# Achievable Ergodic Capacity under $\mathcal{F}$ Composite Fading Conditions

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Abstract—The  $\mathcal{F}$  composite fading model was recently proposed as an accurate and tractable statistical model for the characterization of the composite fading conditions encountered in realistic wireless communication scenarios. In the present contribution we capitalize on the distinct properties of this composite model to evaluate the achievable ergodic capacity over  $\mathcal{F}$  composite fading channels. To this end, we derive an exact closed-form expression for the ergodic capacity, which is subsequently used as a benchmark for the derivation of a tight approximation and a particularly accurate asymptotic representation for large average signal-to-noise ratio values. The derived analytic expressions are provided in closed-form and benefit from their analytical and numerical tractability. This enables the development of meaningful insights on the effect of fading conditions of different severity levels on the overall system performance. Also, it allows the accurate quantification of the signal to noise ratio required in target quality of service requirements under different composite fading conditions.

# I. INTRODUCTION

It is well-known that wireless transmission is subject to multipath fading effects, which are mainly caused by the constructive and destructive interference between two or more versions of the transmitted signal. Since multipath fading is typically detrimental to the performance of wireless communication systems, it is important to characterize and model multipath fading channels accurately so as to understand and improve their behavior. In this context, numerous fading models such as Rayleigh, Rician and Nakagami-*m* have been utilized in an attempt to characterize multipath fading, depending on the nature of the radio propagation environment [1]–[3].

Based on the above, extensive analyses on the performance of various wireless communication systems have been reported in [4]–[13] and the references therein. Specifically, the authors

in [4]–[6] introduced the concepts of capacity analysis under different adaptation policies and carried out an extensive analysis over Rayleigh and Nakagami-m fading channels. Likewise, the ergodic capacity over correlated Rician fading channels and under generalized fading conditions was investigated in [7] and [8], respectively. In the same context, comprehensive capacity analyses over independent and correlated generalized fading channels were performed in [9]–[11] for different diversity receiver configurations. Also, a lower bound for the ergodic capacity of distributed multiple input multiple output (MIMO) systems was derived in [12], while the effective throughput over generalized multipath fading in multiple input single output (MISO) systems was analyzed in [13].

It is recalled that in many practical wireless scenarios, the transmitted signal may not only undergo multipath fading, but also simultaneous shadowing. Shadowing can be typically modeled with the aid of lognormal, gamma, inverse Gaussian and inverse gamma distributions [14]-[19]. Following from this, the simultaneous occurrence of multipath fading and shadowing can be taken into account using any one of the composite fading models introduced in the open technical literature [20]-[25]. Capitalizing on this, the performance of digital communication systems over composite fading channels has been analyzed in [26]-[39]. The majority of these contributions are concerned with analyses relating to outage probability and error analyses in conventional and diversity based communication scenarios. A corresponding analysis of the channel capacity has only been relatively partially addressed. Many of the existing studies are either limited to an ergodic capacity analysis for the case of independent and correlated fading channels in conventional, relay and multiantenna communication scenarios or to the effective capacity and channel capacity under different adaptation policies for the case of conventional communication scenarios. In addition, these analyses have been comprehensively addressed only for the case of gamma distributed shadowing and partially for composite models based on lognormal or IG shadowing effects.

Motivated by this, the authors in [40] recently proposed the use of the Fisher-Snedocor  $\mathcal{F}$  distribution to describe the composite fading conditions encountered during realistic wireless transmission. This composite model is based on the key assumption that the root mean square (rms) power of a Nakagami-m signal is subject to variation induced by an inverse Nakagami-m random variable (RV). It was shown in [40] that this assumption renders the  $\mathcal{F}$  fading model capable of providing a better fit to measurement data than the widely used generalized-K fading model. Additionally, the algebraic representation of the  $\mathcal{F}$  composite fading distribution is fairly tractable and simpler than that of the generalized-K distribution, which until now has been largely considered the most analytically tractable composite fading model.

