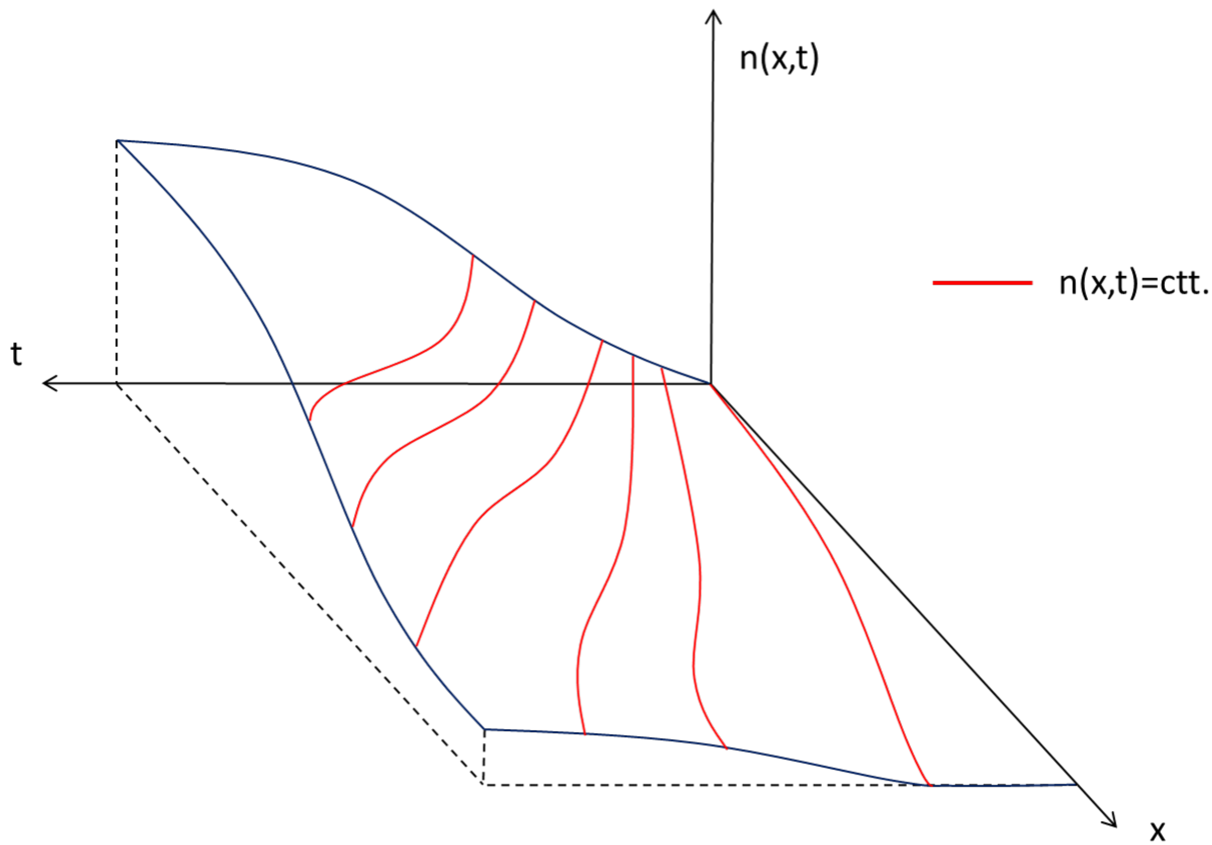


Figure 2: 3D-plot of Newell's cumulative flow function

The FIFO assumption can be expressed as

$$F(r_i) + n_0 = n(0, r_i) = n(l, s_i) = G(s_i) = i. \tag{7}$$

Thus, if we can find all the pairs (x, t) that correspond to the mentioned contour line we will have the full trajectory of vehicle i. To do it we need to solve equation

$$i = n(x, t) = \min \left\{ F(t - \frac{x}{V}) + n_0, G(t - \frac{l - x}{W}) + K(l - x) \right\}. \tag{8}$$

3.2 Method

To solve the above mentioned problem, we will start by formulating an important result regarding the behaviour of the previous equation. Let's first introduce the notation $N_1(x,t)$ to refer to the uncongested part of the Newell's equation and $N_2(x,t)$ to refer to the congested part. We have

$$N_1(x,t) = F(t - \frac{x}{V}) + n_0, \tag{9}$$

and

$$N_2(x,t) = G(t - \frac{l-x}{W}) + K(l-x). \tag{10}$$

Then, we can state the following theorem.

Theorem 3.1. *Fixed $t = T$, the location of the vehicle i in the road segment, $X_i(T;V,W,K)$, is given by*

$$X_i(T;V,W,K) = \min\left\{X_i^1(T;V,W,K), X_i^2(T;V,W,K)\right\}, \tag{11}$$

where $X_i^1 = X_i^1(T;V,W,K)$ satisfies

$$N_1(X_i^1,T) = F(T - \frac{X_i^1}{V}) + n_0 = i \tag{12}$$

and $X_i^2 = X_i^2(T;V,W,K)$ satisfies

$$N_2(X_i^2,T) = G(T - \frac{l-X_i^2}{W}) + K(l - X_i^2) = i. \tag{13}$$

Proof. If we take partial derivatives of the functions $N_1(x,t)$ and $N_2(x,t)$ with respect to x we obtain

$$\frac{\partial N_1(x,t)}{\partial x} = F'(t - \frac{x}{V}) \cdot (-\frac{1}{V}), \tag{14}$$

$$\frac{\partial N_2(x,t)}{\partial x} = G'(t - \frac{l-x}{W}) \cdot (\frac{1}{W}) - K. \tag{15}$$

Using the triangular fundamental diagram $q = \phi(k) \equiv \min\{Vk, W(K-k)\}$, plotted in figure 1, we denote by C the capacity or maximum flow and by k_C the critical density, which corresponds to the capacity flow. Then, it is true that

$$C = Vk_C = W(K - k_C). \tag{16}$$

Besides, the terms $F'(t)$ and $G'(t)$ correspond to the road's entry and exit flows, respectively, so they have to satisfy

$$0 \leq \min\{F'(t), G'(t)\} \leq \max\{F'(t), G'(t)\} \leq C. \tag{17}$$

Using the above relations we can write

$$\frac{\partial N_1(x,t)}{\partial x} = F'(t - \frac{x}{V}) \cdot (-\frac{1}{V}) \geq -\frac{C}{V} = -k_C \tag{18}$$

and

$$\frac{\partial N_2(x,t)}{\partial x} = G'(t - \frac{l-x}{W}) \cdot (\frac{1}{W}) - K \leq \frac{C}{W} - K = K - k_C - K = -k_C. \tag{19}$$

So not only we know that both functions are decreasing with x, but we also have a relation between the decreasing ratios of the two functions

$$-K \leq \frac{\partial N_2(x,t)}{\partial x} \leq -k_C \leq \frac{\partial N_1(x,t)}{\partial x} \leq 0. \tag{20}$$

Besides, at the starting and end points $x = 0$ and $x = l$ we know by definition that

$$N_1(0,t) = F(t)+n_0 = n(0,t) = \min\left\{F(t) + n_0, G(t - \frac{l}{W}) + Kl\right\} \leq G(t-\frac{l}{W})+Kl = N_2(0,t) \tag{21}$$

and

$$N_2(l,t) = G(t) = n(l,t) = \min\left\{F(t - \frac{l}{V}) + n_0, G(t)\right\} \leq F(t - \frac{l}{V}) + n_0 = N_1(l,t). \tag{22}$$

From now onwards, let's assume a fixed time $t = T$. Considering the results given by (20), (21) and (22), and taking into account that both $N_1(x,T)$ and $N_2(x,T)$ are continous functions, we can state that

$$\exists [a,b] \subseteq [0,l] \mid \{\forall c \in [a,b], N_1(c,T) = N_2(c,T)\}. \tag{23}$$

Note that in general we will have $a = b$ and only one intersection point, but it could be possible that $N_1(x,T)$ and $N_2(x,T)$ had a whole interval of common points, since their decreasing ratios can be equal in some cases (see equation (20)).

With all the above information about $N_1(x,T)$ and $N_2(x,T)$, we can make an estimated plot of these space-dependent functions to have a better idea of their behaviour and properties. The result would be something similar to figure 3.

Furthermore, based on Newell's model we know that the solution for the cumulative flow is given by the lower envelope of the functions $N_1(x,T)$ and $N_2(x,T)$ plotted in figure 3. I.e:

$$n(x,T) = \min\{N_1(x,T), N_2(x,T)\}. \tag{24}$$

Now, if we want to find the location of vehicle i in the road segment at time $t = T$, $X_i(T;V,W,K)$ (or X_i to simplify the notation), we have to consider 3 different escenarios:

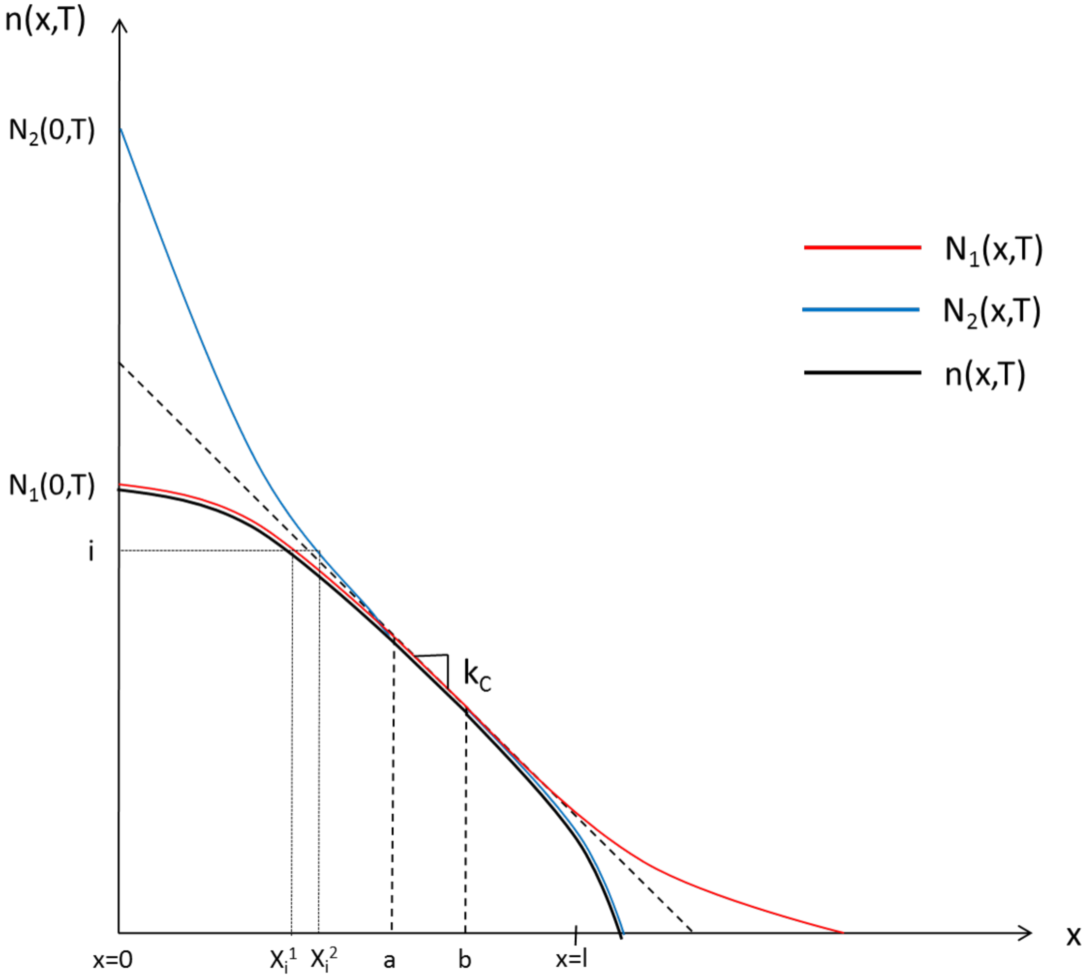


Figure 3: Representation of the congested and uncongested parts of Newell's equation for a fixed t

- **1st case: $i > n(a,T)$, or what is the same $0 \le x < a$**
  If this is true, then $N_1(x,T) < N_2(x,T)$, so we have

$$n(x,T) = \min\{N_1(x,T), N_2(x,T)\} = N_1(x,T), \tag{25}$$

  and the location of vehicle i is given by $X_i = X_i^1$, where X_i^1 satisfies

$$N_1(X_i^1,T) = F(T - \frac{X_i^1}{V}) + n_0 = i. \tag{26}$$

  Besides, $N_2(X_i^1,T) > N_1(X_i^1,T) = i$, so if we take X_i^2 satisfying $N_2(X_i^2,T) = i$ then the decreasing property of the $N_2(x,T)$ function let's us state that

$$X_i^1 < X_i^2, \tag{27}$$

  and we get to the theorem result

$$X_i = X_i^1 = \min\left\{X_i^1, X_i^2\right\}. \tag{28}$$

- **2nd case: $i < n(b,T)$, or what is the same $b < x \le l$**
  Using a similar reasoning, now we have $N_2(x,T) < N_1(x,T)$, so

$$n(x,T) = \min\{N_1(x,T), N_2(x,T)\} = N_2(x,T), \tag{29}$$

  and the location of vehicle i is given by $X_i = X_i^2$, where X_i^2 satisfies

$$N_2(X_i^2,T) = G(T - \frac{l - X_i^2}{W}) + K(l - X_i^2) = i. \tag{30}$$

  Besides, $N_1(X_i^2,T) > N_2(X_i^2,T) = i$, so if we take X_i^1 satisfying $N_1(X_i^1,T) = i$ then the decreasing property of the $N_1(x,T)$ function let's us state that

$$X_i^2 < X_i^1, \tag{31}$$

  and we get to the theorem result

$$X_i = X_i^2 = \min\left\{X_i^1, X_i^2\right\}. \tag{32}$$

- **3rd case:** $n(a,T) \leq i \leq n(b,T)$**, or what is the same** $a \leq x \leq b$
  This is a trivial situation, since $N_1(x,T) = N_2(x,T)$ and then the location of the vehicle i is given by either X_i^1 or X_i^2, so we have

$$X_i = X_i^1 = X_i^2 = \min \left\{ X_i^1, X_i^2 \right\}. \tag{33}$$

Therefore, we have shown that the location of vehicle i at time $t = T$ is in any escenario given by

$$X_i(T; V, W, K) = \min \left\{ X_i^1(T; V, W, K), X_i^2(T; V, W, K) \right\}, \tag{34}$$

and hence the theorem is proved.

■

Using this result, we can now implement an efficient method to estimate the individual trajectories of vehicles crossing a given road segment. Basically, we can find as many pairs (x,t) in a trajectory as we desire just by fixing values for t and applying the theorem to determine the location x.

In the next section we go deeper into the implementation of this method.

3.3 Implementation

We have already established the foundations needed in order to solve the problem of finding the trajectory of a single vehicle i. Having the cumulative functions $F(t)$ and $G(t)$, together with the values of n_0, V, W and K, the steps to take are the following:

- Split the time interval into time steps of size $\Delta t = 0.1 sec$. For each time point $t = T$:
- Compute the location of the vehicle i in the uncongested case:

$$X_i^1(T; V, W, K) = [T - F^{-1}(i - n_0)] \cdot V. \tag{35}$$

- Compute the location of the vehicle i in the congested case. In this case, we need to use a bisection method, since we have an implicit equation given by the function

$$H(x) = G(t - \frac{l - x}{W}) + K(l - x) - i. \tag{36}$$

  Then, we need to find $X_i^2 = X_i^2(T; V, W, K)$ so that

$$H(X_i^2) = G(T - \frac{l - X_i^2}{W}) + K(l - X_i^2) - i = 0. \tag{37}$$

- Take the minimum of both space values to obtain the actual location

$$X_i(T; V, W, K) = \min \left\{ X_i^1(T; V, W, K), X_i^2(T; V, W, K) \right\}. \tag{38}$$

4 An estimation method with overtaking

4.1 Motivation

One of the most determining assumptions made in the previous approach to estimate a vehicle's trajectory was the FIFO condition. Recall that given the entry and exit times (r_i and s_i respectively) of the i-th re-identified vehicle crossing a fixed road segment, the FIFO condition was given by (7).

Based on that hypothesis, we were able to find the trajectories of different vehicles by obtaining the contour lines of the function $n(x,t)$. Hence, we reduced our problem to solving for all pairs (x,t) the equation in (8).

However, accepting the FIFO condition obviously introduces a significant source of error in the results obtained. With that adoption, we are considering that there are no overtakings between vehicles, a situation that is far from being realistic in most traffic contexts. Later in this paper we will have empirical data supporting this statement and showing how big the error is.

For this reason, in the next section we are going to introduce a new method to estimate the trajectories of single vehicles without considering the FIFO condition.

4.2 Method

From now on, we stop assuming FIFO. As we know the entry and exit times for all the vehicles through re-identification, we can easily know what are the entry and exit positions for a given vehicle within the whole group of vehicles. With this information, we also know the number of overtakings that each vehicle has carried out. This is going to be our starting point.

Let's focus on the vehicle entering the road segment in the i-th place (vehicle i). Let M_i be the number of overtakings that vehicle i has suffered. In particular, $M_i > 0$ means that the vehicle has been passed by other drivers, while $M_i < 0$ represents that the vehicle in question has advanced other vehicles in the road section. Now, the position of vehicle i at the exit point of the segment will be $G(s_i) = i + M_i$.

As we can see, the expression of the FIFO condition we had in previous sections is now different:

$$i = F(r_i) + n_0 = n(0, r_i) \neq n(l, s_i) = G(s_i) = i + M_i. \tag{39}$$

Let's now make a reasonable assumption, which is that the number of overtakings M_i happen equally distributed in the time interval of vehicle i in the road segment, $[r_i, s_i]$. Hence, we can introduce the following definition.

Definition 4.1. *The* ***overtaking function*** *of vehicle i is a time-dependent function $\theta_i(t)$ that makes a linear approximation of the place of the vehicle in the segment. I.e:*

$$\theta_i(t) = a_i t + b_i, \tag{40}$$

with initial conditions

$$\theta_i(r_i) = i \tag{41}$$

and

$$\theta_i(s_i) = i + M_i. \tag{42}$$

Remark 1. *The real overtaking function should be a stepwise function, but we construct it using a linear interpolation to avoid having discontinuities in the subsequent operations.*

Remark 2. *Taking into account the initial conditions, we can rewrite the function $\theta_i(t)$ using just one parameter a_i. The result would be*

$$\theta_i(t) = a_i(t - r_i) + i. \tag{43}$$

Remark 3. *The parameter a_i contains information about the vehicle's performance on road and can be useful for other microscopic studies regarding individual driver's behavior. It can be calculated as*

$$a_i = \frac{M_i}{s_i - r_i}. \tag{44}$$

As we can see, it represents a measure of the number of overtakings suffered by vehicle i each time unit. So the greater a_i, the lower is the aggressiveness of the driver, and vice versa.

Finally, in this new context the problem of estimating the trajectory of vehicle i is equivalent to find all pairs (x, t) with $t \in [r_i, s_i]$ satisfying the equation

$$\theta_i(t) = n(x, t) = \min\left\{F(t - \frac{x}{V}) + n_0, G(t - \frac{l - x}{W}) + K(l - x)\right\}. \tag{45}$$

To do so, we will first state a similar result as the one given by theorem (3.1) and then apply the same estrategy than in the FIFO case, fixing values for t and finding the correspondent x values.

Theorem 4.2. *Fixed $t = T$, the location of the vehicle i in the road segment, $X_i(T; V, W, K)$, is given by*

$$X_i(T; V, W, K) = \min\left\{X_i^1(T; V, W, K), X_i^2(T; V, W, K)\right\}, \tag{46}$$

where $X_i^1 = X_i^1(T; V, W, K)$ satisfies

$$N_1(X_i^1, T) = F(T - \frac{X_i^1}{V}) + n_0 = \theta_i(T) \tag{47}$$

and $X_i^2 = X_i^2(T; V, W, K)$ satisfies

$$N_2(X_i^2, T) = G(T - \frac{l - X_i^2}{W}) + K(l - X_i^2) = \theta_i(T). \tag{48}$$

Proof. The first part of the proof is exactly the same as in theorem (3.1), since it is only an study of the functions $N_1(x,t)$ and $N_2(x,t)$, not involving the positioning of the vehicle i. Therefore, we will take for granted what the behaviour of these functions is.

Let's now try to find the location of vehicle i in the road segment at time $t = T$, $X_i(T; V, W, K)$ (or simply X_i). Recall that with "vehicle i" we are referring to the vehicle that enters the segment in the i-th place, and that the overtaking function $\theta_i(t)$ gives us the actual location of this vehicle at time t. Hence, if we want to find the location at time $t = T$ we have to consider 3 different escenarios:

- **1st case:** $\theta_i(T) > n(a,T)$**, or what is the same** $0 \leq x < a$
  If this is true, then $N_1(x,T) < N_2(x,T)$, so we have

$$n(x,T) = \min\{N_1(x,T), N_2(x,T)\} = N_1(x,T), \tag{49}$$

  and the location of vehicle i is given by $X_i = X_i^1$, where X_i^1 satisfies

$$N_1(X_i^1, T) = F(T - \frac{X_i^1}{V}) + n_0 = \theta_i(T). \tag{50}$$

  Besides, $N_2(X_i^1, T) > N_1(X_i^1, T) = \theta_i(T)$, so if we take X_i^2 satisfying $N_2(X_i^2, T) = \theta_i(T)$ then the decreasing property of the $N_2(x,T)$ function let's us state that

$$X_i^1 < X_i^2, \tag{51}$$

  and we get to the theorem result

$$X_i = X_i^1 = \min\left\{X_i^1, X_i^2\right\}. \tag{52}$$

- **2nd case:** $\theta_i(T) < n(b,T)$**, or what is the same** $b < x \leq l$
  Using a similar reasoning, now we have $N_2(x,T) < N_1(x,T)$, so

$$n(x,T) = \min\{N_1(x,T), N_2(x,T)\} = N_2(x,T), \tag{53}$$

  and the location of vehicle i is given by $X_i = X_i^2$, where X_i^2 satisfies

$$N_2(X_i^2, T) = G(T - \frac{l - X_i^2}{W}) + K(l - X_i^2) = \theta_i(T). \tag{54}$$

  Besides, $N_1(X_i^2, T) > N_2(X_i^2, T) = \theta_i(T)$, so if we take X_i^1 satisfying $N_1(X_i^1, T) = \theta_i(T)$ then the decreasing property of the $N_1(x,T)$ function let's us state that

$$X_i^2 < X_i^1, \tag{55}$$

and we get to the theorem result

$$X_i = X_i^2 = \min\left\{X_i^1, X_i^2\right\}. \tag{56}$$

- **3rd case:** $n(a,T) \leq \theta_i(T) \leq n(b,T)$**, or what is the same** $a \leq x \leq b$
  This is a trivial situation, since $N_1(x,T) = N_2(x,T)$ and then the location of the vehicle i is given by either X_i^1 or X_i^2, so we have

$$X_i = X_i^1 = X_i^2 = \min\left\{X_i^1, X_i^2\right\}. \tag{57}$$

Therefore, we have shown that the location of vehicle i at time $t = T$ is in any escenario given by

$$X_i(T;V,W,K) = \min\left\{X_i^1(T;V,W,K), X_i^2(T;V,W,K)\right\}, \tag{58}$$

and hence the theorem is proved.

