Inventory Control Policy of Preventive Lateral Transshipment between Retailers in Multi Periods

Qingren He, Bin Dan*, Ru Liu

School of Economics & Business Administration of Chongqing University and Key Laboratory of Logistics at Chongqing University (China)

*qingrenhe@cqu.edu.cn, * Corresponding author danbin@cqu.edu.cn, 756885541@qq.com*

Abstract:

Purpose: Preventive lateral transshipment can respond to customers who will choose a substitute or to give up when the product is out of stock. Motivated by the common practice, a decision-making model of preventive lateral transshipment with multi selling periods is developed. The purpose of the paper is to explore the optimal preventive lateral transshipment policy with multi selling periods.

Design/methodology/approach: With a discrete-time dynamic programming model, we take a dynamic programming approach and adopt backward induction to analyze two retailers' preventive lateral transshipment policy.

Findings: The optimal preventive lateral transshipment policy is a threshold policy which depends on both the remaining selling periods and inventory level. The above properties ensure that two retailers can control inventory with preventive lateral transshipment.

Practical implications: The retailer can adjust inventory via the threshold type policy. The simple decision rule which compares on-hand inventory level with the critical inventory level can be used to control inventory by preventive lateral transshipment.

Originality/value: A discrete-time decision-making model of preventive lateral transshipment policy is formulated. This model takes consideration of multi selling periods, which is different from most existing researches on preventive lateral transshipment.

1. Introduction

Customers' request for higher product variety in perishable product categories intensifies the uncertainty of supply and demand for the market (Mahto & Kumar, 2008; Chen & Zhang, 2010). As a result, perishable products are often out of stock or overstock. To cope with this problem, some retailers try to adopt the reactive lateral transshipment and achieve the so-called "risk-pooling" effect. However, there are some limitations when the reactive lateral transshipment serves as an important tool in response to customer need. For example, a survey of more than 71,000 customers showed that customers have little patience for stock-outs, and 85% of them will choose a substitute or to give up when they cannot find the precise products they are looking for (Gruen & Corsten, 2004). Reactive lateral transshipment might fail to reply to customers who have little patience for stock-outs. While some leading fashion enterprises, e.g. ZARA and H&M, try to adopt preventive lateral transshipment to reduce stock losses (Caro, Gallien, Díaz, & García, 2010; Sen, 2008). In such situations, preventive lateral transshipment provides the retailer with more opportunities to control inventory after the selling season begins. In this case, a portion of demand was observed in advance, which might reduce some risk of stock-out or overstock and can achieve a "win-win" situation.

When the selling season includes two or more periods, the retailer can implement preventive lateral transshipment in response to the risk of stock-out or overstock. For example, representative published work by Lee and Whang (2002) considered a system of multi retailers, and obtained the optimal lateral transshipment policy. Rong, Snyder and Sun (2010) adopted a two-period model and proved the optimal transshipment policy is composed of the transship-down-to level and the transship-up-to level. Extending this, Summerfield and Dror (2012) introduced a general framework of decision-making of two periods and summarized the policy of preventative lateral transshipment. In addition, research by Seidscher and Miner (2013) established a cost optimization model and showed the optimal transshipment is composed of hold back levels. However, the above literatures focused on the two-period model of preventative lateral transshipment. In practice, the retailer can implement preventive lateral transshipment more times. Furthermore, Agrawal, Chao and Seshadri (2004) found that, as the number of opportunities for transshipping increases, the retailer may get more profit. Therefore, some researchers tried to transfer attention from two periods to more periods. To address these questions, Zhao, Ryan and Deshpande (2008) developed a cost-minimization model by adopting discrete event dynamic programming and demonstrated that the optimal transship-up-to level is decreasing in inventory level. Alternatively, Paterson, Teunter and

Glazebrook (2012) presented an optimal policy which is a function of cost as a correction for the expected cost. Glazebrook, Paterson, Rauscher and Archibald (2014) proposed hybrid transshipments to prevent future shortages and developed an easy-to-compute heuristic for determining which transshipment should be adopted. In addition, Seidscher and Minner (2013) formulated an optimal threshold function for a preventive lateral transshipment, and demonstrated the threshold function is monotonic function of transshipment costs, warehousing costs and stock losses. Roodbergen (2013) established a cost optimization model by the stochastic dynamic programming method and got the optimal transshipment quantity. Similarly, Yousuk and Luong (2013) stated the properties for the optimal transship-up-to level and transship-down-to level in a replenishment cycle by adopting the optimization objective function. Tai and Ching (2014) presented a two-echelon inventory system consists of a supply plant with infinite capacity and proposed an aggregated inventory model for the central warehouse and the local warehouses. Meanwhile, Liu and Liang (2013) studied the medical resources allocation problem in a discrete time-space network model, and got the optimal allocation of medical resources by a proactive mechanism. Although they converted attention from two periods to more selling periods, they still focused on the transshipment which has only been implemented one time at each replenishment cycle. However, in practice, some products cannot be replenished two times (or more), especially for perishable products. Taking this path, Çömez, Stecke and Çakanyıldırım (2012) developed an optimal holdback policy for two retailers in a multi-period model and proved that the policy is characterized by the transship-down-to level. However, the prerequisite is that one retailer is stocked out, while another has excess inventory, which cannot fully reflect the characteristic of preventive lateral transshipment to reduce the risk of future stock-out. So, in order to avoid the risk of stock-out or overstock, the retailer has to transship in product before his inventory level drops to zero.