As a result, this model is characterized by its distinct combination of accurate modeling capability and algebraic tractability. In this contribution, we first present additional analytic expressions for the key statistical metrics of the  $\mathcal{F}$  composite fading model. These formulations are generic and thus, well suited to information-theoretic analyses since they do not result to constrained results that are practically problematic and unreliable. Capitalizing on them, we derive a novel exact analytic expression for the channel capacity with optimum rate adaptation over  $\mathcal{F}$  composite fading channels. Based on this, we also derive an accurate and simple closedform approximation along with a particularly tight asymptotic representation. The expressions provide meaningful insights on the impact of the involved parameters on the overall system performance and to associated limitations. This is useful in numerous emerging wireless applications, such as body area networks and vehicular communications, which are largely characterized by stringent quality of service and low latency requirements.

The remainder of the paper is organized as follows: In Section II, we briefly review the  $\mathcal F$  composite fading model and present its aforementioned redefinition. Then the comprehensive capacity analysis for optimum rate adaptation over  $\mathcal F$  composite fading channels is carried out in Section III, whereas Section IV provides corresponding numerical results and discussions. Finally, Section V presents some concluding remarks.

## II. THE $\mathcal{F}$ COMPOSITE FADING MODEL

Similar to the physical signal model proposed for the Nakagami-m fading channel [41], the received signal in an  $\mathcal{F}$  composite fading channel is composed of separable clusters of multipath, in which the scattered waves have similar delay times, with the delay spreads of different clusters being relatively large. However, in contrast to the Nakagami-m

signal, in an  $\mathcal{F}$  composite fading channel the rms power of the received signal is subject to random variation induced by shadowing. Based on this, the received signal envelope, R, can be expressed as

$$R = \sqrt{\sum_{i=1}^{m} \alpha^2 I_i^2 + \alpha^2 Q_i^2}$$
 (1)

where m represents the number of clusters of multipath,  $I_i$  and  $Q_i$  are independent Gaussian RVs, which denote the in-phase and quadrature phase components of the multipath cluster i, where  $\mathbb{E}[I_i] = \mathbb{E}[Q_i] = 0$  and  $\mathbb{E}[I_i^2] = \mathbb{E}[Q_i^2] = \sigma^2$ , with  $\mathbb{E}[\cdot]$  denoting statistical expectation. In (1),  $\alpha$  is a normalized inverse Nakagami-m RV, where  $m_s$  is the shape parameter and  $\mathbb{E}[\alpha^2] = 1$ , such that

$$f_{\alpha}(\alpha) = \frac{2(m_s - 1)^{m_s}}{\Gamma(m_s) \alpha^{2m_s + 1}} \exp\left(-\frac{m_s - 1}{\alpha^2}\right), \qquad (2)$$

where  $\Gamma(\cdot)$  represents the gamma function [42, eq. (8.310.1)]. Following the approach in [40], we can obtain the corresponding PDF<sup>1</sup> of the received signal envelope, R, in an  $\mathcal{F}$  composite fading channel, namely

$$f_R(r) = \frac{2 m^m (m_s - 1)^{m_s} \Omega^{m_s} r^{2m - 1}}{B(m, m_s) \left[ mr^2 + (m_s - 1) \Omega \right]^{m + m_s}}, \quad (3)$$

which is valid for  $m_s>1$ , while  $B(\cdot,\cdot)$  denotes the beta function [42, eq. (8.384.1)]. The form of the PDF in (3) is functionally equivalent to the  $\mathcal F$  distribution<sup>2</sup>. In terms of its physical interpretation, m denotes the fading severity whereas  $m_s$  controls the amount of shadowing of the rms signal power. Moreover,  $\Omega=\mathbb E[r^2]$  represents the mean power. As  $m_s\to 0$ , the scattered signal component undergoes heavy shadowing. In contrast, as  $m_s\to \infty$ , there exists no shadowing in the wireless channel and therefore it corresponds to a standard Nakagami-m fading channel. Furthermore, as  $m\to\infty$  and  $m_s\to\infty$ , the  $\mathcal F$  composite fading model becomes increasingly deterministic, i.e., it becomes equivalent to an additive white Gaussian noise (AWGN) channel.

