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Digital Twin: Multi-dimensional Model Reduction Method for Performance Optimization of the Virtual Entity

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Abstract

Digital Twin (DT) is an emerging technology that allows manufacturers to simulate and predict states of complex machine systems during operation. This requires that the physical machine state is integrated in a virtual entity, instantaneously. However, if the virtual entity uses computationally demanding models like physics-based finite element models or data driven prediction models, the virtual entity may become asynchronous with its physical entity. This creates an increasing lag between the twins, reducing the effectiveness of the virtual entity. Therefore, in this article, a model reduction method is described for a graph-based representation of multi-dimensional DT model based on spectral clustering and graph centrality metric. This method identifies and optimizes high-importance variables from computationally demanding models to minimize the total number of variables required for improving the performance of the DT.

Keywords: : Digital Twins, Multi-Physics Simulation, Model Fusion, Model Reduction, Spectral Clustering

1. Introduction

Currently Digital Twin (DT) is a leading research topic in manufacturing. The definition of a DT is flexible as this is a highly multidisciplinary approach and depends strongly on the application area. However, the common aspect of all DTs is the cyber-physical fusion. This cyber-physical fusion needs to be defined in the context of the application to produce functional and meaningful living models, which are precise copies of machines or systems based on advanced simulation and industrial internet of things (IIoT) data. In the context of smart manufacturing, DT is often defined as a virtual machine tool system, which aims to reflect the status of its physical object through the integration of manufacturing information. The integration of the physical and virtual entities is the key to a DT. However, in present times, it is not well understood, how to realize this physical and virtual fusion [1].

A large quantity of recent literature exists on DTs describing methods of building the twin and its fundamental components. In [2,3], the authors present a reference model of an industrial DT. The virtual entity (VE) representation of the DT is presented as a collection of various models such as geometric models (G_v), physics-based models (P_v), behavior models (B_v) and prediction or rule-based models (R_v). This is mathematically represented as; VE = $\{G_v, P_v, B_v, R_v\}$. These models do not operate in isolation in the virtual entity of the DT representation. Rather, these models exchange data and information amongst each other to enrich the prediction outcome and simulation results of a physical equipment or a manufacturing process. Such a concept for DT is used to predict the remaining useful life (RUL) of machine components in [4]. In this article, a method of fusing these virtual entity models is presented with graph-based system representation developed in [5]. This method transforms the system variables in to a causal network that models the cause-effect relationship between

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variable with the help of a conceptual modeling framework known as dimensional analysis conceptual modeling [6]. The fusion of models for virtual entity representation of a DT overcomes the problem of information silos created by various simulation models using different software packages.

A graph-based representation is chosen to construct the fusion model of the virtual entity because graphs have emerged as powerful data analytics and representation tool in the recent years. This is because graphs are simple to understand, and several types of analyses become possible for data represented in graphical format. The fusion graph of the virtual entity contains the system variables on its vertices and the relationship between the variables on the edges. This graph is a weighted or unweighted directed graph in a tuple G = (V, E, A) format, where V = {1, 2, ..., n} are the vertices set of the graph, E (i, j) are the edge set of the directed graph where {i,j} ϵ E such that i is the tail of the edge and j is the head. A = [a_{ij}] is the adjacency matrix with non-negative values for weighted graph and 'one' for unweighted graph. This graph G serves as an input to several machine learning algorithms for clustering or classification of data. However, the size of such graph-based representation of the VE models will be high dimensional. That means knowledge of many system variables and their relationship is needed, in order to obtain accurate representation and predictions by the DT. This high-dimensional representation makes the fusion model of the virtual entity asynchronous with the physical entity and makes virtual entity computationally expensive. Therefore, a graph-based model reduction (dimensionality reduction) method is proposed to obtain a reduced set of system variables, which are responsible for maximum information flow in the system and are needed to obtain fast and accurate simulation of the virtual entity.

This article is organized as follows; section 2 introduces the reference model of the VE with the help of a surface grinding case study. After that, a model fusion methodology is proposed for the case study in section 3 with graph-based system representation. Section 4 proposes a model reduction (dimensionality reduction) method for the high dimensional fusion model and section 5 concludes the article.