With respect to the literatures in this article, instead, we focus on a discrete-time model of two retailers when customers have little patience for stock-outs. A key feature of our model is that the transship-up-to level and the transship-down-to level change with the remaining selling periods and the two retailers' inventory status. So, the existence of the two thresholds is proved. Based on the existence, we will also prove that the optimal transshipment control policy is a threshold type and the structural properties of two thresholds are obtained. Finally, With regards to the complex nature of the optimal policy, we propose the corresponding solutional algorithm by the structural properties of the thresholds.

The remainder of this paper is organized as follows. In view of above questions, this article is divided into 5 sections. The description and notations are presented and the discrete-time dynamic programming mode framework is developed in Section 2. Next, we will show the existence of the transship-up-to level and the transship-down-to level and get the properties of the threshold function in Section 3. Based on the properties of the thresholds, the corresponding solution algorithm is proposed in the Section 4. We will present a numerical study in Section 5. In Section 6, we conclude our findings.

2. Problem description and model

2.1. Problem description and assumptions

In this section, we consider a system consisting of two retailers who sell a same kind of perishable products during the selling season. Each of them faces an independent market. Before the selling season, retailer i gets a inventory level of size Q_i with unit cost w, where $I = 1, 2$. There is no replenishment during the whole selling season after the season begins because the procurement lead time is long. The selling horizon is finite, and be divided into N periods of equal length. In this time discretization, the periods are short enough so that there can be at most one unit demand within each period, either at retailer 1 or 2, or neither. In each period, a demand arrives at retailers 1 and 2 as a homogeneous Poisson process with arrival rate λ_1 and λ_2, respectively, where $0 \leq \lambda_1 + \lambda_2 \leq 1$. Following the general assumptions in the published works, each arriving customer is assumed to purchase at most one unit of the product. When a demand arrives at retailer i and there is no positive on-hand inventory, the customer is unwilling to delay purchase. The number of periods remaining until the end of the selling season is denoted by k, $k = N, N - 1, ..., 1$, and the products ordered by retailers arrive at period $k = N$. The sequence of events at the selling period k is illustrated as follows.

Step 1. At the beginning of period k, one unit of product transshipped in at the previous period arrives.

Step 2. At period k, upon a demand at retailer i, if there is positive on-hand inventory, it earns a per-unit selling price p_i, which remains unchanged during the entire selling season. Otherwise, a per-unit shortage cost m_i is incurred.

Step 3. After Step 2, the transshipment decision whether to transship in one unit of product from retailer j, $j = 3 - i$, must be made. One unit of product arrives at retailer i with the transshipment cost c_t before the next selling period begins if the transshipment is necessary. Transshipment cost includes the transportation cost, as well as any other administrative costs associated with transshipment.

Step 4. If there is excess inventory at the end of selling season, i.e., $k = 1$, then excess inventory is salvaged with a unit salvage value of s_i.

2.2. Mathematical model

A dynamic programming model is developed to explore the optimal policy of preventive lateral transshipment to solve the problem described above. When the initial inventory Q_1 and Q_2 are exogenous, a three-dimensional vector is defined to characterize the inventory levels and the

remaining selling periods. The transshipment decision must be made after a demand is realized regardless of whether there is a demand within each period except for period 1. The formulation for this process can be expressed as follows:

$$v_k(x_1, x_2) = \lambda_1 H_1 v_{k-1}(x_1, x_2) + \lambda_2 H_2 v_{k-1}(x_1, x_2) + (1 - \lambda_1 - \lambda_2) H_3 v_{k-1}(x_1, x_2) \tag{1}$$

$v_k(x_1, x_2)$ represents the expected system profit over k selling periods, given current inventory level (x_1, x_2), with x_1, x_2 indicating the inventory level for retailer 1,2, and $0 \le x_1 \le Q_1$, $0 \le x_2 \le Q_2$. Considering all of the possible events that might happen during the entire selling season, we have three events. One unit of demand arrives either at retailer 1 or 2, or neither. $H_1 v_{k-1}(x_1, x_2)$, $H_2 v_{k-1}(x_1, x_2)$, and $H_3 v_{k-1}(x_1, x_2)$ are the expected profit operators of three events respectively, and defined by (2)~(4).

$$H_1 v_{k-1}(x_1, x_2)$$
$$= \begin{cases} p_1 - w + \max\{v_{k-1}(x_1 - 1, x_2) + h_1, -c_t + v_{k-1}(x_1, x_2 - 1) + h_2\} - h_1 x_1 - h_2 x_2 & x_1 \ge 1, x_2 \ge 1 \\ p_1 - w + v_{k-1}(x_1 - 1, x_2) - h_1(x_1 - 1) & x_1 \ge 1, x_2 = 0 \\ \max\{v_{k-1}(x_1, x_2), -c_t + v_{k-1}(x_1 + 1, x_2 - 1) - h_1 + h_2\} - h_2 x_2 - m_1 & x_1 = 0, x_2 \ge 1 \\ v_{k-1}(x_1, x_2) - m_1 & x_1 = 0, x_2 = 0 \end{cases} \tag{2}$$

$$H_2 v_{k-1}(x_1, x_2)$$
$$= \begin{cases} p_2 - w + \max\{v_{k-1}(x_1, x_2 - 1) + h_2, -c_t + v_{k-1}(x_1 - 1, x_2) + h_1\} - h_1 x_1 - h_2 x_2 & x_1 \ge 1, x_2 \ge 1 \\ \max\{v_{k-1}(x_1, x_2), -c_t + v_{k-1}(x_1 - 1, x_2 + 1) + h_1 - h_2\} - m_2 - h_1 x_1 & x_1 \ge 1, x_2 = 0 \\ p_2 - w + v_{k-1}(x_1, x_2 - 1) - h_2(x_2 - 1) & x_1 = 0, x_2 \ge 1 \\ v_{k-1}(x_1, x_2) - m_2 & x_1 = 0, x_2 = 0 \end{cases} \tag{3}$$

