

Latent force models in autonomous GNSS satellite orbit prediction

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Abstract—In this paper, we present a method for improving autonomous GNSS satellite orbit prediction accuracy by including latent forces, i.e. forces that are estimated with broadcast ephemeris data. The purpose of autonomous prediction is to reduce the Time to First Fix of a stand-alone positioning device. Our orbit model includes gravity and solar radiation forces and initial state estimation algorithm. We present a state-space model for the latent forces, which are meant to correct the deficiencies of our force model, and we describe how latent forces are incorporated as a part of our prediction algorithm. Using a novel algorithm, where multiple ephemerides are used to estimate the latent forces, the orbit prediction accuracy for GPS, GLONASS and Beidou is significantly improved. For example, for 7-day prediction of GPS satellites, the 68% quantile of SISRE, which is an estimate of positioning accuracy, reduced 37.1%.

I. INTRODUCTION

A GNSS satellite transmits the data required for positioning in the form known as Broadcast Ephemeris (BE). Transmitting the required data takes some amount of time depending on the constellation. For example with GPS satellites, it takes roughly 30 seconds to transmit the data necessary for positioning purposes. In certain environments, for example in a city centre, where buildings can interfere and block the signal partially or wholly, it may take even several minutes to get a position estimate. From the point of view of the general user, this delay is annoying; for some applications it can be costly or even fatal.

The delay can be reduced by obtaining the BE data from another source. In this case, the receiver only needs a signal from the satellite for pseudorange measurement, and Time to First Fix (TTFF), which is the time required for any position estimate, can be reduced to about 5 seconds. The BE data can be sent to positioning device using assisting data servers. This is not, however, always possible since some devices may lack a network connection or transmitting large amounts of data may be expensive. An alternative to server-based assistance is autonomous positioning, in which the ephemeris is computed for the current time instant based on previously received ephemerides. That is, the device runs an algorithm that predicts the position and clock offset of the satellite from existing and previous BE data. These predictions can then be interpreted as BE to reduce TTFF. Our group has previously presented algorithms for predicting the orbit [1], [2] and the clock offset [1], [3] from the perspective of autonomous positioning. Our research on estimation of model parameters and initial state is reported in [4]. Others have also published methods for

extending the broadcast data in standalone devices, e.g. [5]. In this work, we will focus on orbit prediction.

The orbit of a satellite can be predicted by forming an equation of motion for the satellite and then integrating the orbit starting from initial conditions. The equation of motion is a differential equation that includes the forces acting on a satellite. In this work, we consider the four largest forces: gravitation of the Earth, the Sun and the Moon, and solar radiation pressure (SRP). When we predict the orbits based on these forces, we find that the prediction error shows systematic errors, which suggests that our model has incorrect or missing forces. In [2] we found that adding smaller forces, for example solid tide, ocean tide or Earth albedo, to the force model improves the predictions only slightly, at best only 1 or 2 percent.

In this work we consider latent force models, where the missing forces are estimated based on the data. Latent force models have been applied to orbit prediction in [6], and we continue the work here with similar methods. We will use the abbreviation LFM for latent force models. Latent forces are estimated with a state-space approach, where latent forces are treated as part of the satellite’s dynamic state and are then inferred together with the satellite’s position and velocity. The novelty in our study is that we consider the problem in the context of autonomous positioning. Therefore we will have some restrictions, for example we may only use BE data, which is received at arbitrary times, for the estimation.

The broadcast is only valid for a certain time. For GPS satellites, this time interval is 4 hours. This is not long enough to estimate the latent forces accurately and therefore we will need to collect more than one ephemeris. In this article we present a novel adaptive algorithm, where the latent forces are updated with each received ephemeris, and can then be used for prediction purposes. We do not need to store more than one ephemeris per satellite to the device, so additional memory usage comes only from storing the latent force components. The downside is the increase in computation time. We shall show that the considered approach works with different satellite constellations. We shall consider GPS in detail, and present only general results for GLONASS and Beidou.

