OmniaScience

Journal of Industrial Engineering and Management

JIEM, 2013 – 6(2): 477-494 – Online ISSN: 2013-0953 – Print ISSN: 2013-8423

http://dx.doi.org/10.3926/jiem.403

Integration of graph theory and matrix approach with fuzzy AHP for equipment selection

Hossein Safari, Alireza Faghih, Mohammad Reza Fathi

[1] *Faculty of Management, University of Tehran, (Iran)*

reza.fathi@ut.ac.ir

Received: September 2011
Accepted: March 2013

Abstract:

Purpose: The main purpose of this paper is proposing a new integrated method to equipment selection. Proposed approach is based on fuzzy Analytic Hierarchy Process (FAHP) and GTMA (graph theory and matrix approach) methods that are used for equipment selection.

Design/methodology/approach: In this paper, a two-step fuzzy-AHP and GTMA methodology is structured here that GTMA uses fuzzy-AHP result weights as input weights. Then a real case study is presented to show applicability and performance of the methodology. It can be said that using linguistic variables makes the evaluation process more realistic. Because evaluation is not an exact process and has fuzziness in its body. Here, the usage of fuzzy-AHP weights in GTMA makes the application more realistic and reliable. Proposed approach is applied to a problem of selecting CNC machines to be purchased in a company.

Findings: The outcome of this research is ranking and selecting equipment based on Fuzzy AHP and GTMA techniques. According to this method, the first CNC machine (CNC_1) is the best machine among other machines.

Originality/value: This paper offers a new integrated method for equipment selection that can be used in other areas such as supplier selection, facility location selection and etc.

Keywords: graph theory and matrix approach, analytic hierarchy process, fuzzy set, equipment selection

1. Introduction

The equipment selection problem is essential in manufacturing today because improper equipment selection can negatively affect the overall performance and productivity of a manufacturing system. The outputs of manufacturing system (i.e., the rate, quality and cost) mostly depend on what kinds of properly selected and implemented equipment are used. Selecting the new equipment is a time-consuming and difficult process, requiring advanced knowledge and experience deeply. So, the process can be a hard task for engineers and managers, and also for equipment manufacturer or vendor, to carry out. For a proper and effective evaluation, the decision maker may need a large amount of data to be analyzed and many factors to be considered (Ayag & Ozdemir, 2006). Although equipment selection plays an important role in the design of an effective manufacturing system, the publications on this subject are limited (Kulak, Durmusoglu & Kahrama, 2005). The studies performed could be classified in to two groups as equipment selection and machine selection. One of the recent studies is by Standing, Flores and Olson (2001) which uses multi-attribute utility theory to quantify the contribution of various structural and infrastructural factors for an equipment selection decision. Tabucanon, Batanov and Verma (1994) developed a decision support system for multi-criteria machine selection problem for flexible manufacturing systems (FMS), and used the AHP technique for the selection process. Chen (1999) develops an integer programming model and a heuristic algorithm to solve the problem of multiple time periods. Lagrange an relaxation is used to generate lower bounds for the integer programming model to evaluate the quality of the heuristic solution. Machine selection from fixed number of available machines is also considered by Atmani and Lashkari (1998), who developed a model for machine tool selection and operation allocation in FMS. Wang, Shaw and Chen (2000) proposed a fuzzy multi-attribute decision making model to assist the decision maker to deal with the machine selection problem for a FMS. Dellurgio, Foster and Dickerson (1997) presents a Monte Carlo simulation model for designing and selecting integrated circuit (IC) inspection systems and equipment choices. Beaulieu, Gharbi and Kadi (1997) consider the cell formation and the machine selection problems for the design of a new cellular manufacturing system using a heuristic algorithm. In addition, the articles for an equipment replacement decisions are presented by Oeltjenbruns, Kolarik and Kirschner (1995) and Sullivan, Mcdanold and Van Aken (2002). Yilmaz and Dagdeviren (2011) used a combined approach for equipment selection. Their approach is based on F-PROMETHEE method and zero–one goal programming. Safari, Fathi and Faghih (2011) applied fuzzy analytic hierarchy process (AHP) and the fuzzy technique for order preference by similarity to ideal solution (TOPSIS) methods for the selection of Machine. The proposed methods have been applied to Machine selection problem of an Electerofan company in Iran. Li, Wang, Hu, Lin and Abell (2011) utilizes such a hierarchical composition in generating system configurations with equipment selection for optimal assembly system design. A recursive algorithm is developed to generate feasible assembly sequences and the initial configurations including hybrid configurations. The generated configurations are embedded in an optimal assembly system design problem for simultaneous equipment selection and task assignment by minimizing equipment investment cost. Tuzkaya, Gulsun, Kahraman and Ozgen (2011) proposed an integrated fuzzy multi-criteria decision making methodology for MHESP. The proposed approach is utilized from fuzzy sets, Analytic Network Process (ANP) and Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE) approaches. Evaluation criteria for the MHESP is weighted by fuzzy-ANP (FANP) approach, then, alternative material handling equipment are evaluated by fuzzy-PROMETHEE (FPROMETHEE) approach. The methodology is applied for a manufacturing company to prove its effectiveness. The rest of the paper is organized as follows: The following section presents a concise treatment of the basic concepts of fuzzy set theory. Section 3 presents the methodology. The application of the proposed method is addressed in Section 4. Finally, conclusions are provided in Section 5.

2. Fuzzy sets and Fuzzy Numbers

Fuzzy set theory, which was introduced by Zadeh (1965) to deal with problems in which a source of vagueness is involved, has been utilized for incorporating imprecise data into the decision framework. A fuzzy set $\tilde{A}$ can be defined mathematically by a membership function $\mu_{\tilde{A}}(X)$, which assigns each element x in the universe of discourse X a real number in the interval [0,1]. A triangular fuzzy number $\tilde{A}$ can be defined by a triplet (a, b, c) as illustrated in Figure 1.