■

Using this result, we can now implement an efficient method to estimate the individual trajectories of vehicles crossing a given road segment. Basically, we can find as many pairs (x,t) in a trajectory as we desire just by fixing values for t and applying the theorem to determine the location x.

In the next section we go deeper into the implementation of this method.

4.3 Implementation

In a similar way we did in a previous section, we are going to describe the steps to follow in order to find the trajectories for all the reidentified vehicles:

- Based on entry and exit times, assign entry and exit positions for each vehicle in the dataset, so that we have r_i, s_i, $F(r_i) = i$ and $G(s_i) = i + M_i$ for every vehicle. Then, for each vehicle i:

- Find the parameter a_i corresponding to the overtaking function of the vehicle, $\theta_i(t) = a_i(t - r_i) + i$.

- Split the time interval $[r_i, s_i]$ into time steps of size $\Delta t = 0.1 sec$. For each time point $t = T$:

- Compute the location of the vehicle i in the uncongested case:

$$X_i^1(T;V,W,K) = \left[T - F^{-1}(\theta_i(T) - n_0)\right] \cdot V. \tag{59}$$

- Compute the location of the vehicle i in the congested case. In this case, we need to use a bisection method, since we have an implicit equation given by the function

$$H(x) = G(t - \frac{l - x}{W}) + K(l - x) - \theta_i(t). \tag{60}$$

  Then, we need to find $X_i^2 = X_i^2(T; V, W, K)$ so that

$$H(X_i^2) = G(T - \frac{l - X_i^2}{W}) + K(l - X_i^2) - \theta_i(T) = 0. \tag{61}$$

- Take the minimum of both space values to obtain the actual location

$$X_i(T; V, W, K) = \min \left\{ X_i^1(T; V, W, K), X_i^2(T; V, W, K) \right\}. \tag{62}$$

5 Comparison with NGSIM data

The performance of the proposed method was evaluated in terms of its ability to estimate the trajectories of some especific vehicles based on real world data.

The data are extracted from *Next Generation Simulation* (NGSIM). The dataset consists of the trajectories of vehicles traveling on a stretch of the US 101 freeway in Los Angeles, CA, from 7:50 AM to 8:35 AM on June 15, 2005. The study site is a linear stretch of freeway between the Ventura Blvd and the Cahuenga Blvd off-ramps on the southbound US 101 freeway as shown in Figure 4. The segment is about 0.13 miles with five lanes and one auxiliary lane. On average, it takes around 25 seconds for vehicles to go through this segment in congestion. The 45-min period is split into three 15-minute intervals. Due to the setting of the detection zone, only the vehicles passing the upstream point after the time origin are tracked. A 2-minute warm-up period is used to ensure that all vehicle trajectories in the segment are tracked in the dataset.

There are 1894 vehicle trajectories in the first time period, 1842 in the second time period and 1698 in the third period. We deploy virtual detectors at both ends of the road segment and convert trajectory data into detector volume counts at these locations.

Let's now explain in detail the steps followed to process the data and obtain the results.

5.1 Data Preparation

5.1.1 Time Interval

The sampling frequency of NGSIM is 10Hz. For vehicle i, its location at time step j is $X_i(j\Delta t)$, where $\Delta t = 0.1s$. As an overview, we need to prepare two datasets for the estimation method, namely, the vehicle reidentification measurements and detector measurements.

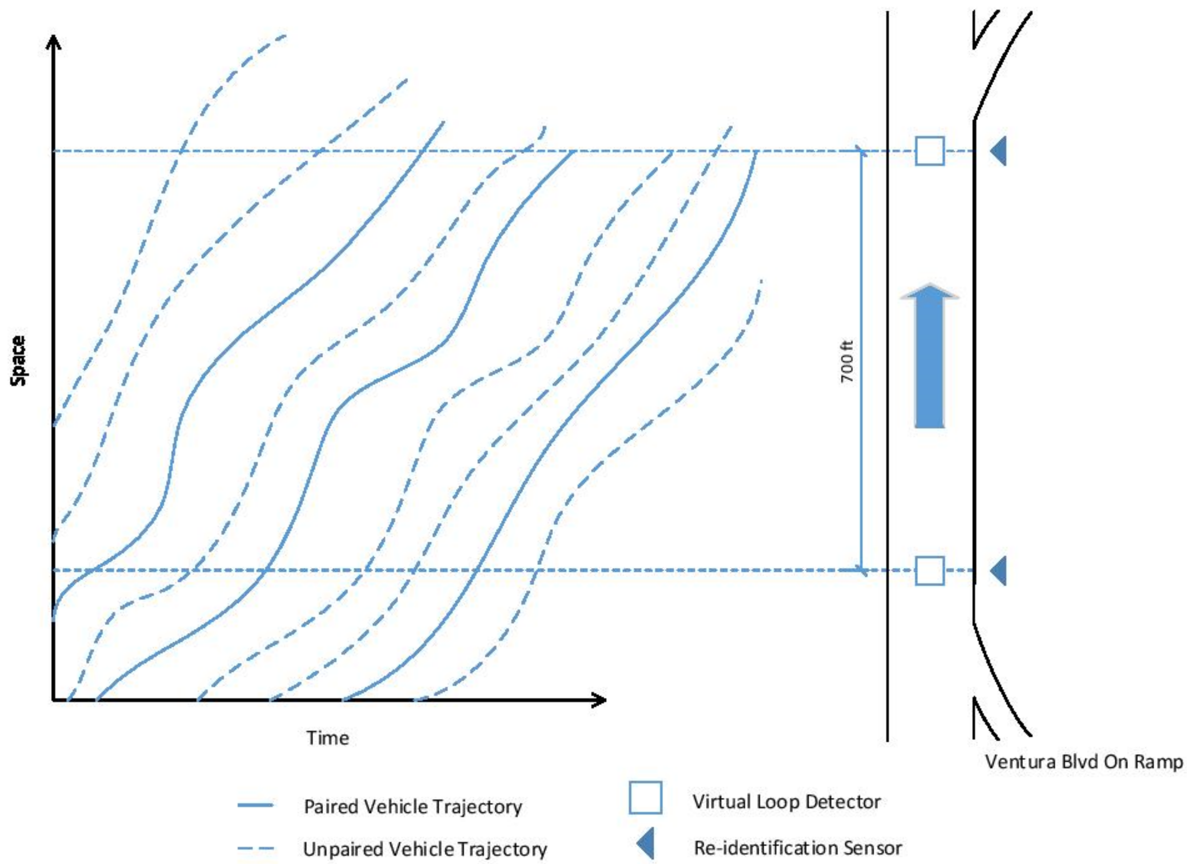


Figure 4: Illustration of the Study Site

5.1.2 Vehicle Reidentification Measurements

The vehicle reidentification data contains the entry/exit time of individual vehicles on the study site. Let x_0 and x_l denote the entry and exit point of the segment respectively. We used a linear interpolation function to find the entry time r_i, formally,

$$r_i = j\Delta t + \frac{x_0 - X_i(j\Delta t)}{X_i(j\Delta t + \Delta t) - X_i(j\Delta t)}\Delta t,$$

where j satisfies $X_i(j\Delta t) < x_0 < X_i(j\Delta t + \Delta t)$. Similarly, the exit time is

$$s_i = j'\Delta t + \frac{x_l - X_i(j'\Delta t)}{X_i(j'\Delta t + \Delta t) - X_i(j'\Delta t)}\Delta t,$$

where j' is chosen such that $X_i(j'\Delta t) < x_l < X_i(j'\Delta t + \Delta t)$.

So based on these expressions, we create a dataset storing the entry and exit times for all the vehicles crossing the road segment.

5.1.3 Flow Measurements

We can compute the cumulative flow based on the entry and exit time, s_i and r_i, for all vehicle i. The cumulative flow functions $F(t)$ and $G(t)$ are practically step functions. However, it is preferable to approximate them with continuous functions which are easier to evaluate. Especially, we can find the instantaneous upstream flow rate at time t by evaluating $f(t) = \lim_{\Delta t \to \infty} \frac{F(t+\Delta t) - F(t)}{\Delta t}$ at almost all points, and the same applies for downstream flow rate $g(t)$. Here, we used a method that approximates the step function with a piecewise linear curve passing through the crests, as illustrated in 5. In the plot, the cumulative flow function at the upstream, $F(t)$, is approximated by $\tilde{F}(t)$. The resulting function is differentiable almost everywhere except at the transition points.

Using the same process of linear approximation we also implement the inverse functions of the cumulative flows, $F^{-1}(t)$ and $G^{-1}(t)$.

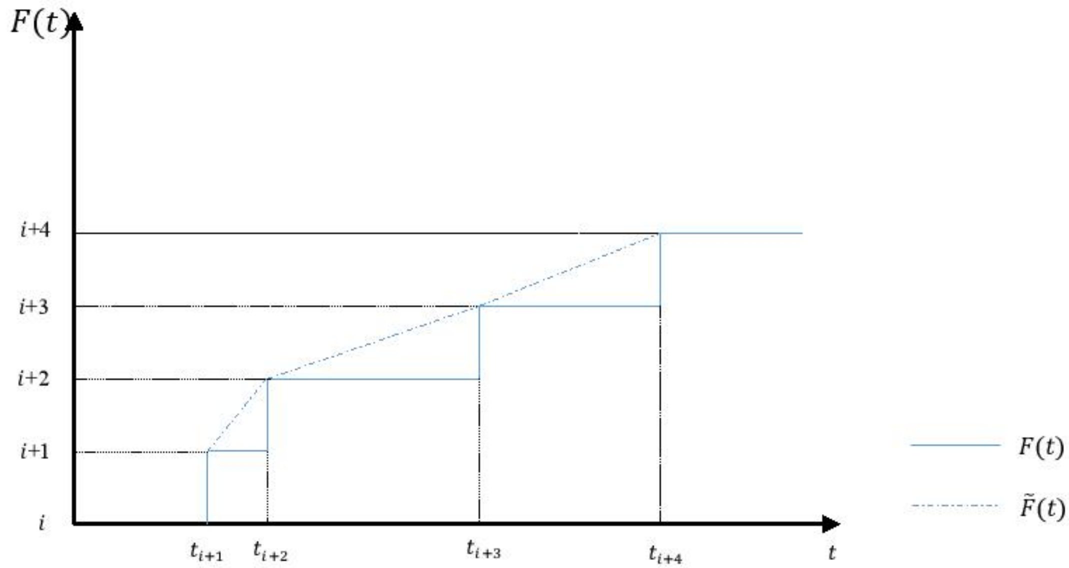


Figure 5: Linear Approximation of Cumulative Flow

5.2 Trajectory computation

Once we have implemented the corresponding cumulative flow functions, as well as their respective inverse functions, we can start with the estimation of the trajectories. To do so, we use the method explained in a previous section.

We will store the results of the trajectories in a big matrix, where the first column contains the different time steps ($\Delta t = 0.1s$) in which we divided the total period of time and the rest of the columns (one for each vehicle) have the locations of the vehicles corresponding to the time step indicated in the first column. In other words, the cell located in row j and column $i+1$ contains $X_i(j\Delta t)$. For each vehicle in the NGSIM dataset, and for each time step, we find the location of the vehicle in the road segment as

$$X_i(t) = \min\left\{X_i^1(t), X_i^2(t)\right\}. \tag{63}$$

where $X_i^1(t)$ and $X_i^2(t)$ are obtained using the congested and uncongested parts of Newell's equation as explained in theorem (3.1). Considering FIFO we have

$$X_i^1(t) = [t - F^{-1}(i - n_0)] \cdot V \tag{64}$$

and

$$G(t - \frac{l - X_i^2(t)}{W}) + K(l - X_i^2(t)) - i = 0, \tag{65}$$

and introducing FIFO violation we have

$$X_i^1(t) = [t - F^{-1}(\theta_i(t) - n_0)] \cdot V \tag{66}$$

and

$$G(t - \frac{l - X_i^2(t)}{W}) + K(l - X_i^2(t)) - \theta_i(t) = 0. \quad (67)$$

In particular, $X_i^1(t)$ is obtained directly from the first equation and $X_i^2(t)$ is approximated using the bisection method with a tolerance value of $0.1ft$ and a maximum number of iterations equal to 20.

As for the parameters (V, W, K) and the initial number of vehicles n_0, we will use the estimations given by Sun's study (Sun et al., 2015) in each of the 3 time intervals in which the NGSIM dataset is divided. The values of these constants will be specified in the next section.

Note that if the values obtained for the vehicles's location are $X_i(t) < 0$ or $X_i(t) > l = 698$ we ignore them, since we are only focused on the study site defined by the road segment $0 \leq x \leq l$.

All the code is implemented using R language and it is fully included in appendix B.

Figure 6 includes a plot of the estimated trajectory for one vehicle. In this plot we can see what do the estimated trajectories look like and distinguish between the congested and uncongested part of the trajectory. Note that in all the cases there will be an initial uncongested state and then it will turn into a congested state, which is consistent with the no transonic rarefraction waves condition inherent to the model used.

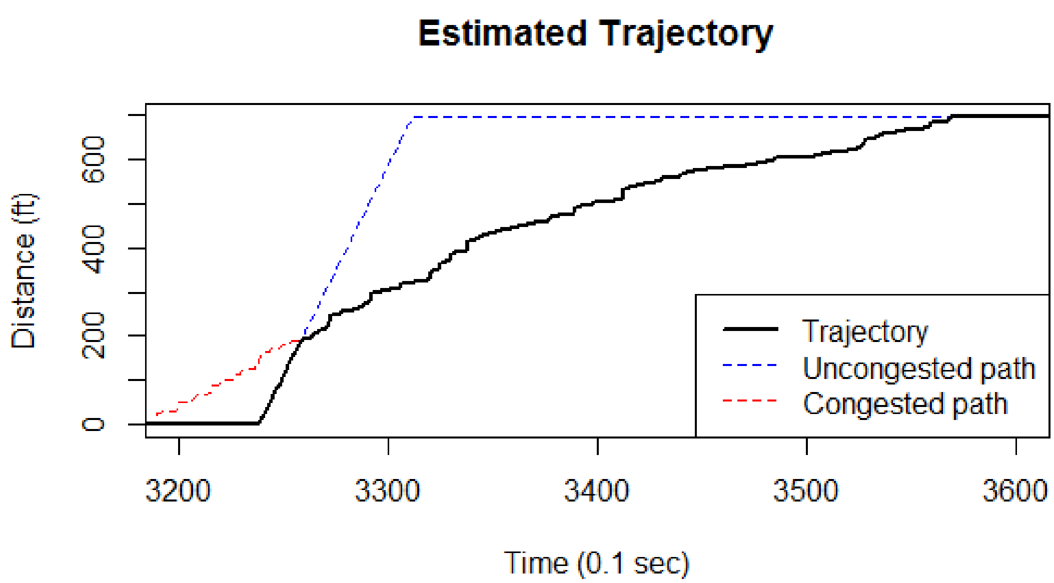


Figure 6: Example of estimated trajectory

5.3 Validation

To examine the performance of the estimation method, the estimated trajectories are compared with the real trajectories, given in the NGSIM datasets. To find the error for each vehicle i, we plot both trajectories and find the area enclosed between the curves. Finally, we divide this area by the total area under the real trajectory curve. It is calculated as

$$ERROR_i = \frac{A_{1i}}{A_{2i}} = \frac{\int_{r_i}^{s_i} |X_i(t) - \hat{X}_i(t)|\, \mathrm{d}t}{\int_{r_i}^{s_i} |\hat{X}_i(t)|\, \mathrm{d}t} \approx \frac{\sum_{t=r_i}^{s_i} |X_i(t) - \hat{X}_i(t)|}{\sum_{t=r_i}^{s_i} |\hat{X}_i(t)|}, \tag{68}$$

where $X_i(t)$ represents the estimated location of vehicle i at time t and $\hat{X}_i(t)$ is the real location of the vehicle at that time.

Figure 7 is an example to have a visual idea of what these method is doing.

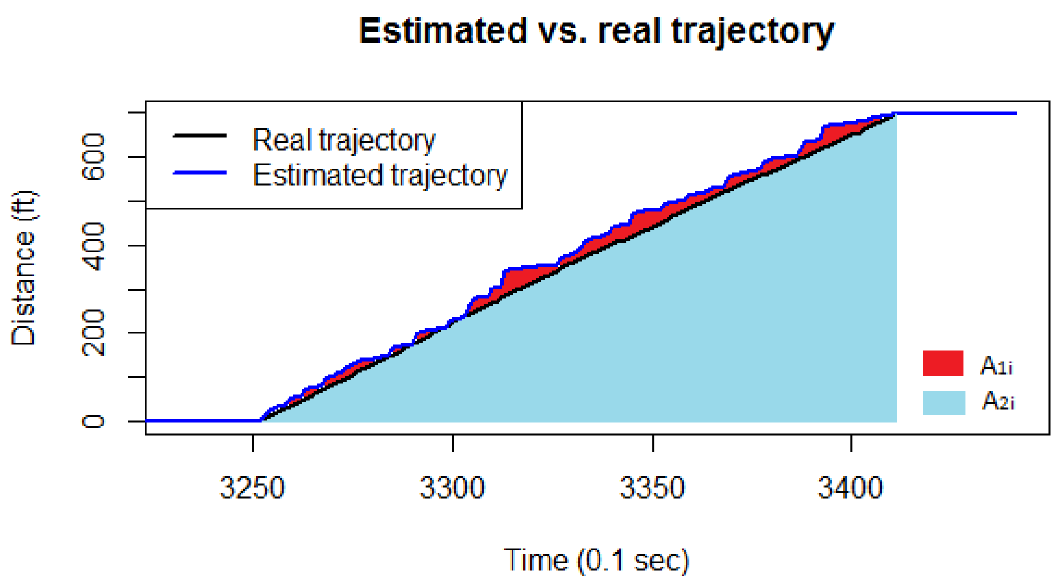


Figure 7: Areas used to validate the estimation method

5.4 Results

For each of the three datasets obtained from NGSIM, we have followed the above steps and obtained the estimated trajectories of all the vehicles. The values of the parameters (V, W, K) and constants used in equations (64), (65), (66) and (67) are summarized in table 2.

In figures 8 and 9 we can compare the real trajectories of two specific vehicles in datasets 3 and 1 with the correspondent estimated trajectories, with and without FIFO assumption.

Table 2: Parameters of the equations

Time interval	n_0**(veh)**	V**(mph)**	W**(mph)**	K **(vpm)**
7:50 AM - 8:05 AM	39	62.00	20.00	156.51
8:05 AM - 8:20 AM	39	50.00	25.67	141.46
8:20 AM - 8:35 AM	51	47.00	21.15	157.98

On the one hand, in the first case we can observe that the vehicle has been overtaken by several other vehicles in the road segment, since the real exit time is quite greater than the estimated exit time under the FIFO hypothesis (which means no overtakings). Actually, the number of overtakings suffered by this vehicle is 15. On the other hand, the second case shows a vehicle with very similar exit times for real and estimated trajectories. In fact, the real exit time is slightly lower than the estimated exit time with FIFO violation. It means that this second vehicle has overtaken a few other vehicles. The real number of overtakings is 2, as we could expect.

In general, the greater the number of overtakings, the greater the difference between the estimated trajectories with and without FIFO assumption. Furthermore, we appreciate really accurate approximations of the trajectories when FIFO hypothesis is not considered.

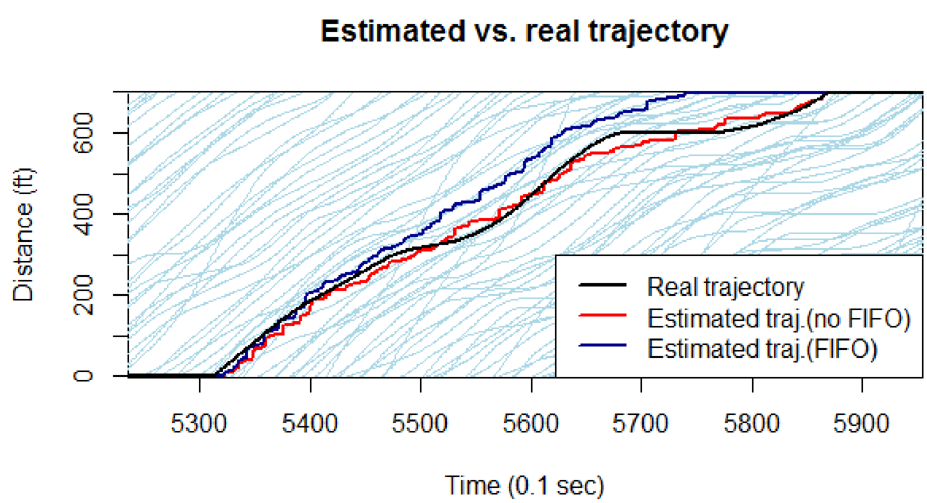


Figure 8: Example of trajectory estimation

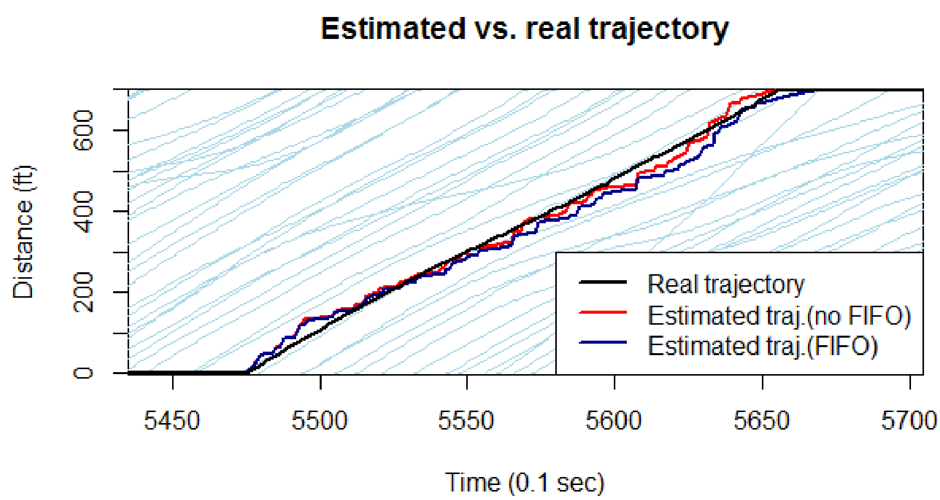


Figure 9: Example of trajectory estimation

Appendix A contains a more exhaustive list of information and plots of different vehicle trajectories.

