Poverty measures and poverty orderings*

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Abstract

We examine the conditions under which unanimous poverty rankings of income distributions can be obtained for a general class of poverty indices. The "per-capita income gap" and the Shorrocks and Thon poverty measures are particular members of this class. The conditions of dominance are stated in terms of comparisons of the corresponding TIP curves and areas.

1 Introduction

Following the publication of Sen's (1976) influential work on poverty measurement, much has been written on this topic and related issues. Because an important reason for measuring poverty is to make poverty comparisons, part of the literature on poverty measurement has developed by focusing on partial poverty orderings.

Given the poverty line (that is, the income level below which one is considered poor), the simplest way of comparing two income distributions in terms of poverty is by comparing some associated poverty measure. However, the choice of a single measure can be arbitrary and, hence, so can the conclusions based on this measure. In addition, different measures may produce contradictory conclusions. As pointed out by various authors (see, for example, Atkinson (1970, 1987) and Foster (1984)), this arbitrariness can be reduced by using a class of poverty measures rather than a single measure. This approach yields partial orders, by making judgements only if all members of a wide class of measures lead to the same conclusion.

Several authors, including Atkinson (1970, 1987), Foster and Shorrocks (1988a, 1988b), Spencer and Fisher (1992), Howes (1993), Jenkins and Lambert (1993, 1997, 1998) and Zheng (1999) have examined the conditions under which unanimous poverty orderings of income random variables are implied by large classes of indices. A comprehensive review of this topic is given by Zheng (2000). If C denotes a class of poverty measures and $I(X, z) \in C$ indicates the degree of poverty associated with the income random variable, X, when the poverty line is at an income level $z > 0$, the results are usually of the form

$$I(X, z) \leq I(Y, z) \text{ for all } I \in C, \text{ for all } z \in \Theta \subseteq \mathbf{R}^+ \tag{1}$$

if and only if

$$X \prec_P Y$$

where $\prec_P$ denotes the ordering that is induced by some comparison principle, P. Because (1) yields a multitude of inequalities, the potential applications of these characterizations are obvious, particularly when $X \prec_P Y$ is easy to verify.

In this work, we consider a comparison principle P based on comparing TIP (Three I's of Poverty) curves and areas. The TIP curve is a graphical device (also called the cumulative poverty gap (CPG) curve or the poverty profile curve) due to Jenkins and Lambert (1997) (see also Spencer and Fisher (1992) and Shorrocks (1995)). In order to introduce this curve, let X be a non-negative income random variable with a distribution function, F, and let F^{-1} be the right continuous quantile function of F, which is defined by

$$F^{-1}(t) = \sup \{x : F(x) \leq t\}, \quad t \in [0, 1].$$

Suppose that a poverty line is established at an income level $z > 0$. The proportion of poor people is denoted by r_z^X; that is,

$$r_z^X = \sup \{F(x) : x < z\}.$$

Let $X_z^* = \min\{X, z\}$ be the random variable, X, censored at z, with a distribution function, F_z. Its corresponding quantile function is F_z^{-1}, where

$$F_z^{-1}(t) = \begin{cases} F^{-1}(t) & \text{if } t < r_z^X, \\ z & \text{if } t \geq r_z^X, \end{cases}$$

for all $t \in [0, 1]$. The relative poverty gap associated with income $F^{-1}(t)$ is defined as

$$\frac{z - F_z^{-1}(t)}{z}$$

and the corresponding TIP curve (see Spencer and Fisher (1992), Shorrocks (1995) and Jenkins and Lambert (1998a)) is given by

$$G_X(p, z) = \int_0^p \left(z - F_z^{-1}(t) \right) dt, \quad p \in [0, 1].$$

The curve $G_X(p, z)$ is increasing and concave and begins at the origin and rises continuously over the interval $[0, r_z^X]$. At $p = r_z^X$ the curve becomes horizontal at a height equal to the mean poverty gap. As Jenkins and Lambert (1997) have pointed out, this curve summarizes three aspects of poverty: incidence, given by r_z^X; intensity, given by the height of the curve at $p = 1$; and inequality, represented by the degree of concavity of the non-horizontal section of the curve. Applications of this curve to the study of the evolution of poverty in Spain during the 1980s can be found in Del Río y Ruiz-Castillo (2001a, 2001b).