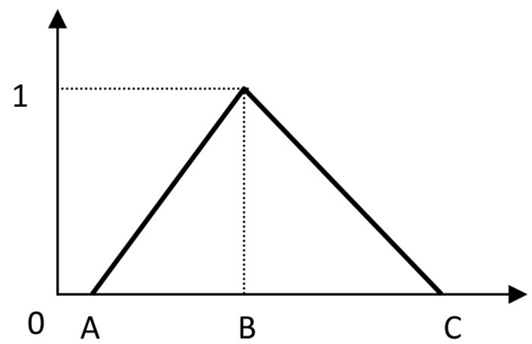

Figure 1. A triangular fuzzy number $\tilde{A}$

The membership function $\mu_{\tilde{A}}(X)$ is defined as

$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{x-a}{b-a} & a\leq x\leq b\\ \dfrac{x-c}{b-c} & b\leq x\leq c\\ 0 & oterwise\end{cases} \tag{1}$$

Basic arithmetic operations on triangular fuzzy numbers $A_1 = (a_1,b_1,c_1)$, where $a_1 \leq b_1 \leq c_1$, and $A_2 = (a_2,b_2,c_2)$, where $a_2 \leq b_2 \leq c_2$, can be shown as follows:

Addition:

$$A_1 \oplus A_2 = \left(a_1+a_2,\ b_1+b_2,\ c_1+c_2\right) \tag{2}$$

Subtraction:

$$A_1 \ominus A_2 = \left(a_1-a_2,\ b_1-b_2,\ c_1-c_2\right) \tag{3}$$

Multiplication: if k is a scalar

$$K \otimes A_1 = \begin{cases} \left(ka_1, kb_1, kc_1\right), k>0 \\ \left(kc_1, kb_1, ka_1\right), k<0 \end{cases} \tag{4}$$

Division:

$$A_1 \varnothing A_2 \approx \left(\frac{a_1}{c_2}, \frac{b_1}{b_2}, \frac{c_1}{a_2}\right) \tag{5}$$

Although multiplication and division operations on triangular fuzzy numbers do not necessarily yield a triangular fuzzy number, triangular fuzzy number approximations can be used for many practical applications (Kaufmann & Gupta, 1988). Triangular fuzzy numbers are appropriate for quantifying the vague information about most decision problems including personnel selection (e.g. rating for creativity, personality, leadership, etc.). The primary reason for using triangular fuzzy numbers can be stated as their intuitive and computational-efficient representation (Karsak, 2002). A linguistic variable is defined as a variable whose values are not numbers, but words or sentences in natural or artificial language. The concept of a linguistic variable appears as a useful means for providing approximate characterization of phenomena that are too complex or ill-defined to be described in conventional quantitative terms (Zadeh, 1975).

3. Research methodology

In this paper, the weights of each criterion are calculated using fuzzy AHP. After that, GTMA is utilized to rank the alternatives. Finally, we select the best equipment based on these results.

3.1. Fuzzy AHP

Despite of its wide range of applications, the conventional AHP approach may not fully reflect a style of human thinking. One reason is that decision makers usually feel more confident to give interval judgments rather than expressing their judgments in the form of single numeric values. As a result, fuzzy AHP and its extensions are developed to solve alternative selection and justification problems. Although FAHP requires tedious computations, it is capable of capturing a human's appraisal of ambiguity when complex multi-attribute decision making problems are considered. In the literature, many FAHP methods have been proposed ever since the seminal paper by Van Laarhoven and Pedrycz (1983). In his earlier work, Saaty (1980) proposed a method to give meaning to both fuzziness in perception and fuzziness in meaning.

This method measures the relativity of fuzziness by structuring the functions of a system hierarchically in a multiple attribute framework. Later on, Buckley (1985) extends Saaty's AHP method in which decision makers can express their preference using fuzzy ratios instead of crisp values. Chang (1996) developed a fuzzy extent analysis for AHP, which has similar steps as that of Saaty's crisp AHP. However, his approach is relatively easier in computation than the other fuzzy AHP approaches. In this paper, Chang's fuzzy extent analysis used for AHP. Kahraman, Cebeci and Ulukan (2003) applied Chang's (1996) fuzzy extent analysis in the selection of the best catering firm, facility layout and the best transportation company, respectively.

Let $O = \{o_1,o_2,\ldots,o_n\}$ be an object set, and $U = \{g_1,g_2,\ldots,g_m\}$ be a goal set. According to the Chang's extent analysis, each object is considered one by one, and for each object, the analysis is carried out for each of the possible goals, g_i. Therefore, m extent analysis values for each object are obtained and shown as follows:

$$\widetilde{M}^{1}_{g_i},\ \widetilde{M}^{2}_{g_i},\ldots,\ \widetilde{M}^{m}_{g_i},\ i=1,2,\ldots,n$$

Where $\widetilde{M}^{j}_{g_i}$ (j=1,2,3,…, m) are all triangular fuzzy numbers. The membership function of the triangular fuzzy number is denoted by $M_{(x)}$. The steps of the Chang's extent analysis can be summarized as follows:

Step 1: The value of fuzzy synthetic extent with respect to the i-th object is defined as:

$$S_i = \sum_{j=1}^{m} \widetilde{M}^{j}_{g_i} \otimes \left[\sum_{i=1}^{n}\sum_{j=1}^{m} \widetilde{M}^{j}_{g_i}\right]^{-1} \tag{6}$$

Where $\otimes$ denotes the extended multiplication of two fuzzy numbers. In order to obtain

$$\sum_{j=1}^{m} \widetilde{M}^{j}_{g_i}$$

We perform the addition of m extent analysis values for a particular matrix such that,

$$\sum_{j=1}^{m} \widetilde{M}^{j}_{g_i} = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j\right) \tag{7}$$

And to obtain $\left[\sum_{i=1}^{n}\sum_{j=1}^{m} \widetilde{M}^{j}_{g_i}\right]^{-1}$ we perform the fuzzy addition operation of $\widetilde{M}^{j}_{g_i}$ (j =1,2,…,m) values such that,

$$\sum_{i=1}^{n}\sum_{j=1}^{m} \widetilde{M}^{j}_{g_i} = \left(\sum_{i=1}^{n} l_i, \sum_{i=1}^{n} m_i, \sum_{i=1}^{n} u_i\right) \tag{8}$$

Then, the inverse of the vector is computed as,

$$\left[\sum_{i=1}^{n}\sum_{j=1}^{m}\widetilde{M}_{g_i}^{j}\right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n}u_i}, \frac{1}{\sum_{i=1}^{n}m_i,}, \frac{1}{\sum_{i=1}^{n}l_i}\right) \tag{9}$$