After implementing the estimation algorithm on every vehicle of the three datasets and computing the errors using the validation method explained before, we obtain the general results included in table 3. The notation used is the following:

- $\bar{v}$: Average speed of the vehicles
- $\sigma(v)$: Standard deviation of the speeds along the road segment
- $\bar{E}_1$: Average error under FIFO hypothesis
- $\sigma(E_1)$: Standard deviation of the error under FIFO hypothesis
- $\bar{E}_2$: Average error under FIFO violation hypothesis
- $\sigma(E_2)$: Standard deviation of the error under FIFO violation hypothesis
- $\bar{a}_i$: Average value of the parameter a_i in the overtaking function
- $\sigma(a_i)$: Standard deviation of the parameter a_i

Table 3: Verification Results

Dataset	$\bar{v}$(ft/s)	$\sigma(v)$	$\bar{E}_1$(%)	$\sigma(E_1)$	$\bar{E}_2$(%)	$\sigma(E_2)$	$\bar{a}_i$(veh/sec)	$\sigma(a_i)$
1	37.918	12.332	16.91	11.67	10.52	6.40	-0.080	0.477
2	33.001	11.177	17.14	13.27	9.53	5.68	-0.096	0.546
3	27.825	9.419	17.58	13.91	9.88	5.30	-0.108	0.619

It is important to notice that the mean of the errors is significantly lower when FIFO is not considered, about $40-50\%$ lower to be more specific. Furthermore, the standard deviation of the errors is also inferior in this case, showing that the errors obtained under FIFO violation are not subject to great fluctuations, which means that the method is more reliable.

Finally, we present some plots to better understand and analyze the results obtained. Figures 10, 11 and 12 show, for each dataset, the distribution of the errors made in estimating the trajectories, both under FIFO assumption and under FIFO violation.

To have a better interpretation, each group of errors has been adjusted using a gamma distribution curve. This type of function is characterized by two parameters, the *shape parameter* α and the *scale parameter* β. They can be calculated from the mean μ and the standard deviation σ according to the relationships

$$\alpha = (\frac{\mu}{\sigma})^2 \tag{69}$$

and

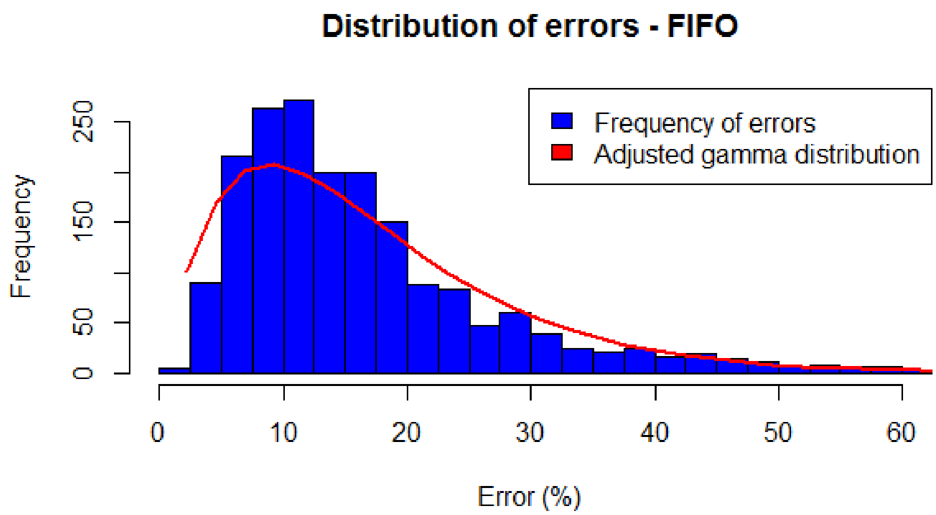
$$\beta = \frac{\sigma^2}{\mu}. \tag{70}$$

The values obtained for the distributions plotted in figures 10, 11 and 12 are contained in table 4.

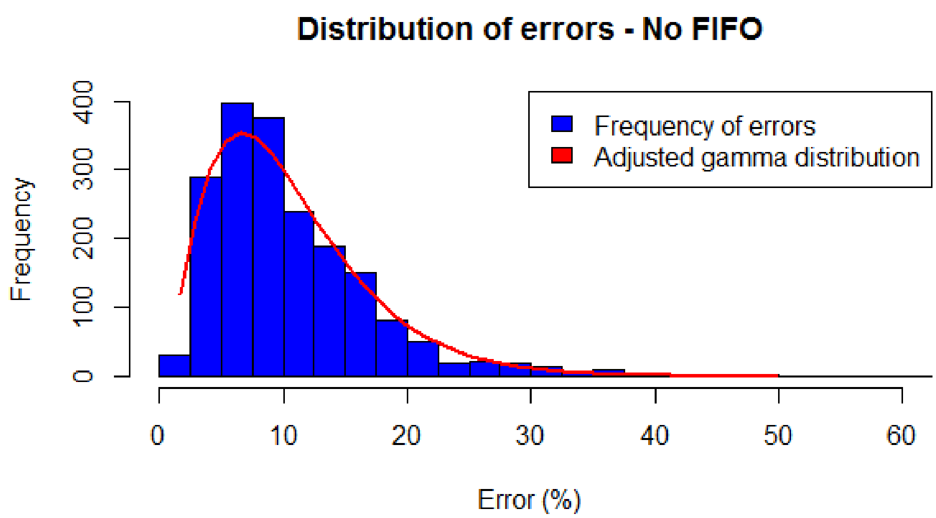
Table 4: Parameters of the Gamma Distributions

Dataset	Case	α	β
1	FIFO	2.10	8.05
	NO FIFO	2.70	3.89
2	FIFO	1.67	10.28
	NO FIFO	2.82	3.38
3	FIFO	1.60	11.02
	NO FIFO	3.48	2.84

Regarding the aggressiveness of the drivers, measured by the parameter a_i, we obtain the distribution presented in figure 13. The high presence of values very close to 0 means that we have a general pattern of a congested traffic situation, where vehicles don't have the freedom to overtake each others and they tend to move as a uniform mass.

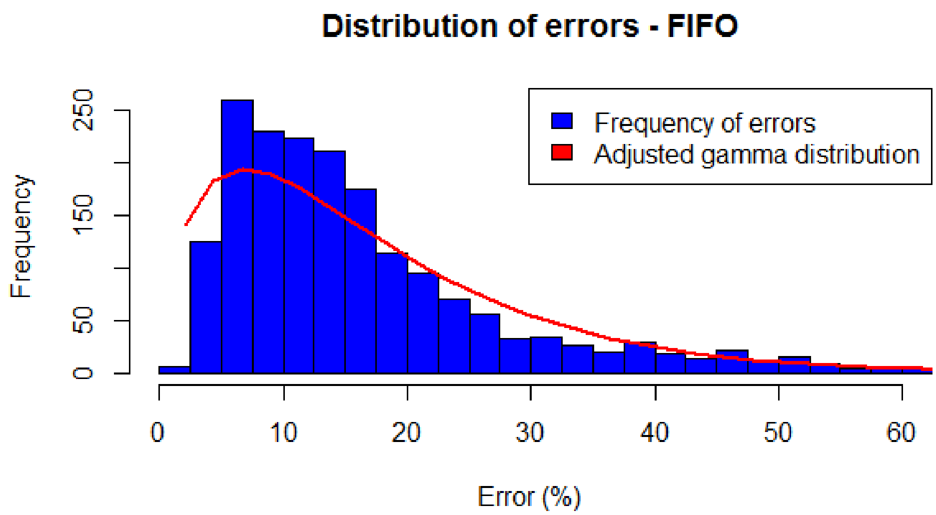


(a) With FIFO hypothesis

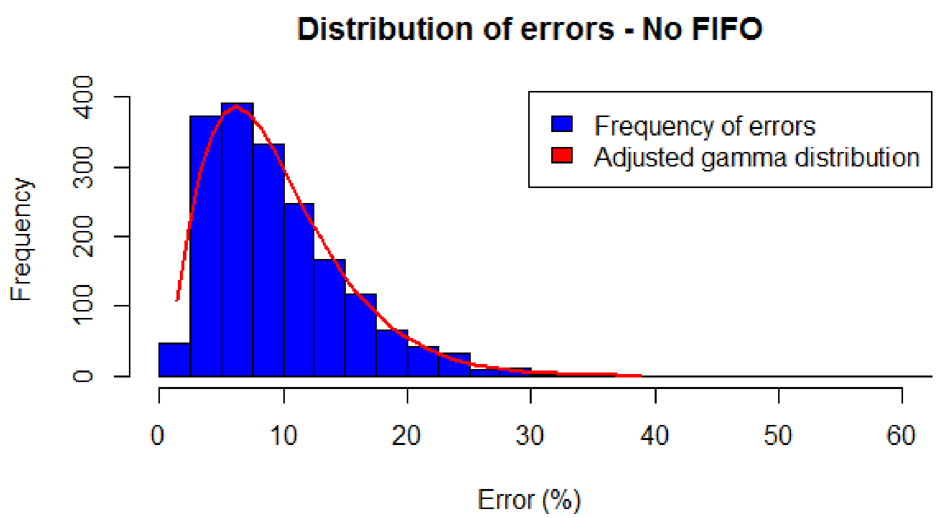


(b) With FIFO violation

Figure 10: Distribution of the errors in Dataset 1

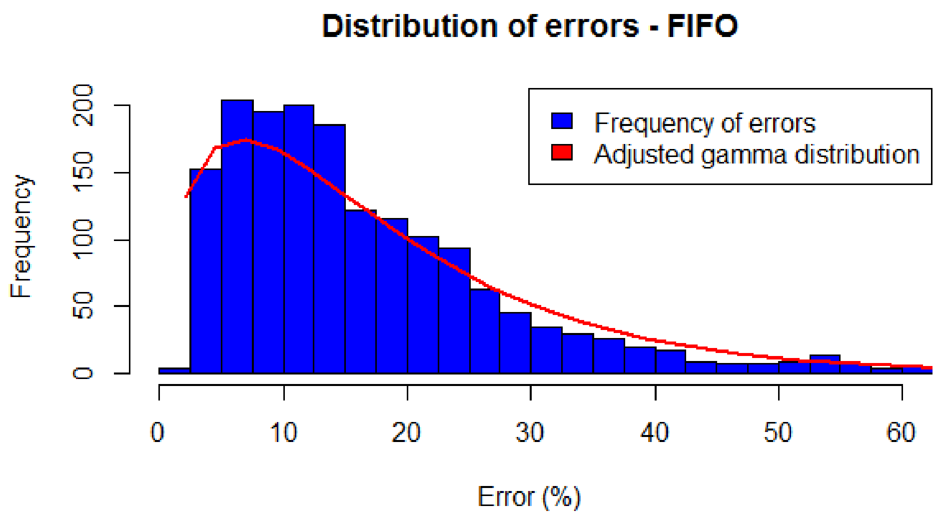


(a) With FIFO hypothesis

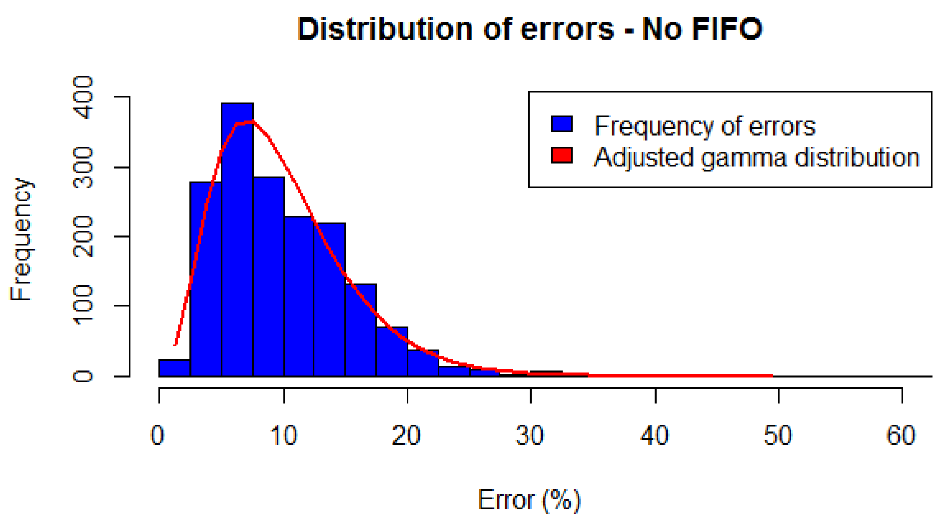


(b) With FIFO violation

Figure 11: Distribution of the errors in Dataset 2



(a) With FIFO hypothesis



(b) With FIFO violation

Figure 12: Distribution of the errors in Dataset 3

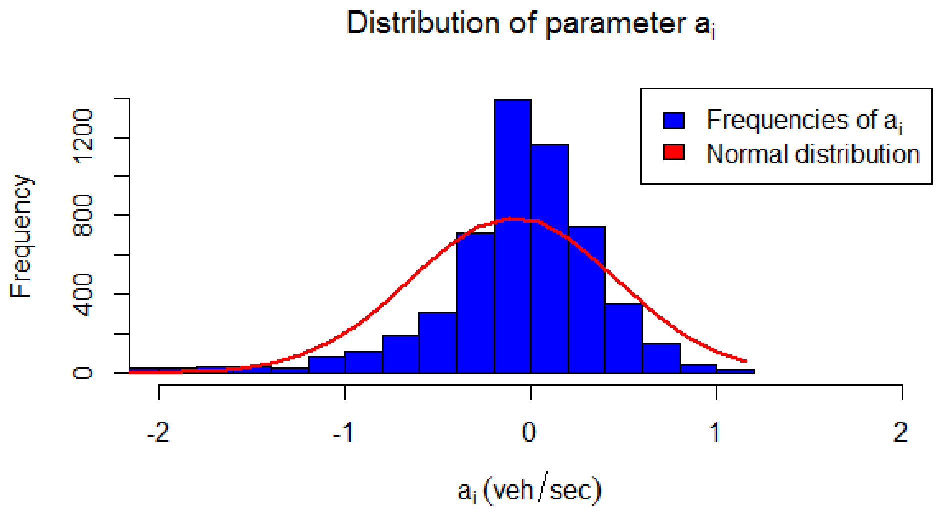


Figure 13: Distribution of the parameter a_i for all the vehicles

6 Conclusion and discussions

This research has presented two major contributions to the traffic flow theory. First, it proposed an innovative and effective trajectory estimation method based on Newell's kinematic wave theory, which is a simplified model that uses cumulative flow to describe traffic condition. Next, the study introduced the FIFO violation condition in Newell's model to obtain an improved and more consistent method. This means an important breakthrough in this area, since it is the first study including FIFO violation in Newell's model. Finally, the estimation method was tested using NGSIM datasets, and the results obtained showed a very good approximation of the real trajectories. However, there are still some little inconsistencies between estimation and observation, and here are some possible error sources:

- low accuracy in the set of parameters (V, W, K) used,
- wrong trajectories recorded in the NGSIM datasets, and
- peak flows that cause the cumulative functions $F(t)$ and $G(t)$ to grow faster than the theoretical capacity of the road, making the estimated trajectories look less smooth.

Besides, these are suggested directions for further research in the topic:

- studying an iterative method for adjusting the parameters (V, W, K) with the estimated trajectories,

- modifying the cumulative functions smartly to obtain smoother trajectories,
- trying different overtaking functions $\theta_i(t)$ to obtain a better performance in the FIFO violation case,
- extending the current method to accommodate other data types (for example, GPS data, Bluetooth data), and
- exploring other applications, including emission estimation and travel time estimation.

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Appendices

A Estimated Trajectories

We have selected a group of significant vehicles in each dataset and here we present some relevant information regarding their trajectories. In particular, the chosen vehicles are the first 300 vehicles entering the road segment after the minute 7 within the 15-minutes interval.

Several plots containing some of the trajectories are also included.

A.1 Dataset 1

ID	Entry Time (s)	Exit Time (s)	Av.Sp (ft/s)	St.Dev	Overtakes	A_i (veh/s)
1	4215	4391	39.59	12.612	-2	-0.011343
2	4215	4398	38.21	15.810	-1	-0.005475
3	4217	4416	35.20	12.502	0	0.000000
4	4226	4418	36.47	15.132	1	0.005225
5	4232	4408	39.77	12.925	-3	-0.017093
6	4234	4433	35.14	11.464	1	0.005034
7	4236	4416	38.83	5.957	-3	-0.016691
8	4236	4450	32.73	13.038	5	0.023447
9	4250	4438	37.16	15.256	0	0.000000
10	4257	4474	32.17	13.101	9	0.041474
11	4257	4446	36.92	12.832	-1	-0.005289
12	4258	4437	38.90	12.148	-4	-0.022294
13	4266	4451	37.83	15.551	1	0.005420
14	4268	4449	38.39	6.963	-3	-0.016502
15	4272	4493	31.61	13.409	9	0.040762
16	4276	4464	37.17	11.365	0	0.000000
17	4279	4460	38.44	12.425	-2	-0.011015
18	4280	4450	41.29	13.293	-6	-0.035491
19	4287	4473	37.61	17.910	-1	-0.005388
20	4298	4482	37.88	11.121	0	0.000000
21	4300	4469	41.40	7.444	-4	-0.023726
22	4303	4521	32.02	13.428	10	0.045873
23	4311	4497	37.51	15.834	2	0.010749
24	4313	4483	40.99	13.005	-3	-0.017619
25	4316	4537	31.49	13.314	9	0.040606
26	4319	4507	37.19	12.254	2	0.010657
27	4323	4508	37.67	15.931	2	0.010794

ID	Entry Time (s)	Exit Time (s)	Av.Sp (ft/s)	St.Dev	Overtakes	A_i (veh/s)
28	4323	4492	41.29	12.416	-5	-0.029575
29	4330	4499	41.12	6.675	-2	-0.011783
30	4332	4519	37.27	13.132	1	0.005340
31	4334	4558	31.10	14.268	10	0.044551
32	4338	4514	39.58	12.003	-2	-0.011341
33	4352	4583	30.33	14.643	14	0.060828
34	4354	4540	37.56	12.591	1	0.005381
35	4354	4544	36.81	13.104	3	0.015822
36	4359	4489	53.76	14.288	-14	-0.107830
37	4362	4532	40.86	15.927	-4	-0.023416
38	4367	4558	36.59	13.145	2	0.010484
39	4368	4499	53.61	13.695	-13	-0.099848
40	4369	4558	37.00	12.538	-1	-0.005301
41	4375	4607	30.09	14.192	12	0.051733
42	4380	4571	36.58	13.226	2	0.010481
43	4384	4559	39.91	16.573	-1	-0.005718
44	4386	4575	37.08	13.573	1	0.005312
45	4400	4586	37.41	11.990	3	0.016081
46	4402	4543	49.61	9.908	-9	-0.063971
47	4403	4579	39.73	15.894	-1	-0.005692
48	4404	4560	44.70	6.896	-5	-0.032017
49	4406	4643	29.45	15.476	15	0.063283
50	4413	4591	39.06	15.157	0	0.000000
51	4416	4604	37.06	11.739	1	0.005310
52	4416	4671	27.39	15.973	19	0.074551
53	4416	4543	55.34	19.465	-17	-0.134782
54	4419	4594	39.89	12.340	-3	-0.017144
55	4430	4620	36.70	11.454	3	0.015772
56	4432	4709	25.21	16.003	25	0.090305
57	4432	4620	37.20	12.505	0	0.000000
58	4433	4611	39.18	14.957	-2	-0.011225
59	4444	4634	36.65	11.913	3	0.015751
60	4445	4628	38.14	11.864	0	0.000000
61	4448	4628	38.90	14.279	0	0.000000
62	4452	4611	43.70	6.772	-7	-0.043825
63	4454	4747	23.81	17.582	27	0.092096
64	4459	4641	38.30	14.467	-1	-0.005486
65	4466	4769	22.98	18.303	31	0.102046
66	4467	4590	56.64	15.791	-17	-0.137958

ID	Entry Time (s)	Exit Time (s)	Av.Sp (ft/s)	St.Dev	Overtakes	A_i (veh/s)
67	4478	4624	47.85	8.121	-8	-0.054843
68	4485	4655	40.90	11.596	0	0.000000
69	4485	4653	41.44	11.656	-2	-0.011875
70	4486	4679	36.00	10.894	5	0.025786
71	4495	4643	47.01	6.586	-6	-0.040407
72	4495	4609	61.41	14.434	-18	-0.158363
73	4500	4672	40.52	12.811	-1	-0.005805
74	4501	4694	36.11	11.922	4	0.020692
75	4501	4821	21.79	18.159	32	0.099913
76	4502	4673	40.89	12.173	-3	-0.017576
77	4517	4688	40.92	13.474	-1	-0.005862
78	4523	4664	49.49	7.050	-8	-0.056720
79	4526	4860	20.86	19.051	36	0.107596
80	4532	4726	35.90	12.543	4	0.020574
81	4532	4648	60.07	12.471	-15	-0.129083
82	4533	4703	40.95	13.508	-3	-0.017601
83	4533	4688	44.92	7.228	-6	-0.038615
84	4535	4676	49.37	4.912	-10	-0.070728
85	4542	4737	35.80	13.999	2	0.010257
86	4543	4733	36.80	13.606	0	0.000000
87	4543	4877	20.89	18.630	31	0.092767
88	4548	4714	42.04	9.204	-6	-0.036137
89	4550	4663	61.72	11.139	-20	-0.176844
90	4559	4901	20.40	19.914	34	0.099370
91	4560	4751	36.54	13.590	0	0.000000
92	4561	4728	41.81	8.167	-7	-0.041929
93	4562	4755	36.08	12.835	-1	-0.005169
94	4565	4705	49.98	5.913	-14	-0.100249
95	4570	4765	35.81	15.105	0	0.000000
96	4571	4915	20.30	20.401	32	0.093061
97	4574	4724	46.74	8.882	-14	-0.093745
98	4579	4739	43.55	10.044	-10	-0.062397
99	4582	4936	19.73	21.325	35	0.098934
100	4582	4776	36.14	13.002	-3	-0.015531
101	4591	4744	45.69	8.450	-12	-0.078553
102	4598	4960	19.27	21.332	37	0.102146
103	4601	4762	43.40	9.478	-10	-0.062178
104	4604	4791	37.25	15.272	-4	-0.021347
105	4611	4801	36.68	13.749	-3	-0.015765