As shown by Jenkins and Lambert (1998a, 1998b), orderings of distributions by non-intersecting TIP curves correspond to unanimous orderings according to the class Γ of generalized poverty gap (GPG) poverty indices, which are increasing Schur-convex functions of absolute poverty gaps. Members of Γ satisfy the Focus, Monotonicity, Transfer, Symmetry and Replication invariance axioms (as defined, for example, by Foster 1984). However, it is well-known that TIP curves often intersect, so that clear rankings of income distributions would not be possible by simple TIP curve comparisons. Although, as shown by Jenkins and Lambert (1998a), unambiguous results are still possible when TIP curves cross once, little has been known about the exact ordering conditions when TIP curves present multiple crossings.

In this paper, we suggest a poverty comparison principle based on comparing TIP areas, which can be used when curves intersect more than once. The normative significance of using this comparison principle is analyzed in terms of a class of poverty indices, C, that are linear in incomes and given by the following functional forms:

$$I_X(\Phi, z) = \int_0^1 \left(\frac{z - F_z^{-1}(t)}{z} \right) d\Phi(t), \tag{2}$$

where the relative poverty gaps are weighted using a continuous probability distribution, Φ, with support $supp(\Phi) \subseteq [0, 1]$ (the integral is interpreted in the Riemann–Stieltjes sense). The class, C, considered recently by Davidson and Duclos (2000), Duclos and Grégoire (2002) and Duclos and Araar (2006), is analogous to the class of linear inequality measures proposed by Mehran (1976) and contains some poverty measures that are well known from the literature. It includes the so-called "per-capita income gap" proposed by Foster et al. (1984) and is obtained when $\Phi(t)$ is the uniform distribution on $(0, 1)$. The Thon (1979) and Shorrocks (1995) poverty indices are members of C, given that $\Phi(t) = 1 - (1 - t)^2$, and the general class of poverty indices proposed by Thon (1983) is also obtained from (2) by choosing

$$\Phi(t) = \frac{c^2}{4(c-1)} - \frac{1}{c-1}\left(\frac{c}{2} - t\right)^2, \ c > 2.$$

Hagenaars (1987) and Shorrocks (1998) also evaluate indices of the form (2) at the income distribution of a finite population.

   The desirability of a poverty measure is evaluated by the axioms it satisfies. In this sense, it can be easily proven that each member $I_X(\Phi, z) \in C$ satisfies the following reasonable axioms: monotonicity ($I_X(\Phi, z)$ increases if a poor person's income decreases), scale invariance ($I_X(\Phi, z)$ is not affected if we multiply income and poverty line by a common factor $a > 0$), focus ($I_X(\Phi, z)$ is not affected by changes in nonpoor incomes) and symmetry ($I_X(\Phi, z)$ is not affected if two people switch their incomes). All these axioms are well-known in the literature and have been discussed thoroughly (see, for example, Foster (1984) and Zheng (2000)). In addition, as proved by Mehran (1976) in the context of income inequality, members of the class C_1 given by

$$C_1 = \{I(\Phi, z) \in C \text{ such that } \Phi \text{ is concave}\}$$

satisfy the Pigou-Dalton Principle of Transfers (any mean-preserving transfer from a poor person to a poorer person that leaves unchanged their relative rank in the distribution, must decrease poverty) and members of C_2, given by

$$C_2 = \{I(\Phi, z) \in C_1 \text{ such that } \phi \text{ is convex, where } \Phi'(t) = \phi(t) \text{ a.e.}\}.$$

satisfy the stronger Diminishing Transfer Principle, which requires that a small transfer from a poor person to a poorer person, with a given proportion of the population in between them, decreases poverty and the decrease is larger the poorer the recipient. In other words, the relative ethical weight assigned to the effect of income changes occurring at the bottom of the distribution is higher in C_2 than in C_1.