Nomenclature

acc	accuracy
a_e	Actual depth of cut (mm)
AE	Acoustic emission intensity (V)
a_{gmax}	maximum un-deformed chip thickness (mm)
a_p	programmed or set depth of cut (mm)
A_r	Real area of contact (mm^2)
b_w	Width of grind (mm)
c	workpiece heat capacity (J/K)
C	Active grit density (mm^{-2})
d_e	equivalent wheel diameter (mm)
d_g	mean grain size (mm)
d_s	diameter of grinding wheel (mm)
d_w	diameter of work piece (mm)
E*	Combined elastic Modulus (MPa)
E_1	Elastic Modulus of Wheel (MPa)
E_2	Elastic Modulus of workpiece (MPa)
e_c	Specific grinding energy (J/mm^3)
f_g	force per grit (N)
F_n	Normal G. Force (N)
F'_n	Normal G. Force per unit width (N/mm)
F_t	Tangential G. Force (N)
F_x	grinding force in x-direction (N)
F_y	grinding force in y-direction (N)
F_z	grinding force in z-direction (N)
G_{Ratio}	Volume of material ground per unit wheel width by volume of wheel worn per unit wheel width (mm^3)
H	Hardness of grinding grain (Rockwell hardness)
h_{cu}	Uncut chip thickness (mm)
K	Archard's constant
k	workpiece thermal conductivity (WK^{-1})
k_g	Grain thermal conductivity (WK^{-1})
L	Sliding distance (mm)
l_c	real contact length (mm)
l_f	deflection contact length (mm)
l_g	geometrical contact length (mm)
L_s	variation of the contact length (mm)
L_w	Grinding distance (mm)
n_b	number of parts
N_s	Rotational wheel speed (m/min)
P	Grinding power (KW)
P_{Ratio}	Volume of metal ground per unit area of wheel surface (mm^3)
Q'	Specific removal rate
r	Grit cutting point shape factor
r_0	grain contact radius (mm)
R_r	Roughness factor
R_t	Maximum Surface Roughness (μm)
R_{ws}	Workpiece partition ratio
t	total grinding contact time (min)
t_b	total time (min)
t_c	Cycle time (min)
t_g	grinding time (min)
T_{max}	Maximum temperature (°C)
T_{mp}	Temperature approaching the melting point (°C)
V	Volume of wear of the wheel (mm^3)
v_s	Wheel speed (m/min)
V_s	Volume of wheel worn per unit wheel width (mm^3)
v_w	Workpiece speed (m/min)
V_w	volume of material ground per unit wheel width (mm^3)
V_x	velocity in x-direction (mm/min)
V_y	velocity in y-direction (mm/min)
V_z	velocity in z-direction (mm/min)
w_r	Wear life cycle
ʌ	Stock removal parameter
μ	Coefficient of Grinding
ν	Poisson's Ratio
ρ	density (kg/m^3)
φ	abrasive angle (°)

2. Reference Model

A reference model for DT is proposed by the authors in [7]. This reference model describes a DT as a fusion of five components working synchronously with each other. These five components are; (1) the physical equipment (PE) consisting of the actual physical device at the shop floor, (2) the virtual entity consisting of the entire virtual representation of the PE, (3) DT data, which is operational data collected from the PE, (4) connection and networking (CN), and (5) Services provided by the DT at the shop floor (Ss). The authors describe these five components as five dimensions of the DT. Each of the five dimensions can be individually modelled for every complex equipment on the shop floor.

In this article, the reference model is implemented on a cylindrical surface grinding case study. In grinding operation, an important concern is the life of the grinding wheel. Efforts to develop a low cost digital twin for grinding operation with RFID tags and single board computers is demonstrated in [8].

The PE consists of various subsystems of the grinding machine, several sensors and IIoT devices are integrated with the machine. Acoustic emission sensors, accelerometers, temperature sensors and sensors for current and voltage measurement are used to capture data from the PE. The VE is the entire virtual representation of the grinding system unifying several models such as the G_v, P_v, B_v and R_v.

