Finite element mapping for efficient image reconstruction in rotational electrical impedance tomography

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Abstract. Electrical impedance tomography (EIT) is a label free harmless imaging method capable of imaging differences in electrical conductivity of a sample. In EIT, a low frequency current is injected into the sample, voltage differences on sample surface are measured, and from these measurements, interior conductivity distribution is reconstructed. To increase the accuracy of reconstruction, rotational EIT (rEIT) has been proposed where independent measurements are taken from multiple rotational positions around the sample. However, the benefit of conventional electrode configurations are limited to small number of rotational positions. We have presented an approach called Limited Angle Full Revolution rEIT (LAFR-rEIT) that uses a small number of electrodes and large number of rotational measurement position measurements over 360-degrees. The results are comparable to previous rotational EIT implementations, and furthermore, the limited EIT boundary access provides space for simultaneous attachment of other measurement modalities. On the other hand, the increased number of measurement positions cause an increase in computational complexity, and optimization is required until 3D applications are feasible. This work presents modifications into finite element mesh presentation of the imaging domain and outlines an optimization, that enables sufficiently light rotation for 3D LAFR-rEIT computations.

1 Introduction

Electrical impedance tomography (EIT) is a non-destructive electrical imaging modality used both in industrial and medical applications. In EIT, a current is injected into the sample of interest and potential differences are measured on

the sample domain [1]. From these measurements, the electrical conductivity distribution is reconstructed. This reconstruction problem is severely ill-posed inverse boundary value problem where the accuracy of the solution benefits from additional independent data. Hence, adding measurement positions through rotating the sample (rotational EIT, rEIT) [2–4] and joint reconstruction of EIT in conjunction with an additional modality have been proposed including ultrasonic imaging and magnetic resonance imaging [5–7].

In our previous work [8], we presented a rEIT design principle in 2D using only a limited section of the sample surface for electrode positioning. The motivation for developing such an approach was to enable both of the mentioned modifications into EIT system: rEIT and simultaneous multimodal imaging. The approach we call Limited Angle Full Revolution rEIT (LAFR-rEIT) benefits from a large number of rotational measurement positions, but consequently, there is an increase of the computational complexity due to the increase of the number of measurements.

Previously rEIT has been modeled either by including modeling electrodes to account for all rotational positions, or by computing position nodes for electrodes in each rotational position. An in-detail comparison of reconstruction approaches used for modeling rotation in EIT is presented in [4]. However, for increased number of rotational positions, these approaches are not feasible: the modeling electrodes might overlap or the node structure on the boundary may become overly complex. To overcome these issues, we implemented weight-based mapping to model rotation [8], where mesh structure is created for initial position and element values are rotated inside this fixed mesh. The advantage of the weight-based mapping is that mesh rotation is independent from its construction. On the downside, the need of optimization persists as the mapping is computationally time and memory inefficient. Additionally, the weight-mapping produces local deformations of object during the rotation, because element boundaries vary in each rotational position.

For more efficient implementation of rEIT reconstructions, this paper proposes a novel approach for creating 2D finite element meshes for rEIT image reconstruction. The proposed mesh construction is composed of an inner boundary within computational domain, and rotational mapping is performed using this inner boundary instead of rotating whole domain with respect to electrodes. The implementation of the rotation inside this mesh is matrix free and, therefore, expected to perform computationally efficiently. We evaluate the computational speed of the proposed method against the weighted method in simulations. The efficacy of the method is verified using experimental LAFR-rEIT data.

2 Methods

The finite element mesh for efficient rotational computations was formed for circular 2D computational domain using triangular elements; an example is shown in Fig. 1. The mesh was composed of rotational boundary nodes at given distance from domain boundary, electrod nodes, and nodes out- and inside of rotational

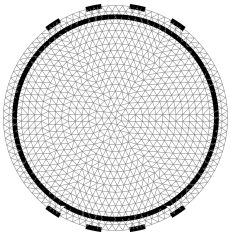


Fig. 1. Rotational boundary, marked with darker, thicker, circular line, is inside the computational domain. Only nodes on and inside the boundary are rotated. Electrodes are marked with darker, thicker short lines.

boundary. The basis of the proposed construction is that rotational boundary is rotation invariant for angles that are multiples of the intended angular step $360°/N$, where N is the total number of rotational measurement positions. Hence, there are no deformations in the mesh structure during rotation of the inside nodes. The rotation procedure itself can be considered as "clicking" the nodes on the boundary: The xy-coordinates of the rotational boundary and inside nodes are rotated with desired multiple of the step and, since the rotational boundary is invariant to these rotations, each rotated node on the boundary can be paired with a node on same coordinates in initial position. According to these pairs, the elements on the rotational boundary are the remapped to static outer elements.

For the rotational EIT problem, solved in the complete computational domain, we use the stacked forward model [8]

$$V_r = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_m \end{bmatrix} = \begin{bmatrix} U(M_1\sigma) \\ U(M_2\sigma) \\ \vdots \\ U(M_m\sigma) \end{bmatrix} + n = U_r(\sigma) + n, \tag{1}$$

where U is the forward operator with complete electrode model [9], M_i is the mapping operator for each rotational position, V_i is the measurement data corresponding to conductivity distribution σ inside the sample and n is additive noise term. Inverse problem is solved using Taylor approximation

$$U_r(\sigma) = U_r(\sigma_0) + J_r(\sigma - \sigma_0), \tag{2}$$

where Jacobian is a stack of blocks: $J_r = (JM_i)_j$.