Where u_i, m_i, $l_i>0$

Finally, to obtain the S_j, we perform the following multiplication:

$$S_i = \sum_{j=1}^{m}\widetilde{M}_{g_i}^{j}\otimes[\sum_{i=1}^{n}\sum_{j=1}^{m}\widetilde{M}_{g_i}^{j}]^{-1} = \left(\sum_{j=1}^{m}l_j\otimes\sum_{i=1}^{n}l_i, \sum_{j=1}^{m}m_j\otimes\sum_{i=1}^{n}m_i, \sum_{j=1}^{m}u_j\otimes\sum_{i=1}^{n}u_i\right) \tag{10}$$

Step 2: The degree of possibility of $\widetilde{M}_2$ = (l_2,m_2,u_2) ≥ $\widetilde{M}_1$ = (l_1,m_1,u_1) is defined as

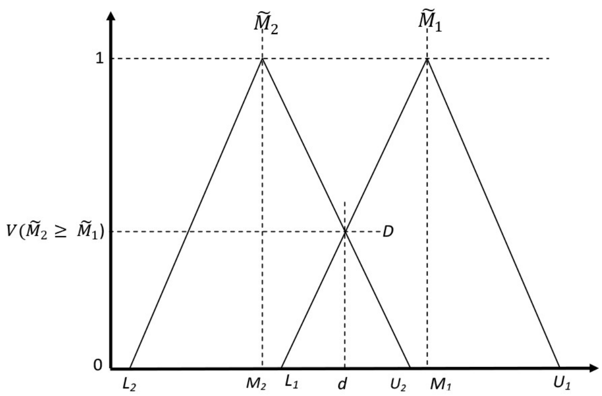

Figure 2. The degree of possibility of $\widetilde{M}_1 \geq \widetilde{M}_2$

$$V(\widetilde{M}_2 \geq \widetilde{M}_1) = \text{Sup}[min(\widetilde{M}_1(x), \widetilde{M}_2(y))] \tag{11}$$

This can be equivalently expressed as,

$$V(\widetilde{M}_2 \geq \widetilde{M}_1) = \text{hgt}(\widetilde{M}_2 \cap \widetilde{M}_1) = \widetilde{M}_2(d) = \begin{cases} 1 & if\ m_2 \geq m_1 \\ 0 & if\ l_1 \geq u_2 \\ \dfrac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)}, & otherwise \end{cases} \tag{12}$$

Figure 2 illustrates V ($\widetilde{M}_2$ ≥ $\widetilde{M}_1$) for the case d for the case $m_1 < l_1 < u_2 < m_1$, where d is the abscissa value corresponding to the highest cross over point D between $\widetilde{M}_1$ and $\widetilde{M}_2$,To compare $\widetilde{M}_1$ and $\widetilde{M}_2$, we need both of the values V($\widetilde{M}_1$ ≥ $\widetilde{M}_2$) and V($\widetilde{M}_2$ ≥ $\widetilde{M}_1$).

Step 3: The degree of possibility for a convex fuzzy number to be greater than k convex fuzzy numbers M_i (I=1, 2… K) is defined as

$$V(\widetilde{M} \geq \widetilde{M}_1, \widetilde{M}_2, ..., \widetilde{M}_k) = \min V(\widetilde{M} \geq \widetilde{M}_i),\ i = 1,2,...,k$$

Step 4: Finally, W=(min V($s_1 \geq s_k$) min V($s_2 \geq s_k$),….,min V($s_n \geq s_k$))^T, is the weight vector for k = 1,...,n.

In order to perform a pairwise comparison among the parameters, a linguistic scale has been developed. Our scale is depicted in Figure 3 and the corresponding explanations are provided in Table 1. Similar to the importance scale defined in Saaty's classical AHP (Saaty, 1980), we have used five main linguistic terms to compare the criteria: "equal importance", "moderate importance", "strong importance", "very strong importance" and "demonstrated importance". We have also considered their reciprocals: "equal unimportance", "moderate unimportance", "strong unimportance", "very strong unimportance" and "demonstrated unimportance". For instance, if criterion A is evaluated "strongly important" than criterion B, then this answer means that criterion B is "strongly unimportant" than criterion A.

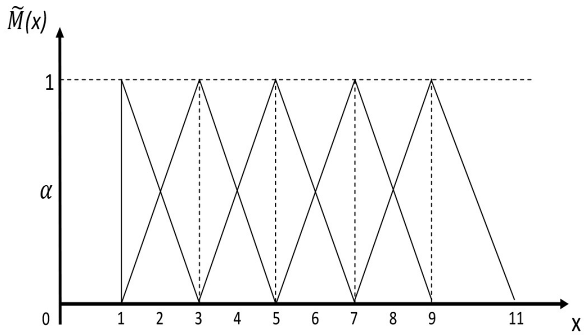

Figure 3. Membership functions of triangular fuzzy numbers corresponding to the linguistic scale

Linguistic scale	triangular fuzzy numbers	inverse of triangular fuzzy numbers
Equal Importance	(1, 1, 1)	(1, 1, 1)
Moderate Importance	(1, 3, 5)	(1/5, 1/3, 1)
Strong importance	(3, 5, 7)	(1/7, 1/5, 1/3)
Very strong importance	(5, 7, 9)	(1/9, 1/7, 1/5)
Demonstrated importance	(7, 9, 11)	(1/11, 1/9, 1/7)

Table 1. The linguistic scale and corresponding triangular fuzzy numbers

3.2. The GTMA method

Graph theory is a logical and systematic approach. The advanced theory of graphs and its applications are very well documented. Rao (2007) in his book presents this methodology and shows some of its applications. Graph/digraph model representations have proved to be useful for modeling and analyzing various kinds of systems and problems in numerous fields of science and technology (Darvish, Yasaei & Saeedi, 2009). The matrix approach is useful in analyzing the graph/digraph models expeditiously to derive the system function and index to meet the objectives (Rao, 2007). The graph theory and matrix methods consist of the digraph representation, the matrix representation and the permanent function representation. The digraph is the visual representation of the variables and their interdependencies. The matrix converts the digraph into mathematical form and the permanent function is a mathematical representation that helps to determine the numerical index (Faisal, Banwet & Shankar, 2007).

The step by step explanation of the methodology is as follows:

Step 1: Identifying equipment selection attributes. In this step all the criteria which affect the decision is determined. This can be done by using relevant criteria available in the literature or getting information from the decision maker.

Step 2: Determine equipment alternatives. All potential alternatives are identified.