   We prove in this paper that non-intersecting TIP curves principle is equivalent to unambiguous poverty ranking by all measures in C_1 (this result is well-known and it appears, for instance, in Duclos and Araar (2006)). In order to obtain unambiguous

poverty ranking by all measures in C_2, we use a weaker comparison principle based on comparing TIP areas. This weaker principle is, therefore, more sensitive to the distribution of income among the poorest.

The plan of the paper is as follows. In Section 2, we show that two income random variables can be unanimously ranked by all poverty indices in C with Φ concave if and only if their TIP curves do not intersect. In this section, we also provide a condition for stochastic equality of the censored random variables, X_z^* and Y_z^*, under the hypothesis of non-intersecting TIP curves. More precisely, we prove that, if the TIP curves do not intersect and if $I_X(\Phi, z) = I_Y(\Phi, z)$ for some strictly concave distribution function Φ, then X_z^* and Y_z^* are stochastically equal. Section 2 includes examples. In Section 3, we show that, when the TIP curves intersect, unambiguous rankings are still possible. In this case, the ordering condition is based on comparisons of the respective TIP areas. Section 4 contains concluding remarks.

2 Characterization in terms of TIP curves

Denote

$$C_1 = \{I(\Phi, z) \in C \text{ such that } \Phi \text{ is concave}\}.$$

The following result connects the unambiguous poverty ordering of two income random variables based on the class C_1 of poverty indices with the non-intersection of the corresponding TIP curves.

Theorem 1 *Let X and Y be two non-negative income random variables and let z > 0 be a fixed poverty line. Then,*

$$G_X(p, z) \leq G_Y(p, z) \text{ for all } p \in [0, 1] \tag{3}$$

if and only if

$$I_X(\Phi, z) \leq I_Y(\Phi, z), \text{ for all } I(\Phi, z) \in C_1. \tag{4}$$

Proof ($\Longrightarrow$) Let F and G be the distribution functions of X and Y, respectively, and let F^{-1} and G^{-1} be the corresponding quantile functions. Note that (2) can be written as

$$I_X(\Phi, z) = \int_0^{r_z^X} \left(\frac{z - F^{-1}(t)}{z} \right) d\Phi(t). \tag{5}$$

Integration by parts in (5) for Riemann–Stieltjes integrals, given that $\Phi(0) = 0$, yields

$$I_X(\Phi, z) = \frac{1}{z} \int_0^{r_z^X} \Phi(t)\, dF^{-1}(t) \tag{6}$$

and, analogously,

$$I_Y(\Phi, z) = \frac{1}{z} \int_0^{r_z^Y} \Phi(t)\, dG^{-1}(t). \tag{7}$$

On the other hand, condition (3) is equivalent to the condition

$$\int_0^p \frac{F_z^{-1}(t)}{z}\, dt \geq \int_0^p \frac{G_z^{-1}(t)}{z}\, dt, \text{ for all } p \in [0, 1]. \tag{8}$$

Since the functions, $F_z^{-1}(t)/z$ and $G_z^{-1}(t)/z$, can be considered analogous to two distribution functions defined on $\left[0, r_z^X\right]$ and $\left[0, r_z^Y\right]$, respectively, it follows from Theorem 1.4.1 of Stoyan (1983) that (8) holds if and only if

$$\int_0^{r_z^X} \Phi(t)\, d\left[\frac{F_z^{-1}(t)}{z}\right] \leq \int_0^{r_z^Y} \Phi(t)\, d\left[\frac{G_z^{-1}(t)}{z}\right]$$

