Approaching the Shannon Limit by Means of Optimal FTN Signals with Increased Size of PAM Signal Constellation

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***Abstract*— Application of Faster than Nyquist (FTN) signals allows achieving high spectral and energy characteristics of the communication system. Further approaching the Shannon limit that determines the channel throughput is related to increasing the size of the signal constellation. However, an increase in the size of the signal constellation results in sharp growth of energy losses for FTN signals. To reduce these energy losses optimal FTN pulses obtained as a result of solving the optimization problem with the constraint on signal constellation size M may be used. In this work, optimal pulse shapes for pulse-amplitude modulation (PAM) with M=4, 16, 64, 256 were found. It was shown that applying optimal FTN signals with PAM allows reducing energy losses by 4 dB regarding the FTN signals based on root-raised-cosine pulses for coherent symbol-by-symbol detection algorithm. In terms of approaching the Shannon limit, the closest results are provided by determining the bandwidth containing 99% of signal energy.**

Keywords—faster than Nyquist signaling, optimal signals, optimization problem, Shannon limit, pulse-amplitude modulation, root-raised-cosine pulses

I. Introduction

In the works [1]-[3] the properties of FTN signals are considered and estimations of their bit error rate (BER) performance in channels with additive white Gaussian noise (AWGN) with power spectral density $N_0/2$ are made. It is shown that the application of such signals allows overcoming the Nyquist limit [4] and achieving high spectral and energy characteristics of the communication system. Further approaching the throughput of a transmission channel or the Shannon limit is related to increasing the size of the signal constellation. Due to this, the rate of information transmission may be increased without widening occupied frequency bandwidth ΔF [5].

Random FTN signal $y(t)$ may be formed with the use of root raised cosine (RRC) pulses [2]-[3] which duration T_s is much longer than the transmission time of one symbol of channel alphabet. Such FTN signals provide overcoming the Nyquist limit (2 bit/s/Hz for one-sided bandwidth) while energy losses for binary signal constellation remain relatively low [2]-[3]. An increase in the size of the signal constellation results in sharp growth of energy losses [6]-[7]. It can be explained by a greater impact of intersymbol interference caused by long duration of $s(t)$ on correlation properties of received signal sequence.

These energy losses may be reduced due to the application of optimal FTN pulses $s_{opt}(t)$ that also provide overcoming the Nyquist limit. The duration of optimal FTN pulses exceeds the transmission time T of one bit of the original binary channel message significantly. When the optimization problem is being solved for the increased size of signal constellation corresponding to pulse-amplitude manipulation (PAM), it is possible to add the constraint on the level of intersymbol interference between adjacent transmitting pulses. This constraint is especially important for large sizes (up to 256) of the signal constellation. Setting the controllable level of intersymbol interference makes reducing energy losses during the detection of optimal FTN signals possible and provides bringing spectral and energy efficiency of the system closer to the Shannon limit [8]-[9].

In this paper, the possibilities of using optimal FTN signals with an increased size of PAM signal constellation (up to 256) are analyzed and numerical estimations of approaching the Shannon limit are given.

II. Optimal FTN Signals with Increased Size of PAM Signal Constellation

To analyze the possibilities of using the signal constellation with increased size for optimal FTN pulses let us consider an example of the simplest pulse-amplitude manipulation. A random sequence consisting of N single optimal FTN pulses $s_{opt}(t)$ with duration $T_s=LT$, where L=2,3,…, and with energy E_s may be represented as follows:

$$y(t)=\sqrt{E_s/T}\sum_{p=-N/2}^{N/2} c_j^{(n)} s_{opt}\left(t-p\xi T\right). \qquad (1)$$

Here the value ξ defines the rate of signal transmission over a communication channel and lies in the range $0<\xi\le 1$. For instance, when binary data are transmitted at the rate $R=1/\xi T$, the Nyquist limit will be achieved at $\xi=1$ and $\Delta F=1/2T$ for signals without the carrier.

For PAM signal constellation with an increased size the values $c_j^{(p)}$ in (1) are calculated by the next formula (M is the volume of channel alphabet or the size of the signal constellation) [4]:

$$c_j^{(p)}=\frac{M-2j+1}{M-1},\ j=1...M .$$

If M=2, then j=1,2 and $c_1^{(p)}=1$, $c_2^{(p)}=-1$. For M=4 we have j=1,2,3,4 and $c_1^{(p)}=1$, $c_2^{(p)}=0.33$, $c_3^{(p)}=-0.33$, $c_4^{(p)}=-1$. The values $c_j^{(p)}$ are uniformly distributed in the area from –1 to +1.