Step 3: Graph representation of the criteria and their inter dependencies. Equipment selection criterion is defined as a factor that influences the selection of an alternative. The equipment selection criteria digraph models the alternative selection criteria and their inter relationship. This digraph consists of a set of nodes N = {n_i}, with i = 1, 2,...,M and a set of directed edges E = {e_{ij}}. A node n_i represents i-th alternative selection criterion and edges represent the relative importance among the criteria. The number of nodes M considered is equal to the number of alternative selection criteria considered. If a node 'i' has relative importance over another node 'j' in the alternative selection, then a directed edge or arrow is drawn from node i to node j (i.e. e_{ij}). If 'j' has relative importance over 'i' directed edge or arrow is drawn from node j to node i (e_{ji}) (Rao, 2007).

Step 4: Develop equipment selection criteria matrix of the graph. Matrix representation of the alternative selection criteria digraph gives one-to-one representation. A matrix called the equipment selection criteria matrix. This is an M in M matrix and considers all of the criteria (i.e. A_i) and their relative importance (i.e. a_{ij}). Where A_i is the value of the i-th criteria represented by node n_i and a_{ij} is the relative importance of the i-th criteria over the j-th represented by the edge e_{ij} (Rao, 2007; Faisal et al., 2007).

The value of A_i should preferably be obtained from available or estimated data. When quantitative values of the criteria are available, normalized values of a criterion assigned to the alternatives are calculated by v_i/v_j, where v_i is the measure of the criterion for the i-th alternative and v_j is the measure of the criterion for the j-th alternative which has a higher measure of the criterion among the considered alternatives. This ratio is valid for beneficial criteria only. A beneficial criteria means its higher measures are more desirable for the given application. Whereas, the non-beneficial criterion is the one whose lower measures are desirable and the normalized values assigned to the alternatives are calculated by v_j/v_i.

$$CS\,\text{Matrix} = \begin{bmatrix} A_1 & a_{12} & a_{13} & a & a & a_{1.m} \\ a_{21} & A_2 & a_{23} & \cdots & \cdots & a_{2.m} \\ a_{31} & a_{32} & A_3 & \cdots & \cdots & a_{3.m} \\ \vdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \vdots & \ddots & \cdots & \cdots & \cdots & \vdots \\ a_1 & a_1 & a_1 & \cdots & \cdots & A_m \end{bmatrix} \tag{13}$$

Step 5: Obtaining alternative selection criteria function for the matrix. The permanent of this matrix, is defined as the alternative selection criteria function. The permanent of a matrix was introduced by Cauchy in 1812. At that time, while developing the theory of determinants, he also defined a certain subclass of symmetric functions which later Muir named permanents (Nourani & Andresen, 1999). The permanent is a standard matrix function and is used in combinatorial mathematics (Faisal et al., 2007; Rao, 2006). The permanent function is obtained in a similar manner as the determinant but unlike in a determinant where a negative sign appears in the calculation, in a variable permanent function positive signs replace these negative signs (Faisal et al., 2007; Rao, 2006). Application of the permanent concept will lead to a better appreciation of selection attributes. Moreover, using this no negative sign will appear in the expression (unlike determinant of a matrix in which a negative sign can appear) and hence no information will be lost (Rao, 2006).

The per(CS) contains terms arranged in (M+1) groups, and these groups represent the measures of criteria and the relative importance loops. The first group represents the measures of M criteria. The second group is absent as there is no self-loop in the digraph. The third group contains 2- criterion relative importance loops and measures of (M-2) criteria. Each term of the fourth group represents a set of a 3-criterion relative importance loop, or its pair, and measures of (M-3) criteria. The fifth group contains two sub-groups. The terms of the first sub-group is a set of two 2-criterion relative importance loops and the measures of (M-4) criteria. Each term of second sub-group is a set of a 4-attribute relative importance loop, or its pair, and the measures of (M-4) criteria. The sixth group contains two subgroups. The terms of the first sub-group are a set of a 3-criterion relative importance loop, or its pair, and 2-criterion importance loop and the measures of (M-5) criteria. Each term of the second sub-group is a set of a 5-criterion relative importance loop, or its pair, and the measures of (M-5) criteria. Similarly other terms of the equation are defined. Thus, the CS fully characterizes the considered alternative selection evaluation problem, as it contains all possible structural components of the criteria and their relative importance. It may be mentioned that this equation is nothing but the determinant of an M _ M matrix but considering all the terms as positive.

Step 6: Evaluation and ranking of the alternatives, in this step all alternatives are ranked according to their permanent values calculated in the previous step.

$$
\begin{aligned}
\text{per (Cs)} = \; & \prod_{i=1}^{M} A_i + \sum_{i=1}^{M-1} \sum_{j=i+1}^{M} \dots \sum_{M=t+1}^{M} (a_{ij} a_{ji}) A_k A_l A_m A_n A_o \dots A_t A_M \\
& + \sum_{i=1}^{M-2} \sum_{j=i+1}^{M-1} \sum_{k=i+1}^{M} \dots \sum_{M=t+!}^{M} \left(a_{ij} a_{jk} a_{ki} + a_{ik} a_{kj} a_{ji} \right) A_l A_m A_n A_o \dots A_t A_M \\
& + \sum_{i=1}^{M-3} \sum_{j=i+1}^{M} \sum_{k=i+1}^{M-1} \sum_{l=i+2}^{M} \dots \sum_{M=t+1}^{M} \left(a_{ij} a_{ji} + a_{kl} a_{lk} \right) A_m A_n A_o \dots A_t A_M \\
& + \sum_{i=1}^{M-3} \sum_{j=i+1}^{M} \sum_{k=i+1}^{M-1} \sum_{l=i+2}^{M} \dots \sum_{M=t+1}^{M} \left(a_{ij} a_{jk} a_{kl} a_{li} + a_{il} a_{lk} a_{kj} a_{ji} \right) A_m A_n A_o \dots A_t A_M \; +
\end{aligned}
\tag{14}
$$

$$\sum_{i=1}^{M-2}\sum_{j=1}^{M-1}\sum_{j=i+1}^{M}\sum_{l=1}^{M-1}\sum_{m=l+1}^{M-2}\dots\dots\sum_{m=t+1}^{M}\left(a_{ij}a_{jk}a_{ki}+a_{ik}a_{kj}a_{ji}\right)\left(a_{lm}a_{m}\right)A_nA_o\dots A_tA_m+$$