$$H_3 v_{k-1}(x_1, x_2) =$$
$$\begin{cases} \max\{v_{k-1}(x_1, x_2), -c_t + v_{k-1}(x_1 + 1, x_2 - 1) - h_1 + h_2, -c_t + v_{k-1}(x_1 - 1, x_2 + 1) + h_1 - h_2\} - h_1 x_1 - h_2 x_2 & x_1 \ge 1, x_2 \ge 1 \\ \max\{v_{k-1}(x_1, x_2), -c_t + v_{k-1}(x_1 + 1, x_2 - 1) + h_2 - h_1\} - h_2 x_2 & x_1 = 0, x_2 \ge 1 \\ \max\{v_{k-1}(x_1, x_2), -c_t + v_{k-1}(x_1 - 1, x_2 + 1) + h_1 - h_2\} - h_1 x_1 & x_1 \ge 1, x_2 = 0 \\ v_{k-1}(x_1, x_2) & x_1 = 0, x_2 = 0 \end{cases} \tag{4}$$

The formula (2) represents the expected profit when a demand arrives at retailer 1 with arrival rate λ_1 at selling period k, where h_1, h_2, is the unit holding cost per unit of selling period. The transshipment decision is whether to transship in one unit of product from retailer 2 after one unit of demand is fulfilled. For $x_1 \ge 1$, $x_2 \ge 1$, and when the inequality $v_{k-1}(x_1 - 1, x_2) \le v_{k-1}(x_1, x_2 - 1) - c_t - h_1 + h_2$ holds, transshipping in one unit of product is optimal from retailer 2. For $x_1 = 0$, $x_2 \ge 1$, and the inequality $- c_t - h_1 + h_2 + v_{k-1}(x_1 + 1, x_2 - 1) \ge v_{k-1}(x_1, x_2)$, transshipping in one unit of product is necessary from retailer 2. For $x_2 = 0$, retailer 1 neither transships in nor out one unit of product. Formula (3) and (4) are illustrated analogously.

For $k = 1$, the excess inventory is salvaged besides the sale, and the expected profit of retailer 1 is as follows:

$$v_1(x_1, x_2) =$$
$$\begin{cases} \lambda_1\big[p_1 + s_1(x_1 - 1) + s_2 x_2\big] + \lambda_2\big[p_2 + s_1 x_1 + s_2(x_2 - 1)\big] + (1 - \lambda_1 - \lambda_2)(s_1 x_1 + s_2 x_2) - w(x_1 + x_2) & x_1 \geq 1, x_2 \geq 1 \\ -\lambda_1 m_1 + \lambda_2\big[p_2 + s_2(x_2 - 1)\big] + (1 - \lambda_1 - \lambda_2)s_2 x_2 - w x_2 & x_1 = 0, x_2 \geq 1 \\ \lambda_1\big[p_1 + s_1(x_1 - 1)\big] - \lambda_2 m_2 + (1 - \lambda_1 - \lambda_2)s_1 x_1 - w x_1 & x_1 \geq 1, x_2 = 0 \\ -\lambda_1 m_1 - \lambda_2 m_2 & x_1 = 0, x_2 = 0 \end{cases} \tag{5}$$

3. Analysis of model of preventive lateral transshipment

3.1. Performance of preventive lateral transshipment

Two retailers do not influence each other when there is no preventive lateral transshipment. So, the expected profit at period k is as follow:

$$v_k^N(x_1, x_2) = \lambda_1 H_1 v_{k-1}^N(x_1, x_2) + \lambda_2 H_2 v_{k-1}^N(x_1, x_2) + (1 - \lambda_1 - \lambda_2) v_{k-1}^N(x_1, x_2) \tag{6}$$

where

$$H_1 v_{k-1}^N(x_1, x_2) = \begin{cases} p_1 - w + v_{k-1}^N(x_1 - 1, x_2) - h_1(x_1 - 1) - h_2 x_2 & x_1 \geq 1 \\ -m_1 + v_{k-1}^N(x_1, x_2) - h_2 x_2 & x_1 = 0 \end{cases} \tag{7}$$

$$H_2 v_{k-1}^N(x_1, x_2) = \begin{cases} p_2 - w + v_{k-1}^N(x_1, x_2 - 1) - h_1 x_1 - h_2(x_2 - 1) & x_2 \geq 1 \\ -m_2 + v_{k-1}^N(x_1, x_2) - h_1 x_1 & x_2 = 0 \end{cases} \tag{8}$$

The formula (7) represents the expected profit at period k when a customer arrives at retailer 1. The formula (8) represents the profit when customer arrives at retailer 2. The question is whether two retailers can get more profits with preventive lateral transshipment than without it. So, we have the following theorem 1.

Theorem 1. More profit can be collected with preventive lateral transshipment than without it under the condition of equal initial inventory.

Proof. For $k = 1$, there is no transshipment. So, $v_k^N(x_1, x_2) = v_k(x_1, x_2)$ holds from formula (5). For $k = 2$, we need to prove $v_2(x_1, x_2) - v_2^N(x_1, x_2) \geq 0$ by the induction. For $x_1 \geq 1$, the inequality $\max\{v_1(x_1 - 1, x_2), -c_t + v_1(x_1, x_2 - 1) - h_1 + h_2\} \geq v_1^N(x_1 - 1, x_2)$ always holds. For $x_1 = 0$, the inequality $\max\{v_1(x_1, x_2), -c_t + v_1(x_1 + 1, x_2 - 1) - h_1 + h_2\} \geq v_1^N(x_1, x_2)$ holds. Overall, for $k = 2$, we can prove $v_2(x_1, x_2) - v_2^N(x_1, x_2) \geq 0$. For $k \geq 3$, we need to prove that the inequality $v_k(x_1, x_2) - v_k^N(x_1, x_2) \geq 0$ holds. When $x_1 \geq 1$, we can prove the inequality

$\max\{v_{k-1}(x_1 - 1, x_2) - h_2, - c_t + v_{k-1}(x_1, x_2 - 1) - h_1\} \geq v_{k-1}^{N}(x_1 - 1, x_2) - h_2$ always holds by the induction. Similarly, we can prove the second term and the third term. When $x_1 = 0$, the inequality $v_k(x_1, x_2) - v_k^{N}(x_1, x_2) \geq 0$ holds.