for all non-decreasing and concave functions, Φ, or, equivalently, if and only if

$$\frac{1}{z} \int_0^{r_z^X} \Phi(t)\, dF^{-1}(t) \leq \frac{1}{z} \int_0^{r_z^Y} \Phi(t)\, dG^{-1}(t) \tag{9}$$

for all non-decreasing and concave functions, Φ. Combining (9), (6) and (7) implies (4). ($\Longleftarrow$) For $p = 0$, (3) is obvious because $G_X(0, z) = G_Y(0, z) = 0$. Now, for each $p \in (0, 1]$, the distribution function defined by

$$\Phi_p(t) = \begin{cases} t/p & \text{if} \quad 0 \leq t < p \\ 1 & \text{if} \quad t \geq p \end{cases}$$

is concave. Hence,

$$I_X(\Phi_p, z) \leq I_Y(\Phi_p, z), \text{ for all } p \in (0, 1]. \tag{10}$$

Since $I_X(\Phi_p, z) = (pz)^{-1} G_X(p, z)$ and $I_Y(\Phi_p, z) = (pz)^{-1} G_Y(p, z)$, (3) follows directly from (10). $\qquad\square$

Example 2 The Pareto income distribution has become one of the most popular and widely used models for empirical income data. Thus, it would be interesting to compare the TIP curves of such distributions in terms of their parameters. In this way, we can

obtain their unanimous poverty rankings based on the class C_1. The corresponding distribution function of a Pareto random variable with parameters ε and α is

$$F(x) = 1 - \left(\frac{\varepsilon}{x}\right)^\alpha, \quad x \geq \varepsilon, \quad \alpha > 0, \quad \varepsilon > 0.$$

From straightforward computation, the corresponding TIP curve is

$$G(p, z) = \begin{cases} zp + \dfrac{\varepsilon\alpha}{(\alpha - 1)}\left((1 - p)^{\frac{\alpha-1}{\alpha}} - 1\right), & \text{if } 0 \leq p < 1 - \left(\dfrac{\varepsilon}{z}\right)^\alpha \\[4mm] z + \dfrac{z}{\alpha - 1}\left(\dfrac{\varepsilon}{z}\right)^\alpha - \dfrac{\varepsilon\alpha}{(\alpha - 1)}, & \text{if } 1 - \left(\dfrac{\varepsilon}{z}\right)^\alpha \leq p \leq 1. \end{cases} \tag{11}$$

Let $X_i(i = 1, 2)$ be two Pareto variables with parameters $(\varepsilon_i, \alpha_i)$. Let $z > 0$ be a fixed poverty line. If $\alpha_1 = \alpha_2 > 1$ and $\varepsilon_1 \leq \varepsilon_2$, it can be easily verified that $G_{X_2}(t; z) \leq G_{X_1}(t; z)$ for $0 \leq t \leq 1$. Using Theorem 1, we have $I_{X_2}(\Phi, z) \leq I_{X_1}(\Phi, z)$, for all $I(\Phi, z) \in C_1$. Note that the distribution with a lower minimum income implies higher poverty according to all members of C_1. If we fixed $\varepsilon_1 = \varepsilon_2 > 0$ and $\alpha_1 \geq \alpha_2 > 1$, we reach the same conclusion. Since $E[X_i] = \alpha_i\varepsilon_i/(\alpha_i - 1)$, the distribution with the lower average value implies higher poverty according to all members of C_1.

If X and Y are income random variables such that $G_X(p, z) \leq G_Y(p, z)$ for all $p \in [0, 1]$, it is interesting and natural to ask what simple sufficient condition would imply the stochastic equivalence of their corresponding censored random variables, X_z^* and Y_z^*. In the existing literature, conditions implying equality in the distributions of random variables under various stochastic orderings can be found in Baccelli and Makowski (1989), Bhattacharjee and Sethuraman (1990), Scarsini and Shaked (1990), Bhattacharjee (1991), Jun (1994), Li and Zhu (1994), Cai and Wu (1997), Denuit et al. (2000) and Bhattacharjee and Bhattacharya (2000). In this tradition, we obtain the following theorem.