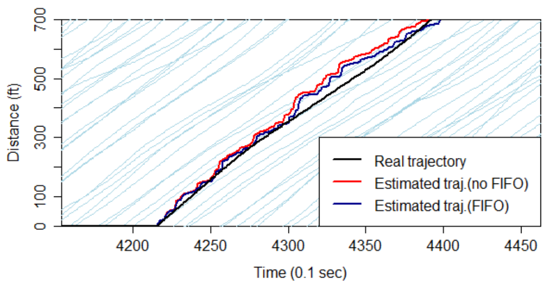
ID	Entry Time (s)	Exit Time (s)	Av.Sp (ft/s)	St.Dev	Overtakes	A_i (veh/s)
106	4612	4764	45.97	8.663	-12	-0.079025
107	4624	4821	35.31	19.080	-1	-0.005059
108	4624	4780	44.81	8.783	-10	-0.064198
109	4627	4984	19.55	19.604	35	0.098013
110	4634	4818	37.95	10.689	-5	-0.027187
111	4639	4794	45.05	8.120	-10	-0.064536
112	4644	4997	19.79	20.758	36	0.102047
113	4646	4844	35.26	18.615	-2	-0.010102
114	4646	4801	44.93	8.084	-11	-0.070811
115	4652	4833	38.48	10.628	-5	-0.027562
116	4659	4813	45.37	10.657	-12	-0.077996
117	4664	4822	44.29	8.595	-9	-0.057102
118	4669	4880	33.05	16.846	2	0.009469
119	4670	5024	19.69	20.803	37	0.104397
120	4672	4853	38.74	11.623	-6	-0.033299
121	4673	4789	59.86	12.297	-22	-0.188658
122	4673	4827	45.37	10.408	-13	-0.084500
123	4685	4861	39.84	16.432	-7	-0.039957
124	4691	4878	37.51	20.018	-5	-0.026871
125	4692	4852	43.77	11.122	-12	-0.075246
126	4693	5054	19.33	21.998	38	0.105248
127	4699	4847	47.14	7.185	-15	-0.101304
128	4705	4897	36.34	19.009	-5	-0.026028
129	4705	4902	35.46	12.016	-4	-0.020320
130	4710	5082	18.77	22.200	41	0.110255
131	4712	4891	39.10	11.685	-9	-0.050417
132	4720	4909	36.95	16.228	-5	-0.026468
133	4721	4870	46.87	6.065	-16	-0.107440
134	4722	4930	33.51	17.033	-4	-0.019202
135	4730	5101	18.80	22.632	39	0.105057
136	4743	4935	36.18	13.158	-3	-0.015549
137	4743	4886	48.79	12.610	-16	-0.111848
138	4743	4915	40.54	11.593	-9	-0.052276
139	4744	5116	18.79	22.442	40	0.107698
140	4746	4952	33.84	16.699	-3	-0.014546
141	4756	4930	40.14	13.190	-10	-0.057504
142	4762	5078	22.10	25.208	28	0.088652
143	4767	4904	50.77	12.988	-17	-0.123647
144	4772	4973	34.74	14.661	-1	-0.004977

ID	Entry Time (s)	Exit Time (s)	Av.Sp (ft/s)	St.Dev	Overtakes	A_i (veh/s)
145	4778	4955	39.49	11.033	-7	-0.039603
146	4781	4952	40.98	11.940	-10	-0.058708
147	4786	4934	47.12	7.191	-15	-0.101258
148	4790	4988	35.21	22.566	-3	-0.015131
149	4790	5139	20.00	21.417	36	0.103162
150	4794	4970	39.74	10.283	-9	-0.051243
151	4798	4967	41.27	11.819	-11	-0.065038
152	4803	4951	47.02	7.845	-17	-0.114517
153	4803	5001	35.26	16.254	-4	-0.020209
154	4814	5162	20.05	20.621	38	0.109175
155	4816	4989	40.29	10.538	-8	-0.046177
156	4818	4989	40.94	11.967	-10	-0.058654
157	4827	4972	48.34	8.000	-15	-0.103879
158	4827	5007	38.76	13.139	-7	-0.038866
159	4830	5033	34.38	15.628	-1	-0.004926
160	4838	5187	20.05	21.198	39	0.112006
161	4846	5048	34.55	16.895	1	0.004949
162	4849	5013	42.72	10.134	-10	-0.061201
163	4855	5005	46.25	8.853	-13	-0.086131
164	4858	5023	42.37	12.015	-10	-0.060697
165	4861	5024	42.76	9.387	-10	-0.061266
166	4861	5207	20.21	19.658	40	0.115841
167	4868	5017	46.73	9.029	-14	-0.093722
168	4871	5061	36.58	17.101	-1	-0.005240
169	4874	5037	42.76	12.178	-10	-0.061268
170	4880	5040	43.64	9.278	-10	-0.062521
171	4885	5033	47.31	7.909	-14	-0.094890
172	4898	5052	45.40	8.852	-9	-0.058545
173	4898	5045	47.48	8.175	-12	-0.081624
174	4905	5113	33.51	17.088	4	0.019205
175	4907	5058	46.07	10.997	-10	-0.066004
176	4912	5058	47.68	8.895	-10	-0.068302
177	4920	5064	48.56	8.947	-9	-0.062608
178	4924	5074	46.56	10.378	-9	-0.060040
179	4928	5097	41.50	8.531	-6	-0.035671
180	4928	5084	44.81	10.966	-8	-0.051354
181	4936	5140	34.23	13.871	5	0.024522
182	4946	5102	44.82	10.140	-7	-0.044946
183	4952	5229	25.18	17.419	30	0.108222

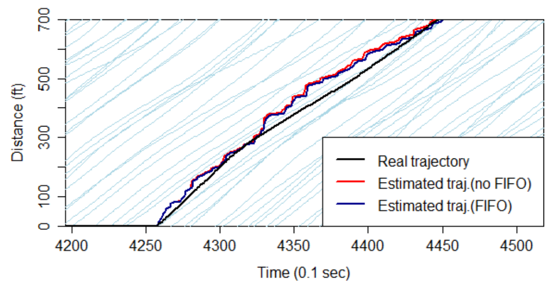
ID	Entry Time (s)	Exit Time (s)	Av.Sp (ft/s)	St.Dev	Overtakes	A_i (veh/s)
184	4957	5120	42.86	8.164	-4	-0.024560
185	4958	5154	35.57	14.073	5	0.025481
186	4959	5112	45.55	10.480	-9	-0.058736
187	4968	5121	45.52	11.404	-6	-0.039130
188	4970	5112	49.36	12.743	-12	-0.084853
189	4975	5134	43.82	8.322	-5	-0.031393
190	4978	5165	37.37	14.167	3	0.016063
191	4983	5253	25.83	17.022	29	0.107332
192	4990	5146	44.66	8.285	-4	-0.025592
193	4991	5141	46.64	11.200	-6	-0.040089
194	5003	5154	46.38	11.266	-5	-0.033223
195	5007	5157	46.45	11.763	-4	-0.026620
196	5009	5186	39.63	15.876	2	0.011355
197	5012	5127	60.53	4.124	-15	-0.130075
198	5015	5170	45.15	8.279	-2	-0.012938
199	5019	5169	46.54	10.801	-4	-0.026669
200	5019	5133	61.20	21.210	-17	-0.149055
201	5024	5276	27.68	17.687	24	0.095166
202	5034	5188	45.36	7.747	-1	-0.006498
203	5036	5178	49.39	12.007	-6	-0.042458
204	5037	5187	46.32	11.379	-4	-0.026545
205	5046	5212	42.11	17.566	3	0.018097
206	5051	5168	59.77	2.806	-12	-0.102753
207	5052	5199	47.35	12.326	-4	-0.027137
208	5053	5192	49.99	11.653	-6	-0.042970
209	5056	5292	29.54	16.834	21	0.088880
210	5057	5208	46.36	8.123	-3	-0.019927
211	5061	5228	41.85	16.553	1	0.005995
212	5062	5200	50.37	11.938	-8	-0.057731
213	5072	5226	45.49	10.854	-2	-0.013036
214	5073	5225	45.83	8.024	-4	-0.026263
215	5078	5205	55.12	15.658	-10	-0.078973
216	5086	5314	30.49	16.810	18	0.078626
217	5088	5239	46.14	9.515	-2	-0.013219
218	5090	5251	43.39	16.222	0	0.000000
219	5092	5240	47.24	8.709	-3	-0.020305
220	5094	5230	51.59	10.960	-6	-0.044351
221	5099	5213	61.28	4.525	-12	-0.105352
222	5108	5252	48.49	8.773	-3	-0.020842

ID	Entry Time (s)	Exit Time (s)	Av.Sp (ft/s)	St.Dev	Overtakes	A_i (veh/s)
223	5115	5275	43.39	9.216	1	0.006216
224	5116	5287	41.01	17.932	4	0.023503
225	5118	5264	47.90	10.253	-4	-0.027448
226	5120	5337	32.09	17.221	14	0.064372
227	5124	5269	48.38	8.741	-4	-0.027727
228	5127	5286	44.01	9.613	-1	-0.006305
229	5133	5302	41.22	17.727	4	0.023622
230	5134	5249	60.99	3.566	-13	-0.113593
231	5136	5352	32.23	17.620	13	0.060026
232	5137	5277	49.84	9.485	-6	-0.042843
233	5139	5300	43.46	10.748	-1	-0.006227
234	5147	5291	48.30	7.979	-5	-0.034602
235	5147	5266	58.82	16.798	-13	-0.109547
236	5154	5316	43.18	10.660	-1	-0.006186
237	5155	5371	32.29	17.910	10	0.046266
238	5157	5337	38.88	16.492	1	0.005570
239	5163	5295	52.58	10.398	-8	-0.060262
240	5168	5330	43.00	10.951	-2	-0.012322
241	5176	5352	39.60	16.433	2	0.011348
242	5177	5319	49.27	7.726	-6	-0.042351
243	5187	5390	34.41	18.246	8	0.039438
244	5197	5340	48.79	8.384	-3	-0.020969
245	5198	5330	52.95	11.549	-8	-0.060684
246	5202	5364	43.06	12.372	0	0.000000
247	5202	5389	37.43	14.541	3	0.016089
248	5211	5415	34.11	17.081	9	0.043979
249	5215	5346	53.21	11.849	-7	-0.053358
250	5215	5362	47.33	9.472	-5	-0.033906
251	5215	5378	42.92	11.096	-2	-0.012298
252	5224	5416	36.23	13.946	6	0.031145
253	5232	5394	42.88	10.962	-1	-0.006144
254	5233	5441	33.47	18.466	9	0.043161
255	5237	5373	51.01	8.965	-7	-0.051152
256	5239	5439	34.89	15.634	5	0.024992
257	5246	5400	45.30	17.066	-3	-0.019472
258	5247	5456	33.46	19.086	9	0.043146
259	5250	5397	47.43	9.792	-6	-0.040768
260	5261	5407	47.68	9.316	-5	-0.034155
261	5265	5408	48.66	9.125	-5	-0.034855

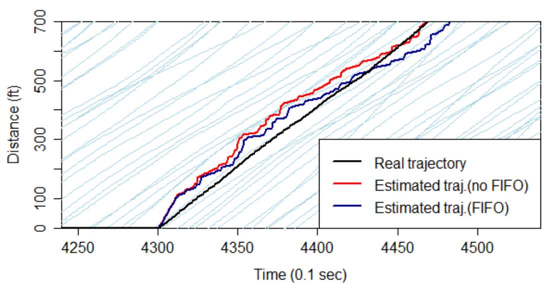
ID	Entry Time (s)	Exit Time (s)	Av.Sp (ft/s)	St.Dev	Overtakes	A_i (veh/s)
262	5269	5485	32.26	19.536	10	0.046221
263	5278	5426	47.10	7.533	-4	-0.026992
264	5282	5433	46.30	7.947	-4	-0.026533
265	5282	5454	40.60	22.622	0	0.000000
266	5283	5439	44.62	7.845	-4	-0.025571
267	5297	5444	47.30	7.101	-3	-0.020328
268	5298	5486	37.08	15.388	6	0.031873
269	5299	5463	42.51	8.311	0	0.000000
270	5301	5515	32.64	18.892	10	0.046763
271	5304	5455	46.29	8.181	-5	-0.033158
272	5314	5533	31.82	19.287	12	0.054713
273	5318	5475	44.51	4.599	-3	-0.019128
274	5318	5501	38.15	15.657	3	0.016397
275	5321	5479	44.01	8.665	-4	-0.025220
276	5321	5496	39.95	7.903	-1	-0.005723
277	5329	5486	44.39	7.382	-4	-0.025438
278	5333	5509	39.72	8.264	1	0.005691
279	5335	5459	56.51	9.197	-11	-0.089059
280	5338	5501	42.96	8.482	-4	-0.024618
281	5340	5565	31.06	18.364	10	0.044494
282	5350	5533	38.26	14.850	1	0.005482
283	5354	5534	38.78	9.998	2	0.011110
284	5357	5506	46.88	5.722	-6	-0.040300
285	5360	5596	29.57	19.969	16	0.067779
286	5361	5530	41.27	6.464	-4	-0.023648
287	5373	5555	38.34	14.346	1	0.005492
288	5377	5546	41.30	5.905	-2	-0.011834
289	5378	5561	38.15	8.795	1	0.005466
290	5382	5529	47.38	7.340	-9	-0.061089
291	5387	5618	30.15	18.342	16	0.069106
292	5388	5546	44.03	5.447	-5	-0.031542
293	5388	5574	37.69	14.252	0	0.000000
294	5394	5579	37.75	7.882	1	0.005408
295	5401	5556	45.26	6.587	-6	-0.038906
296	5401	5569	41.63	7.090	-4	-0.023854
297	5406	5594	37.02	13.509	2	0.010606
298	5406	5590	37.85	8.254	0	0.000000
299	5410	5580	41.15	7.563	-3	-0.017687
300	5413	5576	42.68	7.668	-6	-0.036689



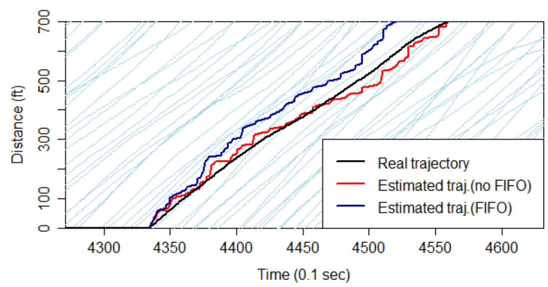
(a) Vehicle with $ID = 1$



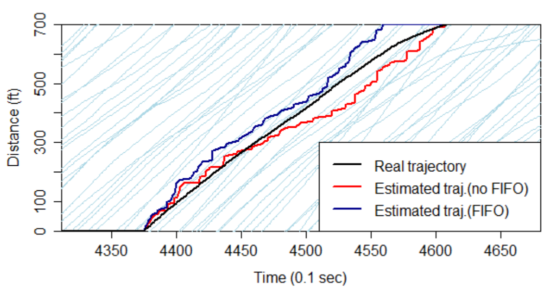
(b) Vehicle with $ID = 11$



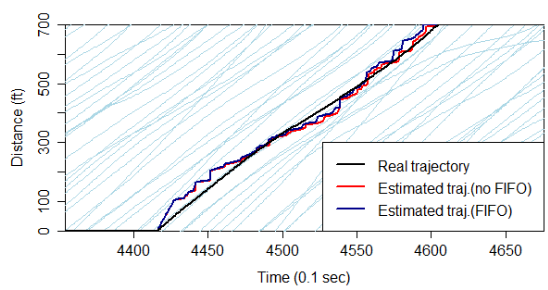
(c) Vehicle with $ID = 21$



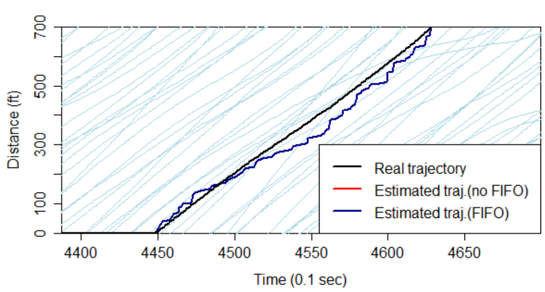
(d) Vehicle with $ID = 31$



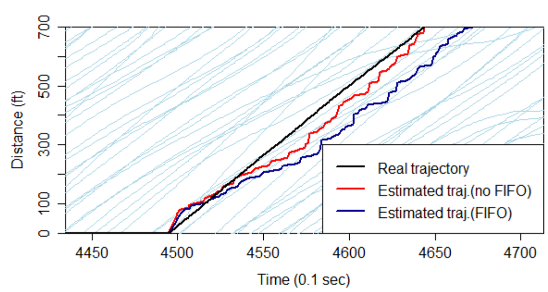
(e) Vehicle with $ID = 41$



(f) Vehicle with $ID = 51$

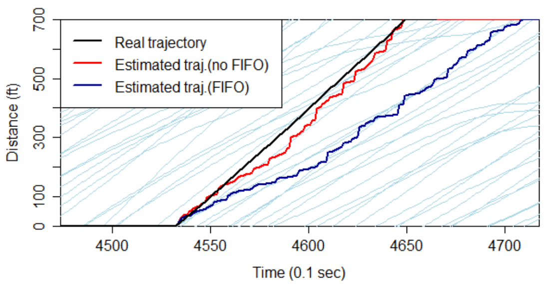


(g) Vehicle with $ID = 61$

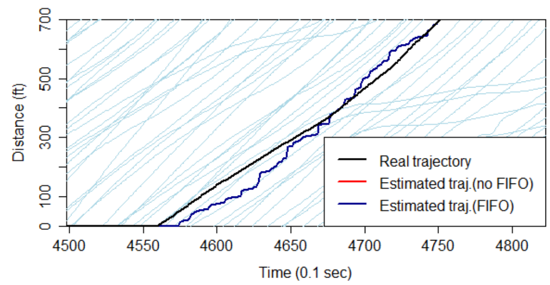


(h) Vehicle with $ID = 71$

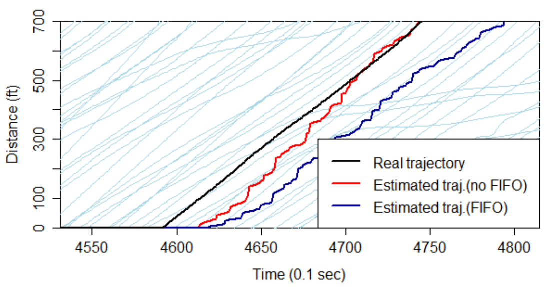
Figure 14: Plots of diferent trajectories in Dataset 1



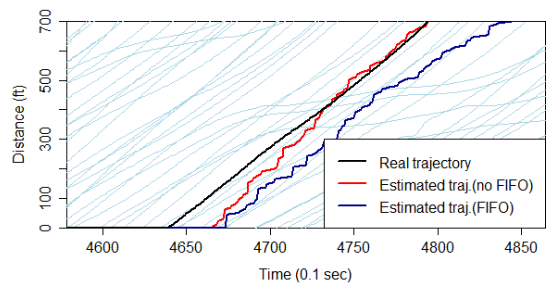
(a) Vehicle with $ID = 81$



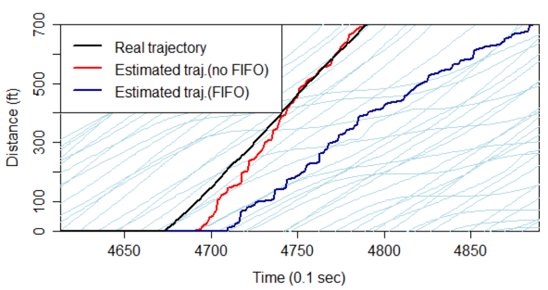
(b) Vehicle with $ID = 91$



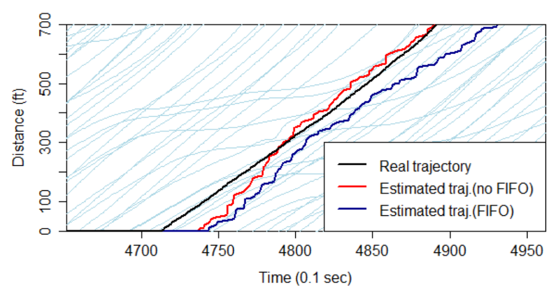
(c) Vehicle with $ID = 101$



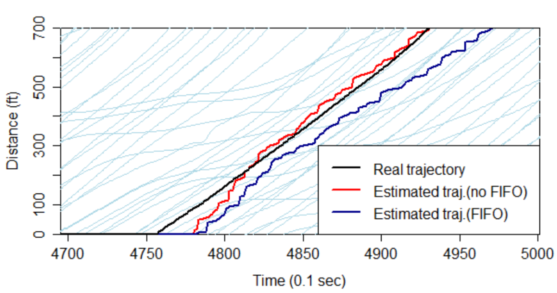
(d) Vehicle with $ID = 111$



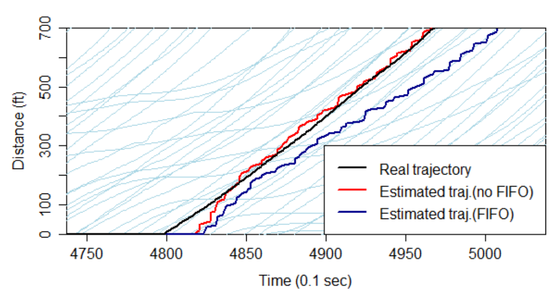
(e) Vehicle with $ID = 121$



(f) Vehicle with $ID = 131$

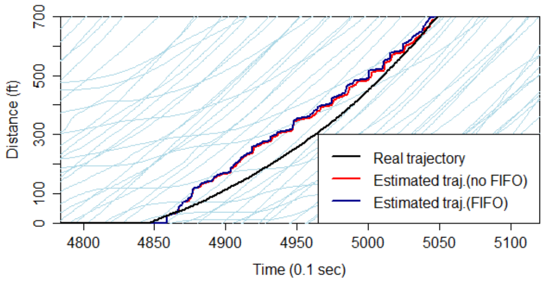


(g) Vehicle with $ID = 141$

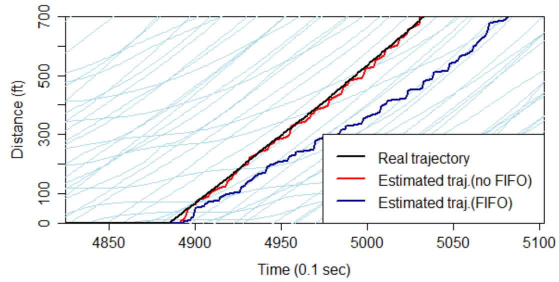


(h) Vehicle with $ID = 151$

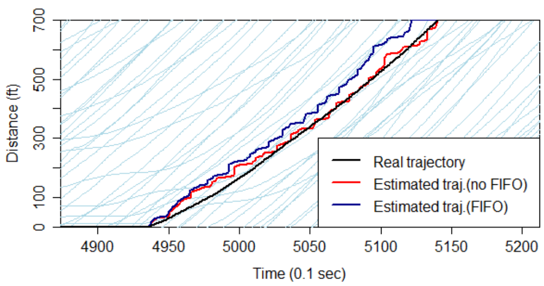
Figure 15: Plots of diferent trajectories in Dataset 1