Theorem 1 indicates that two retailers can collect more profit when the preventive lateral transshipment policy is adopted. Two retailers can control inventory by transshipping in or transshipping out products even if no demand arrives.

3.2. Analysis of policy of preventive lateral transshipment

At each period, transshipment decision must be made after a demand is realized except for period 1. For $x_1 = Q_1$, retailer 1 cannot transship in product. Similarly, for $x_1 = 0$, transshipping out one unit of product from retailer 1 cannot be implemented. A control policy of the system specifies the transship-up-to level and the transship-down-to level of the system at any period and any inventory state. Therefore, a simple decision rule composed of the critical inventory levels of the transship-up-to level and the transship-down-to level is developed, and the critical inventory level is calculated and stored in advance. So, the retailers can control preventive lateral transshipment by comparing on-hand inventory level with the critical inventory level stored in advance. So, we have the following theorem 2. Without loss of generality, we focus our analysis on retailer 1.

Theorem 2. For $k \geq 2$, for the inventory of retailer 2, $x_2 \in \{1, 2, …, Q_2\}$, there exist some transship-up-to levels $IT_k(x_2) \in \{0, 1, …, Q_1 -1\}$ for retailer 1. Similarly, for the inventory of retailer 2 $x_2 \in \{0,1, …, Q_2 -1\}$, there exist some transship-down-to levels $OT_k(x_2) \in \{1, 2, …, Q_1 -1, Q_1\}$ for retailer 1. $IT_k(x_2)$ and $OT_k(x_2)$ can be obtained from formula (9) and (10).

$$IT_k(x_2) = \max\{x_1: v_k(x_1 + 1, x_2) - v_k(x_1, x_2 + 1) \geq c_t + h_1 - h_2\} \qquad (9)$$

$$OT_k(x_2) = \min\{x_1: v_k(x_1, x_2 + 1) - v_k(x_1 + 1, x_2) \geq c_t + h_2 - h_1\} \qquad (10)$$

Proof. Let $g_k(x_1, x_2) = v_k(x_1 + 1, x_2) - v_k(x_1, x_2 + 1)$, and we need to prove the $g_k(x_1, x_2)$ is non-increasing in x_1 by the induction. When $k = 2$, we can prove the inequality $v_2(x_1 + 2, x_2) - v_2(x_1 + 1, x_2 + 1) - v_2(x_1 + 1, x_2) + v_2(x_1 + 1, x_2) \leq 0$ holds by formula (5). When $k \geq 3$, and $x_1 = 0$, $x_2 = 0$ holds, $\Delta g_{1,k}(0, 0) = v_k(1, 1) - v_k(0, 2) - v_k(1, 0) + v_k(0, 1)$. We follow Zhuang and Li (2010) to prove the inequality $\Delta g_{1,k}(0, 0) \leq 0$ holds. When $x_1 = 0$, $x_2 \geq 1$, $x_1 = 1$, $x_2 \geq 0$ and $x_1 \geq 1$, $x_2 \geq 1$, we also can prove that the inequality $\Delta g_{1,k}(x_1, x_2) \leq 0$ holds by the induction. Therefore, there exist the transship-up-to level $IT_k(x_2)$ and the transship-down-to level $OT_k(x_2)$.

Theorem 2 indicates that the transship-down-to level and the transship-up-to level are used to determine whether inventory should be transshipped between the retailers or not. For the inventory level of retailer 2 within each period, there exist some critical inventory thresholds, which at most divide the set $\{0, 1, …, Q_1\}$ into three subsets. In the first subset, where the inventory is below the transship-up-to level, retailer 1 can increase the inventory to the threshold by transshipping in products. In the second subset, where the inventory lies between the band defined by the transship-up-to level and the transship-down-to level, retailer 1 does nothing. In the third subset, where the inventory is above the transship-down-to level, retailer 1 can reduce the inventory to the threshold by transshipping out products. Therefore, two retailers can control inventory by transshipping in or out products, and we have the following theorem 3.

Theorem 3. For $k \geq 2$, in each selling period, it is optimal for retailer 1 to transship in one unit of product from retailer 2 if the inventory of retailer 2 satisfies the condition $OT_k(x_1) \leq x_2 \leq Q_2$. When the inequality $0 \leq x_2 \leq IT_k(x_1)$ holds, retailer 1 transships out one unit of product to retailer 2. Otherwise, both retailers do nothing.

Proof. When the inequality $OT_k(x_1) \leq x_2 \leq Q_2$ holds, we can obtain the inequality $v_k(x_1, x_2 + 1) - h_2 \leq c_t + v_k(x_1 + 1, x_2) - h_1$. Following formula (9) and the inequality $\Delta g_{2,k}(x_1, x_2) \leq 0$, we can prove the inequality $\max\{x_1: v_k(x_1 + 1, x_2) - v_k(x_1, x_2 + 1) \geq - c_t + h_2 - h_1\} \leq IT_k(x_2)$ holds. Therefore, it is optimal for retailer 1 to transship in one unit of product from retailer 2. Similarly, when the inequality $0 \leq x_2 \leq OT_k(x_1)$ holds, it is optimal for retailer 1 to transship out one unit of product from retailer 2.