In difference mode EIT, the observation model $\Delta V_r = J_r\Delta\sigma + \Delta n_r$ is the difference between two measurement sets $V_r^{(1)}$ and $V_r^{(2)}$. Reconstruction of the change in conductivity distribution $\Delta\sigma$ inside the sample between first and second measurement is computed from boundary measurements as Tikhonov regularized solution to minization problem

$$\hat{\Delta\sigma} = \arg\min_{\Delta\sigma} \left\{ \|\Delta V_r - J_r\Delta\sigma\|^2 + \alpha\|L\Delta\sigma\|^2 \right\},$$

where α is the weight given for regularization and L is smoothness promoting regularization matrix [8, 10].

For the weighted method, mapping matrix $M = (M_i)$ is a linear mapping that produces conductivity distribution in rotated coordinates $M_i\sigma$ through matrix multiplication. Elements of $M_i = (j, k)_i$ are the areas of intersection of elementes e_j and e_k, where j-coordinate is in the rotated while k-coordinate is in the original position [8].

We evaluated the speed of weighted and proposed rotations through simulation of rotational acquisitions having 8, 16, 32, 64, 128 or 256 measurement positions. The qualitative accuracy of the reconstructions were assessed using experimental data of a manually rotated gelatin phantom (250mm in total diameter) in saline bath tank (280mm in diameter). First measurement for difference EIT reconstruction was homogeneous gelatin and for the second measurement, resisting inclusion of 29mm in diameter was included. Measurement was conducted with eight electrodes in use on KIT4 EIT-device [11] and data was acquired from 64 equidistant rotational measurement positions. Experimental protocol is described in detail in our previous work [8].

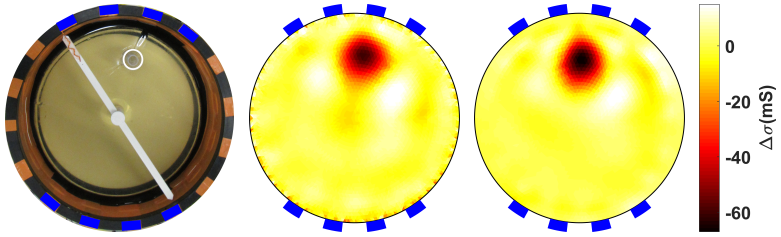
The speed comparison was performed using EIDORS [12] v3.9 compatible MATLAB R2017a (The MathWorks, Inc.) implementation in Lenovo P51 laptop with Intel i7-7820HQ CPU, 32 GB of RAM and 64-bit Windows 10 operating system. We used complete electrode model with electrode impedance value of $z = 0.01$, and a Laplacian prior with hyperparameter value of 0.06 was chosen for the regularization.

3 Results

Regarding computation times in weighted mapping, most costly process is the formation of weighted mapping $M = (M_i)$. In forward solution, each M_i is implemented as matrix free rotation operation, and for inverse solution, complete M is computed at once and then used for the Jacobian $J_r = (JM_i)$, where J is the Jacobian in initial measurement position, i.e., no rotation. For proposed rotation method, computation of M is not required; however, each rotational

Table 1. Comparison of computation times between weighted and proposed rotation.

Rotational measurement positions	8	16	32	64	128	256
Forward weighted (2x)[1]	4 min	9 min	18 min	39 min	75 min	149 min
Forward proposed (2x)[1]	7 sec	16 sec	35 sec	67 sec	2 min	4 min
Weighted inversion: computation of M	16 sec	32 sec	66 sec	136 sec	4 min	11 min
Weighted inversion (including Jacobian)	6 sec	12 sec	23 sec	45 sec	71 sec	128 sec
Proposed inversion (including Jacobian)	6 sec	12 sec	23 sec	47 sec	96 sec	218 sec
[1]) Solved for homogenous and non-homogenous samples.						
Size of full M in memory	0.2 GB	0.4 GB	0.9 GB	1.6 GB	2.8 GB	9.6 GB



(a) (b) (c)

Fig. 2. (a) Photo of the gelatin sample and measurement tank, (b) reconstruction using the weighted method and (c) reconstruction using the proposed method.

Jacobian block JM_i is computed separately as a single unit from "click" rotated mesh which constitutes for the most time in the inversion procedure.

A comparison of computation times for weighted and proposed methods is shown in Table 1. The advantage of proposed method is imminent in the computing times of the forward problem: proposed method uses only about 3 % of the computing time compared to the weighted method. For inversion problem, the computation of Jacobian J_r and inversion itself are about the same order, but with the proposed method, time for creating M is saved.

The meshes used in the computations were composed of from 7290 to 7418 elements for the forward solution and from 1840 to 2160 elements for the inverse solution depending on the number of rotational measurement positions. The sizes of full weighted mapping matrices M are shown in Table 1 as it corresponds to the size of the Jacobian in both methods. The sparsity of M is of order 0.15 %.

In Fig. 2, reconstructions from the data of experimental measurements agree qualitatively very well, but are not exactly matching. Using the weighted method, reconstruction in Fig. 2 (b) shows slightly more variation even though smoothness promoting Laplacian prior was used. This is an indication of the local deformations caused by the weighted mapping. On the contrary, the proposed method is smoother with the same hyperparameter value, indicating robustness against noise.

4 Conclusions

In this work, we have presented a rotational mesh construction that removes the dependency between electrode positions and rotational measurement positions in rEIT. Further, we have significantly decreased the computation time when compared to our previous rotational algorithm. Presented rotation procedure is applicable to any computational domain, where rotated object is bounded during rotation to a radius that is less than distance from center-of-rotation to closest electrode. Proposed method is efficiently extensible into 3D. Further optimiza-

tion may be achieved by keeping the rotational mapping M and/or Jacobian matrix J_r in memory for consecutive uses, or through matrix-free implementation exploiting the block-structure of J_r.

Acknowledgements and conflicts of interest

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