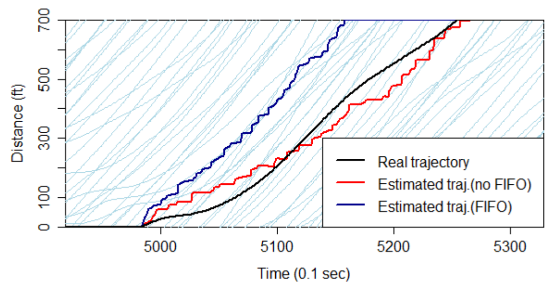
(a) Vehicle with $ID = 161$



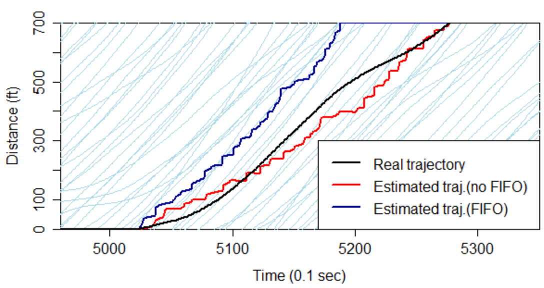
(b) Vehicle with $ID = 171$



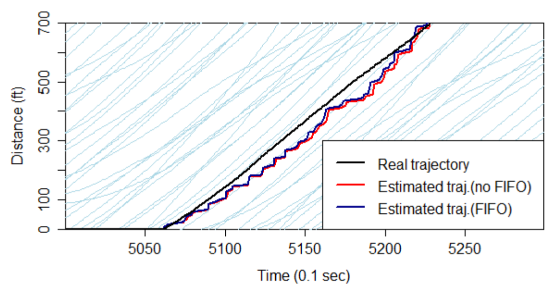
(c) Vehicle with $ID = 181$



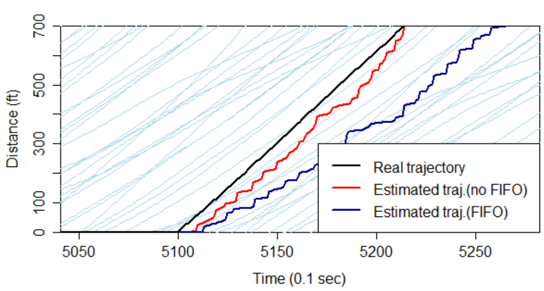
(d) Vehicle with $ID = 191$



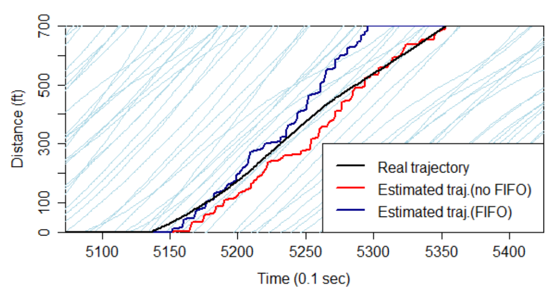
(e) Vehicle with $ID = 201$



(f) Vehicle with $ID = 211$

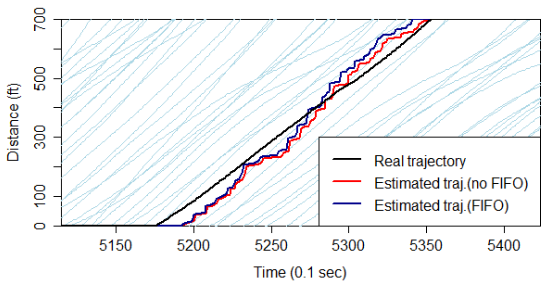


(g) Vehicle with $ID = 221$

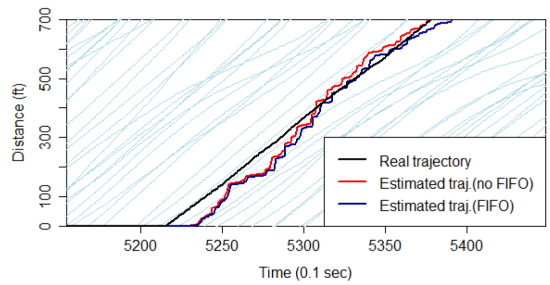


(h) Vehicle with $ID = 231$

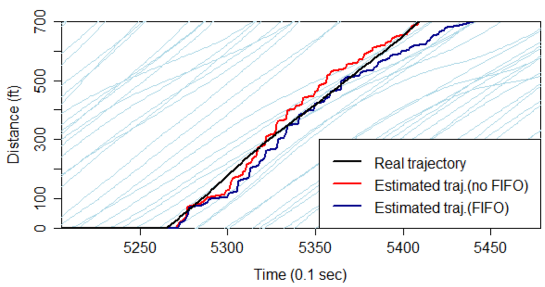
Figure 16: Plots of diferent trajectories in Dataset 1



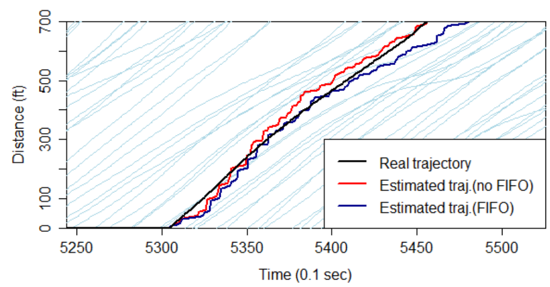
(a) Vehicle with $ID = 241$



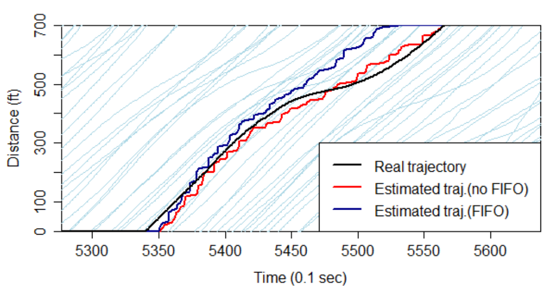
(b) Vehicle with $ID = 251$



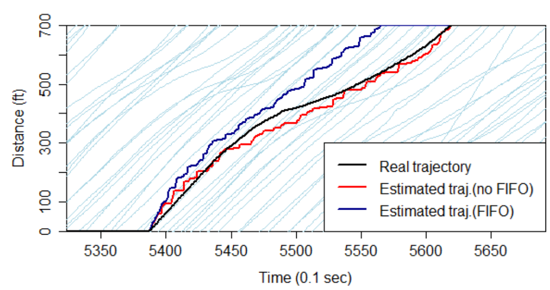
(c) Vehicle with $ID = 261$



(d) Vehicle with $ID = 271$



(e) Vehicle with $ID = 281$



(f) Vehicle with $ID = 291$

Figure 17: Plots of diferent trajectories in Dataset 1

A.2 Dataset 2

ID	Entry Time (s)	Exit Time (s)	Av.Sp (ft/s)	St.Dev	Overtakes	A_i (veh/s)
1	4218	4442	31.18	14.415	9	0.040205
2	4222	4426	34.21	16.433	5	0.024506
3	4223	4404	38.51	19.430	0	0.000000
4	4223	4419	35.71	15.197	0	0.000000
5	4224	4397	40.32	20.500	-4	-0.023108
6	4232	4380	47.30	26.280	-11	-0.074547
7	4244	4452	33.49	15.754	5	0.023993
8	4249	4421	40.74	20.421	-2	-0.011672
9	4253	4441	37.10	16.515	0	0.000000
10	4253	4386	52.58	9.595	-13	-0.097936
11	4255	4457	34.54	15.219	4	0.019794
12	4268	4456	37.03	13.247	2	0.010612
13	4274	4470	35.54	16.284	3	0.015273
14	4276	4475	35.17	14.849	3	0.015114
15	4281	4449	41.73	22.325	-4	-0.023914
16	4284	4456	40.66	18.948	-3	-0.017474
17	4293	4429	51.36	29.263	-9	-0.066228
18	4295	4487	36.23	15.824	4	0.020763
19	4299	4476	39.38	14.102	-1	-0.005642
20	4313	4482	41.33	21.344	-1	-0.005922
21	4313	4484	40.97	18.887	-1	-0.005870
22	4316	4504	37.05	15.824	2	0.010616
23	4316	4487	40.87	16.175	-2	-0.011709
24	4327	4515	37.22	15.093	3	0.015997
25	4333	4500	41.82	20.519	-2	-0.011981
26	4335	4528	36.25	14.962	5	0.025965
27	4343	4526	38.13	16.899	2	0.010926
28	4343	4509	42.15	20.233	-3	-0.018114
29	4345	4519	40.14	12.568	-1	-0.005751
30	4356	4560	34.09	4.869	6	0.029304
31	4357	4546	36.88	11.334	4	0.021134
32	4363	4531	41.58	12.661	1	0.005957
33	4366	4528	43.14	20.307	-3	-0.018541
34	4373	4580	33.75	9.298	5	0.024176
35	4379	4513	52.01	8.763	-9	-0.067064
36	4387	4529	49.12	16.479	-4	-0.028152
37	4391	4545	45.30	20.502	-3	-0.019470

ID	Entry Time (s)	Exit Time (s)	Av.Sp (ft/s)	St.Dev	Overtakes	A_i (veh/s)
38	4393	4562	41.40	18.954	-1	-0.005931
39	4394	4592	35.28	9.413	3	0.015162
40	4398	4601	34.35	8.987	5	0.024603
41	4410	4595	37.92	6.706	2	0.010865
42	4413	4569	44.61	13.195	-4	-0.025565
43	4420	4584	42.63	19.549	-3	-0.018322
44	4421	4622	34.72	8.507	7	0.034818
45	4425	4585	43.46	19.032	-4	-0.024903
46	4437	4638	34.75	8.405	10	0.049787
47	4438	4597	43.76	19.850	-3	-0.018806
48	4441	4642	34.69	9.012	9	0.044723
49	4441	4605	42.51	16.917	-2	-0.012181
50	4441	4619	39.27	4.386	0	0.000000
51	4445	4601	44.65	13.213	-5	-0.031984
52	4451	4652	34.65	4.484	7	0.034754
53	4454	4610	44.53	19.884	-5	-0.031901
54	4458	4665	33.81	7.819	8	0.038752
55	4463	4616	45.46	13.028	-6	-0.039074
56	4463	4629	41.97	10.041	-2	-0.012026
57	4468	4629	43.43	19.081	-4	-0.024890
58	4470	4626	44.88	4.946	-6	-0.038577
59	4471	4659	37.05	9.390	2	0.010616
60	4478	4678	34.75	7.594	8	0.039827
61	4482	4637	45.05	12.178	-6	-0.038728
62	4485	4651	42.02	8.897	-4	-0.024081
63	4492	4654	43.18	18.451	-3	-0.018560
64	4492	4675	38.22	8.875	2	0.010952
65	4497	4703	33.99	6.792	11	0.053565
66	4506	4669	42.69	11.429	-1	-0.006116
67	4507	4711	34.19	11.400	11	0.053884
68	4507	4667	43.66	11.156	-4	-0.025021
69	4512	4693	38.58	5.085	3	0.016583
70	4513	4666	45.50	19.581	-7	-0.045632
71	4517	4689	40.52	8.129	0	0.000000
72	4520	4676	44.62	11.149	-5	-0.031965
73	4524	4686	43.12	11.145	-3	-0.018534
74	4525	4732	33.73	7.028	12	0.057986
75	4528	4679	46.27	19.961	-6	-0.039773
76	4530	4695	42.39	8.227	-3	-0.018219

ID	Entry Time (s)	Exit Time (s)	Av.Sp (ft/s)	St.Dev	Overtakes	A_i (veh/s)
77	4532	4710	39.06	5.805	0	0.000000
78	4540	4699	43.80	9.784	-4	-0.025098
79	4543	4715	40.57	9.194	2	0.011625
80	4546	4744	35.18	7.365	8	0.040325
81	4549	4715	42.21	7.349	1	0.006048
82	4552	4727	39.86	10.307	2	0.011420
83	4553	4712	44.05	9.158	-4	-0.025244
84	4554	4701	47.51	19.616	-9	-0.061253
85	4557	4729	40.74	6.802	0	0.000000
86	4566	4712	47.81	19.075	-6	-0.041095
87	4572	4755	38.12	6.485	4	0.021844
88	4573	4765	36.44	8.476	4	0.020884
89	4577	4720	48.75	19.469	-6	-0.041903
90	4577	4739	43.14	7.915	-3	-0.018542
91	4593	4779	37.41	5.936	4	0.021436
92	4594	4753	43.80	7.497	-2	-0.012550
93	4601	4748	47.46	19.125	-4	-0.027198
94	4606	4784	39.12	6.262	3	0.016812
95	4607	4793	37.50	5.720	4	0.021492
96	4611	4798	37.41	7.205	4	0.021437
97	4612	4771	44.04	6.965	-3	-0.018930
98	4624	4808	37.88	5.811	6	0.032565
99	4627	4769	49.09	8.644	-6	-0.042202
100	4627	4786	43.90	6.449	-2	-0.012579
101	4637	4809	40.63	7.923	4	0.023284
102	4641	4825	38.01	6.619	7	0.038120
103	4645	4784	50.17	9.177	-7	-0.050313
104	4649	4806	44.49	6.471	-1	-0.006374
105	4653	4800	47.62	12.869	-4	-0.027288
106	4659	4835	39.76	6.548	4	0.022786
107	4662	4803	49.39	8.050	-5	-0.035377
108	4666	4822	44.65	6.561	-1	-0.006396
109	4673	4849	39.55	6.609	6	0.033997
110	4673	4822	46.70	9.500	-2	-0.013382
111	4679	4863	38.06	6.664	6	0.032712
112	4680	4842	43.25	5.562	0	0.000000
113	4682	4818	51.28	10.081	-7	-0.051427
114	4695	4842	47.50	9.303	-1	-0.006806
115	4695	4878	38.14	5.941	6	0.032789

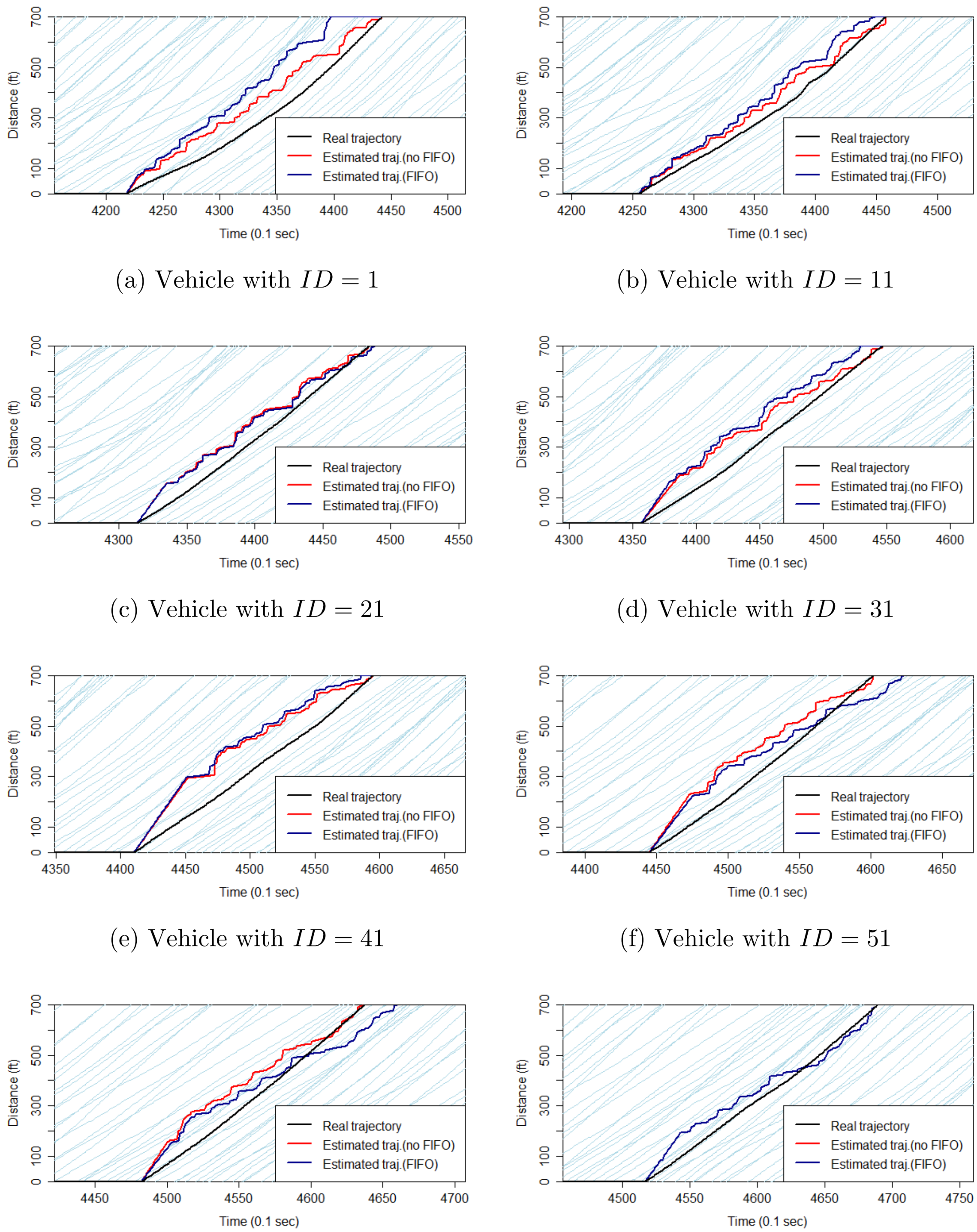
ID	Entry Time (s)	Exit Time (s)	Av.Sp (ft/s)	St.Dev	Overtakes	A_i (veh/s)
116	4697	4879	38.41	6.806	7	0.038522
117	4699	4899	34.99	4.882	10	0.050127
118	4701	4836	51.75	10.363	-7	-0.051898
119	4709	4893	37.91	5.605	6	0.032583
120	4710	4860	46.64	7.189	-4	-0.026726
121	4715	4868	45.67	14.395	-1	-0.006543
122	4716	4865	46.76	7.879	-4	-0.026798
123	4721	4904	38.06	6.341	5	0.027267
124	4727	4849	57.25	18.318	-10	-0.082027
125	4729	4866	50.68	9.937	-6	-0.043565
126	4730	4912	38.34	6.984	4	0.021972
127	4734	4919	37.63	6.033	5	0.026958
128	4740	4924	37.94	6.487	5	0.027179
129	4740	4924	37.84	6.307	5	0.027106
130	4742	4878	51.03	12.135	-8	-0.058482
131	4749	4889	49.76	7.864	-7	-0.049900
132	4755	4940	37.62	5.884	4	0.021559
133	4758	4895	50.72	10.045	-7	-0.050866
134	4763	4950	37.34	5.934	5	0.026747
135	4768	4953	37.81	5.885	5	0.027084
136	4771	4918	47.44	7.279	-5	-0.033983
137	4773	4910	51.05	10.191	-8	-0.058513
138	4776	4925	46.95	11.449	-3	-0.020177
139	4777	4961	37.99	5.748	3	0.016327
140	4784	4968	37.94	5.345	4	0.021741
141	4792	4943	46.23	5.939	-4	-0.026491
142	4797	4980	38.21	5.988	6	0.032849
143	4801	4981	38.82	5.759	6	0.033365
144	4806	4944	50.28	13.783	-6	-0.043219
145	4808	4959	46.32	6.584	-4	-0.026546
146	4809	4996	37.44	7.345	7	0.037551
147	4812	4961	46.86	6.684	-4	-0.026856
148	4813	4992	39.04	6.856	2	0.011187
149	4824	4975	46.17	5.821	-4	-0.026461
150	4825	4977	45.83	8.935	-4	-0.026263
151	4828	5010	38.42	9.068	7	0.038531
152	4834	5010	39.64	7.267	5	0.028398
153	4836	4978	48.98	7.708	-6	-0.042103
154	4841	4997	44.71	8.124	0	0.000000

ID	Entry Time (s)	Exit Time (s)	Av.Sp (ft/s)	St.Dev	Overtakes	A_i (veh/s)
155	4842	4995	45.65	6.871	-3	-0.019619
156	4849	5032	38.23	8.770	6	0.032865
157	4850	5015	42.41	6.759	2	0.012152
158	4855	5003	47.22	8.298	-2	-0.013530
159	4857	4995	50.60	12.274	-8	-0.057990
160	4857	5002	48.35	6.337	-5	-0.034638
161	4866	5034	41.59	7.283	3	0.017876
162	4879	5034	45.01	7.485	1	0.006448
163	4882	5031	46.90	7.110	-2	-0.013439
164	4887	5025	50.82	9.344	-4	-0.029122
165	4894	5052	44.07	8.507	3	0.018941
166	4897	5073	39.69	5.801	6	0.034120
167	4899	5051	46.00	8.172	-2	-0.013181
168	4904	5051	47.20	6.751	-1	-0.006762
169	4906	5051	48.07	6.276	-3	-0.020660
170	4912	5063	46.39	7.390	-1	-0.006646
171	4919	5073	45.21	10.311	2	0.012954
172	4920	5069	46.84	7.961	-1	-0.006711
173	4921	5075	45.41	5.852	1	0.006506
174	4921	5069	47.38	7.030	-4	-0.027153
175	4929	5077	47.15	9.612	0	0.000000
176	4946	5104	44.12	6.619	2	0.012640
177	4946	5107	43.41	4.798	2	0.012437
178	4948	5098	46.58	7.081	-1	-0.006673
179	4948	5097	46.97	10.790	-3	-0.020187
180	4956	5117	43.38	6.177	1	0.006215
181	4958	5118	43.50	4.403	1	0.006232
182	4968	5142	40.05	4.375	3	0.017215
183	4971	5115	48.40	10.386	-3	-0.020800
184	4974	5130	44.63	6.357	0	0.000000
185	4982	5120	50.93	6.523	-2	-0.014593
186	4992	5167	39.79	4.628	7	0.039907
187	4992	5145	45.51	9.588	-1	-0.006520
188	4997	5158	43.44	6.181	1	0.006224
189	5002	5149	47.35	5.126	-2	-0.013567
190	5004	5161	44.27	7.161	0	0.000000
191	5004	5178	40.11	7.439	4	0.022985
192	5007	5164	44.66	7.963	-1	-0.006399
193	5012	5171	43.95	6.858	1	0.006297