$$\sum_{i=1}^{M-4}\sum_{j=i+1}^{M-1}\sum_{k=j+1}^{M}\sum_{l=1}^{M}\sum_{m=l+1}^{M}\dots\dots\sum_{M=t+1}^{M}\left(a_{ij}a_{jk}a_{kl}a_{lm}a_{mi}+a_{\Im}a_{mj}a_{lk}a_{kj}a_{ji}\right)A_nA_o\dots..A_tA_m \quad +$$

$$\sum_{i=1}^{M-3}\sum_{j=i+1}^{M-1}\sum_{k=j+1}^{M}\sum_{l=1}^{M}\sum_{m=l+1}^{M-1}\sum_{n=m+1}^{M}\dots\dots.\sum_{M=t+1}^{M}\left(a_{ij}a_{jk}a_{ki}+a_{ik}a_{kj}a_{ji}\right)\left(a_{lm}a_{mn}a_{nl}+a_{\ln}a_{nm}a_{ml}\right)A_o\dots.A_tA_m$$

$$+\sum_{i=1}^{M-5}\sum_{j=i+1}^{M-1}\sum_{k=j+1}^{M}\sum_{l=1}^{M-2}\sum_{m=l+1}^{M-1}\sum_{n=m+1}^{M}\dots\dots.\sum_{M=t+1}^{M}\left(a_{ij}a_{jk}a_{ki}+a_{ik}a_{kj}a_{ji}\right)+\left(a_{lm}a_{mn}a_{nl}+a_{\ln}a_{nm}a_{ml}\right)A_o\dots A_tA_M$$

$$+\sum_{i=1}^{M-5}\sum_{j=i+1}^{M-1}\sum_{k=j+1}^{M}\sum_{l=1}^{M}\sum_{m=l+1}^{M}\sum_{n=m+1}^{M}\dots\dots.\sum_{M=t+1}^{M}\left(a_{ij}+a_{jk}a_{kl}a_{lm}a_{mn}a_{nj}+a_{¿}a_{nm}a_{ml}a_{lk}a_{kj}a_{ji}\right)A_o\dots A_tA_M$$

4. A numerical application of proposed approach

The proposed approach is applied in a manufacturing company, located in Qom, Iran. The company wants to purchase a few CNC machines to reduce the work in- process inventory and to replace its old equipment. The high technology equipment make significant improvements in the manufacturing processes of the firms and the correct decisions made at this stage brings the companies competitive advantage. Therefore, selecting the most proper CNC machines is of great importance for the company. But it is hard to choose the most suitable one among the machines which dominate each other in different characteristics. In the application, firstly through the literature investigation and studying other papers that are related to equipment selection, six criteria are selected. These criteria include weight (C_1), power(C_2), price (C_3), stroke (C_4),spindle (C_5) and diameter (C_6). In addition, there are six alternatives include CNC_1, CNC_2, CNC_3, CNC_4, CNC_5 and CNC_6. Figure 4 shows the inter relationships between the criteria.

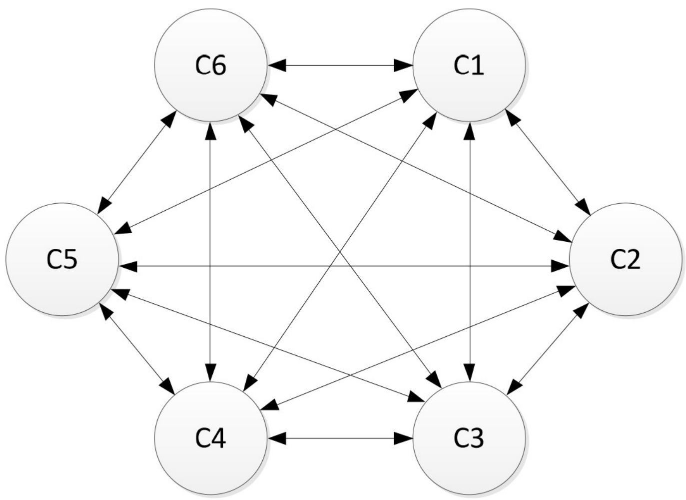

Figure 4. The inter relationships between the criteria

4.1. Fuzzy AHP

In fuzzy AHP, firstly, the criteria and alternatives' importance weights must be compared. Afterwards, the comparisons about the criteria and alternatives, and the weight calculation need to be made. Thus, the evaluation of the criteria according to the main goal and the evaluation of the alternatives for these criteria must be realized. Then, after all these evaluation procedure, the weights of the alternatives can be calculated. In the second step, these weights are used to GTMA calculation for the final evaluation. Decision makers from different backgrounds may define different weight vectors. They usually cause not only the imprecise evaluation but also serious persecution during decision process. For this reason, we proposed a group decision based on FAHP to improve pair-wise comparison. Firstly each decision maker (DM), individually carry out pair-wise comparison and you can see them in Table 2, Table 3 and Table 4.

DM_1	C_1	C_2	C_3	C_4	C_5	C_6
C_1	(1,1,1)	(1,1.2,1.4)	(2, 2.5,3.7)	(2,2.1,2.6)	(1.5,2.4,3.8)	(2.1,3.4,3.7)
C_2	(.71,.83,1)	(1,1,1)	(1,1.5,2)	(.25,.33,.5)	(1,2.3,3)	(1.24,2.3,3.6)
C_3	(.27,.4,.5)	(.5,.67,1)	(1,1,1)	(.14,.2,.33)	(.98,1.4,2)	(1.4,2.8,3.22)
C_4	(.38,.48,.5)	(2,3,4)	(3,5,7)	(1,1,1)	(2,3,4.2)	(1.5,3,3.8)
C_5	(2,3,4)	(3,5,7)	(.5,.7,1.02)	(.24,.33,.5)	(1,1,1)	(3.2,3.7,4.25)
C_6	(.27,.3,.48)	(.28,.43,.8)	(.31,.36,.7)	(.26,.3,.67)	(.24,.27,.31)	(1,1,1)