Theorem 3 *Let X and Y be two non-negative income random variables and let $z > 0$ be a fixed poverty line. If*

$$G_X(p, z) \leq G_Y(p, z) \text{ for all } p \in [0, 1] \tag{12}$$

and if

$$I_X(\Phi, z) = I_Y(\Phi, z) \tag{13}$$

for some strictly concave distribution function, Φ, then the censored random variables, X_z^* and Y_z^*, have the same distribution.

Proof. Suppose that Φ is a strictly concave distribution function. Then, there exists a strictly decreasing, non-negative and integrable function, φ, such that

$$\Phi(t) = \int_0^t \varphi(u)\,du, \ t \in [0, 1)$$

(see Zygmund, 1959). Using the properties of the Riemann–Stieltjes integral, we have

$$z \cdot I_X(\Phi, z) = \int_0^1 \left(z - F_z^{-1}(t)\right) d\Phi(t) = \int_0^1 \varphi(t)\,dG_X(t, z). \tag{14}$$

Partial integration of (14) and $G_X(0, z) = 0$ reveals that

$$z \cdot I_X(\Phi, z) = \varphi(1)G_X(1, z) - \int_0^1 G_X(t, z)\,d\varphi(t). \tag{15}$$

Analogously, it can be shown that

$$z \cdot I_Y(\Phi, z) = \varphi(1)G_Y(1, z) - \int_0^1 G_Y(t, z)\,d\varphi(t). \tag{16}$$

Combining (13), (15) and (16) yields

$$\varphi(1)\left[G_Y(1, z) - G_X(1, z)\right] - \int_0^1 \left[G_Y(t, z) - G_X(t, z)\right] d\varphi(t) = 0. \tag{17}$$

Since $G_X(p, z) \le G_Y(p, z)$ for all $p \in [0, 1]$, $\varphi(1) \ge 0$ and

$$d\varphi(t) < 0, \tag{18}$$

from (17), it follows that

$$\int_0^1 \left[G_Y(t, z) - G_X(t, z)\right] d\varphi(t) = 0. \tag{19}$$

Given (12), the continuity of the TIP curves and (18), from (19), we obtain

$$G_Y(t, z) = G_X(t, z) \text{ for all } t \in [0, 1]. \tag{20}$$

Taking the derivative completes the proof. □

Example 4 Let X and Y be two non-negative income random variables and let $z > 0$ be a fixed poverty line. If $G_X(p, z) \le G_Y(p, z)$ for all $p \in [0, 1]$ and if $T(X, z) = T(Y, z)$ (where $T(\cdot, z)$ denotes Thon's poverty index), then X_z^* and Y_z^* have the same distribution. In particular, $I_X(\Phi, z) = I_Y(\Phi, z)$ for all $I(\Phi, z) \in C$.

3 Characterization in terms of TIP areas

We have shown in Theorem 1 that non-intersecting TIP curves are equivalent to a unanimous poverty ordering by all indices in C_1. However, in practical applications, TIP curves often intersect. In this case, we can still order the distributions by a subclass of C_1. Taking into account that, for a concave Φ, its derivative, Φ', exists (except possibly at a countable number of points), we consider the class, C_2, in which functionals are of the form of (2), where Φ is concave and differentiable almost everywhere (a.e.) with a convex derivative; that is,

$$C_2 = \{I(\Phi, z) \in C_1 \text{ such that } \phi \text{ is convex, where } \Phi'(t) = \phi(t) \text{ a.e.}\}.$$