ID	Entry Time (s)	Exit Time (s)	Av.Sp (ft/s)	St.Dev	Overtakes	A_i (veh/s)
194	5018	5165	47.42	6.549	-2	-0.013587
195	5021	5182	43.16	5.014	1	0.006183
196	5028	5155	54.77	14.754	-8	-0.062777
197	5031	5205	40.24	5.003	6	0.034586
198	5033	5196	42.86	6.851	1	0.006140
199	5037	5187	46.47	5.364	-2	-0.013316
200	5044	5203	43.72	5.505	2	0.012527
201	5046	5190	48.48	8.698	-3	-0.020836
202	5047	5211	42.66	5.649	2	0.012223
203	5048	5198	46.41	5.351	-3	-0.019946
204	5058	5219	43.35	5.404	2	0.012421
205	5061	5203	49.09	9.561	-4	-0.028131
206	5074	5214	49.86	10.618	-1	-0.007143
207	5076	5240	42.65	4.997	3	0.018331
208	5076	5239	42.98	5.695	1	0.006157
209	5088	5238	46.52	5.126	-1	-0.006665
210	5092	5253	43.34	5.235	1	0.006210
211	5093	5255	43.34	5.104	1	0.006209
212	5094	5234	49.78	8.380	-5	-0.035661
213	5105	5269	42.51	4.706	2	0.012180
214	5105	5278	40.44	1.933	4	0.023177
215	5111	5259	47.38	5.033	-1	-0.006788
216	5112	5256	48.61	7.160	-3	-0.020891
217	5113	5277	42.56	5.839	-1	-0.006097
218	5120	5284	42.57	4.759	2	0.012199
219	5125	5277	45.92	5.740	-2	-0.013158
220	5128	5290	42.94	6.985	3	0.018456
221	5132	5293	43.25	5.525	3	0.018588
222	5134	5281	47.42	6.172	-3	-0.020382
223	5135	5284	46.66	6.975	-2	-0.013369
224	5139	5301	43.04	5.090	3	0.018497
225	5143	5306	42.85	5.318	3	0.018416
226	5148	5298	46.59	7.431	0	0.000000
227	5151	5298	47.50	6.397	-2	-0.013611
228	5151	5288	50.85	10.675	-6	-0.043714
229	5162	5317	45.05	7.254	3	0.019361
230	5163	5314	46.15	4.873	-1	-0.006612
231	5167	5330	42.84	4.083	3	0.018414
232	5169	5316	47.35	3.435	-1	-0.006783

ID	Entry Time (s)	Exit Time (s)	Av.Sp (ft/s)	St.Dev	Overtakes	A_i (veh/s)
233	5172	5331	43.96	5.674	2	0.012596
234	5178	5314	51.10	4.754	-4	-0.029284
235	5180	5344	42.51	4.072	3	0.018269
236	5185	5345	43.41	3.813	3	0.018659
237	5188	5330	49.08	5.312	-4	-0.028129
238	5190	5341	46.41	6.401	-1	-0.006649
239	5194	5354	43.80	4.826	2	0.012551
240	5196	5359	42.64	4.290	2	0.012217
241	5197	5337	49.54	6.211	-5	-0.035490
242	5198	5365	41.86	3.936	1	0.005997
243	5199	5347	47.31	4.743	-3	-0.020335
244	5207	5368	43.14	6.095	0	0.000000
245	5210	5371	43.29	3.449	0	0.000000
246	5210	5371	43.45	5.169	0	0.000000
247	5211	5374	42.67	4.770	0	0.000000
248	5228	5392	42.49	3.538	1	0.006088
249	5229	5389	43.63	5.365	-1	-0.006250
250	5230	5397	41.70	3.793	2	0.011949
251	5232	5396	42.45	3.568	0	0.000000
252	5242	5409	42.00	4.258	3	0.018053
253	5244	5408	42.46	5.383	1	0.006082
254	5246	5401	45.14	3.976	-1	-0.006466
255	5254	5421	41.81	3.906	2	0.011981
256	5261	5434	40.26	4.199	4	0.023072
257	5262	5416	45.45	3.825	-1	-0.006511
258	5267	5433	42.00	5.648	0	0.000000
259	5274	5394	58.49	15.393	-9	-0.075419
260	5275	5446	40.73	4.018	2	0.011669
261	5278	5445	41.74	2.751	0	0.000000
262	5281	5455	40.03	4.167	3	0.017205
263	5283	5434	46.40	2.983	-4	-0.026589
264	5287	5448	43.34	5.477	-1	-0.006210
265	5287	5461	40.14	4.850	2	0.011500
266	5290	5450	43.65	6.967	-2	-0.012506
267	5295	5462	41.65	7.474	1	0.005967
268	5295	5471	39.69	5.655	2	0.011373
269	5296	5463	41.82	3.033	0	0.000000
270	5300	5478	39.37	6.584	1	0.005641
271	5302	5460	44.24	6.293	-5	-0.031691

ID	Entry Time (s)	Exit Time (s)	Av.Sp (ft/s)	St.Dev	Overtakes	A_i (veh/s)
272	5308	5481	40.26	2.561	1	0.005768
273	5309	5486	39.44	7.592	1	0.005651
274	5312	5493	38.49	6.665	3	0.016543
275	5314	5488	40.21	4.475	0	0.000000
276	5317	5480	43.03	5.278	-4	-0.024660
277	5320	5493	40.30	2.761	-1	-0.005773
278	5329	5504	39.82	6.731	0	0.000000
279	5331	5507	39.67	4.400	1	0.005683
280	5331	5532	34.82	6.673	7	0.034915
281	5332	5515	38.10	6.227	0	0.000000
282	5338	5506	41.69	4.222	-3	-0.017920
283	5345	5521	39.59	2.906	0	0.000000
284	5346	5530	38.06	6.537	1	0.005453
285	5349	5515	42.12	6.228	-3	-0.018104
286	5353	5553	34.81	7.140	4	0.019951
287	5354	5522	41.44	4.724	-3	-0.017810
288	5355	5531	39.53	4.064	-2	-0.011327
289	5361	5541	38.79	5.812	0	0.000000
290	5369	5539	40.97	5.641	-2	-0.011740
291	5371	5571	34.89	7.235	3	0.014998
292	5381	5568	37.38	5.383	1	0.005355
293	5385	5587	34.60	7.553	6	0.029745
294	5387	5588	34.77	4.332	6	0.029891
295	5388	5558	40.98	5.502	-4	-0.023483
296	5391	5560	41.15	4.189	-4	-0.023581
297	5394	5576	38.54	3.398	-1	-0.005522
298	5396	5583	37.29	5.465	0	0.000000
299	5399	5572	40.42	5.756	-4	-0.023162
300	5404	5599	35.79	6.627	3	0.015381



(a) Vehicle with $ID = 1$

(b) Vehicle with $ID = 11$

(c) Vehicle with $ID = 21$

(d) Vehicle with $ID = 31$

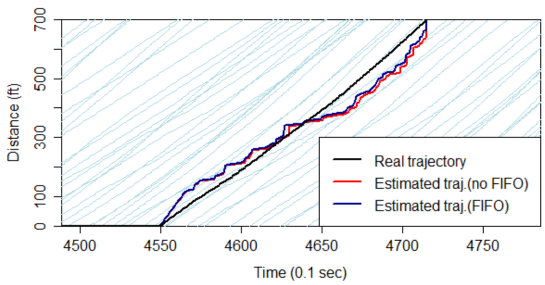
(e) Vehicle with $ID = 41$

(f) Vehicle with $ID = 51$

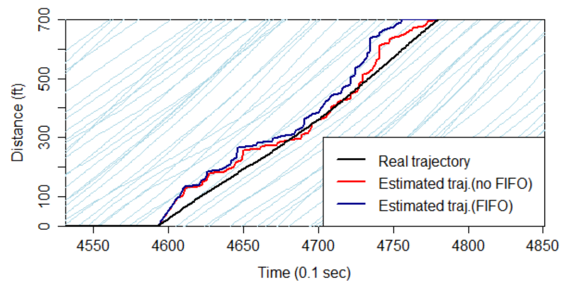
(g) Vehicle with $ID = 61$

(h) Vehicle with $ID = 71$

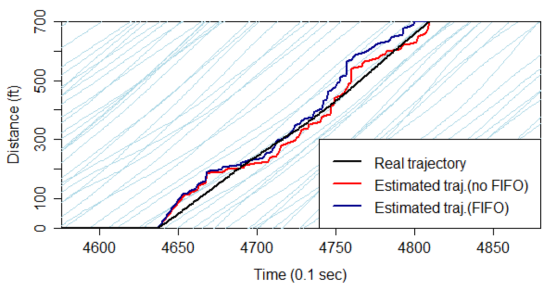
Figure 18: Plots of diferent trajectories in Dataset 2



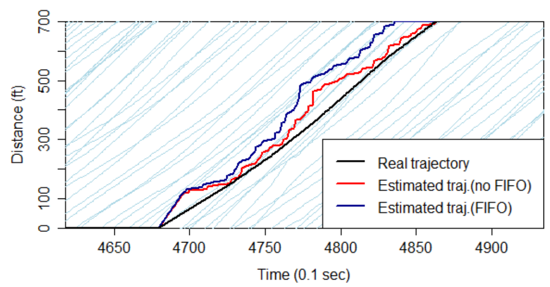
(a) Vehicle with $ID = 81$



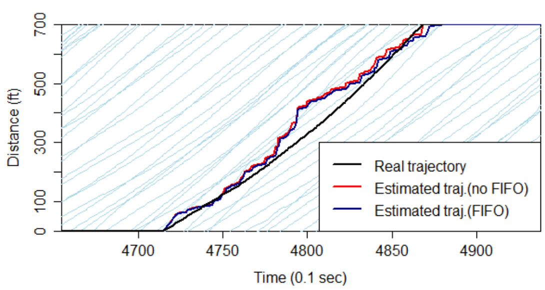
(b) Vehicle with $ID = 91$



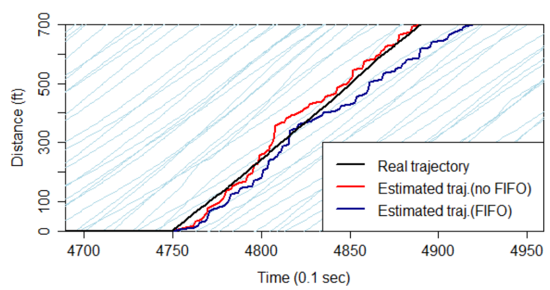
(c) Vehicle with $ID = 101$



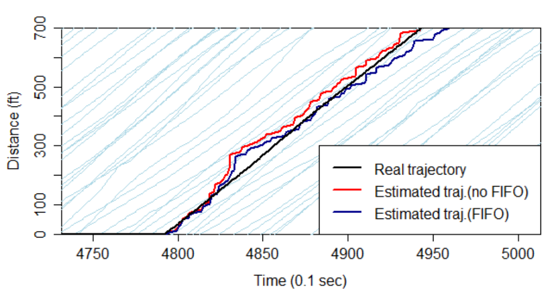
(d) Vehicle with $ID = 111$



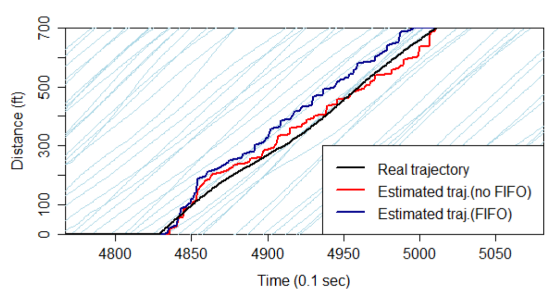
(e) Vehicle with $ID = 121$



(f) Vehicle with $ID = 131$

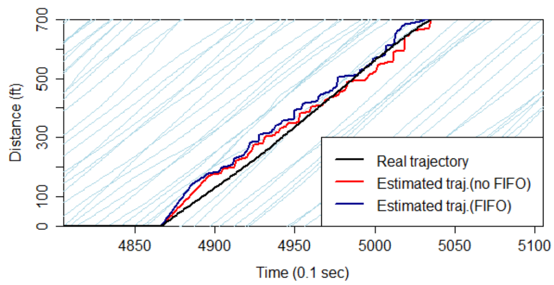


(g) Vehicle with $ID = 141$

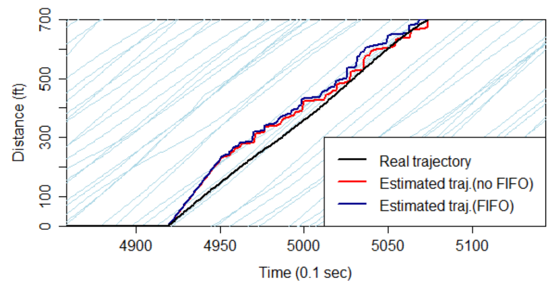


(h) Vehicle with $ID = 151$

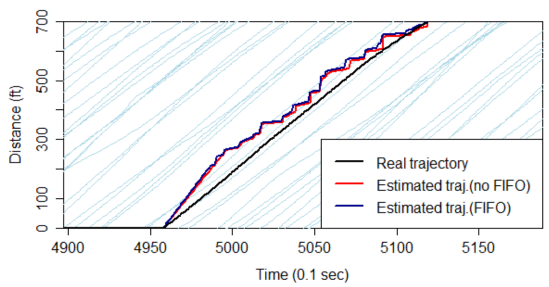
Figure 19: Plots of diferent trajectories in Dataset 2



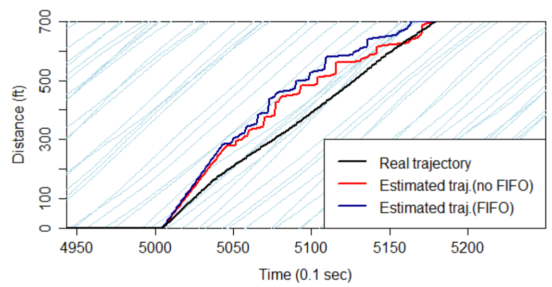
(a) Vehicle with $ID = 161$



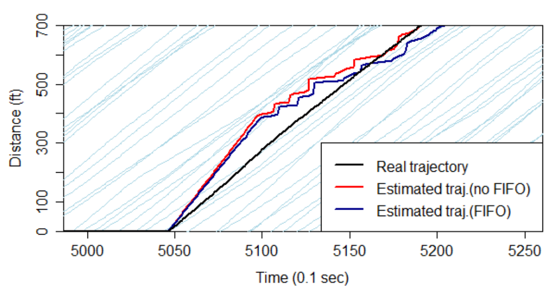
(b) Vehicle with $ID = 171$



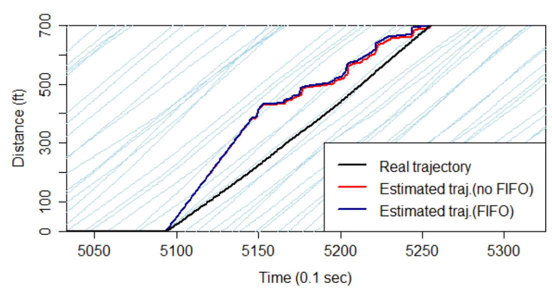
(c) Vehicle with $ID = 181$



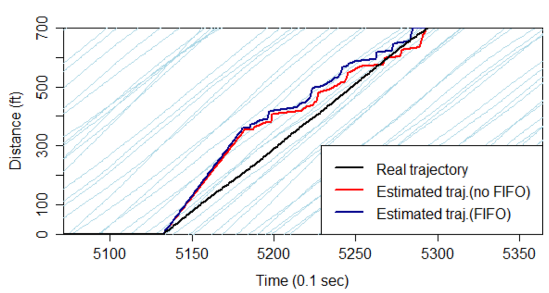
(d) Vehicle with $ID = 191$



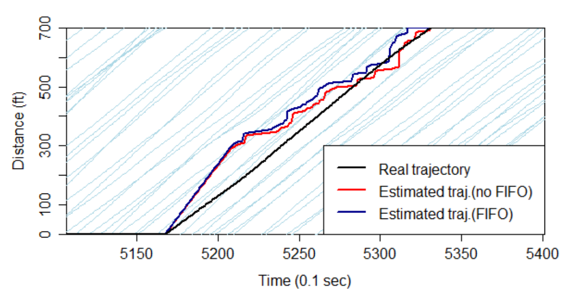
(e) Vehicle with $ID = 201$



(f) Vehicle with $ID = 211$

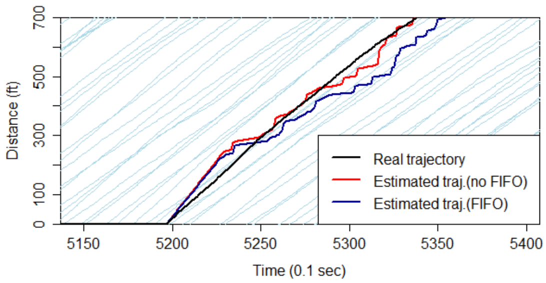


(g) Vehicle with $ID = 221$

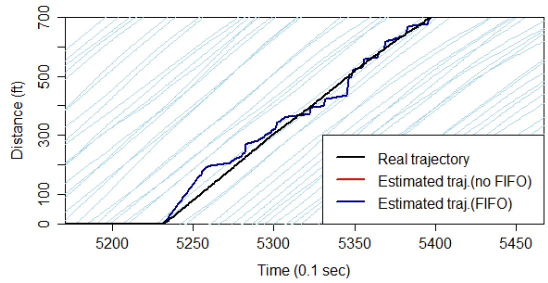


(h) Vehicle with $ID = 231$

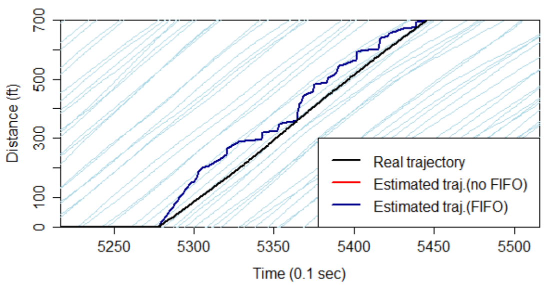
Figure 20: Plots of diferent trajectories in Dataset 2



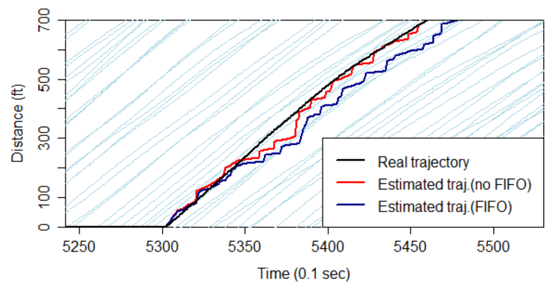
(a) Vehicle with $ID = 241$



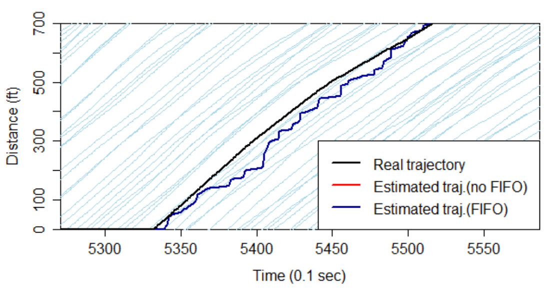
(b) Vehicle with $ID = 251$



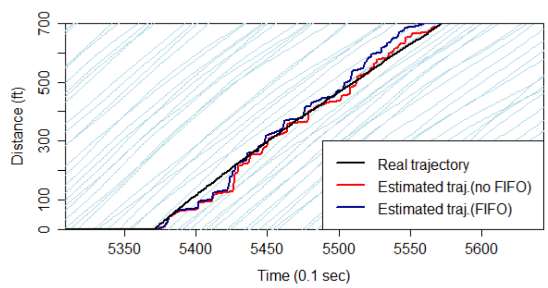
(c) Vehicle with $ID = 261$



(d) Vehicle with $ID = 271$



(e) Vehicle with $ID = 281$



(f) Vehicle with $ID = 291$

Figure 21: Plots of diferent trajectories in Dataset 2

A.3 Dataset 3

ID	Entry Time (s)	Exit Time (s)	Av.Sp (ft/s)	St.Dev	Overtakes	A_i (veh/s)
1	4205	4501	23.58	10.390	15	0.050679
2	4208	4367	43.81	10.760	-13	-0.081593
3	4212	4474	26.59	11.001	8	0.030477
4	4219	4383	42.60	14.305	-12	-0.073244
5	4224	4431	33.72	11.609	-4	-0.019321
6	4228	4402	40.03	17.808	-11	-0.063079
7	4230	4523	23.84	10.850	14	0.047816
8	4241	4444	34.40	6.411	-5	-0.024639
9	4242	4451	33.48	11.262	-5	-0.023981
10	4253	4492	29.20	11.728	4	0.016733
11	4254	4536	24.71	9.991	15	0.053101
12	4257	4453	35.68	8.567	-7	-0.035787
13	4264	4473	33.47	12.182	-3	-0.014384
14	4266	4480	32.63	4.940	-2	-0.009350
15	4272	4463	36.41	8.346	-7	-0.036510
16	4281	4513	30.04	11.510	1	0.004304
17	4282	4550	26.11	10.298	12	0.044883
18	4287	4403	60.52	22.741	-22	-0.190766
19	4296	4496	34.83	5.695	-4	-0.019962
20	4298	4518	31.74	10.114	-1	-0.004547
21	4299	4529	30.33	11.441	2	0.008691
22	4306	4484	39.35	7.704	-9	-0.050736
23	4308	4568	26.80	10.424	12	0.046072
24	4313	4532	31.92	10.738	1	0.004573
25	4324	4514	36.74	6.205	-7	-0.036846
26	4325	4519	36.04	13.005	-6	-0.030978
27	4329	4556	30.76	12.459	5	0.022036
28	4334	4548	32.66	11.079	0	0.000000
29	4340	4531	36.57	8.486	-5	-0.026197
30	4345	4544	34.99	10.292	-3	-0.015041
31	4346	4550	34.19	14.336	-1	-0.004899
32	4350	4586	29.58	10.909	9	0.038142
33	4352	4559	33.74	12.335	0	0.000000
34	4356	4565	33.50	5.218	0	0.000000
35	4362	4554	36.24	7.687	-4	-0.020768
36	4367	4523	44.84	14.089	-14	-0.089940
37	4369	4571	34.43	12.787	0	0.000000

ID	Entry Time (s)	Exit Time (s)	Av.Sp (ft/s)	St.Dev	Overtakes	A_i (veh/s)
38	4374	4583	33.29	13.579	0	0.000000
39	4374	4569	35.79	7.189	-3	-0.015383
40	4376	4610	29.72	7.910	5	0.021289
41	4387	4597	33.14	12.929	2	0.009494
42	4387	4585	35.35	11.000	-2	-0.010130
43	4389	4584	35.74	12.149	-4	-0.020484
44	4389	4622	29.92	8.758	5	0.021431
45	4393	4597	34.31	11.131	-3	-0.014745
46	4399	4610	33.15	14.351	-2	-0.009498
47	4408	4611	34.39	9.560	-1	-0.004926
48	4414	4611	35.38	10.718	-1	-0.005069
49	4415	4629	32.71	7.936	3	0.014057
50	4415	4651	29.56	9.644	8	0.033875
51	4431	4645	32.62	13.900	3	0.014019
52	4431	4669	29.33	9.780	11	0.046216
53	4432	4649	32.15	8.152	4	0.018427
54	4432	4624	36.28	14.938	-3	-0.015591
55	4433	4623	36.73	10.329	-5	-0.026309
56	4435	4620	37.88	8.639	-8	-0.043418
57	4444	4682	29.24	6.435	8	0.033517
58	4444	4657	32.83	14.075	1	0.004704
59	4449	4640	36.56	10.128	-6	-0.031423
60	4450	4660	33.25	6.466	0	0.000000
61	4458	4698	29.05	10.240	9	0.037454
62	4458	4646	37.11	15.137	-6	-0.031899
63	4459	4680	31.65	13.456	1	0.004534
64	4461	4683	31.41	8.731	2	0.009001
65	4465	4661	35.57	14.207	-4	-0.020382
66	4465	4694	30.50	7.486	3	0.013108
67	4474	4715	29.05	8.182	7	0.029132
68	4475	4661	37.54	15.796	-6	-0.032271
69	4476	4646	41.17	11.027	-14	-0.082578
70	4481	4690	33.42	9.031	-2	-0.009577
71	4487	4717	30.40	7.467	4	0.017420
72	4488	4731	28.71	10.591	7	0.028797
73	4498	4741	28.76	7.570	8	0.032961
74	4499	4688	36.83	15.982	-7	-0.036937
75	4504	4712	33.63	12.487	-3	-0.014455
76	4507	4712	33.93	9.216	-3	-0.014581