Theorem 3 demonstrates that, the simple decision rule by comparing on-hand inventory level with the critical inventory level can be used to control preventive lateral transshipment between the retailers. Moreover, the critical inventory level makes up the threshold type policy which is composed of the transship-up-to level and the transship-down-to level from theorem 2. Toward the characterization of the optimal transshipment policy, it is necessary to further obtain the structural properties of the thresholds in the following proposition.

Proposition 1. For $k \geq 3$, the transship-down-to level $OT_k(x_2)$ is non-decreasing in the remaining periods and the transship-up-to level $IT_k(x_2)$ is non-increasing in the remaining periods during the rest of the selling period.

Proof. For $k = 3$, we have $OT_2(x_2) \leq OT_3(x_2)$. When $k \geq 4$ holds, we assume the induction hypothesis holds. When $x_1 = 0$, $OT_{k-1}(x_2) = OT_k(x_2) = 0$ holds, and $OT_{k-1}(x_2) \leq OT_k(x_2)$ holds via the hypothesis. When $x_1 \geq 1$, $x_2 = 0$, the equality $OT_{k-1}(0) = \min\{OT_k(0): v_{k-1}(OT_k(0), x_2 + 1) -$

$v_{k-1}(OT_k(0) + 1, x_2) \geq c_t + h_2 - h_1\}$ holds from (10). Therefore, we have $OT_{k-1}(0) \leq OT_k(0)$. Overall, when $x_1 \geq 1$, $x_2 = 0$, the inequality $OT_k(0) \geq OT_{k-1}(0)$ holds. Similarly, when $x_1 \geq 1$, $x_2 \geq 1$, the inequality $OT_{k-1}(x_2) \leq OT_k(x_2)$ holds. Overall, the inequality $OT_{k-1}(x_2) \leq OT_k(x_2)$ holds. Similarly, the inequality $IT_{k-1}(x_2) \geq IT_k(x_2)$ holds.

Proposition 1 indicates that, two retailers do not tend to implement transshipment as a result of the higher transship-down-to level and lower transship-up-to level earlier during the selling season. On the contrary, two retailers tend to implement transshipment when there are less remaining periods. Two retailers do not worry that they cannot implement transshipment as long as they satisfy the condition of transshipment. It is different from two independent retailers because two retailers always cooperate to balance the inventory, while two independent retailers always maximizes her/his own profit.

Proposition 2. For $k \geq 2$, both the transship-up-to level $IT_k(x_2)$ and the transship-down-to level $OT_k(x_2)$ are non-decreasing in the inventory of retailer 2.

Proof. Recall that inequality $\Delta g_{1,k}(x_1, x_2) \leq 0$ holds. Following Zhuang and Li (2010), we can prove the inequality $\Delta g_{2,k}(x_1, x_2) \geq 0$ holds by the induction. So, we can obtain $IT_k(x_2 + 1) \geq IT_k(x_2)$ and the inequality $OT_k(x_2 + 1) \geq OT_k(x_2)$ from formula (9) and (10).

Proposition 2 demonstrates when the gap of the inventory of two retailers is wider, both retailers tend to balance the inventory. Specifically, when the inventory of retailer 2 is relatively low, in order to balance the inventory, the relatively low transship-down-to level of retailer 1 encourages more transshipment from retailer 1 to retailer 2. Meanwhile, the relatively low transship-up-to level of retailer 1 avoids more transshipment from retailer 2 to retailer 1. This is different from the case when two retailers are independent, because they may fight against each other if the gap of the inventory of two retailers is wider. Similarly, we can get corresponding managerial insights when the inventory level of retailer 2 is relatively high.

4. Algorithm

In the previous sections, we proved that two retailers can control preventive lateral transshipment by the transship-up-to level and the transship-down-to level. However, the thresholds depend on both retailers' inventory states and the remaining periods. The value iteration algorithm is applied intensively as initial inventory level and the selling periods increase. Therefore, in this section we will develop a convenient algorithm based on the structural properties of the thresholds from Proposition 1 and Proposition 2. Given initial

inventory Q_1 and Q_2, $IT_k(x_2)$ and $OT_k(x_2)$ can be obtained via formula (1), (9) and (10). Without loss of generality, the algorithm for retailer 1 is as follows.

Step 1. Initialize $x_1 = Q_1$ and $x_2 = Q_2$.

Step 2. Calculate the profit $v_k(x_1, x_2)$ under all states by formula (1)~(5), and go to Step 3.

Step 3. Set $x_2 = 1$ and $x_1 = Q_1 - 1$. If $v_k(x_1 + 1, x_2 - 1) - v_k(x_1, x_2) \geq c_t + h_1 - h_2$ holds, then set $IT_k(1) = Q_1 - 1$, and go to Step 4. Otherwise, go to Step 5.

Step 4. Set $x_1 = x_1$ and $x_2 = x_2 + 1$. If the inequality $v_k(x_1 + 1, x_2) - v_k(x_1, x_2 + 1) \geq c_t + h_1 - h_2$ holds, then set $IT_k(x_2 + 1) = x_1$. Repeat Step 4 until $x_2 = Q_2$ holds. Otherwise, go to Step 6.

Step 5. Set $x_2 = x_2$ and $x_1 = x_1 - 1$. If the inequality $v_k(x_1, x_2 - 1) - v_k(x_1 - 1, x_2) \geq c_t + h_1 - h_2$ holds, then set $IT_k(1) = x_1 - 1$. Repeat Step 5 until $x_1 - 0$ holds. Otherwise, go to Step 6.