The rest of this article is organized as follows: in Section II we present the model of our orbit predictor, which includes the force model and initial state estimation algorithm. In Section III we describe the form of the latent forces and how they

are estimated as a part of the initial state. In Section IV we gather the main results of the study and Section V concludes the article.

II. ORBIT DETERMINATION

To predict the orbit of a satellite, we use a differential equation for the motion of the satellite that includes models of forces acting on the satellite. This differential equation is integrated with certain initial conditions to obtain the position and velocity of the satellite at a later time. In this section we describe the force model and how we obtain the initial conditions for the prediction.

A. Force model

Our force model includes expressions for the four major forces acting on a satellite. These are the gravity of the Earth, the Sun and the Moon and solar radiation pressure (SRP). The force model also includes the so-called latent forces, which are described further in Section III.

The gravitational potential of the Earth may be calculated with

$$U_E = \frac{GM_E}{r} \sum_{n=0}^{\infty} \sum_{m=0}^n \left[\left(\frac{R_E}{r} \right)^n P_{nm}(\sin \phi) \left(C_{nm} \cos(m\lambda) + S_{nm} \sin(m\lambda) \right) \right], \quad (1)$$

where G is gravitational constant, R_E and M_E are the radius and the mass of the Earth and r is the distance of the satellite from Earth's center. Parameters λ and ϕ are the longitude and the latitude of the satellite. Terms P_{nm} are the associated Legendre polynomials of degree n and order m . We use coefficients up to degree and order 8. Values for C_{nm} and S_{nm} are from EGM2008 [7].

The potential of the Earth is calculated in Earth-centered Earth-fixed (ECEF) coordinate system, which is rotating with the Earth. In inertial coordinate system the acceleration caused by the potential U_E in (1) is

$$\mathbf{a}_{\text{Earth}} = \mathbf{R}_{\text{ECEF}}^{-1} \nabla U_E, \quad (2)$$

where \mathbf{R}_{ECEF} is the transformation matrix from inertial coordinate system to ECEF and ∇ denotes the gradient.

The Sun and the Moon are modelled as point masses. The gravitational acceleration caused by a celestial body relative to the motion of the Earth is

$$\mathbf{a}_{\text{cb}} = GM_{\text{cb}} \left(\frac{\mathbf{r}_{\text{cb}} - \mathbf{r}_{\text{Sat}}}{\|\mathbf{r}_{\text{cb}} - \mathbf{r}_{\text{Sat}}\|^3} - \frac{\mathbf{r}_{\text{cb}}}{\|\mathbf{r}_{\text{cb}}\|^3} \right), \quad (3)$$

where M_{cb} is the mass of the celestial body and \mathbf{r}_{cb} and \mathbf{r}_{Sat} are the positions of the celestial body and the satellite in an Earth-centered inertial reference frame. For the position of the Sun and the Moon, we use DE202 ephemeris data provided by JPL [8].

For the solar radiation pressure, we use a two-parameter model. The total acceleration is presented as a sum of two components. The first acceleration component is in the direction from the Sun to the satellite and the second component is

orthogonal to the first component and is caused by reflection of the radiation from solar panels. Total acceleration by SRP is calculated with

$$\mathbf{a}_{\text{SRP}} = \nu \left(-\alpha_1 \frac{1}{r_{\text{Sun}}^2} \mathbf{e}_{\text{Sat,Sun}} + \alpha_2 \mathbf{e}_{\text{y-bias}} \right), \quad (4)$$

where r_{Sun} is the distance between the Sun and the satellite, $\mathbf{e}_{\text{Sat,Sun}}$ is a unit vector from satellite to Sun and $\mathbf{e}_{\text{y-bias}}$ is a unit vector to the y-direction, which is defined as

$$\mathbf{e}_{\text{y-bias}} = \frac{\mathbf{r}_{\text{Sat}} \times (\mathbf{r}_{\text{Sun}} \times \mathbf{r}_{\text{Sat}})}{\|\mathbf{r}_{\text{Sat}} \times (\mathbf{r}_{\text{Sun}} \times \mathbf{r}_{\text{Sat}})\|}. \quad (5)$$

The parameter $\nu \in [0, 1]$ models the shadows of the Earth and the Moon. We use a conical model for the shadows as described in [9]. The shadow of the Moon is considered only when the satellite is not in Earth's shadow. Parameters α_1 and α_2 are estimated with historical precise ephemeris (PE) data on a server and then delivered to the device as assistance data. The estimation process is described in detail in [4].