Based on (3), the PDF of the instantaneous SNR,  $\gamma$ , in an  $\mathcal{F}$  composite fading channel can be straightforwardly deduced by using the variable transformation  $\gamma = \overline{\gamma}r^2/\Omega$ , such that

$$f_{\gamma}(\gamma) = \frac{m^m (m_s - 1)^{m_s} \overline{\gamma}^{m_s} \gamma^{m-1}}{B(m, m_s) \left[ m\gamma + (m_s - 1) \overline{\gamma} \right]^{m+m_s}} \tag{4}$$

where  $\overline{\gamma} = \mathbb{E}[\gamma]$  denotes the corresponding average SNR. To this effect, the redefined moments

$$\mathbb{E}[\gamma^n] \triangleq \int_0^\infty \gamma^n f_{\gamma}(\gamma) \mathrm{d}\gamma \tag{5}$$

 $^1$ It is worth highlighting that in the present paper, we have modified slightly the underlying inverse Nakagami-m PDF from that used in [40] and subsequently the PDF for the  $\mathcal F$  composite fading model. While the PDF in [40] is completely valid for physical channel characterization, it has some limitations in its admissible parameter range when used in analyses relating to digital communications. The redefined PDF in (3), on the other hand, is well consolidated and hence, more useful in practice.

<sup>2</sup>Letting  $r^2=x$ ,  $m=d_1/2$ ,  $m_s=d_2/2$ ,  $\Omega=d_2/(d_2-2)$  and performing the required transformation yields the  $\mathcal F$  distribution,  $f_X(x)$ , with parameters  $d_1$  and  $d_2$ .

is expressed as

$$\mathbb{E}[\gamma^n] = \frac{(m_s - 1)^n \overline{\gamma}^n \Gamma(m+n) \Gamma(m_s - n)}{m^n \Gamma(m) \Gamma(m_s)} \qquad m_s > n.$$
(6)

Of note, the corresponding redefined moment-generating function (MGF) can be obtained in closed-form similarly to the initial MGF representation in [40]. However, since (6) is only valid for  $m_s > n$  i.e. it is not valid for all values of the involved parameters, its validity and usefulness is limited to a notational representation of the involved integral that appears from the generic representation of the MGF function. This was also how it was used in the evaluation of the average bit error probability of the differential binary phase shift keying in [40], instead of its theoretical use as a statistical measure.

### III. ERGODIC CAPACITY ANALYSIS

Channel capacity is a core performance metric in conventional and emerging communication systems, and its achievable limits are largely affected by the incurred fading conditions during wireless transmission. Ergodic capacity is the most widely used capacity measure and is concerned with CSI knowledge only at the receiver as well as a fixed transmit power. In what follows, we derive novel exact, approximate and asymptotic analytic expressions for this useful measure over  $\mathcal F$  composite fading conditions.

**Theorem 1.** For  $m, \gamma, \overline{\gamma}, B \in \mathbb{R}^+$  and  $m_s > 1$ , the channel capacity per unit bandwidth with optimum rate adaptation under  $\mathcal{F}$  composite fading conditions can be expressed as

$$\frac{C_{\text{ORA}}}{B} = \frac{1}{\ln(2)} \left\{ \psi(m+m_s) - \psi(m_s) + \frac{(m_s-1)\overline{\gamma} - m}{m+m_s} \times_3 F_2 \left( 1, 1, 1+m; 2, 1+m+m_s; 1 - \frac{(m_s-1)\overline{\gamma}}{m} \right) \right\}$$
(7)

where B denotes the channel bandwidth,  $\psi(\cdot)$  is the digamma function,  $\ln(\cdot)$  is the natural logarithm and  ${}_3F_2(\cdot,\cdot,\cdot;\cdot,\cdot;\cdot)$  is a special case of the generalized hypergeometric function  ${}_pF_q(\cdot,\cdot,\ldots,\cdot;\cdot,\ldots,\cdot;\cdot)$ , with p=3 and q=2 [42].