For FTN signals with PAM the values $c_j^{(p)}$ are equiprobable for each p.

In [10] the optimization problem was solved with the constraint on the correlation coefficient. This constraint has to ensure the condition of minimization intersymbol interference on the interval (0, T_s).

Let us take a closer look. The worst sequence in terms of BER performance is a sequence (or its part) (1) consisting of interchanging pulses with minimum and maximum possible amplitudes. If we choose $c_{\min}$ to be equal to the minimum value c_j in absolute value, the constraint on the coefficient of mutual correlation may be written the next way:

$$\max_j \left[\max_{n=1..([L/\xi]-1)} \left\{ (\mathrm{M}-1)^2 I \right\} \right] \le K_0 \text{, where} \qquad (2)$$

$$I = \int_{n\xi T}^{LT} c_{\min} s_{opt}(t)\, c_j s_{opt}(t - n\xi T)\, dt \, .$$

We need to solve the optimization problem with the constraint on correlation coefficient (2) for each size M of the signal constellation and each value of transmission rate (defined by ξ) and pulse duration T_s=LT. The more the value M, the greater the influence of this constraint on the shape of obtained optimal function $s_{opt}(t)$. The step-by-step illustration of the constraint (2) for M=4, pulse duration T_s=4T, and ξ=1 is shown in fig.1.

In fig.1 you can see two optimal FTN pulses with PAM for $c_{\min}$=0.33 and c_j=1 transmitted at the rate R=1/T. The interval of analysis during solving the problem at each step n is equal to 3T (n=1), 2T (n=2) and T (n=1). It is easy to notice that the most significant influence of adjacent pulses on each other takes place at the interval 3T. Obviously, the situation will get worse if M increases.

The shapes of RRC pulses and optimal FTN pulses for M=256 and T_s=8T are compared in fig.2. The pulses match each other on the interval –1.5T<t<1.5T and differ in the remaining area of existence. The important thing is that the polarity of pulse shape may change at the time interval – 4T<t<–T and T<t<4T.

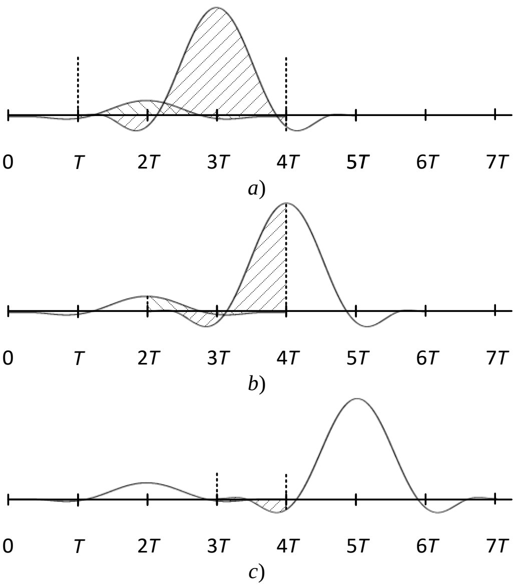

Fig. 1. Step-by-step illustration of applying the constraint (2) during the process of optimizing the shape $s_{opt}(t)$ in (1): a) n=1, b) n=2, c) n=3.

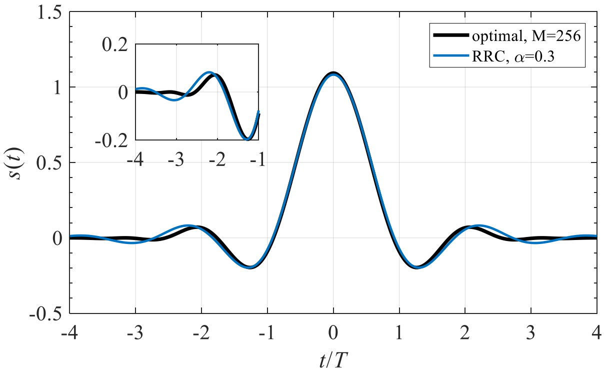

Fig. 2. The shape of optimal FTN pulse for M=256 and RRC pulse.