Table 2. Pair-wise comparison of first decision maker

DM_2	C_1	C_2	C_3	C_4	C_5	C_6
C_1	(1,1,1)	(2.5,3,3.6)	(3.1,3.4,3.6)	(2.21,2.6,3.8)	(2.33,2.9,3.2)	(1.8,2.5,3.21)
C_2	(.28,.3,.4)	(1,1,1)	(.17,.2,.25)	(.25,.33,.50)	(.33,1.3,1.5)	(2.22,3.34,4)
C_3	(.27,.29,.3)	(4,5,6)	(1,1,1)	(1.26,1.7,3.2)	(2.3,2.8,3.47)	(.8,.96,1.3)
C_4	(.26,.3,.45)	(2,3,4)	(.31,.56,.79)	(1,1,1)	(.33,.5,1)	(1.2,1.8,2.6)
C_5	(2,3,4)	(.3,.56,.8)	(.29,.36,.43)	(1,2,3)	(1,1,1)	(2.25,2.5,3)
C_6	(.3,.4,.56)	(.2,.3,.45)	(.77,1.04,1.2)	(.38,.56,.83)	(.33,.4,.44)	(1,1,1)

Table 3. Pair-wise comparison of second decision maker

DM_3	C_1	C_2	C_3	C_4	C_5	C_6
C_1	(1,1,1)	(1,2,3)	(.9,1.2,1.9)	(3.3,3.9,4.6)	(1,2,3)	(.33,.5,1)
C_2	(.33,.5,1)	(1,1,1)	(2.3,2.76,3.8)	(1.45,3,3.5)	(4,5,6)	(1.5,1.8,2.11)
C_3	(.53,.83,1.1)	(.26,.36,.4)	(1,1,1)	(.14,.17,.2)	(2,3,4)	(2.6,3.4,4.1)
C_4	(.2,.26,.3)	(.29,.34,.6)	(5,6,7)	(1,1,1)	(3,4,5)	(.2,.5,1.1)
C_5	(.2,.26,.3)	(5,6,7)	(.25,.33,.5)	(.2,.25,.33)	(1,1,1)	(1.05,2.16,2.9)
C_6	(1,2,3)	(.47,.56,.6)	(.24,.29,.38)	(.91,2,5)	(.34,.46,.9)	(1,1,1)

Table 4. Pair-wise comparison of third decision maker

Then, a comprehensive pair-wise comparison matrix is built as in Table 5.

D	C_1	C_2	C_3	C_4	C_5	C_6
C_1	(1,1,1)	(1.5,2.1,2.67)	(2,2.37,3.09)	(2.51,2.89,3.67)	(1.61,2.43,3.35)	(1.41,2.13,2.6)
C_2	(.44,.55,.8)	(1,1,1)	(1.16,1.49,2.04)	(.65,1.2,1.5)	(1.78,2.87,3.5)	(1.65,2.5,3.23)
C_3	(.36,.51,.65)	(1.59,2,2.48)	(1,1,1)	(.52,.72,1.24)	(1.77,2.4,3.16)	(1.6,2.39,2.8)
C_4	(.29,.37,.42)	(1.43,2.11,2.9)	(2.77,3.85,4.93)	(1,1,1)	(1.78,2.5,3.4)	(.97,1.77,2.5)
C_5	(1.43,2.11,2.9)	(2.77,3.85,4.93)	(.35,.47,.65)	(.48,.86,1.28)	(1,1,1)	(2.17,2.8,3.4)
C_6	(.53,.9,1.34)	(.33,.43,.64)	(.44,.56,.78)	(.52,.96,2.17)	(.3,.38,.57)	(1,1,1)

Table 5. Fuzzy pair-wise comparison matrix

After forming fuzzy pair-wise comparison matrix, we calculate the weight of all criteria. The weight calculation details are given below. Because of the other calculations are similar for each comparison matrix, these are not given here and can be done simply according the computations below. The value of fuzzy synthetic extent with respect to the i-th object (I = 1,2,...,8) is calculated as

$$S_1 = (10.03,\ 12.9,\ 16.4) \otimes (0.0132,\ 0.0171,\ 0.02) = (0.1325,\ 0.2209,\ 0.3808)$$

$$S_2 = (6.68,\ 9.61,\ 12.07) \otimes (0.0132,\ 0.0171,\ 0.02) = (0.0882,\ 0.1643,\ 0.2801)$$

$$S_3 = (6.83,\ 9.02,\ 11.40) \otimes (0.0132,\ 0.0171,\ 0.02) = (0.0902,\ 0.1542,\ 0.2646)$$

$$S_4 = (8.23,\ 11.6,\ 15.15) \otimes (0.0132,\ 0.0171,\ 0.02) = (0.1087,\ 0.1984,\ 0.3515)$$

$$S_5 = (8.2,\ 11.08,\ 14.16) \otimes (0.0132,\ 0.0171,\ 0.02) = (0.1082,\ 0.1895,\ 0.3285)$$

$$S_6 = (3.12,\ 4.23,\ 6.511) \otimes (0.0132,\ 0.0171,\ 0.02) = (0.0412,\ 0.0723,\ 0.1509)$$

Then the V values calculated using these vectors are shown in Table 6.

(V)	S_1	S_2	S_3	S_4	S_5	S_6
S_1	-	1	1	1	1	1
S_2	0.722	-	1	0.834	0.872	1
S_3	0.664	0.945	-	0.779	0.816	1
S_4	0.906	1	1	-	1	1
S_5	0.861	1	1	0.961	-	1
S_6	0.110	0.405	0.425	0.250	0.267	-

Table 6. V values result

Thus, the weight vector from Table 5 is calculated and normalized as

$$W^t = (0.2344,\ 0.1694,\ 0.1557,\ 0.2125,\ 0.2020,\ 0.0258)$$

4.2. The GTMA method

The weights of the alternatives are calculated by fuzzy AHP up to now, and then these values can be used in GTMA. After calculating the weights, we formed the decision matrix that shows in Table 7. This decision matrix is made by Questionnaire. We used the mathematical mean for forming the aggregate decision matrix.

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	18.96	0.31	30.61	1.43	16.25	3.72
A_2	20.76	0.24	16.33	1.70	11.52	5.93
A_3	16.88	0.25	26.55	1.01	15.85	3.72
A_4	11.48	0.31	32.87	1.31	12.01	6.21
A_5	26.26	0.22	28.68	1.03	12.24	3.48
A_6	14.12	0.16	24.58	1.68	9.91	8.22
MAX	26.26	0.31	32.87	1.70	16.25	8.22

Table 7. Decision matrix of GTMA

In the next step, we normalized the decision matrix that shows in Table 8.

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.722	1.000	0.931	0.842	1.000	0.453
A_2	0.791	0.779	0.497	1.000	0.709	0.722
A_3	0.643	0.806	0.808	0.597	0.975	0.453
A_4	0.437	0.989	1.000	0.773	0.739	0.755
A_5	1.000	0.710	0.873	0.604	0.753	0.424
A_6	0.538	0.508	0.748	0.991	0.610	1.000

Table 8. Normalized decision matrix

Then, according to GTMA method, we carry out pair-wise comparison with respect to their weight that shows from Table 9 to Table 15.