ID	Entry Time (s)	Exit Time (s)	Av.Sp (ft/s)	St.Dev	Overtakes	A_i (veh/s)
77	4509	4756	28.21	7.640	8	0.032338
78	4511	4754	28.69	10.613	5	0.020553
79	4517	4709	36.42	15.471	-8	-0.041740
80	4520	4738	32.12	11.745	0	0.000000
81	4523	4729	34.01	9.141	-3	-0.014617
82	4525	4793	25.98	9.522	15	0.055822
83	4529	4769	29.11	11.016	4	0.016683
84	4535	4753	32.01	11.455	-2	-0.009171
85	4537	4726	36.90	4.829	-9	-0.047573
86	4539	4820	24.91	9.432	15	0.053539
87	4543	4727	38.03	17.716	-10	-0.054478
88	4546	4790	28.56	10.832	6	0.024551
89	4552	4780	30.65	12.876	3	0.013171
90	4555	4757	34.56	9.686	-4	-0.019806
91	4560	4802	28.83	11.502	7	0.028913
92	4568	4769	34.79	14.262	-4	-0.019936
93	4573	4777	34.17	4.928	-3	-0.014688
94	4574	4754	38.86	18.513	-10	-0.055667
95	4575	4772	35.46	8.541	-6	-0.030484
96	4581	4807	30.92	10.529	3	0.013289
97	4586	4777	36.53	11.720	-6	-0.031402
98	4589	4843	27.51	9.110	9	0.035475
99	4593	4839	28.44	10.516	6	0.024450
100	4600	4791	36.62	3.902	-4	-0.020985
101	4601	4790	36.96	19.080	-8	-0.042356
102	4603	4857	27.43	10.327	9	0.035370
103	4605	4811	33.93	14.141	-3	-0.014582
104	4612	4830	32.04	12.811	-1	-0.004591
105	4615	4865	28.00	10.165	7	0.028078
106	4616	4790	40.00	15.600	-11	-0.063032
107	4618	4905	24.27	7.786	15	0.052157
108	4619	4835	32.40	15.119	-4	-0.018567
109	4627	4846	31.87	12.940	-1	-0.004566
110	4630	4944	22.22	8.497	19	0.060471
111	4632	4917	24.48	9.676	13	0.045600
112	4637	4822	37.59	15.903	-10	-0.053858
113	4639	4855	32.34	15.021	-3	-0.013899
114	4650	4881	30.22	12.329	1	0.004330
115	4652	4886	29.77	11.789	2	0.008529

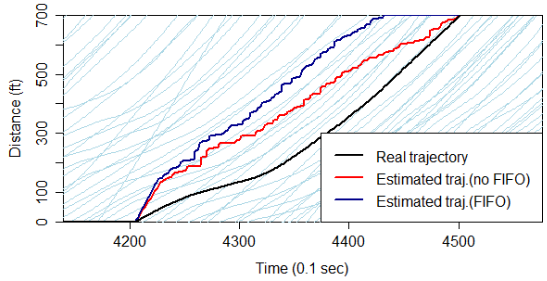
ID	Entry Time (s)	Exit Time (s)	Av.Sp (ft/s)	St.Dev	Overtakes	A_i (veh/s)
116	4653	4868	32.46	13.083	-3	-0.013951
117	4658	4978	21.82	9.752	19	0.059389
118	4663	4841	39.31	17.261	-12	-0.067579
119	4666	4883	32.21	14.333	-3	-0.013843
120	4667	4892	31.02	13.309	-1	-0.004444
121	4667	4855	37.23	15.844	-12	-0.063998
122	4680	4996	22.11	9.841	16	0.050676
123	4682	4901	31.93	14.484	-2	-0.009150
124	4683	4879	35.55	13.965	-10	-0.050929
125	4686	4888	34.56	14.744	-7	-0.034661
126	4690	5007	21.99	10.593	16	0.050397
127	4692	4926	29.79	12.566	0	0.000000
128	4703	4921	31.97	13.907	-3	-0.013742
129	4706	4899	36.08	14.481	-9	-0.046519
130	4707	5073	19.11	11.945	25	0.068459
131	4714	5053	20.62	11.383	19	0.056123
132	4720	4915	35.83	15.887	-9	-0.046195
133	4724	5092	18.97	12.094	26	0.070654
134	4724	4946	31.50	13.714	-4	-0.018054
135	4726	4925	35.14	13.817	-9	-0.045304
136	4727	4962	29.64	11.784	-4	-0.016989
137	4742	4943	34.69	14.872	-9	-0.044732
138	4742	4971	30.58	12.795	-4	-0.017526
139	4743	5112	18.92	12.099	23	0.062339
140	4748	4979	30.18	12.707	-3	-0.012971
141	4748	4949	34.81	13.299	-10	-0.049868
142	4761	5001	29.01	11.994	-2	-0.008312
143	4766	4971	33.97	12.724	-8	-0.038939
144	4767	5137	18.83	12.040	24	0.064743
145	4769	4970	34.66	14.654	-12	-0.059582
146	4771	5008	29.43	12.695	-3	-0.012649
147	4779	5036	27.20	11.372	0	0.000000
148	4784	5160	18.59	12.611	24	0.063923
149	4786	4997	33.14	12.161	-10	-0.047482
150	4787	5023	29.55	13.035	-5	-0.021165
151	4791	5001	33.09	13.867	-10	-0.047411
152	4802	5069	26.15	11.677	2	0.007493
153	4804	5010	33.82	13.623	-9	-0.043607
154	4804	5029	31.01	12.209	-8	-0.035541

ID	Entry Time (s)	Exit Time (s)	Av.Sp (ft/s)	St.Dev	Overtakes	A_i (veh/s)
155	4814	5188	18.66	11.166	25	0.066848
156	4818	5060	28.85	13.492	-5	-0.020665
157	4819	5100	24.85	11.045	4	0.014239
158	4823	5039	32.36	15.923	-10	-0.046359
159	4829	5136	22.71	10.769	8	0.026029
160	4833	5050	32.24	14.125	-11	-0.050801
161	4838	5120	24.76	10.613	3	0.010640
162	4838	5088	27.94	13.991	-5	-0.020013
163	4844	5064	31.70	14.268	-10	-0.045408
164	4849	5063	32.72	12.228	-12	-0.056246
165	4850	5214	19.15	11.917	19	0.052129
166	4851	5137	24.35	10.211	3	0.010467
167	4863	5113	27.92	14.114	-4	-0.015998
168	4863	5092	30.55	13.183	-10	-0.043762
169	4870	5235	19.14	11.792	20	0.054848
170	4874	5160	24.39	9.534	3	0.010484
171	4891	5186	23.68	10.813	7	0.023748
172	4892	5127	29.66	12.028	-6	-0.025492
173	4892	5120	30.59	10.543	-8	-0.035057
174	4897	5078	38.49	11.166	-18	-0.099247
175	4903	5141	29.35	12.327	-5	-0.021025
176	4906	5096	36.84	15.474	-16	-0.084452
177	4913	5212	23.37	11.382	6	0.020086
178	4917	5164	28.31	12.083	-3	-0.012169
179	4921	5184	26.51	7.288	-2	-0.007596
180	4928	5231	23.02	12.566	8	0.026378
181	4928	5151	31.39	15.287	-10	-0.044971
182	4941	5172	30.33	15.095	-6	-0.026071
183	4944	5188	28.57	13.155	-4	-0.016372
184	4944	5223	25.03	6.498	1	0.003586
185	4946	5253	22.71	12.826	7	0.022776
186	4954	5287	20.94	11.249	11	0.033003
187	4958	5210	27.72	12.951	-5	-0.019857
188	4963	5204	29.03	13.733	-7	-0.029114
189	4965	5249	24.51	6.592	2	0.007023
190	4965	5278	22.33	13.238	6	0.019199
191	4967	5310	20.32	11.684	10	0.029115
192	4982	5337	19.68	12.868	14	0.039472
193	4984	5277	23.81	6.361	2	0.006824

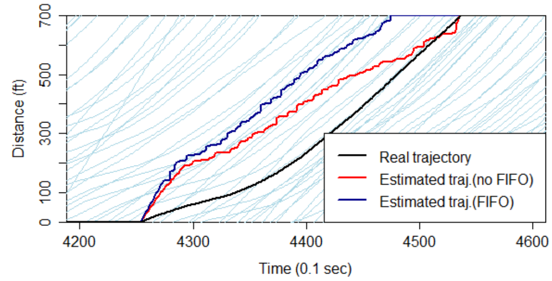
ID	Entry Time (s)	Exit Time (s)	Av.Sp (ft/s)	St.Dev	Overtakes	A_i (veh/s)
194	4985	5236	27.76	12.520	-4	-0.015908
195	4985	5227	28.83	13.407	-9	-0.037168
196	4988	5307	21.90	12.071	4	0.012549
197	5001	5376	18.63	13.156	14	0.037367
198	5001	5335	20.94	12.337	7	0.020995
199	5002	5261	26.91	12.479	-5	-0.019277
200	5003	5304	23.18	6.673	-1	-0.003321
201	5010	5260	27.99	13.039	-8	-0.032086
202	5016	5407	17.87	14.810	14	0.035833
203	5017	5331	22.21	6.810	1	0.003182
204	5018	5162	48.29	11.139	-30	-0.207568
205	5036	5298	26.60	12.446	-7	-0.026672
206	5041	5320	25.07	11.851	-4	-0.014367
207	5042	5365	21.60	8.094	2	0.006189
208	5044	5475	16.23	14.580	19	0.044179
209	5053	5320	26.08	11.917	-6	-0.022415
210	5057	5389	21.01	8.925	2	0.006021
211	5059	5368	22.57	10.798	-1	-0.003234
212	5068	5507	15.90	13.530	20	0.045557
213	5070	5351	24.80	12.181	-6	-0.021315
214	5070	5358	24.25	11.521	-6	-0.020845
215	5073	5420	20.11	10.578	5	0.014406
216	5085	5397	22.32	11.421	-2	-0.006395
217	5091	5529	15.96	13.251	18	0.041159
218	5096	5404	22.64	12.619	-3	-0.009732
219	5097	5392	23.68	11.250	-6	-0.020353
220	5110	5419	22.55	11.867	-1	-0.003231
221	5111	5230	58.78	36.658	-34	-0.286319
222	5112	5560	15.58	14.298	20	0.044637
223	5113	5419	22.86	11.401	-5	-0.016379
224	5118	5443	21.49	12.407	-1	-0.003078
225	5121	5464	20.39	9.628	-1	-0.002921
226	5136	5438	23.10	12.470	-4	-0.013240
227	5144	5497	19.75	9.741	3	0.008487
228	5146	5489	20.34	11.234	1	0.002914
229	5147	5580	16.10	14.071	18	0.041515
230	5147	5473	21.41	11.913	-4	-0.012269
231	5162	5516	19.73	12.249	3	0.008482
232	5168	5507	20.57	11.622	1	0.002947

ID	Entry Time (s)	Exit Time (s)	Av.Sp (ft/s)	St.Dev	Overtakes	A_i (veh/s)
233	5169	5471	23.09	12.765	-8	-0.026459
234	5169	5597	16.31	16.119	15	0.035059
235	5177	5536	19.45	8.676	2	0.005573
236	5182	5540	19.51	12.418	3	0.008384
237	5192	5550	19.47	12.543	3	0.008369
238	5193	5558	19.16	8.971	3	0.008235
239	5199	5505	22.78	12.029	-8	-0.026114
240	5200	5538	20.62	11.610	-2	-0.005910
241	5206	5571	19.10	13.287	3	0.008209
242	5214	5628	16.83	16.149	10	0.024113
243	5214	5720	13.80	12.309	19	0.037564
244	5218	5534	22.09	11.586	-8	-0.025320
245	5224	5565	20.43	12.520	-2	-0.005854
246	5224	5580	19.63	8.590	0	0.000000
247	5227	5438	33.14	16.449	-26	-0.123426
248	5235	5416	38.55	9.682	-31	-0.171204
249	5247	5609	19.26	9.398	2	0.005519
250	5249	5589	20.56	13.155	-2	-0.005891
251	5256	5661	17.23	14.327	5	0.012346
252	5258	5578	21.82	10.939	-7	-0.021879
253	5263	5700	15.99	11.516	8	0.018324
254	5277	5630	19.74	13.363	-1	-0.002828
255	5277	5692	16.82	11.795	3	0.007229
256	5279	5478	35.10	16.108	-28	-0.140803
257	5280	5657	18.49	10.129	-2	-0.005297
258	5287	5739	15.45	13.013	5	0.011065
259	5297	5752	15.34	12.421	7	0.015383
260	5306	5670	19.18	14.238	-3	-0.008244
261	5308	5910	11.59	15.073	27	0.044839
262	5311	5696	18.12	11.880	-3	-0.007786
263	5311	5839	13.23	13.797	11	0.020851
264	5311	5868	12.53	16.579	15	0.026932
265	5324	5697	18.70	15.543	-5	-0.013396
266	5327	5861	13.08	13.920	11	0.020620
267	5328	5825	14.05	13.221	5	0.010068
268	5329	5886	12.53	15.158	15	0.026917
269	5335	5787	15.45	13.728	-1	-0.002213
270	5335	5755	16.65	12.633	-3	-0.007157
271	5340	5739	17.46	14.489	-7	-0.017511

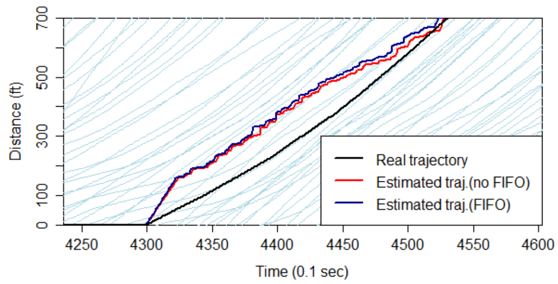
ID	Entry Time (s)	Exit Time (s)	Av.Sp (ft/s)	St.Dev	Overtakes	A_i (veh/s)
272	5352	5869	13.50	13.872	8	0.015473
273	5354	5638	24.51	19.321	-19	-0.066723
274	5367	5799	16.16	8.846	-4	-0.009263
275	5368	5954	11.92	14.752	21	0.035852
276	5370	5897	13.25	14.689	10	0.018976
277	5370	5789	16.67	13.774	-8	-0.019102
278	5373	5897	13.33	13.862	9	0.017182
279	5381	5929	12.75	15.488	13	0.023738
280	5385	5912	13.22	14.313	10	0.018941
281	5386	5843	15.26	15.252	-6	-0.013116
282	5393	5830	15.95	11.406	-9	-0.020566
283	5394	5972	12.08	14.766	16	0.027694
284	5402	5984	11.99	16.626	19	0.032627
285	5406	5881	14.69	15.066	-3	-0.006315
286	5409	5928	13.45	13.277	5	0.009634
287	5415	5597	38.25	19.195	-37	-0.202746
288	5420	5862	15.80	9.312	-10	-0.022640
289	5421	5976	12.59	14.489	12	0.021646
290	5426	6021	11.73	12.620	20	0.033608
291	5427	5911	14.42	15.031	-2	-0.004131
292	5432	5751	21.91	16.008	-27	-0.084764
293	5435	5957	13.37	13.191	4	0.007665
294	5444	5993	12.70	14.725	11	0.020016
295	5445	5890	15.71	11.766	-10	-0.022505
296	5446	6036	11.83	12.728	18	0.030512
297	5448	5950	13.89	15.570	-2	-0.003981
298	5462	6060	11.68	13.293	22	0.036824
299	5465	5973	13.75	15.608	1	0.001970
300	5466	5982	13.52	12.432	2	0.003874



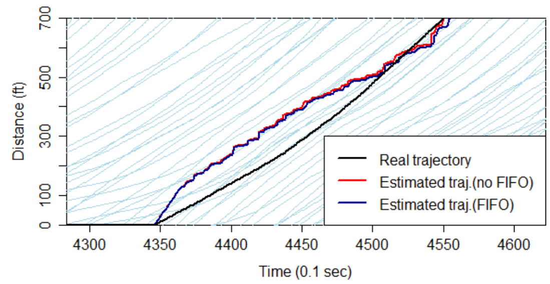
(a) Vehicle with $ID = 1$



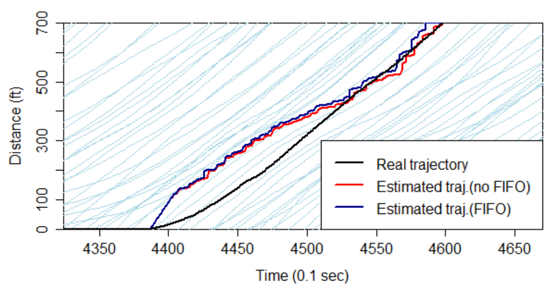
(b) Vehicle with $ID = 11$



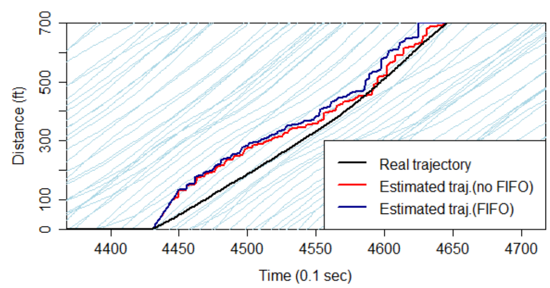
(c) Vehicle with $ID = 21$



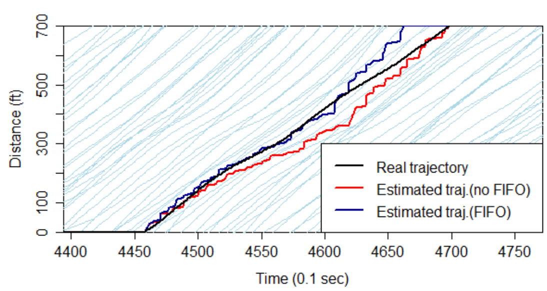
(d) Vehicle with $ID = 31$



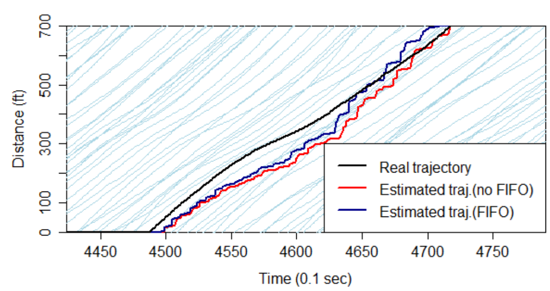
(e) Vehicle with $ID = 41$



(f) Vehicle with $ID = 51$

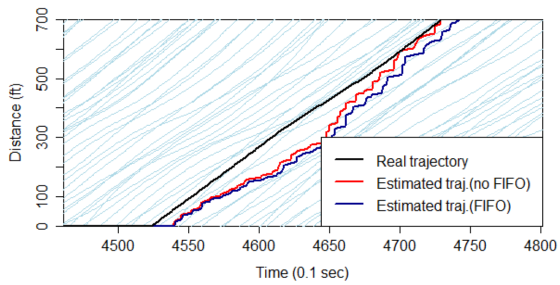


(g) Vehicle with $ID = 61$

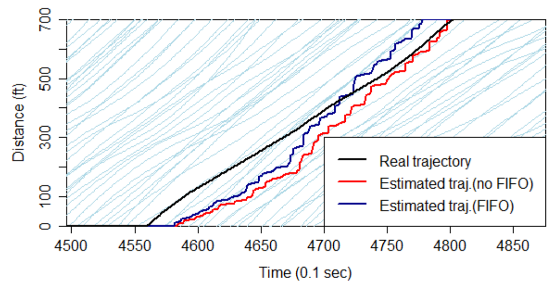


(h) Vehicle with $ID = 71$

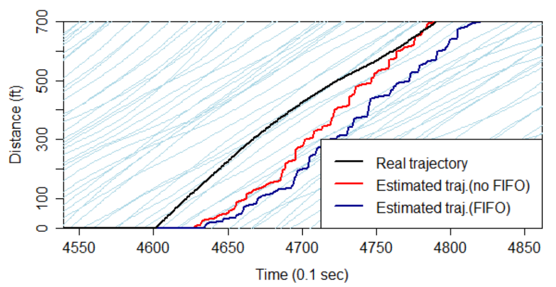
Figure 22: Plots of diferent trajectories in Dataset 3