*Proof.* It is recalled that the channel capacity with optimum rate adaptation in the presence of fading is defined as

$$C_{\text{ORA}} \triangleq B \int_0^\infty \log_2(1+\gamma) f_{\gamma}(\gamma) d\gamma.$$
 (8)

Therefore, by substituting (4) in (8), the  $C_{\rm ORA}$  per unit bandwidth for the case of  ${\cal F}$  composite fading channels is given by

$$\frac{C_{\text{ORA}}}{B} = \frac{m^m (m_s - 1)^{m_s} \overline{\gamma}^{m_s}}{B(m, m_s)} \times \int_0^\infty \frac{\gamma^{m-1} \log_2(1 + \gamma)}{[m\gamma + (m_s - 1)\overline{\gamma}]^{m+m_s}} d\gamma.$$
(9)

The involved integral in (9) can be expressed in closed-form using [44, eq. (2.6.2.7)] and with the aid of logarithmic and hypergeometric function identities [42], [44]. By performing the necessary change of variables and after some algebraic manipulations, (7) is deduced, which completes the proof.

It is worth highlighting that (7) is expressed in terms of widely known functions, which are readily available in most standard scientific software packages. Nonetheless, an accurate closed-form approximation can be also deduced as a special case.

**Proposition 1.** For  $m, \gamma, \overline{\gamma}, B \in \mathbb{R}^+$ ,  $m_s > 1$  and  $m_s >> m$ , the channel capacity per unit bandwidth with optimum rate adaptation over  $\mathcal{F}$  composite fading channels can be tightly approximated as follows:

$$\frac{C_{\text{ORA}}^{\text{appr.}}}{B} \approx 1 + \frac{(m_s - 1)\,\overline{\gamma} - m}{\ln(2)(m + m_s)} \times \times {}_{3}F_{2}\left(1, 1, 1 + m; 2, 1 + m + m_s; 1 - \frac{(m_s - 1)\,\overline{\gamma}}{m}\right).$$
(10)

*Proof.* It is obvious that  $m+m_s\approx m_s$  when  $m_s>>m$ . By recalling (7) and the properties of the digamma function [42], [44], it is evident that  $\psi(m+m_s)\approx \psi(m_s)$ , when  $m_s>>m$ , which yields

$$\psi(m+m_s) - \psi(m_s) \longrightarrow 0. \tag{11}$$

Based on the above and after some manipulations, (7) reduces to (10), which completes the proof.

In the same context, a particularly elegant as well as simple and tight asymptotic expression is derived for the case of high average SNR values.

**Proposition 2.** For  $m, \gamma, \overline{\gamma}, B \in \mathbb{R}^+$ ,  $m_s > 1$  and  $\overline{\gamma} >> 0$ , the channel capacity per unit bandwidth with optimum rate adaptation over  $\mathcal{F}$  composite fading channels can be asymptotically expressed as follows:

$$\frac{C_{\text{ORA}}^{\text{asym.}}}{B} \simeq \frac{\ln(\overline{\gamma}) + \ln(m_s - 1) - \ln(m) + \psi(m) - \psi(m_s)}{\ln(2)}.$$
(12)

*Proof.* The ergodic capacity per unit bandwidth at the high SNR regime can be accurately lower bounded as [45]

$$C_{\text{ORA}}(\overline{\gamma}) \simeq \frac{\ln(\overline{\gamma})}{\ln(2)} + \frac{1}{\ln(2)} \frac{\partial}{\partial n} AF_{\overline{\gamma}}^{(n)} \Big|_{n=0}$$
 (13)

where

$$AF \triangleq \frac{\mathbb{E}[\gamma^n]}{\mathbb{E}[\gamma]^n} - 1 \tag{14}$$

denotes the corresponding amount of fading. Importantly, the above analytic expression can be equivalently re-written in the following simplified form [46]

$$C_{\text{ORA}}(\overline{\gamma}) \simeq \frac{1}{\ln(2)} \frac{\partial}{\partial n} \mathbb{E}[\gamma^n] \Big|_{n=0}.$$
 (15)

To this effect and recalling the provided moments of the  $\mathcal{F}$  composite fading model in (6), the asymptotic capacity for this case can be derived by determining the first derivative of (6) with respect to n and then setting n=0, namely

$$C_{\text{ORA}}^{\text{asym.}} = \frac{\partial}{\partial n} \left[ \frac{(m_s - 1)^n \overline{\gamma}^n \Gamma(m+n) \Gamma(m_s - n)}{\ln(2) m^n \Gamma(m) \Gamma(m_s)} \right]_{n=0}.$$
(16)