Step 6. Set $x_1 = x_1 - 1$ and $x_2 = x_2 + 1$. If the inequality $v_k(x_1, x_2) - v_k(x_1 - 1, x_2 + 1) \geq c_t + h_1 - h_2$ holds, then set $IT_k(x_2 + 1) = x_1 - 1$, and go to step 4. Otherwise, return to Step5.

The calculation of $OT_k(x_2)$ may refer to Step 3~Step 6.

5. Computational experiment

In this section, a computational experiment to supplement the analytical results is further provided. The experiment will explore the existence of transshipment area of two retailers and analyze the impact of the transshipment cost, shortage cost, and salvage value on transshipment and profit. The base parameters used in the computational experiment are as follows. $p_1 = p_2 = 40$, $c_t = 2$, $w = 20$, $s_1 = s_2 = 5$, $m_1 = m_2 = 10$, $h_1 = 0.5$, $h_2 = 0.2$, $\lambda_1 = 0.2$ and $\lambda_2 = 0.1$.

5.1. Region of preventive lateral transshipment

The optimal policy is computed by the algorithm from Section 4, and presented in Figure 1. As shown in Figure 1, both the transship-up-to level and transship-down-to level are non-decreasing in the inventory of retailer 2. Only in Area I and Area II, the preventive transshipment occurs, and no transshipment occurs in Area III. As transshipment cost increases, both the area of transshipping in and transshipping out decrease because marginal profit by transshipment gets lower.
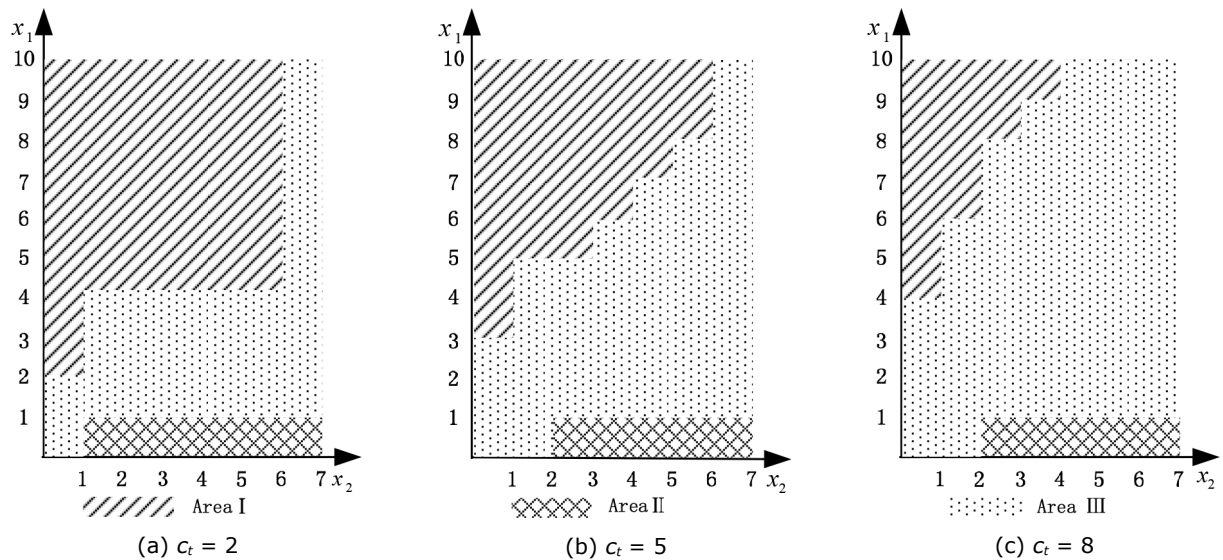
Figure 1. Region of preventive lateral transshipment

In Area I, retailer 2 transships in products from retailer 1. In Area II, retailer 1 transships in products from retailer 2. In Area III, there exists no transshipment. No matter which retailer a demand arrives at, the inventory of retailer 1 decreases when the inventory state of two retailers falls in Area I. When a demand arrives at retailer 1, she immediately satisfies the customer's demand if she has available on-hand inventory. So, the inventory of retailer 1 decreases. When a demand arrives at retailer 2, he transships in one unit from retailer 1, which also makes her inventory decrease. Similarly, no matter which retailer a demand arrives at, the inventory of retailer 2 decreases when the inventory state of two retailers falls in Area II.

5.2. Impact of transshipment cost on transshipment and profit

Under the selling period $k = 40$ and the inventory level of retailer 2 $x_2 = 6$, the impact of transshipment cost on transshipment and profit is as Figure 2 and Figure 3 show.

As transshipment cost increases, the threshold of transship-up-to level keeps unchanged, and the transship-down-to level increases.

As transshipment cost increases, the threshold of transship-up-to level keeps unchanged because the inventory level of retailer 2 is relatively high compared with the initial ordering quantity. Moreover, both retailers face more remaining sale periods. Therefore, the two factors which lead to the lower threshold of transship-up-to level prevent the occurrence of transshipment.