With the forces described above, we obtain the differential equation

$$\begin{aligned} \ddot{\mathbf{r}} &= \mathbf{a}_{\text{Earth}} + \mathbf{a}_{\text{Moon}} + \mathbf{a}_{\text{Sun}} + \mathbf{a}_{\text{SRP}}, \\ \mathbf{r}(t_0) &= \mathbf{r}_0, \quad \dot{\mathbf{r}}(t_0) = \mathbf{v}_0. \end{aligned} \quad (6)$$

We solve the equation numerically by substituting the expressions for different accelerations. Initial conditions \mathbf{r}_0 and \mathbf{v}_0 are estimated with the method described in the following section.

B. Initial state estimation

If the position and velocity obtained directly from the satellite's navigation message are used as initial conditions for the differential equation (6), predictions beyond a few hours have very poor accuracy. It is necessary to determine initial position and velocity by fitting the ephemeris data to our force model.

We use an Extended Kalman filter to estimate the initial state as described in [4]. The state of the satellite consists of position \mathbf{r} and velocity \mathbf{v} , and the dynamical model for the motion of the satellite, (6), can be written as

$$\frac{d}{dt} \begin{bmatrix} \mathbf{r}(t) \\ \mathbf{v}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{v}(t) \\ \mathbf{a}(\mathbf{r}(t), t) \end{bmatrix}. \quad (7)$$

The states can then be inferred with Extended Kalman filter. The acceleration \mathbf{a} for each time instant is calculated as in (6). In Section III we show how this model can be modified to also include the latent forces. This way we can estimate the initial position and velocity together with the missing forces.

The data from the GPS and Beidou broadcast ephemeris is sampled with 15-minute interval from $t_{\text{toe}} - 1.5\text{h}$ to $t_{\text{toe}} + 1.5\text{h}$, which gives us 13 measurement points. The positions are then used as measurements for the Extended Kalman filter algorithm.

For GLONASS we need a different approach. GLONASS broadcast data is valid only for 15 minutes from t_{toe} . This interval is too short to determine the initial state. Thus, for GLONASS we use two broadcast ephemerides, with a time

difference of 12 hours, in the initial state estimation. The algorithm is described in [10].

III. LATENT FORCES

In [6], it was shown how latent forces can be applied to satellite orbit prediction. In that study, it was shown that using latent forces can improve orbit predictions when latent forces are estimated with a few days of continuous precise ephemeris data. Here, we will show that the method can also be used in autonomous positioning, with occasional BE data.

Latent forces are modelled as superpositions of stochastic resonators. These are characterized by differential equation

$$\frac{d^2}{dt^2}c_n(t) = -(2\pi n f)^2 c_n(t) + w_n(t), \quad (8)$$

where $w_n(t)$ is Gaussian white noise with spectral density q_n , n is an integer and f is the base frequency of the oscillation. In [11] it is shown that this model can be written as a linear state-space model

$$\frac{d}{dt}\mathbf{c}_n(t) = \mathbf{F}_n \mathbf{c}_n(t) + \mathbf{L}_n w_n(t), \quad (9)$$

with

$$\mathbf{c}_n = \begin{bmatrix} c_n(t) \\ \frac{dc_n(t)}{dt} \end{bmatrix}, \mathbf{F}_n = \begin{bmatrix} 0 & 1 \\ -(2\pi n f)^2 & 0 \end{bmatrix} \quad (10)$$

and $\mathbf{L}_n = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

for a single resonator. Multiple harmonic components can be included by augmenting the components and constructing the corresponding matrices block-diagonally. If \mathbf{x}_N is vector formed by stacking c_i , $i = 1, \dots, N$, where N is the number of components in the latent force model, then the state space model for latent force in direction X is