III. BER PERFORMANCE

To estimate BER performance of detection of optimal FTN signals we will use simulation modeling in Matlab environment. Optimal FTN signals with PAM are detected with coherent symbol-by-symbol detection algorithm [11]. The values of pulse duration and the start and the end of the packet are assumed to be known. Additive white Gaussian noise (AWGN) channel with average power spectral density N_0/2 is used as a transmission channel. After signal detection error probability is calculated. Each value p_{err} of error probability is determined for the specific ratio of optimal FTN signal energy per bit to noise power spectral density E_b/N_0 with the help of 10^6 transmitted symbols of channel alphabet. Average BER performance depends on BER performance of each realization of a random sequence of pulses.

Fig.3 contains the results of simulation modeling representing the values of error probability in relationship with the ratio E_b/N_0 for optimal FTN signals with M=256. The results for FTN signals based on RRC pulses with roll-off factor α=0.3 (blue line) are shown for comparison. It can be seen that for small values of signal-to-noise ratio (up to 30 dB) error probabilities for optimal FTN signals and signals based on RRC pulses match each other. Then the energy gain provided by optimal FTN signals takes place.

Such behavior of dependencies in fig.3 is explained by the fact that for high values of signal-to-noise ratio (more than 30 dB) the main contribution is made by a combination of adjacent symbols with a great difference in pulse amplitude. For FTN signals based on RRC pulses such influence is stronger (fig.2) than for FTN signals based on optimal pulses which were obtained to minimize this influence (2).

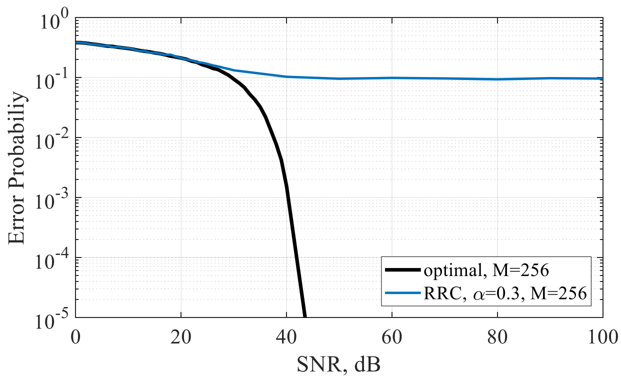

Fig. 3. BER performance of FTN signals with PAM based on optimal FTN pulses and RRC pulses (M=256, T_s=8T)

IV. Estimation of Approaching the Shannon Limit

In this section, we will make numerical estimations of applying optimal FTN signals with PAM signal constellation of increased size (up to M=256) for approaching the Shannon limit. These estimations are mostly affected by the next factors.

The first one is a determination of two-sided occupied frequency bandwidth ΔF. For finite in time optimal pulses the shape of ΔF differs from a rectangular shape. The most frequently used criterion for determination of occupied frequency bandwidth is the concentration of 99% of signal energy $\Delta F_{99\%}$ [12]-[13] or a specified level of energy spectrum (e.g., ΔF_{-60dB}) [10]-[11], [14]. Therefore, the quantitative value of spectral efficiency for specific signals depends on the way of determining ΔF.

Secondly, the energy efficiency of the application of optimal FTN signals is related to detection algorithms. The algorithms of coherent symbol-by-symbol detection have the simplest implementation in conditions of reliable phase and clock synchronization. Nevertheless, maximum likelihood sequence detection algorithms [6], [15]-[16] like the Viterbi algorithm allow improving energy efficiency when signals with intersymbol interference are used.

The values of energy and spectral efficiency of optimal FTN signals and signals based on RRC pulses for different sizes of PAM signal constellation are presented in fig.4. The y-axis corresponds to the ratio of signal energy per bit to noise power spectral density while the x-axis stands for the ratio of transmission rate to the frequency bandwidth containing 99% of signal energy. The Shannon limit is plotted by a black curve. The points on the plane were obtained via simulation modeling for bit error probability $p_{err}=10^{-3}$.

For signals based on RRC pulses with roll-off factor α=0.3 only the results for PAM signal constellation with M=4, 16, 64 were obtained. As it is shown in fig.3, for M=256 the error probability does not become less than $p_{err}=0.05$ when E_b/N_0 exceeds 30 dB.

