OPTIMIZING THE DESIGN AND OPERATION OF MEASUREMENTS FOR CONTROL: DYNAMIC OPTIMIZATION APPROACH

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Abstract: We shall present approaches to optimize the design of a measurement system and to schedule dynamically a versatile measurement resource. The analysis is based on expressing the system management task as a dynamic programming problem in which the system state is partially observable. We shall review the well-known linear-quadratic-Gaussian case, and discuss and give example solutions of discrete state systems with approximate dynamic programming methods to solve such problems in practice. Design problem is then studied by assessing the long term performance of control and how it depends on properties of measurements. A simple design example is presented for a discrete state system.

Keywords: Optimal measurement, Dynamic programming, Decision support.

1. INTRODUCTION

Measurement selection is an important task of system design. As the decisions to operate the system are made based on information available about the system, it is not trivial to integrate the design of the measurements to a overall system design, in particular when measurement subsystem constitutes a considerable amount of the overall system costs. One cost-efficient option of measurement subsystem design in many applications is a multipurpose analyzer. However, choosing such a device leads to a complex dynamic optimization problem of scheduling which of the possible measurements is made at each time instant. The scheduling problem was first phrased and solved in a linear-quadratic system already by Meier, Peschon and Dressler [1] in 1967. However, in more complicated cases the problem is computationally extremely heavy, and has only recently attracted interest in robotics [see e.g. 2], and also in quality control at industrial processes [3, 4]. Analyzing the value of information through dynamically optimized system performance, measurements can be designed and scheduled so that the limited measurement resources provide the most valuable information.

Current industrial problems in which a systematic approach on measurement scheduling and design provides a great potential of improvement include designing laboratory activities supporting on-line measurements and controlling scanning measurements. E.g. in paper industry the quality of the web produced is measured with a scanning device. At present the scanner is operated regularly across the web, but it can be shown that there exist circumstances at which other scan paths will provide a better uniformity of quality [5, 6].

This paper is organized as follows. Section 2 defines the measurement scheduling problem as a dynamic optimization problem. Section 3 discusses both the exact and approximate solution of this problem for discrete state systems. Furthermore Section 3 shows how linear-quadratic-Gaussian case separates into independent measurement scheduling and control problems in this formalism, which was shown already in [1] in somewhat different formulation. Section 4 outlines, how the measurement design problem is solved with the scheduling problem as a subtask. Section 5 shows three examples on operation of measurement system: the exact solution of a binary state system, and approximate solutions of a three state problem and a five state problem, the latter inspired by a paper machine quality management problem based on laboratory measurements. Section 6 presents four case studies of measurement system design problem for a three-state system. Section 7 summarizes the main findings and outlines future research.

2. DYNAMIC PROGRAMMING WITH UNCERTAIN MEASUREMENTS – THE SCHEDULING PROBLEM

The system management task is formulated in terms of system states x_t, the control actions u_t and the measurement choice m_t. The measurement choice is to be understood as any combination of simultaneous measurements that the present measurement system M allows. Given the operational cost, at any time as $h(x_{t+1},u_t,m_{t+1})$ the action optimization with time horizon T and discounting factor $0<\alpha<1$ is:

$$u^*\left(p^{(+)}(x_0)\right) = \underset{\{u_t\}_{t=0}^{T-1},\{m_t\}_{t=1}^{T-1}}{\arg\min} E\left\{\sum_{t=0}^{T}\alpha^t h\left(x_{t+1}, u_t, m_{t+1}\right)\right\} \quad (1),$$

where $p^{(+)}(x_0)$ is the current state information expressed as state probabilities at time t=0, and after the measurement at t=0 has been made (posteriority denoted by '+'). The expectation value $E\{\}$ is to be calculated with respect to the current state information. Hence u^* is a real-valued functional of the t=0 probability distribution if state is continuous, and a function from a d-1 dimensional space to real numbers if state consist of d discrete states. In this paper we concentrate on discrete state systems. Hence $p^{(+)}(x_0)$ is a d-dimensional vector with additional constraints that its values are semipositive and add up to one.

We assume the system to be a Markov chain with action dependent transition probabilities, i.e. a Markov decision process. Thus the system dynamics is described with state transition matrices $p_{ij}(u)$ that give the probability of state j at next time step provided that present action u has been taken and the present state is i. The measurements are described with conditional probabilities $q^{(m)}(z_t^{(m)}|x_t)$: the probability that with measurement m a measurement result $z_t^{(m)}$ is obtained if the true state is x_t.