$$\frac{d}{dt}\mathbf{x}_{N,X}(t) = \mathbf{F}_{N,X}\mathbf{x}_{N,X}(t) + \mathbf{L}_{N,X}\mathbf{w}_{N,X}(t), \quad (11)$$

$$\mathbf{a}_X = \mathbf{H}_{N,X}\mathbf{x}_{N,X} + b_X,$$

where matrices $\mathbf{F}_{N,X}$ and $\mathbf{L}_{N,X}$ are block-diagonal consisting of corresponding matrices in (10) and

$$\mathbf{H}_{N,X} = [1 \ 0 \ 1 \ 0 \ \dots \ 1 \ 0]^T. \quad (12)$$

We included a bias term b_X in (11), so that oscillation does not have to be zero-centered. This parameter is considered to be constant in the model and is estimated together with other latent force components.

The forces are modelled in *Radial-Transverse-Normal* (RTN) coordinate system. This coordinate system is defined by unit vectors

$$\mathbf{e}_R = \frac{\mathbf{r}}{\|\mathbf{r}\|}, \quad \mathbf{e}_N = \frac{\mathbf{r} \times \mathbf{v}}{\|\mathbf{r} \times \mathbf{v}\|}, \quad \mathbf{e}_T = \mathbf{e}_N \times \mathbf{e}_R. \quad (13)$$

We model latent forces in each of these directions separately as a superposition of stochastic resonators. The acceleration caused by latent forces is then transformed into inertial coordinate system with

$$\mathbf{a}_{\text{latent}}(\mathbf{r}(t), \mathbf{v}(t), t) = \mathbf{R}_{\text{RTN}}^{-1}(\mathbf{r}(t), \mathbf{v}(t)) \begin{bmatrix} \mathbf{a}_R(t) \\ \mathbf{a}_T(t) \\ \mathbf{a}_N(t) \end{bmatrix}, \quad (14)$$

where \mathbf{R}_{RTN} is the transformation matrix from inertial coordinate system to RTN and \mathbf{a}_X , where $X \in \{R, T, N\}$, is the latent force component in the corresponding direction.

As we mentioned in Section II-B, the latent force model and the satellite force model can be combined. We modify (7) so that the state also includes latent force components for each direction and we include the acceleration by the latent forces, $\mathbf{a}_{\text{latent}}$, from (14) into the dynamic model function. We then have an augmented state vector

$$\mathbf{x}_a(t) = \begin{bmatrix} \mathbf{r}(t) \\ \mathbf{v}(t) \\ \mathbf{x}_{N,R}(t) \\ \mathbf{x}_{N,T}(t) \\ \mathbf{x}_{N,N}(t) \\ b_R \\ b_T \\ b_N \end{bmatrix} \quad (15)$$

and augmented dynamic model

$$\mathbf{f}(\mathbf{x}_a(t), t) = \begin{bmatrix} \mathbf{v}(t) \\ \mathbf{a}(\mathbf{r}(t), t) + \mathbf{a}_{\text{latent}}(\mathbf{r}(t), \mathbf{v}(t), t) \\ \mathbf{F}_{N,R}\mathbf{x}_{N,R}(t) \\ \mathbf{F}_{N,T}\mathbf{x}_{N,T}(t) \\ \mathbf{F}_{N,N}\mathbf{x}_{N,N}(t) \\ 0 \\ 0 \\ 0 \end{bmatrix}. \quad (16)$$

The noise terms are formed similarly with concatenated vectors and block-diagonally constructed matrices. Small process noise is added for each bias component b_X , $X \in \{R, T, N\}$, to allow small variations with time.

Once the latent forces have been estimated with sufficient amount of data, the term $\mathbf{a}_{\text{latent}}$ can also be added to differential equation (6). The expected value of each latent force component is calculated by simply integrating (11), which can be solved analytically. Total acceleration is then calculated with (14).

IV. RESULTS

Here we present the results for satellite prediction with latent force models. We compare the performance of two approaches. First we consider an approach where no latent force models were used and each received broadcast was used to initialize a new prediction. The second approach is an adaptive process, where we start from no data and start receiving broadcasts at random times. Each broadcast is used to update the state of the satellite which includes the position, velocity and latent forces.