Theorem 5 *Let X and Y be two non-negative income random variables and let $z > 0$ be a fixed poverty line. Then,*

$$\int_0^p G_X(t, z)\, dt \le \int_0^p G_Y(t, z)\, dt, \text{ for all } p \in [0, 1] \text{ and } G_X(1, z) \le G_Y(1, z) \qquad (21)$$

if and only if

$$I_X(\Phi, z) \le I_Y(\Phi, z), \quad \text{for all } \ I(\Phi, z) \in C_2. \qquad (22)$$

Proof. ($\Rightarrow$) Let $I(\Phi, z) \in C_2$. Suppose that $\Phi' = \phi$ a.e. for some non-increasing convex and non-negative function, ϕ. Then, there exists a non-increasing non-negative function, φ, such that

$$\phi(t) - \phi(1) = \int_t^1 \varphi(p)\, dp, \ p \in (0, 1].$$

Using integration by parts, it follows, for $p \in (0, 1]$ that

$$\phi(t) = \phi(1) + (1 - t)\varphi(1) - \int_0^1 (p - t)^+ d\varphi(p). \qquad (23)$$

Because of the properties of the Riemann–Stieltjes integral, we can write

$$z \cdot I_X(\Phi, z) = \int_0^1 \left(z - F^{-1}(t)\right) d\Phi(t) = \int_0^1 \phi(t)\, dG_X(t, z),$$

and using (23), we obtain

$$z \cdot I_X(\Phi, z) = \int_0^1 \left[\phi(1) + (1 - t)\varphi(1) - \int_0^1 (p - t)^+ d\varphi(p)\right] dG_X(t, z). \qquad (24)$$

The additivity properties of integrals, Fubini's Theorem and $G_X(0, z) = 0$ imply that (24) can be rewritten as follows:

$$\phi(1)G_X(1, z) + \varphi(1) \int_0^1 (1 - t)\, dG_X(t, z) - \int_0^1 \int_0^p (p - t)\, dG_X(t, z)\, d\varphi(p). \qquad (25)$$

Using integration by parts, for each p, we obtain

$$\int_0^p (p - t)\, dG_X(t, z) = \int_0^p G_X(t, z)\, dt,$$

which, when combined with (25), implies

$$z \cdot I_X(\Phi, z) = \phi(1)G_X(1, z) + \varphi(1) \int_0^1 G_X(t, z)\, dt - \int_0^1 \int_0^p G_X(t, z)\, dt\, d\varphi(p).$$

We complete the proof by noting that $\phi(1) \geq 0$, $\varphi(1) \geq 0$, $d\varphi(p) \leq 0$ and (21).

($\Leftarrow$) The condition given in (22) implies $G_X(1, z) \leq G_Y(1, z)$ when $\Phi(t)$ is a uniform distribution on $(0, 1)$.

For $p = 0$, it is obvious that the first condition in (21) is satisfied. Now, for each $p \in (0, 1]$, the following distribution function in $[0, 1]$ is concave and has a convex derivative:

$$\Phi_p(t) = \begin{cases} (2pt - t^2)/p^2 & \text{if} \quad 0 \leq t < p \\ 1 & \text{if} \quad t \geq p \end{cases}.$$

Hence, it follows that

$$I_X(\Phi_p, z) \leq I_Y(\Phi_p, z), \ \text{ for all } p \in (0, 1].$$

Integration by parts leads to

$$I_X(\Phi_p, z) = \frac{2}{zp^2} \int_0^p G_X(t; z)\, dt,$$

and the proof is complete. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

4 Concluding remarks

In this paper, we have characterized the comparisons of TIP curves in terms of a class C_1 of linear poverty measures. When TIP curves intersect (possibly more than once) we have shown that unambiguous poverty orderings are still possible, by focusing on

a more restricted class $C_2 \subset C_1$. The criterion, then, is to compare the underlying TIP areas. Because of the axioms that the members of this class must fulfill, it must be the case that, in order to use this criteria, decision-makers should be ready to assume a larger sensitivity of poverty measures to the distribution of income among the poorest.

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