G_v is the 3D model of the grinding wheel geometry. Information regarding the geometry of the grinding wheel can be obtained from these models. The P_v describes the physical phenomena associated with the operations of the PE. Finite element method (FEM) are used to obtain the physics-based modeling and simulation of the grinding wheel parameters during operation. The P_v is a complex representation of all the physical parameters effecting the grinding process and its effect on the change of grinding wheel geometry due to wheel wear. B_v describes the behavior of the grinding machine or the grinding process under certain conditions. For example, what happens when the maximum temperature of grinding exceeds predefined limits or the power consumption increase? B_v is a comprehensive model, which provides an exhaustive explanation on the events experienced by the DT. R_v is the highest-level model, which facilitates prediction and decision-making by the virtual entity with the help of machine learning algorithms. For example, tool wear prediction in manufacturing by application of deep learning methods like Recurrent Neural Networks (RNN) [9] or ensemble classifiers like Random Forest [10].

The other components i.e. DT data, CN and Ss are important auxiliary components of the DT, which are dependent on PE and VE to give context to the definition of a DT. This article focuses categorically on the high dimensional fusion model of the VE to develop a methodology for model reduction that enhances the performance of the twinning process.

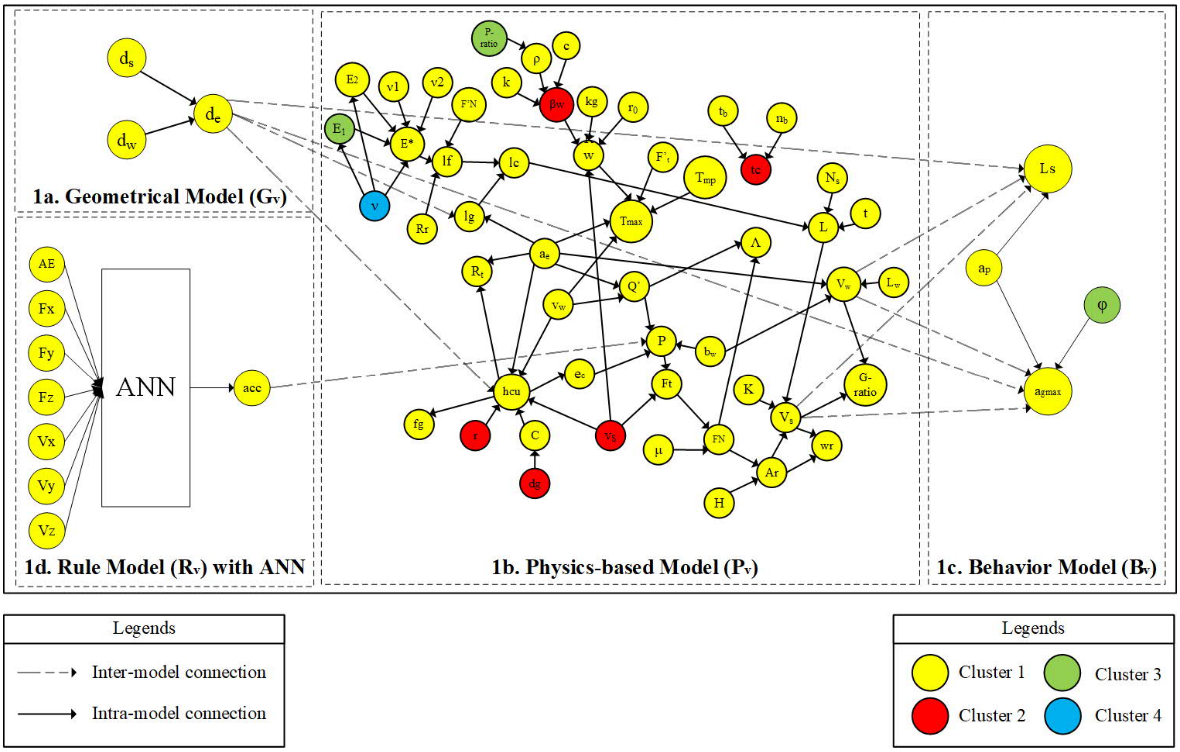

Fig. 1. The fusion model G_F of the VE; (a) Geometry model G_v; (b) Physics-based model P_v; (c) Behavior model B_v; (d) Rule model R_v.

3. Model Fusion

Fusion of the above mentioned multi-dimensional models are an important step in construction and operation of the VE. These models embed large amount of information such as product geometry, process physics, functional behavior or failure rules in different formats. Hence, realization of the fusion process is a challenging but essential task. Different software are used to build different models, which might not be compatible with each other. For example, the geometry models are constructed by 3D computer aided design (CAD) software whereas the physics-based model are built with finite element modeling (FEM) software and the rule-based model are constructed with machine learning algorithms. For this reason, a graph-based approach is used for the fusion model shown in Figure 1.