We shall view the results for GPS in detail, and consider a single satellite for which we view a single time period along

TABLE I

SISRE WEIGHT VALUES FOR DIFFERENT CONSTELLATIONS. VALUES ARE FROM [12].

	GPS	GLONASS	Beidou MEO	Beidou IGSO/GEO
w_R	0.98	0.98	0.98	0.99
$w_{T,N}^2$	$\frac{1}{49}$	$\frac{1}{45}$	$\frac{1}{54}$	$\frac{1}{126}$

with a quantile curves for multiple test runs. For GLONASS and Beidou satellite, we view only the general performance, and compare it to our old approach.

A. Methods and test setup

Our final goal in autonomous positioning is to mimic the broadcast ephemeris data. Therefore, we should choose an error metric that is also suitable for broadcast ephemerides. For this purpose, *Signal-In-Space Range Error*, or SISRE works well.

The term SISRE is used in multiple ways in literature. In [12], SISRE refers to a single statistical value for a larger set of position errors, while in [13], [14] SISRE refers to a measure derived from a single position error of a satellite. Furthermore, another definition is given for SISRE in [15]. Although all definitions are closely related, care should be taken when using the term SISRE.

Our interpretation is that SISRE is the instantaneous RMS of pseudorange errors from every location on the surface of the Earth where the satellite in question is visible, caused by the error in satellite position. This interpretation is also used in Global Positioning System Standard Positioning Service Performance Standard [13]. This means that we can calculate SISRE value for any prediction. We calculate SISRE with

$$\text{SISRE} = \sqrt{w_R^2 \Delta R^2 + w_{T,N}^2 (\Delta T^2 + \Delta N^2)}, \quad (17)$$

where ΔX , $X \in \{R, T, N\}$, denotes the error components in RTN-coordinate system and weights w_R and $w_{T,N}$ depend on the satellite constellation, or in case of Beidou, also on the satellite type. The values of these weights for different satellite types are listed in Table I. Note that in the table one weight is squared while the other is not.

When we consider pseudorange errors, the effect of the clock error should also be included in (17). We omit the clock error in this study and consider only the error from the orbit.

We shall consider only improvements in pseudoranges rather than positioning accuracy improvement. This is mainly because we are only simulating the functionality of a positioning device. Considering positioning accuracy improvements would require consideration of many factors such as satellite geometry, number of satellites, set of used satellites, measurement errors etc. Furthermore, other error factors, such as ionospheric delays would have to be taken into account. For these reasons, we have chosen only to analyze the pseudorange measurement improvements by comparing SISRE values.

The parameters for the model were chosen heuristically. Base frequency f in (8) is taken to be the reciprocal of one orbital period of a satellite and hence varies with constellation

TABLE II

COMPUTATION TIMES WITH DIFFERENT NUMBER OF LATENT FORCE COMPONENTS.

Number of components (N)	Average computation time (s)
1	8.9
2	10.8
3	12.2
4	13.6
5	15.4
6	44.4
7	48.6

and satellite type. The number of components N for each direction was chosen to be 3. The number of components did not largely affect the prediction accuracy, but it did however affect the computation time, which we will consider briefly next.

We will consider increase in computation time as a function of N . We will compute the time it takes for our integrator to propagate the state and covariance matrix for a period of one week with values $N = 1, \dots, 7$. Our integrator was RKF7(8) based on [9].

In Table II, we present the median of computation times of 200 one-week propagations of state and covariance matrix. Simulations were run with 64-bit Matlab (R2016a) on MacBook Pro version 10.12.3 with 8 GB 166 MHz memory and 2.8 GHz Intel Core i7 processor. The most relevant part in Table II is the relative time differences rather than the absolute values of computation time. Computation time grows practically linearly, when $N \leq 5$, but after this we see a large jump in computation time. We also noted that the total number of components did not affect the computation time as much as the maximum number of components in one direction. For example, computation was much faster with configuration $N_R = N_T = N_N = 3$ than with $N_R = 7, N_T = N_N = 1$, even though the total number of components are equal.