Following from this, with the aid of the properties of the gamma function along with some algebraic manipulations, the first derivative of (6) for the case of the  $\mathcal F$  composite fading with respect to n is expressed by the following closed-form representation

$$\frac{\partial}{\partial n} \mathbb{E}[\gamma^n] = \frac{(m_s - 1)^n \overline{\gamma}^n \Gamma(m+n) \Gamma(m_s - n)}{m^n B(m, m_s) \Gamma(m+m_s)} \times \left\{ \psi(m+n) - \psi(m_s - n) - \ln\left(\frac{m}{(m_s - 1)\overline{\gamma}}\right) \right\}.$$

By substituting (17) in (16), setting n=0, i.e.  $(1/\ln(2))\partial\mathbb{E}[\gamma^n]/\partial n|_{n=0}$ , and carrying out some algebraic manipulations, (12) is deduced, which completes the proof.

It is noted that the simple algebraic representation of (12) provides useful insight on the impact of the involved parameters on the overall system performance. This is also evident through the fact that it can be also expressed in terms of  $\overline{\gamma}$ , namely

$$\overline{\gamma}_{\mathrm{ORA}} \approx 2^{\frac{C_{\mathrm{ORA}}^{\mathrm{asym.}}}{B}} e^{\psi(m_s) - \psi(m)} \frac{m}{m_s - 1},$$
 (18)

which provides insights on the value of  $\overline{\gamma}$  for fixed  $\overline{\gamma}_{ORA}$  and fading parameters. Notably, this is useful in quantifying the required  $\overline{\gamma}$  value for meeting target quality of service requirements under different fading conditions.

It is noted here that the tightness of the derived approximation and asymptotic representations against the exact closedform expression in (7) is rather high. This is evident by the fact that the insightful equalities

$$\lfloor C_{\mathrm{ORA}} \rfloor = \lfloor C_{\mathrm{ORA}}^{\mathrm{appr.}} \rfloor = \lfloor C_{\mathrm{ORA}}^{\mathrm{asym.}} \rfloor$$
 (19)

and

$$\lceil C_{\text{ORA}} \rceil = \lceil C_{\text{ORA}}^{\text{appr.}} \rceil = \lceil C_{\text{ORA}}^{\text{asym.}} \rceil$$
 (20)

with  $\lceil \cdot \rceil$  and  $\lfloor \cdot \rfloor$  denoting the ceiling and floor functions, respectively [42], hold for the majority of possible combinations of fading severity at moderate and high average SNR regimes.

# IV. NUMERICAL RESULTS

In this section, we utilize the analytic results obtained in the previous section to quantify the channel capacity with optimum rate adaptation rate for various communication scenarios under realistic multipath fading and shadowing conditions.

Fig. 1 illustrates the exact  $C_{\rm ORA}$  per unit bandwidth over  $\mathcal{F}$  composite fading channels with five different combinations of the m and  $m_s$  parameters, namely heavy shadowing  $(m=50.0, m_s=1.1)$ , severe multipath fading  $(m=0.5, m_s=1.1)$ 

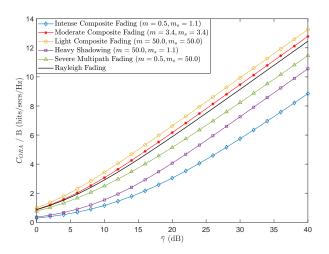


Fig. 1: Exact  $C_{ORA}/B$  versus average SNR under  $\mathcal{F}$  fading conditions for different values of the m and  $m_s$  parameters.

TABLE I: Exact, Approximate and Asymptotic Capacity

Involved Parameters			Ergodic Capacity		
m	$m_s$	$\overline{\gamma}$	Ex. (7)	Appr. (10)	Asym. (12)
1.2	1.2	10dB	1.76	1.40	0.73
2.2	1.2	10dB	1.87	0.91	1.06
2.2	3.2	10dB	2.96	3.11	2.67
4.8	4.8	10dB	3.19	3.11	2.98
1.2	1.2	15dB	2.89	2.53	2.40
2.2	1.2	15dB	3.08	2.12	2.73
2.2	3.2	15dB	4.43	4.58	4.33
4.8	4.8	15dB	4.72	4.64	4.65
1.2	1.2	20dB	4.27	3.92	4.06
2.2	1.2	20dB	4.52	3.56	4.39
2.2	3.2	20dB	6.03	6.17	5.99
4.8	4.8	20dB	6.33	6.25	6.31
1.2	1.2	30dB	7.41	7.05	7.38
2.2	1.2	30dB	7.72	6.76	7.71
2.2	3.2	30dB	9.31	9.46	9.31
4.8	4.8	30dB	9.63	9.55	9.63
1.2	1.2	40dB	10.71	10.35	10.70
2.2	1.2	40dB	11.03	10.07	11.03
2.2	3.2	40dB	12.63	12.78	12.63
4.8	4.8	40dB	12.95	12.87	12.95