The transship-down-to level increases because the expected profit collected from transshipment decreases as transshipment cost increases. When transshipment cost is small, the expected marginal profit with transshipment is high, which makes the transship-down-to level be lower. However, as transshipment cost increases further, the expected profit collected by the preventive lateral transshipment decreases, and the transship-up-to level increases accordingly.
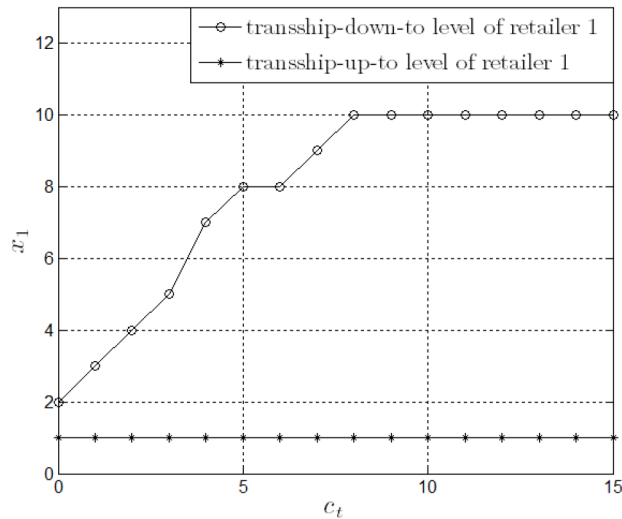

Figure 2. Relation between transshipment and transshipment cost

As transshipment cost increases, the profit with transshipment decreases, and the gap between the profit with transshipment and that without transshipment narrows.

When transshipment cost increases from 0 to 15, profit with transshipment decreases from 116.1 to 78.3, while profit without transshipment keeps unchanged. Moreover, profit with transshipment is always greater than without it. With a lower transshipment cost, a higher expected profit level can be collected by implementing transshipment. Therefore, both retailers choose to transship in order to get more profit. As transshipment cost increases further, the transshipment gets more and more difficult, which makes the expected profit collected by transshipping get less.
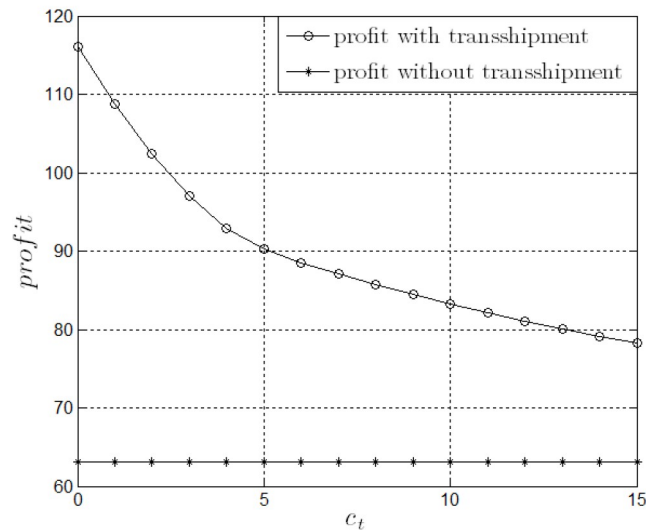
Figure 3. Relation between profit and transshipment cost

5.3. Impact of shortage cost on transshipment and profit

Under the selling period $k = 40$ and the inventory level of retailer 2 $x_2 = 6$, the impact of shortage cost on transshipment and profit is as Table 1 shows.

As shortage cost increases, the transship-up-to level a n d transship-down-to level keep unchanged. As shortage cost increases, the value of preventive transshipment gets larger and larger. Therefore, the transship-down-to level needs to be set lower. Because transshipment time is zero, it is optimal to keep at most one unit product. Two factors make the transship-up-to level and transship-down-to level to be insensitive to the shortage cost.

Shortage cost	Transship-up-to level	Transship-down-to level	Profit with transshipment	Profit without transshipment	Profit increment
0	1	4	109.9332	78.719	31.2142
1	1	4	109.0927	77.1579	31.9348
2	1	4	108.2811	75.5967	32.6844
3	1	4	107.4853	74.0356	33.4497
4	1	4	106.7188	72.4745	34.2443
5	1	4	105.9664	70.9133	35.0531
6	1	4	105.2308	69.3522	35.8786
7	1	4	104.5138	67.7911	36.7227
8	1	4	103.8116	66.2299	37.5817
9	1	4	103.1241	64.6688	38.4553
10	1	4	102.4506	63.1077	39.3429

Table 1. Relation between profit, transshipment and shortage cost

As shortage cost increases, profit with transshipment decreases, but the gap between profit with transshipment and that without transshipment becomes wider. This indicates that adopting preventive lateral transshipment can get more profit by avoiding the stock-out compared with the case without transshipment. As shortage cost increases, the value of preventive transshipment gets larger and larger.

5.4. Impact of salvage value on transshipment and profit

Under the selling period $k = 40$ and the inventory level of retailer 2 $x_2 = 8$, the impact of salvage value on transshipment and profit is as Table 2 shows.

As shortage cost increases, the transship-up-to level and transship-down-to level keep unchanged. The excess inventory is salvaged only at the end of selling period, which makes salvage value to have less effect on the transship-up-to level and transship-down-to level.

Salvage value	Transship-up-to level	Transship-down-to level	Profit with transshipment	Profit without transshipment	Profit increment
0	1	4	100.5369	60.6138	39.9231
1	1	4	100.9019	61.1126	39.7893
2	1	4	101.2824	61.6113	39.6711
3	1	4	101.6664	62.1101	39.5563
4	1	4	102.0579	62.6089	39.449
5	1	4	102.4506	63.1077	39.3429
6	1	4	102.8434	63.6064	39.2370
7	1	4	103.2365	64.1052	39.1313
8	1	4	103.7759	64.604	39.1719
9	1	4	104.5476	65.1027	39.4449
10	1	4	105.3487	65.6015	39.7472
11	1	4	106.1499	66.1003	40.0496
12	1	4	106.9511	66.5991	40.352
13	1	4	107.7967	67.0978	40.6989
14	1	4	108.8209	67.5966	41.2243
15	1	4	109.9259	68.0954	41.8305

Table 2. Relation between profit, transshipment and salvage value

As salvage value increases, profit with transshipment increases, and profit with transshipment is always greater than that without transshipment. Moreover, when salvage value increases from 0 to 7, the gap between profit with transshipment and profit without transshipment narrows. In fact, retailer 2 can obtain all salvage revenue by transshipment as a result of a larger gap between two retailers' salvage value. So, the gap narrows. Similarly, when salvage

value increases from 8 to 15, the gap between profit with transshipment and profit without transshipment increases.