We have reason to assume the forces missing from our force model to cause acceleration of magnitude $\sim 10^{-9} \frac{m}{s^2}$. This is stated in [2], [9]. We use this (squared) as prior variance for our latent force components; the prior mean is taken to be zero. Process noise for both resonators and bias terms is assumed to be small compared to amplitude of the oscillation. Different values were tried for the variance of both parameters and the ones that yielded best results were chosen. Values for the variances in our model were $(10^{-11} \frac{m}{s^2})^2$ for the resonators and $(10^{-12} \frac{m}{s^2})^2$ for the bias terms.

We point out that LFM assumes the reliability of previous broadcasts with respect to current orbit. Therefore, if an anomaly is detected, e.g. the position is in clear conflict with the predicted position, we reset our current estimates of latent forces and re-initialize the filter. Also, latent forces are not used in prediction until at least two broadcast ephemerides have been received. This is because estimates of latent forces from a single broadcast are too imprecise.

For our test scenario, we choose a period of 20 weeks, where we receive broadcasts randomly. By random, we mean

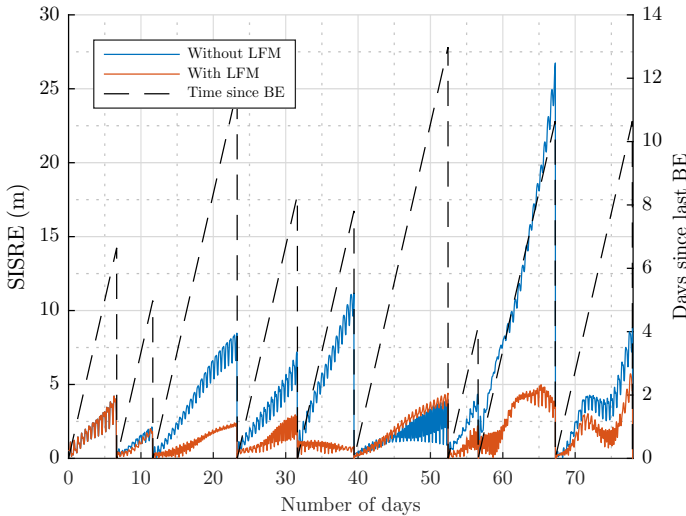


Fig. 1. One example of LFM performance compared to traditional approach. After a few broadcasts, we see vast improvements in prediction accuracy.

that the next broadcast is chosen from a uniform distribution of some length. This 20-week period is then repeated 50 times and results are combined. Our starting point was the start of GPS week 1878, which is roughly the start of 2016. So we simulate roughly 1000 weeks of predictions. This also models a real-life situation, where the device does not have any data, then it is taken into use and turned on every now and then.

B. Example GPS satellite

With GPS satellites, LFM improved the prediction accuracy for all satellites. We do not show collective results for the whole constellation but take an example satellite (PRN 16) for which we present detailed results. Figure 1 shows a 20-week period where broadcasts are received randomly and predictions are made between two broadcasts.

We used a maximum interval between two consecutive broadcasts of two weeks. That is, we simulate a device that is used in a way that it receives a signal from this particular satellite at least once every two weeks. This is a reasonable assumption in many applications.

We show 68% and 95% quantiles of the standard approach and LFM for our example satellite in Figure 2. For this particular satellite LFM worked exceptionally well. We cut off the comparison during the last day of two weeks, because we have fewer predictions during this time and so error quantiles are not reliable. This is also the reason we see rapid changes during the last days especially in 95% quantile.

Generally, the improvement for one week predictions was around 20 – 30%, and for some satellites even 50%. This is a huge improvement compared to 1–2% we previously achieved by adding physics-based expressions to the force model [2]. General performance level and improvement in accuracy for GPS constellation can be viewed from Table III.