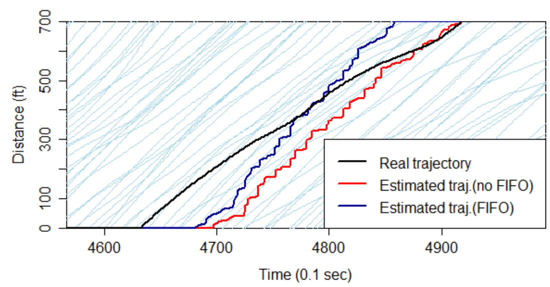
(a) Vehicle with $ID = 81$



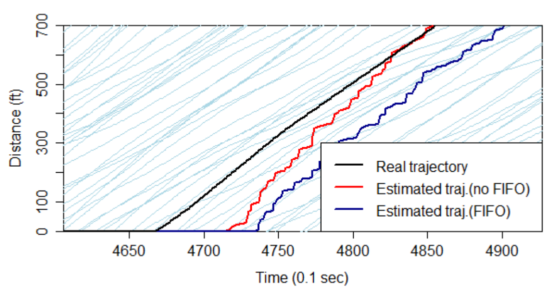
(b) Vehicle with $ID = 91$



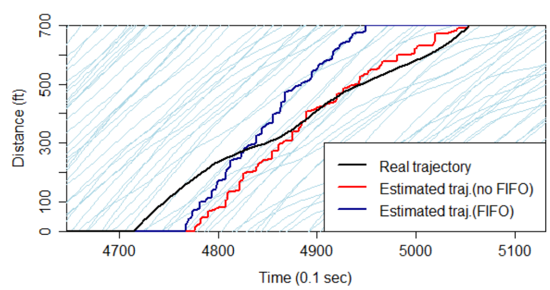
(c) Vehicle with $ID = 101$



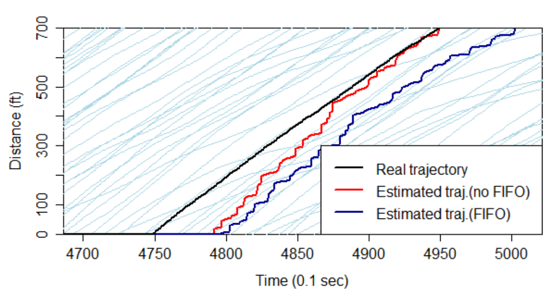
(d) Vehicle with $ID = 111$



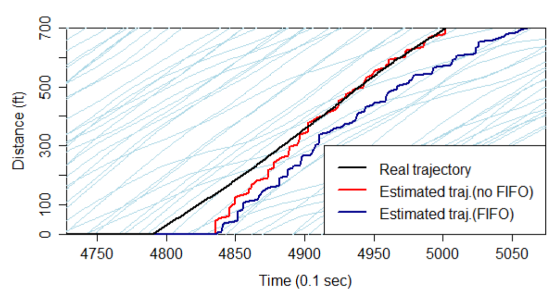
(e) Vehicle with $ID = 121$



(f) Vehicle with $ID = 131$

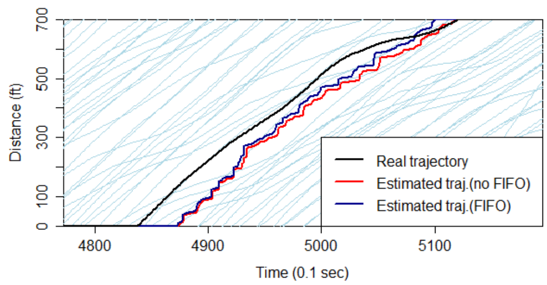


(g) Vehicle with $ID = 141$

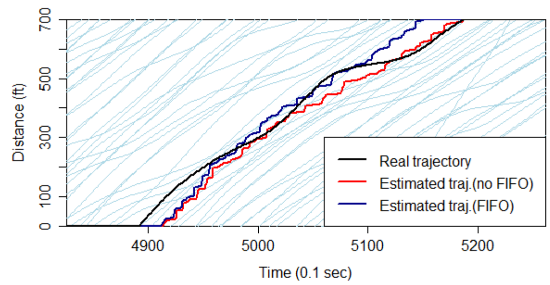


(h) Vehicle with $ID = 151$

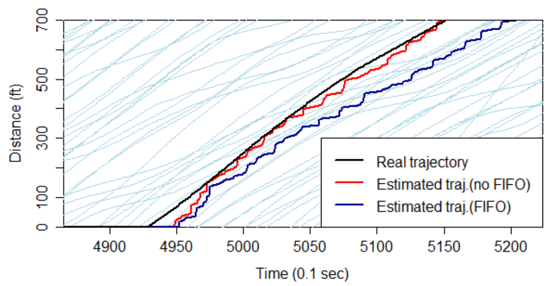
Figure 23: Plots of diferent trajectories in Dataset 3



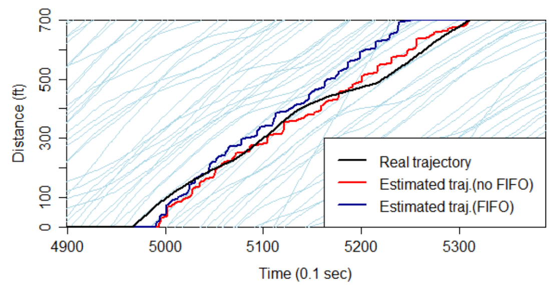
(a) Vehicle with $ID = 161$



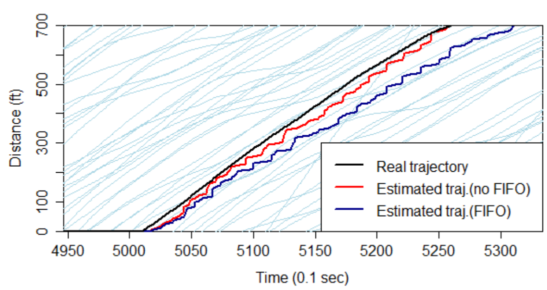
(b) Vehicle with $ID = 171$



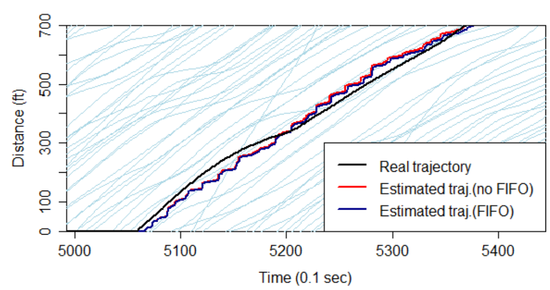
(c) Vehicle with $ID = 181$



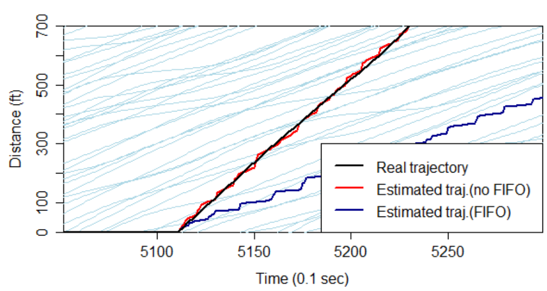
(d) Vehicle with $ID = 191$



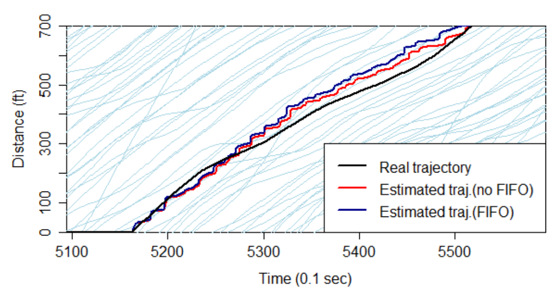
(e) Vehicle with $ID = 201$



(f) Vehicle with $ID = 211$

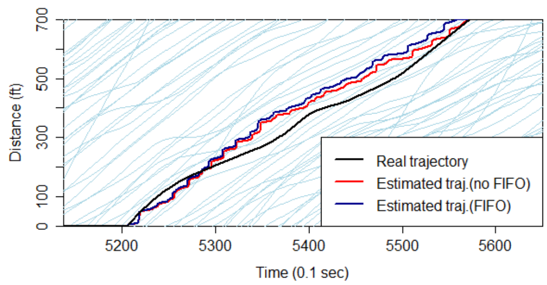


(g) Vehicle with $ID = 221$

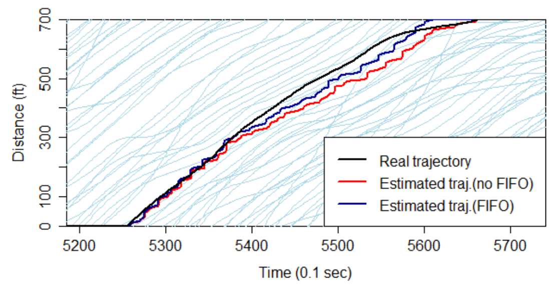


(h) Vehicle with $ID = 231$

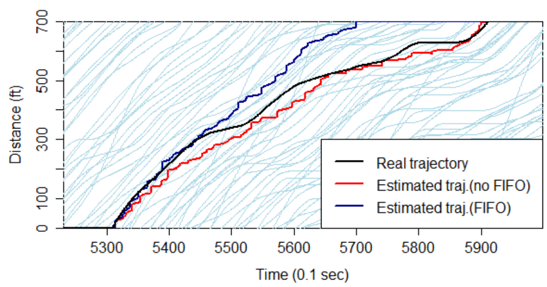
Figure 24: Plots of diferent trajectories in Dataset 3



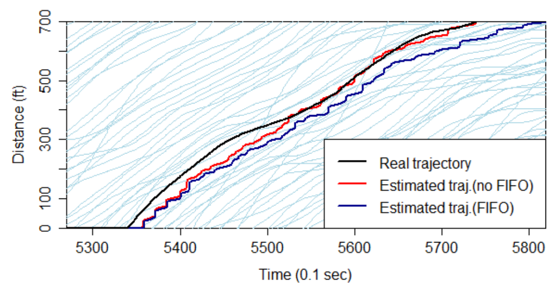
(a) Vehicle with $ID = 241$



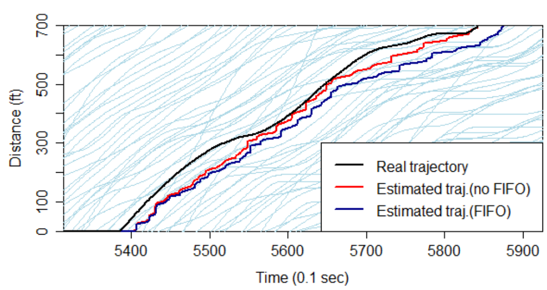
(b) Vehicle with $ID = 251$



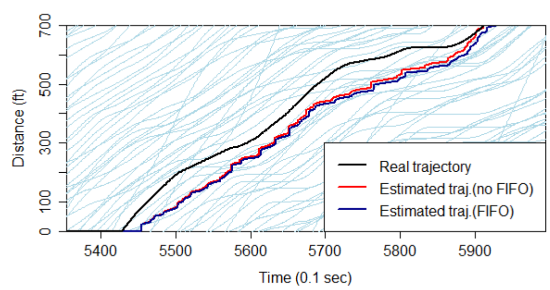
(c) Vehicle with $ID = 261$



(d) Vehicle with $ID = 271$



(e) Vehicle with $ID = 281$



(f) Vehicle with $ID = 291$

Figure 25: Plots of diferent trajectories in Dataset 3

B R Workspace

```
################### FUNCTIONS ###################

#Returns the entry and exit times of a vehicle in the road segment considered
summary.veh<-function(traj, start, end){
  result<-approx(traj$L_y,traj$Frame, xout=c(start,end))
  return(result[[2]])
}

#Returns a matrix with entry and exit times for all the vehicles, as well as the cumulative functions #F(t), G(t), their inverses and initial number of vehicles in the road segment
sumNGSIM<-function(traj1){
  start_y<-578
  end_y<-578+698
  l<-end_y-start_y
  traj1<-traj1[,c(1,2,6)]
  n=length(unique(traj1$ID))
  pb<-txtProgressBar(max=n,style=3)
  i=1
  t.travel=matrix(rep(-1,3*n),ncol=3)
  for (id in unique(traj1$ID)){
    t.travel[i,]<-c(id,summary.veh(traj1[traj1$ID==id,],start_y, end_y))
    i=i+1
    setTxtProgressBar(pb,i)
  }
  t.travel<-as.data.frame(t.travel);
  colnames(t.travel)<-c("ID","Enter","Leave")
  #construct cumulative flow at entrance/exit
  f.raw<-approxfun(sort(t.travel$Enter), seq(0:(sum(!is.na(t.travel$Enter))-1)),rule=2)
  g.raw<-approxfun(sort(t.travel$Leave), seq(0:(sum(!is.na(t.travel$Leave))-1)),rule=2)
  invf.raw<-approxfun(seq(0:(sum(!is.na(t.travel$Enter))-1)), sort(t.travel$Enter), rule=2)
  invg.raw<-approxfun(seq(0:(sum(!is.na(t.travel$Leave))-1)), sort(t.travel$Leave), rule=2)

  #initial number of vehicles (estimation starts at t=120s, the warm-up period)
  n0<-f.raw(1200)-g.raw(1200)
  #observed cumulative flow at entrance/exit
  ffun<-function(t){return(ifelse(t<1200,0,f.raw(t)-f.raw(1200)))}
  gfun<-function(t){return(ifelse(t<1200,0,g.raw(t)-g.raw(1200)))}
  invffun<-function(n){return(invf.raw(n+f.raw(1200)))}
  invgfun<-function(n){return(invg.raw(n+g.raw(1200)))}
  t.travel<-t.travel[t.travel$Enter>1200 & !is.na(rowSums(t.travel)),]
```

```
  return(list(t.travel,ffun,gfun,invffun,invgfun,n0,l,start_y,end_y))
}

#Applies the congested part of Newell's equation (useful to implement the bisection method)
form<-function(gfun, t, l, p, x){
  return(gfun(t-(l-x)/(20*5280/36000))+(156.51*5/5280)*(l-x)-p)
}

#Implementation of the bisection method to find "x" in the congested part of Newell's equation
bisection<-function(gfun, t, l, p, tol, max_iter){
  x1<-0
  x2<-l
  if (sign(form(gfun,t,l,p,x1))==sign(form(gfun,t,l,p,x2))){
    ifelse (sign(form(gfun,t,l,p,x1))>0, return(l), return(0))
  }
  else{
    it<-1
    while(it<=max_iter){
      x3<-(x1+x2)/2
      if(form(gfun,t,l,p,x3)==0 | (x2-x1)/2<tol){return(x3)}
      it<-it+1
      ifelse (sign(form(gfun,t,l,p,x3))==sign(form(gfun,t,l,p,x1)), x1<-x3, x2<-x3)
    }
    return(x3)
  }
}

################### MAIN CODE ###################

#Read and prepare data

traj1<-read.table("trajectories-0750am-0805am.txt",)
traj2<-read.table("trajectories-0805am-0820am.txt",)
traj3<-read.table("trajectories-0820am-0835am.txt",)
colnames(traj2)<-c("ID","Frame","Tot_frame","G_time","L_x","L_y","G_x","G_y","L","W","Class","V","Acc","Lane","P_veh","F_veh","Spacing","Headway")

sum.result<-sumNGSIM(traj1)
#vehicle entry/exit table
all.veh1<-sum.result[[1]]
#cumulative flow at upstream
ffun<-sum.result[[2]]
#cumulative flow at downstream
gfun<-sum.result[[3]]
#inverse cumulative flow at upstream
invffun<-sum.result[[4]]
```

```
#inverse cumulative flow at downstream
invgfun<-sum.result[[5]]
#initial number of vehicles
n0<-sum.result[[6]]
#length of the road segment
l<-sum.result[[7]]
start_y<-sum.result[[8]]
end_y<-sum.result[[9]]

#Data analysis for each vehicle (av.speed,st.dev,max,min,overtakes)

for (i in 1:length(all.veh3$ID)){
  id<-all.veh3[i,1]
  all.veh3$AvSpeed[i]<-10*l/(all.veh3$Leave[i]-all.veh3$Enter[i])
  all.veh3$StDev[i]<-sqrt(sum((traj3[traj3$ID==id,12]-all.veh3$AvSpeed[i])^2)/
      length(traj3[traj3$ID==id,12]))
  all.veh3$MaxV[i]<-max(traj3[traj3$ID==id,12])
  all.veh3$MinV[i]<-min(traj3[traj3$ID==id,12])
}
all.veh3=all.veh3[order(all.veh3[,2]),]
all.veh3$Entrypos=seq(1:n)
all.veh3=all.veh3[order(all.veh3[,3]),]
all.veh3$Exitpos=seq(1:n)
all.veh3$overtakes=all.veh3$Exitpos-all.veh3$Entrypos
all.veh3$Ai=all.veh3$overtakes/(all.veh3$Leave-all.veh3$Enter)

#Dimensions of the discretized segment

a<-floor(l)+1
b<-ceiling(max(all.veh1$Leave))
n=length(all.veh1$ID)

#Compute estimated trajectories of all the vehicles with the FIFO hypothesis

all.traj1.beta=matrix(nrow=b-1,ncol=n+1)
all.traj1.beta[,1]=seq(1:(b-1))
pb<-txtProgressBar(max=n,style=3)
for (p in 1:n){
  traj.result=matrix(nrow=b-1,ncol=4)
  traj.result[,1]=all.traj1.beta[,1]
  entrytime=all.veh1[p,2]
  p1=ffun(entrytime)
  for (i in 1200:(b-1)){
    traj.result[i,2]<-(62*5280/36000)*(i-invffun(p1))
    if(traj.result[i,2]<0){traj.result[i,2]<-0}
    else if(traj.result[i,2]>l){traj.result[i,2]<-l}
    traj.result[i,3]<-bisection(gfun,i,l,p1+n0,0.1,20)
    traj.result[i,4]<-min(traj.result[i,2],traj.result[i,3])
  }
```

```
  all.traj1.beta[,p+1]=traj.result[,4]
  setTxtProgressBar(pb,p)
}
all.traj1.beta<-as.data.frame(all.traj1.beta)

#Compute estimated trajectories of all the vehicles without the FIFO hypothesis

all.traj.nofifo1.beta=matrix(nrow=b-1,ncol=n+1)
all.traj.nofifo1.beta[,1]=seq(1:(b-1))
pb<-txtProgressBar(max=n,style=3)
for (p in 1:n){
  traj.result=matrix(nrow=b-1,ncol=4)
  traj.result[,1]=all.traj.nofifo1.beta[,1]
  entrytime=all.veh1[p,2]
  exittime=all.veh1[p,3]
  p1=ffun(entrytime)
  theta1=all.veh1$overtakes[p]/(exittime-entrytime)
  theta2=(p1*exittime-entrytime*(p1+all.veh1$overtakes[p]))/(exittime-entrytime)
  for (i in 1200:(b-1)){
    theta=theta1*i+theta2
    traj.result[i,2]<-(62*5280/36000)*(i-invffun(theta))
    if(traj.result[i,2]<0){traj.result[i,2]<-0}
    else if(traj.result[i,2]>l){traj.result[i,2]<-l}
    traj.result[i,3]<-bisection(gfun,i,l,theta+n0,0.1,20)
    traj.result[i,4]<-min(traj.result[i,2],traj.result[i,3])
  }
  all.traj.nofifo1.beta[,p+1]=traj.result[,4]
  setTxtProgressBar(pb,p)
}
all.traj.nofifo1.beta<-as.data.frame(all.traj.nofifo1.beta)

#Compute real trajectories for all vehicles using NGSIM datasets

real.traj1=matrix(0,nrow=9363,ncol=1895)
real.traj2=matrix(0,nrow=9688,ncol=1843)
real.traj3=matrix(0,nrow=9683,ncol=1699)
real.traj1[,1]=seq(1:9363)
real.traj2[,1]=seq(1:9688)
real.traj3[,1]=seq(1:9683)
pb<-txtProgressBar(max=1698,style=3)
for (p in 1:1698){
  id=all.veh3[p,1]
  traj.single<-traj3[traj3$ID==id,c(2,6)]
  traj.single$L_y<-traj.single$L_y-578
  traj.single$L_y[traj.single$L_y<0]<-0
  traj.single$L_y[traj.single$L_y>698]<-698
  frameinit=traj.single$Frame[1]
```

```
  framefinal=min(traj.single$Frame[length(traj.single$Frame)],9683)
  real.traj3[frameinit:framefinal,p+1]=traj.single[traj.single$Frame<=framefinal,2]
  setTxtProgressBar(pb,p)
}
real.traj3=as.data.frame(real.traj3)

#Compute errors in the estimated trajectories

pb<-txtProgressBar(max=1698,style=3)
for(p in (1:1698)){  #select a vehicle
  entrytime=all.veh3[p,2]
  exittime=all.veh3[p,3]

  traj.single=real.traj3[,c(1,p+1)]  #Real trajectory
  traj.single=as.data.frame(traj.single)
  colnames(traj.single)<-c("Frame", "L_y")
  traj.single=traj.single[traj.single$Frame>(entrytime) & traj.single$Frame<(exittime+40),]

  traj.result=all.traj.nofifo3.beta[,c(1,p+1)]  #Estimated traj. (no FIFO)
  traj.result=as.data.frame(traj.result)
  colnames(traj.result)<-c("Frame", "L_y")
  traj.result=traj.result[traj.result$Frame>(entrytime) & traj.result$Frame<(exittime+40),]

  er=seq(from=traj.result$Frame[1], to=traj.result$Frame[length(traj.result$Frame)], by=1)
  er<-as.data.frame(er)
  for(i in 1:length(er[,1])){
    c=traj.result$L_y[traj.result$Frame==er[i,1]]
    d=traj.single$L_y[traj.single$Frame==er[i,1]]
    er[i,2]=abs(c-d)
    er[i,3]=d
  }
  colnames(er)<-c("Frame","Error","Real")
  mp=sum(er$Error)/sum(er$Real)
  errors3[p,2]=mp
  setTxtProgressBar(pb,p)
}

##################  PLOTS  ##################

#Plots of the individual trajectories for all the vehicles in the dataset

pb=txtProgressBar(max=1698,style=3)
for (p in 1:1698){  #select a vehicle
  entrytime=all.veh3[p,2]
  exittime=all.veh3[p,3]
```

```
  traj.single=real.traj3[,c(1,p+1)]  #Real trajectory
  traj.single=as.data.frame(traj.single)
  colnames(traj.single)<-c("Frame", "L_y")
  traj.single=traj.single[traj.single$Frame>(entrytime-30) & traj.single$Frame<(exittime+30),]

  traj.result=all.traj.nofifo3.beta[,c(1,p+1)]  #Estimated traj. (no FIFO)
  traj.result=as.data.frame(traj.result)
  colnames(traj.result)<-c("Frame", "L_y")
  traj.result=traj.result[traj.result$Frame>(entrytime-30) & traj.result$Frame<(exittime+30),]

  traj.result.fifo=all.traj3.beta[,c(1,p+1)]  #Estimated traj. (FIFO)
  traj.result.fifo=as.data.frame(traj.result.fifo)
  colnames(traj.result.fifo)<-c("Frame", "L_y")
  traj.result.fifo=traj.result.fifo[traj.result.fifo$Frame>(entrytime-30) & traj.result.fifo$Frame<(exittime+30),]

  plot(traj.single$Frame, traj.single$L_y, type='l', lwd=2, xlab="Time (0.1 sec)", ylab="Distance (ft)", xlim=c(entrytime-30, exittime+30))
  lines(traj.result[,1],traj.result[,2],lty=1, lwd=1.5, col=100)
  lines(traj.result.fifo$Frame, traj.result.fifo$L_y, lty=1, lwd=1.5, col="red")
  legend("bottomright",c("Real trajectory","Estimated traj.(no FIFO)","Estimated traj.(FIFO)"),col=c(1,100,"red"),lty=c(1,1,1),lwd=c(2,2,2))

str = sprintf('plot%i.png', p)
dev.copy(png,str,width=607,height=356)
dev.off()
setTxtProgressBar(pb,p)
}

#Plots of errors in histograms

errors3[,3]=errors3[,3]*100
v=seq(0,100,by=2.5)

h1=hist(errors3[,2], breaks=v)
plot(h1, col="blue", main="Distribution of errors - No FIFO", xlab="Error (%)", xlim=range(seq(0,60,by=10)))
plot(h1, density=1000, col="blue", angle=45, main="Distribution of errors - No FIFO", xlab="Error")
x=errors3[,2]
xfit<-seq(min(x),max(x),length=40)
yfit<-dnorm(xfit,mean=mean(x),sd=sd(x))  #Adjusted normal curve
yfit <- yfit*diff(h1$mids[1:2])*length(x)
lines(xfit, yfit, col="red", lwd=2)
```