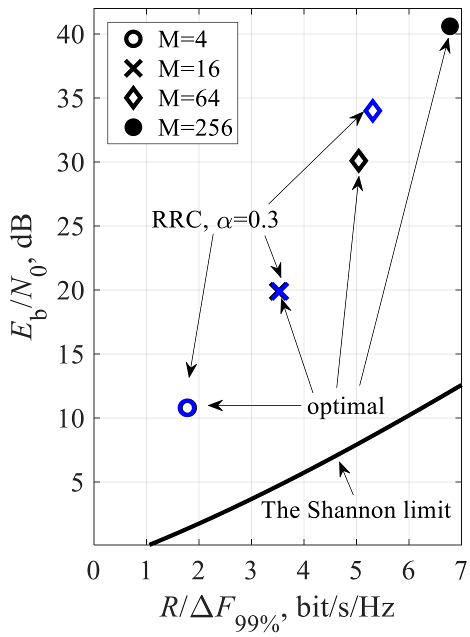

Fig. 4. Energy and spectral efficiency of FTN signals ($\Delta F_{99\%}$)

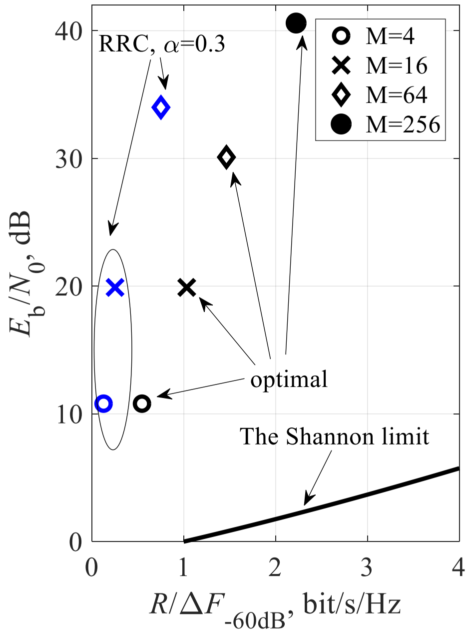

Fig. 5. Energy and spectral efficiency of FTN signals (ΔF_{-60dB})

We found the results for optimal FTN signals with M=4, 16, 64, 256. Energy and spectral efficiency of such signals are close to the efficiency of signals based on RRC pulses for M=4, 16, 64. When M=256, the efficiency of optimal FTN signals outreaches the efficiency of RRC signals significantly. The more the size of signal constellation, the more the gain in energy efficiency provided by optimal signals. For example, for M=64 energy losses reduce by more than 4 dB.

Optimal FTN signals with an increased size of PAM signal constellation may approach the Shannon limit only due to improving energy efficiency.

Now we should find out how the determination of occupied frequency bandwidth for the level of –60 dB of energy spectrum influences the results (fig.5).

Optimal FTN signals with PAM (M=4, 16, 64, 256) have much higher spectral efficiency than FTN signals based on RRC pulses. Besides, it becomes greater when M increases. For instance, if M=64, spectral efficiency increases by two times. This is due to the condition of providing a high reduction rate of out-of-band emissions (higher than for RRC pulses) used during the process of solving the optimization problem. Regarding the Shannon limit (fig.5) optimal FTN signals with PAM are much further for ΔF_{-60dB} than for $\Delta F_{99\%}$. Comparing the values of efficiency shown in fig.4 and fig.5, we can note that for $\Delta F_{99\%}$ spectral efficiency is about 3.5 times greater than for ΔF_{-60dB}.

V. Conclusions

In this paper, we have analyzed the opportunities of application of optimal FTN signals with PAM signal constellation of increased size (up to M=256) for approaching the Shannon limit and made quantitative estimations. It was shown that applying optimal FTN signals with PAM allows reducing energy losses by 4 dB regarding the signals based on RRC pulses for coherent symbol-by-symbol detection algorithm.

In practice, it is interesting to apply optimal FTN signals with PAM signal constellation of greater size (more than 256). It is especially important for using optimal FTN signals in digital TV and radio broadcasting systems.

The difficulty of comparing the spectral and energy efficiency of existing finite signals to the Shannon limit is illustrated. We have considered the determination of occupied frequency bandwidth according to the criterion of concentration 99% of signal energy and the criterion of a specified level of energy spectrum (–60 dB). The difference in estimations of the spectral efficiency of optimal FTN signals on the Shannon plane reaches about 3.5 times depending on the way of determining occupied frequency bandwidth. In terms of approaching the Shannon limit, the closest results are provided by determining the bandwidth containing 99% of signal energy. However, we can assume that the difference in estimations may change if occupied frequency bandwidth is determined for the level of energy spectrum less than –60 dB.

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