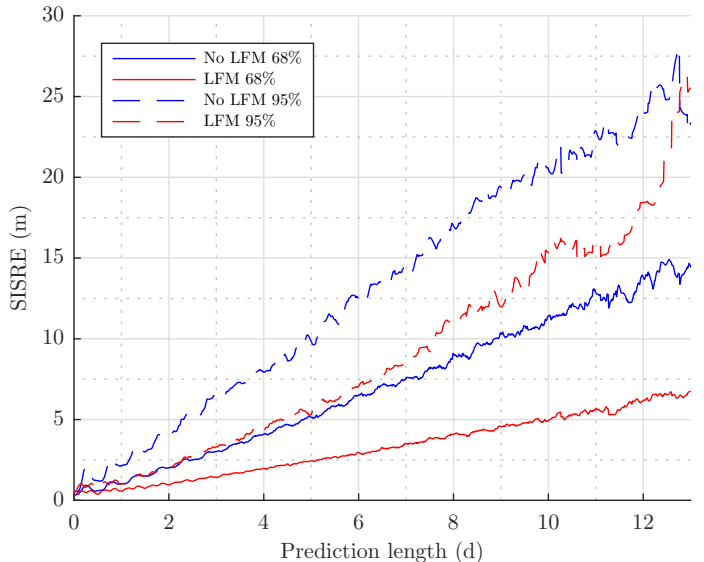


Fig. 2. SISRE quantile curves for GPS PRN 16 with and without LFM.

C. General results

We do not present detailed results for separate satellites here, but instead, we summarize results for each constellation. For Beidou, we present results for each orbit type separately. We, however, note that prediction accuracy levels and improvements may vary between satellites. The maximum time interval between two consecutive ephemerides was set to two weeks for GLONASS and Beidou MEO and IGSO. For Beidou GEO, it was set to one week. In Table III we gather results for each constellation and compare predictions with and without LFM. Constellation-wise, improvement in percentages was significant in many cases.

We chose a different maximum interval between two broadcasts and prediction length comparison in Table III for Beidou GEO satellites, and we also show only 68% quantile. This is because Beidou geostationary satellites are frequently maneuvered [16], and in this kind of situation, where typical error is thousands of meters or even more, it is not sensible to perform comparison.

All satellite constellations show improvements at the chosen prediction length in 68% quantile. In the 95% quantile, deficiencies of the model become apparent, and for example change in prediction accuracy of GLONASS satellites is less than 1%. However, the level of improvement is still significant with GPS and Beidou.

As stated earlier, improvements vary between satellites and have different prediction accuracies. To establish the true improvement level of the method, improvements in prediction accuracies should be viewed for each satellite separately. For full details about prediction accuracy for each tested satellite, and for more precise description of the methods, one may refer to [17].

TABLE III
COMPARISON OF METHODS: SISRE OF OLD METHOD AND LFM, 68%
AND 95% QUANTILES AT FIXED PREDICTION LENGTH

68% quantile			95% quantile		
GPS/7 days					
old	new	change	old	new	change
6.88 m	4.33 m	-37.1%	17.66 m	9.73 m	-44.9%
GLONASS/7 days					
old	new	change	old	new	change
12.16 m	9.32 m	-23.4%	28.11 m	27.85 m	-0.95%
Beidou MEO/7 days					
old	new	change	old	new	change
13.09 m	5.44 m	-58.48%	26.35 m	14.53 m	-44.86%
Beidou IGSO/7 days					
old	new	change	old	new	change
15.09 m	12.16 m	-19.39%	32.85 m	25.81 m	-21.4%
Beidou GEO/3 days					
old	new	change	old	new	change
15.11 m	7.21 m	-52.3%	-	-	-

V. CONCLUSIONS

In this paper we presented a method to combine the traditional mechanistic force models with data-driven latent force models in autonomous GNSS satellite prediction. Even though model is rather simple, and the same generic model is used with all satellite constellations and types, improvements in prediction accuracy were significant. The combined mechanistic model and data-driven approach gave much bigger improvement in accuracy than previously had been achieved with a more complex force model. GPS satellites and all Beidou satellite types showed significant improvements, while GLONASS predictions did not improve as much. Noting that prediction accuracy can be significantly improved with a hybrid method of mechanistic model and data-driven approach, more complex models could be tested in the future.

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