3.1. The Fusion Model based on Reference Model

Firstly, directed graphs are generated to construct individual models with the help of dimensional analysis conceptual modeling framework. Thereafter, these models are interlinked to form the unified model of the VE, and it is denoted as a multi-dimensional graph, G_F. Figure 1 shows the individual models as well as the inter model connections to obtain the larger fusion model for building a DT for grinding system.

- G_v is the simplest graphical model. It contains geometric information such as dimensions. G_v contains information like diameter of the grinding wheel (d_s), bore to hole ratio, width of the bore and work piece diameter (d_w). Another parameter that can be obtained from the geometry models of the grinding wheel and the work piece is equivalent diameter (d_{eq}). This measure allows the comparison between two grinding applications. From the relationship of the equivalent grinding diameter, the graphical model G_v is constructed. G_v is connected to P_v through the geometry information it contains regarding the equivalent diameter.
- P_v is the physics-based model generated based on dimensional analysis conceptual modeling framework along with well understood grinding physics available in literature. P_v is the graphical representation of the multi-physical phenomena in the grinding process and it is solved with finite element method to obtain wheel wear and volume of material ground.
- B_v represents the dynamic behavior model of the grinding process. B_v determines the variation of dynamic response such as the grinding force, vibration, and grinding power. This dynamic behavior model effects grinding quality parameters such as roughness or residual stresses. In [9], the dynamic behavior of grinding process is defined mathematically as a function of chip thickness and contact length between wheel and workpiece. According to the authors, in high speed grinding, variation in vibration or work piece runout are dynamic behavior patterns that influences the grinding quality. B_v can be built as a function of maximum uncut chip thickness and contact length between the wheel and workpiece. B_v is connected to P_v through common parameters governing process physics and dynamic behavior such as V_s, V_w and d_{eq}.
- R_v is a data-driven model which defines the deduction rule between the grinding wheel wear and the input parameters. Data collection is done from the grinding process with the help of acoustic emission sensors and accelerometers. The input parameters are rotational speed, tangential force and vibration along X, Y and Z. An artificial neural network (ANN) is implemented similar to [10] to associate specific levels of the sensor signals with the grinding wheel wear. The feed forward propagation of the ANN is a logistical regression method, but the back propagation is the actual learning event when the weights of the ANN are learnt to predict grinding wheel wear based on sensor data. In order to connect R_v to the rest of the models, accuracy of the ANN is chosen. When the accuracy of the ANN is sufficiently high, R_v can be connected to P_v through grinding power [11].

4. Model Reduction

The fusion model G_F of the VE contains of large number of variables whose states and values must be known to obtain correct prediction and simulation results from the VE. In this section, a novel method of model reduction (or dimensionality reduction) is introduced to obtain a reduced model of the graph G_F so that knowledge of a smaller number of variables is required to compute and optimize the target variables. This improves the performance of the simulation process of the VE making it fast and computationally less expensive. This method uses spectral clustering techniques to group similar variables of graph G_F together. Spectral clustering is an unsupervised machine learning technique where a graph is partitioned into clusters based on the topology of the graph. Spectral clustering uses graph Laplacian to construct the clusters. The following sections describe the subcomponents of the model reduction method.

4.1. Spectral Clustering

A spectral clustering algorithm is implemented to obtain the graph partition of G_F. G_F consists of 62 variables in total. The first step in the algorithm is to define a similarity matrix. The similarity matrix is chosen as the adjacency of G_F. From this adjacency matrix, the graph Laplacian is computed, and the eigenvalues and eigenvectors are computed from the Laplacian. As G_F is a directed graph, the Laplacian is normalized to obtain the graph cuts. The number of clusters are decided based on the eigenvectors. Presence of sharp peaks in the eigenvector graph indicate the presence of a partition. The cluster is reconstructed with the graph along with k-means clustering where k is decided by the eigenvector values. For G_F, k=4 was obtained. Figure 1 shows the clusters from spectral clustering algorithm for G_F. The clusters are colour coded. The cluster marked in yellow is the first and biggest cluster with 53 variables in it, which cannot be subdivided further when the value of k is 4. The variables marked in red belong to the second cluster that contains many variables whose values depend upon the variables in the first cluster. The green and blue clusters contain other less important variables that do not contribute significantly to the prediction and simulation results of the VE. The spectral clustering already sets a hierarchy of variables. To find the relative importance of these clusters to each other, page rank algorithm is used to find the cluster hierarchy. The cluster importance is defined in the next section.