	C_1	C_2	C_3	C_4	C_5	C_6
C_1	-	0.580	0.601	0.524	0.537	0.901
C_2	0.420	-	0.521	0.444	0.456	0.868
C_3	0.399	0.479	-	0.423	0.435	0.858
C_4	0.476	0.556	0.577	-	0.513	0.892
C_5	0.463	0.544	0.565	0.487	-	0.887
C_6	0.099	0.132	0.142	0.108	0.113	-
w_j	0.234	0.169	0.156	0.213	0.202	0.026

Table 9. Pair-wise comparison of criteria with respect to each other

CNC_1	C_1	C_2	C_3	C_4	C_5	C_6
C_1	0.722	0.580	0.601	0.524	0.537	0.901
C_2	0.420	1.000	0.521	0.444	0.456	0.868
C_3	0.399	0.479	0.931	0.423	0.435	0.858
C_4	0.476	0.556	0.577	0.842	0.513	0.892
C_5	0.463	0.544	0.565	0.487	1.000	0.887
C_6	0.099	0.132	0.142	0.108	0.113	0.453

Table 10. Pair-wise comparison of criteria with respect to A_1

CNC_2	C_1	C_2	C_3	C_4	C_5	C_6
C_1	0.791	0.580	0.601	0.524	0.537	0.901
C_2	0.420	0.779	0.521	0.444	0.456	0.868
C_3	0.399	0.479	0.497	0.423	0.435	0.858
C_4	0.476	0.556	0.577	1.000	0.513	0.892
C_5	0.463	0.544	0.565	0.487	0.709	0.887
C_6	0.099	0.132	0.142	0.108	0.113	0.722

Table 11. Pair-wise comparison of criteria with respect to A_2

CNC_3	C_1	C_2	C_3	C_4	C_5	C_6
C_1	0.643	0.580	0.601	0.524	0.537	0.901
C_2	0.420	0.806	0.521	0.444	0.456	0.868
C_3	0.399	0.479	0.808	0.423	0.435	0.858
C_4	0.476	0.556	0.577	0.597	0.513	0.892
C_5	0.463	0.544	0.565	0.487	0.975	0.887
C_6	0.099	0.132	0.142	0.108	0.113	0.453

Table 12. Pair-wise comparison of criteria with respect to A_3

CNC_4	C_1	C_2	C_3	C_4	C_5	C_6
C_1	0.437	0.580	0.601	0.524	0.537	0.901
C_2	0.420	0.989	0.521	0.444	0.456	0.868
C_3	0.399	0.479	1.000	0.423	0.435	0.858
C_4	0.476	0.556	0.577	0.773	0.513	0.892
C_5	0.463	0.544	0.565	0.487	0.739	0.887
C_6	0.099	0.132	0.142	0.108	0.113	0.755

Table 13. Pair-wise comparison of criteria with respect to A_4

CNC_5	C_1	C_2	C_3	C_4	C_5	C_6
C_1	1.000	0.580	0.601	0.524	0.537	0.901
C_2	0.420	0.710	0.521	0.444	0.456	0.868
C_3	0.399	0.479	0.873	0.423	0.435	0.858
C_4	0.476	0.556	0.577	0.604	0.513	0.892
C_5	0.463	0.544	0.565	0.487	0.753	0.887
C_6	0.099	0.132	0.142	0.108	0.113	0.424

Table 14. Pair-wise comparison of criteria with respect to A_5

CNC_6	C_1	C_2	C_3	C_4	C_5	C_6
C_1	0.538	0.580	0.601	0.524	0.537	0.901
C_2	0.420	0.508	0.521	0.444	0.456	0.868
C_3	0.399	0.479	0.748	0.423	0.435	0.858
C_4	0.476	0.556	0.577	0.991	0.513	0.892
C_5	0.463	0.544	0.565	0.487	0.610	0.887
C_6	0.099	0.132	0.142	0.108	0.113	1.000

Table 15. Pair-wise comparison of criteria with respect to A_6

After that we calculate the permanent matrix. The permanent matrix of each alternative is indicated in Table 16.

Alternative	Permanent matrix
CNC_1	10.7761
CNC_2	10.0513
CNC_3	8.5713
CNC_4	10.7022
CNC_5	8.7418
CNC_6	10.1887

Table 16. Permanent matrix of each alternative

Finally, we rank all machines with respect to their permanent matrix that shows in Table 17.

Alternative	Permanent matrix	Rank
CNC_1	10.7761	1
CNC_2	10.0513	4
CNC_3	8.5713	6
CNC_4	10.7022	2
CNC5	8.7418	5
CNC_6	10.1887	3

Table 17. Ranking alternative

According to Table 17, the first CNC machine (CNC_1) is the best machine among other machines.

5. Conclusion

A proper equipment selection is a very important activity for manufacturing systems due to the fact that improper equipment selection can negatively affect the overall performance and productivity of a manufacturing system. In this paper, a two-step fuzzy-AHP and GTMA methodology is structured here that GTMA uses fuzzy-AHP result weights as input weights. Then a real case study is presented to show applicability and performance of the methodology. It can be said that using linguistic variables makes the evaluation process more realistic. Because evaluation is not an exact process and has fuzziness in its body. Here, the usage of fuzzy-AHP weights in GTMA makes the application more realistic and reliable. The proposed model has only been implemented on an equipment selection problem in the company; however, company management has found the proposed model satisfactory and implementable in others equipment selection decisions. As a future direction, other decision-making methods such as fuzzy ELECTRE, fuzzy GTMA and interval GTMA can be used in this area.

Acknowledgement

The authors would like to thank the anonymous reviewers and the editor for their insightful comments and suggestions.