6. Conclusions

In this paper, we considered a system of two retailers with preventive lateral transshipment. First, we showed the existence of the transship-up-to level and the transship-down-to level. Second, we demonstrated that the optimal lateral transshipment control policy is a threshold type. Furthermore, based on the structural properties of the thresholds, the corresponding algorithm has been proposed as well. Besides, by analyzing the optimal control policy, we obtained some managerial insights as follows.

1. Two retailers can control the preventive lateral transshipment policy by the transship-up-to level and the transship-down-to level during the selling season. When a retailer's inventory is below the threshold, it is optimal to transship in one unit of product. In contrast, when the inventory of a retailer is above the threshold, it is optimal to transship out one unit of product. Otherwise, it is optimal to do nothing.

2. At the beginning of the season, two retailers do not tend to implement transshipment as a result of the higher transship-down-to level and lower transship-up-to level.

Acknowledgment

References

Agrawal, V., Chao, X., & Seshadri, S. (2004). Dynamic balancing of inventory in supply chains. *European Journal of Operational Research*, 159(2), 296-317.
http://dx.doi.org/10.1016/j.ejor.2003.08.017

Caro, F., Gallien, J., Díaz, M., & García, J. (2010). Zara uses operations research to reengineer its global distribution process. *Interfaces*, 40(1), 71-84. http://dx.doi.org/10.1287/inte.1090.0472

Chen, X, & Zhang, J. (2010). Production control and supplier selection under demand disruptions. *Journal of Industrial Engineering and Management*, 3(3), 421-446. http://dx.doi.org/10.3926/jiem.2010.v3n3.p421-446

Çömez, N., Stecke, K.E., & Çakanyıldırım, M. (2012). In-season transshipments among competitive retailers. *Manufacturing & Service Operations Management*, 14(2), 290-300. http://dx.doi.org/10.1287/msom.1110.0364

Glazebrook, K., Paterson, C., Rauscher, S., & Archibald, T. (2014). Benefits of hybrid lateral transshipments in multi-item inventory systems under periodic replenishment. *Production and Operations Management*, 23(6), 1-14.

Gruen, T., & Corsten, D. (2004). Stock-outs cause walkouts. *Harvard Business Review*, 82(5), 26-27. http://hbr.org/2004/05/stock-outs-cause-walkouts/ar/1

Lee, H., & Whang, S. (2002). The impact of the secondary market on the supply chain. *Management science*, 48(6), 719-731. http://dx.doi.org/10.1287/mnsc.48.6.719.189

Liu, M., & Liang, J. (2013). Dynamic optimization model for allocating medical resources in epidemic controlling. *Journal of Industrial Engineering and Management*, 6(1), 73-88. http://dx.doi.org/10.3926/jiem.663

Mahto, D., & Kumar, A. (2008). Application of root cause analysis in improvement of product quality and productivity. *Journal of Industrial Engineering and Management*, 1(2), 16-53. http://dx.doi.org/10.3926/jiem.2008.v1n2.p16-53

Paterson, C., Teunter, R., & Glazebrook, K. (2012). Enhanced lateral transshipments in a multi-location inventory system. *European Journal of Operational Research*, 221(2), 317-327. http://dx.doi.org/10.1016/j.ejor.2012.03.005

Rong, Y., Snyder, L.V., & Sun, Y. (2010). Inventory sharing under decentralized preventive transshipments. *Naval Research Logistics*, 57(6), 540-562. http://dx.doi.org/10.1002/nav.20420

Roodbergen, K. (2013). Transshipment and rebalancing policies for library books. *European Journal of Operational Research*, 228(2), 447-456. http://dx.doi.org/10.1016/j.ejor.2013.02.008

Seidscher, A., & Minner, S. (2013). A Semi-Markov decision problem for proactive and reactive transshipments between multiple warehouses. *European Journal of Operational Research*, 230(1), 42-52. http://dx.doi.org/10.1016/j.ejor.2013.03.041

Sen, A. (2008). The US fashion industry: A supply chain review. *International Journal of Production Economics,* 114(2), 571-593. http://dx.doi.org/10.1016/j.ijpe.2007.05.022

Summerfield, N.S., & Dror, M. (2012). Stochastic programming for decentralized newsvendor with transshipment. *International Journal of Production Economics*, 137(2), 292-303. http://dx.doi.org/10.1016/j.ijpe.2012.02.014

Tai, A., & Ching, W. (2014). Optimal inventory policy for a Markovian two-echelon system with returns and lateral transshipment. *International Journal of Production Economics*, 151(1), 48-55. http://dx.doi.org/10.1016/j.ijpe.2014.01.010

Yousuk, R., & Luong, T. (2013). Modelling a two–retailer inventory system with preventive lateral transshipment using expected path approach. *European Journal of Industrial Engineering*, 7(2), 248-274. http://dx.doi.org/10.1504/EJIE.2013.052571

Zhao, H., Ryan, J., & Deshpande, V. (2008). Optimal dynamic production and inventory transshipment policies for a two-location make-to-stock system. *Operational Research*, 56(2), 400-410. http://dx.doi.org/10.1287/opre.1070.0494

Zhuang, W., & Li, Z. (2010). A new method of proving structural properties for certain class of stochastic dynamic control problems. *Operations Research Letters*, 38(5), 462-467. http://dx.doi.org/10.1016/j.orl.2010.05.008