4.2. Cluster Importance

Page rank algorithm is used to find the relative importance of the clusters. This algorithm has shown stability in node ranking and has been popularly used for variable screening in physical networks such as chemical networks, protein networks and power grids. Page rank algorithm is a modified form of eigenvector centrality measure of vertices in a graph. Page rank computes the principal eigenvector of the matrix describing the edges in the graph using the power method. It is a probabilistic ranking technique to obtain which variables in the graph could be more important based on the degree distribution and fitness function. The nodes with higher Page rank are more central, meaning they tend to have a higher influence on the objective function, or they are connected to the variables which have higher influence on the objective. The principle of Page rank algorithm holds for directed unipartite networks, complex or small scale, as it generates a hierarchy of nodes in the network based on the probabilities of the nodes at a given state of the system obtained from the degree distribution. Page rank is computed for G_F. The Page rank scores with power iteration method are shown in Figure 2. It is found that higher page rank scores are obtained for variables lying in the first cluster. Whereas the page rank scores of variables lying in other clusters are low and fail to qualify as important variables when a threshold of 0.025 is selected. This threshold of 0.025 is selected based on the percentile Page rank score of all the variables and applying classical Pareto 80/20 principle on the percentile scores.

It can be observed from the spectral clustering pattern that cluster 1 contains V_s and V_w, which are the target variables of the physics-based model P_v. At this point, it is desirable to reformulate the VE model based only on the variables belonging to cluster 1. Variables in other clusters can be neglected as they contain dependent variables and their knowledge itself does not alter the computing or simulation time of the VE by a large amount. Clusters 2, 3 and 4 contain variables, which have a low page rank score, and it could be inferred that they do not contribute significantly to the target variables V_s and V_w. Cluster 1 is therefore the most important cluster containing all the important variables that should be optimized to obtain higher performance of the twin. In other words, Cluster 1 is the reduced model of G_F that contains the knowledge of all the variables, which are critical to represent the VE with a reasonable accuracy.

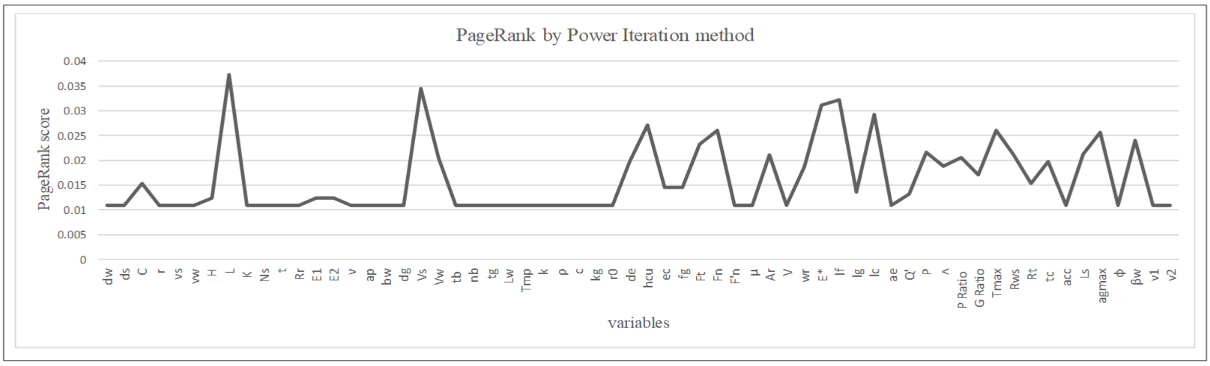

Fig. 2. Page Rank score of G_F by power iteration method

To validate this approach, performance variables V_s and V_w are optimized based only on the variables from cluster 1 and the values are compared with other grinding case studies from the literature. There are conflicting objectives in the VE as the variable V_s should be minimized and V_w should be maximized in the grinding operation. Hence, in the next section, a multi-objective optimization problem is formulated and solved to demonstrate the significance of cluster 1 variables on the performance of the VE.