References

Atmani, A., & Lashkari, R.S. (1998). A model of machine tool selection and operation allocation in flexible manufacturing system. *International Journal of Production Research,* 36, 1339-1349. http://dx.doi.org/10.1080/002075498193354

Ayag, Z., & Ozdemir, R.G. (2006). A fuzzy AHP approach to evaluating machine tool alternatives. *Journal of Intelligent Manufacturing,* 17, 179-190. http://dx.doi.org/10.1007/s10845-005-6635-1

Beaulieu, A., Gharbi, A., & Kadi, A. (1997). An algorithm for the cell formation and the machine selection problems in the design of a cellular manufacturing system. *International Journal of Production Research*, 35, 1857-1874. http://dx.doi.org/10.1080/002075497194958

Buckley, J.J. (1985). Fuzzy hierarchical analysis. *Fuzzy Sets and Systems*, 17, 233-247. http://dx.doi.org/10.1016/0165-0114(85)90090-9

Chang, D.Y. (1996). Applications of the extent analysis method on fuzzy AHP. *European Journal of Operational Research*, 95, 649-655. http://dx.doi.org/10.1016/0377-2217(95)00300-2

Chen, M.A. (1999). Heuristic for solving manufacturing process and equipment selection problems. *International Journal of Production Research*, 37, 359-374. http://dx.doi.org/10.1080/002075499191814

Darvish, M., Yasaei, M. & Saeedi, A.(2009). Application of the graph theory and matrix methods to contractor ranking. *International Journal of Project Management*, 27(6), 610-619. http://dx.doi.org/10.1016/j.ijproman.2008.10.004

Dellurgio, S.A., Foster, S.T., & Dickerson, G. (1997). Utilizing simulation to develop economic equipment selection and sampling plans for integrated circuit manufacturing. *International Journal of Production Research*, 35, 137-155. http://dx.doi.org/10.1080/002075497196028

Faisal, M.N., Banwet, D. & Shankar, R. (2007). Quantification of risk mitigation environment of supply chains using graph theory and matrix methods. *European Journal of Industrial Engineering*, 1(1), 22-39. http://dx.doi.org/10.1504/EJIE.2007.012652

Kahraman, C., Cebeci, U., & Ulukan, Z. (2003). Multi-criteria supplier selection using fuzzy AHP. *Logistics Information Management*, 16(6), 382-394. http://dx.doi.org/10.1108/09576050310503367

Karsak, E.E. (2002). Distance-based fuzzy MCDM approach for evaluating flexible manufacturing system alternatives. *International Journal of Production Research,* 40(13), 3167-3181. http://dx.doi.org/10.1080/00207540210146062

Kaufmann, A., & Gupta, M.M. (1988). *Fuzzy mathematical models in engineering and management science.* Amsterdam: North-Holland.

Kulak, O., Durmusoglu, M.B., & Kahraman, C. (2005). Fuzzy multi attribute equipment selection based on information axiom. *Journal of Materials Processing Technology*, 169, 337-345. http://dx.doi.org/10.1016/j.jmatprotec.2005.03.030

Li, S., Wang, H., Hu, S., Lin, Y., & Abell, J. (2011). Automatic generation of assembly system configuration with equipment selection for automotive battery manufacturing. *Journal of Manufacturing Systems*, 30, 188-195. http://dx.doi.org/10.1016/j.jmsy.2011.07.009

Nourani, Y., & Andresen, B. (1999). Exploration of NP-hard enumeration problems by simulated annealing -the spectrum values of permanents. *Theoretical computer science*, 215(1-2), 51-68. http://dx.doi.org/10.1016/S0304-3975(99)80002-4

Oeltjenbruns, H., Kolarik, W.J., & Kirschner, R.S. (1995).Strategic planning in manufacturing Systems-AHP application to an equipment replacement decision. *International Journal of Production Economics*, 38, 189-197. http://dx.doi.org/10.1016/0925-5273(94)00092-O

Rao, R.V. (2007). *Decision making in the manufacturing environment: using graph theory and fuzzy multiple attribute decision making methods*. London: Springer.

Rao, R.V. (2006). A decision-making framework model for evaluating flexible manufacturing systems using digraph and matrix methods. *The International Journal of Advanced Manufacturing Technology*, 30(11), 1101-1110. http://dx.doi.org/10.1007/s00170-005-0150-6

Saaty, T.L. (1980). *The analytic hierarchy process.* New York: McGraw- Hill.

Safari, H., Fathi, M.R., & Faghih, A. (2011). Applying fuzzy AHP and fuzzy TOPSIS to Machine Selection. *Journal of American Science*, 7(9), 755-765.

Standing, G., Flores, B., & Olson, D. (2001).Understanding managerial preferences in selection equipment. *Journal of Operation Management*, 19, 23–37. http://dx.doi.org/10.1016/S0272-6963(00)00047-4

Sullivan, G.W., Mcdanold, T.N., & Van Aken, E.M. (2002). Equipment replacement decisions and lean manufacturing. *Robotics and Computer-Integrated Manufacturing*, 18, 255-265. http://dx.doi.org/10.1016/S0736-5845(02)00016-9

Tabucanon, M.T., Batanov, D.N., & Verma, D.K. (1994). Intelligent decision support system (DSS) for the selection process of alternative machines for flexible manufacturing systems (FMS). *Computers in Industry*, 25, 131-143. http://dx.doi.org/10.1016/0166-3615(94)90044-2

Tuzkaya, G., Gulsun, B., Kahraman, C., & Ozgen, D. (2010). An integrated fuzzy multi-criteria decision making methodology for material handling equipment selection problem and an application. *Expert Systems with Applications,* 37, 2853-2863. http://dx.doi.org/10.1016/j.eswa.2009.09.004

Van Laarhoven, P.J.M., & Pedrcyz, W. (1983). A fuzzy extension of Saaty's priority theory. *Fuzzy Sets and Systems*, 11, 229-241. http://dx.doi.org/10.1016/S0165-0114(83)80082-7

Wang, T.Y., Shaw, C.F., & Chen, Y.L (2000). Machine selection in flexible manufacturing cell: A fuzzy multiple attribute decision making approach. *International Journal of Production Research*, 38, 2079-2097. http://dx.doi.org/10.1080/002075400188519

Yilmaz, B., & Dagdeviren, M. (2011). A combined approach for equipment selection: F-PROMETHEE method and zero–one goal programming. *Expert Systems with Applications,* 38, 11641-11650. http://dx.doi.org/10.1016/j.eswa.2011.03.043

Zadeh, L.A. (1965). Fuzzy sets. *Information and Control,* 8(3), 338-353. http://dx.doi.org/10.1016/S0019-9958(65)90241-X

Zadeh, L.A. (1975). The concept of a linguistic variable and its application to approximate reasoning-I. *Information Sciences*, 8(3), 199-249. http://dx.doi.org/10.1016/0020-0255(75)90036-5