 $m_s=50.0$ ), intense ( $m=0.5,\ m_s=1.1$ ), moderate ( $m=3.4,\ m_s=3.4$ ), and light ( $m=50.0,\ m_s=50.0$ ) composite fading. The  $C_{\rm ORA}$  per unit bandwidth over Rayleigh fading channels is also illustrated in Fig. 1 for comparison. As anticipated, the lowest spectral efficiency occurs when the channel is subject to simultaneous severe multipath fading and heavy shadowing, i.e., intense composite fading. On the contrary, the highest spectral efficiency appears in the light composite fading scenarios. This is largely due to the fact that the  $\mathcal F$  composite fading channel tends to become more deterministic, i.e., approaches an AWGN channel, as the m and  $m_s$  parameters approach infinity i.e. large m and  $m_s$  in reality. Also, the difference between the two scenarios

is substantial across all SNR regimes, since the achieved channel capacity in the case of light composite fading is over 50% higher than the capacity for the case of intense composite fading. Interestingly, it is noted that the spectral efficiency is higher when the channel is subject to severe multipath fading compared to the channel is subject undergoing heavy shadowing. This suggests that the shadowing constitutes a more dominating influence on the performance of wireless communications systems, compared to the multipath fading. Furthermore, the severe multipath fading (m=0.5,  $m_s=50.0$ ) case is equivalent to the Nakagami-m fading, for m=0.5. Accordingly, as shown in Fig. 1, the Rayleigh fading case (m=1) exhibits a higher spectral efficiency compared to the severe multipath fading case considered in this paper.

The tightness of the proposed approximation and the asymptotic expression against the derived exact analytic expression are depicted in Table I. It is evident that the accuracy of the proposed closed-form approximation is high across all fading conditions and average SNR values. A similar behavior is observed for the proposed asymptotic expression since its accuracy is poor in low average SNR values, as expected, but remarkably high in moderate and particularly in high average SNR regimes. It is also noted that the two proposed expressions exhibit a useful complementarity, while it is recalled that the identities  $\lfloor C_{\mathrm{ORA}} \rfloor = \lfloor C_{\mathrm{ORA}}^{\mathrm{asym}} \rfloor = \lfloor C_{\mathrm{ORA}}^{\mathrm{asym}} \rfloor$  and  $\lceil C_{\mathrm{ORA}} \rceil = \lceil C_{\mathrm{ORA}}^{\mathrm{appr.}} \rceil = \lceil C_{\mathrm{ORA}}^{\mathrm{asym}} \rceil$  hold for the majority of possible combinations of fading severity at moderate and high average SNR regimes. This verifies the usefulness of these simplified analytic expressions.

## V. CONCLUSION

In this paper, we presented a comprehensive ergodic capacity analysis over  $\mathcal{F}$  composite fading channels, which have been shown to characterize multipath fading and shadowing conditions more accurately than the widely used generalized-K fading model. In this context, we derived a novel exact analytic expressions along with simple and accurate approximate and asymptotic expressions. It was shown that the achievable channel capacity varies considerably even at slight changes of the average SNR and of the severity of the incurred multipath fading and shadowing conditions. The impact of different types of  $\mathcal{F}$  composite fading was also investigated through comparisons with the respective capacity for the case of a Rayleigh fading channel. This has highlighted that different types of composite fading can have a profound effect, which is beyond the range of the fading conditions experienced in a conventional Rayleigh fading environment. Finally, the provided results and insights will be useful in the design and deployment of future communications systems. For example when assessing technologies such as channel selection and spectrum aggregation for use in heterogeneous networks, telemedicine and vehicular communications, among others.

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