4.3. Multi objective optimization

Spectral clustering and graph centrality metric define variables in cluster 1 as the variables, which have the highest impact on the target variables i.e. V_s and V_w. R_t is also considered in the optimization problem as maximum roughness should be minimized. But, R_t contains v_s in its equation, which is a variable from cluster 2. While formulating the R_t optimization equation, v_s is replaced by 1 as it is in the denominator. This replacement is done for all the variables belonging to cluster 2, 3 and 4. Due to the presence of conflicting objectives in the VE, a multi-objective optimization problem is formulated based on the mathematical equations for grinding defined in [12,13]. The optimization problem contains three objectives however, several other objectives can be selected depending on the application of the VE. The optimization formulation is as follows:

f(cluster 1)

objective 1

min : $f(V_s)$

where, $V_s = b_w\, a_s\, \pi\, d_s$

objective 2

max $f(V_w)$

where, $V_w = b_w\, a_e\, L_w$

objective 3

min $f(R_t)$

$$\text{where, } R_t \approx \left(\frac{v_w}{v_s} \cdot \frac{1}{C\, r \sqrt{d_e}} \right)^{\frac{2}{3}} \text{ and } d_e = \frac{d_s d_w}{d_s \pm d_w}$$

$$\text{with model reduction } (v_s \approx 1);\ R_t \approx \left(\frac{v_w}{C\, r \sqrt{d_e}} \right)^{\frac{2}{3}}$$

subjected to;

$200 \le d_s \le 355\,\text{mm}$

$150 \le L_w \le 300\,\text{mm}$

$0.0505 \le a_e \le 0.1\,\text{mm}$

In objective 1, V_s is defined as a function of a_s, which is the wear depth of the grinding wheel. As no standard value for a_s

was found in literature, it is assumed to be the lowest value of the variable a_e. In objective 3, the value of C is chosen as 32 μ/mm^2 and grit size r is chosen as 500 based on ANSI B74.16, 1995. Also, for surface grinding, $d_e \approx d_s$. The optimization problem is solved with the help of a MATLAB based genetic algorithm solver known as 'gamultiobj' and data collected from the digital environment for grinding with IIoT devices. The values of the variables, whose data are not collected from the digital grinding environment, are derived from previous work on grinding parameter optimization such as the case study presented in [14]. Acoustic emission data is collected from the grinding machine with acoustic emission sensors. The acoustic emission intensity is related to the grinding power P to compute volume of wheel wear V_w, if a significant accuracy is obtained from the rule-based model R_v. The fitness function for multi-objective optimization is defined according to the mathematical expression of the performance variables. The optimization result is shown in Table 1, for $n_b = 20$.

Table 1. Optimization results.

Variables	Optimized value
V_s	24.485 mm^3
V_w	1419.4 mm^3
R_t	0.8 μm

To compare the optimization results with the case study from literature, G_{Ratio} is used. G_{Ratio} is a measure of the ability of a grinding wheel to remove materials and it is given by the ratio of V_w/V_s. G_{Ratio} obtained from Table 1 is 57.95 and G_{Ratio} obtained from the case study is 60. Hence, less than 10% error is obtained. This indicates that the variables in cluster 1 is sufficient to provide meaningful information on performance optimization of the VE. Thus, cluster 1 containing 53 variables provides an equivalent representation of the high dimensional model G_F.

5. Conclusion

In this article, a model reduction method is presented for multidimensional graphical representation of the variables in the VE models. This model reduction method uses spectral clustering to group similar variables together and analyses their importance in optimization of the performance variables that governs the overall performance of the DT. An important performance indicator of the DT is the time required by the simulation models to update itself based on industrial internet of things (IIoT) data from the shop floor. This model reduction method can identify the less sensitive variables in the VE so that data collection or complicated analytics of these variables can be eliminated. This makes the VE representation of the DT faster and more efficient.

6. Future Work

In the future, a detailed graphical representation of the grinding system will be built, and the model reduction method will be integrated with the functional DT for daily operations of the grinding machine to make the simulation models faster and more sensitive to the working environment. Collecting and processing all information needed to update simulation runs of the PE remains a practical challenge for high-fidelity simulation models, which try to mimic the reality in greater detail. Model reduction methods, such as the method described in this article, will facilitate in understanding which parameters have the most influence on the process and selectively optimize those parameters in the virtual entity to enhance the performance of the DT.

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