Let us assume firstly that the state information after measurement at time t is $p^{(+)}(x_t)$; secondly, that an action u_t is taken; and thirdly, a measurement m_{t+1} made with a result of z_{t+1}. Then by probability propagation of the Markov chain and the Bayesian interpretation of measurement gives:

$$\begin{aligned} p^{(+)}(x_{t+1} = j \mid u, m, z_{t+1}^{(m)}, p^{(+)}(x_t)) &= \sum_i \frac{q^{(m)}(z_{t+1}^{(m)} \mid x_{t+1} = j)\, p_{ij}(u)\, p^{(+)}(x_t = i)}{P(z_{t+1}^{(m)})} \\ P(z_{t+1}^{(m)}) &= \sum_{i,j} q^{(m)}(z_{t+1}^{(m)} \mid x_{t+1} = j)\, p_{ij}(u)\, p^{(+)}(x_t = i) \end{aligned} \quad (2)$$

$p(z_{t+1}^{(m)})$ is the probability of getting the measurement result $z_{t+1}^{(m)}$ when previous state information is $p^{(+)}(x_t)$. Then the optimization problem of Eq.(1) can be rewritten as an iteration (a dynamic programming problem) with V_T being the optimal T-horizon cost:

$$V_T(p^{(+)}(x_t)) = \min_u \min_m \left[E_{x_t}\left\{h(x_{t+1}, u, m)\right\} + \alpha \mathrm{E}_{z_{t+1}^{(m)}}\left\{V_{T-1}(p^{(+)}(x_{t+1} \mid u, m, z_{t+1}^{(m)}, p^{(+)}(x_t)))\right\}\right] \quad (3).$$

If the measurement system is not of multiuse, the minimization with respect to m_{t+1} is trivial and the problem reduces to the standard form of partially observed Markov decision process (POMDP), see e.g. [2].

In the infinite horizon case the iteration of Eq. (3) turns into a functional equation:

$$V(p^{(+)}(x_t)) = \min_u \min_m \left[E_{x_t}\left\{h(x_{t+1},u,m)\right\} + \alpha E_{z_{t+1}^{(m)}}\left\{V(p^{(+)}(x_{t+1} \mid u,m,z_{t+1}^{(m)}, p^{(+)}(x_t)))\right\}\right] \quad (4).$$

The T-horizon measurement choice problem at time $t=0$ given state information prior $p^{(-)}(x_0)$ to measurement is then formulated as:

$$\begin{aligned}
&m_T{}^*(p^{(-)}(x_0),u_{-1}) = \arg\min_{m_0}\left[E_{x_0^{(-)}}\left\{h(x_0,u_{-1},m_0)\right\} + E_{z_0^{(m)}}\left\{V_T(p^{(+)}(x_0 \mid m, z_0^{(m)}, p^{(-)}(x_0))\right\}\right] \\
&p^{(+)}(x_0 = j \mid m, z_0^{(m)}, p^{(-)}(x_0)) = \frac{q^{(m)}(z_{0|}^{(m)} \mid x_0 = i)\,p^{(-)}(x_0 = j)}{\sum_i q^{(m)}(z_{0|}^{(m)} \mid x_0 = i)\,p^{(-)}(x_0 = j)} \\
&P(z_0^{(m)}) = \sum_i q^{(m)}(z_{0|}^{(m)} \mid x_0 = i)\,p^{(-)}(x_0 = j)
\end{aligned} \quad (5).$$

If the measurement cost is additive to the cost of state and action, the optimal measurement choice will not depend on the previous action u_{-1} other than through $p^{(-)}(x_0)$.

In case of infinite horizon, V_T is replaced by V. This completes the definition of the measurement scheduling problem.

3. METHODS TO SOLVE THE MEASUREMENT SCHEDULING PROBLEM

3.1 Exact solution of discrete state case

Sondik proved in 1971 [7, 8] that the solution to the problem in the Eq.(3) can be reformulated as

$$V_t\left(\overline{p}\right) = \min_{\alpha\in\Gamma_t} \alpha^T p \quad (6),$$

where $\left\lfloor \overline{p} \right\rfloor_i = p\left(x_t = i\right)$, with $\sum_i \left\lfloor \overline{p} \right\rfloor_i = 1$ and α is a $|x|$-dimensional vector and Γ_t is a set of α-vectors that can be constructed iteratively. Therefore the exact solution of Eq. (3) is always piecewise linear and concave in probability $\overline{p}$. Thus each of the α-vectors defines the optimal actions, in our case the control action and the choice of measurement, for a certain region of probability $\overline{p}$. The solution to the problem for a given time horizon t can then be presented as the collection of α-vectors, Γ_t.

The recursion of α-model collection Γ_t from the collection Γ_{t-1} is presented e.g. in [2]. The method is rather straightforward. The difficulty lies in that as the time horizon increases, the number of α-vectors increases rapidly, more rapidly than exponentially. The size of the problem can be decreased by pruning α-vectors which do not contribute to optimality at any probability $\overline{p}$. The method to find these pruned vectors is, however, computationally time-consuming.

This exact solution can be directly used only for small problems with few states and/or short optimization horizon.

3.2 Point-based approximate solution for discrete case

The exact method finds the optimal solution for all state information probabilities. The difficulty is the increasing number of α-vectors as optimization time horizon increases. The idea of the point-based solution is to solve the problem in a fixed set of probability points. This is based on fact that at any probability point only one α-vector is active in the minimization of Eq. (6). As a result only the number of α-vectors that need to be considered over any time horizon is the number of points at which the solution is constructed. Obviously, the selection of probability points is critical for the point-based method, but if the points are selected properly, the solution is close to exact optimal solution even for high number of discrete states and for a long time horizon.

Several methods to select points have been presented in [2]. In our case studies points are selected randomly or set in a regular grid, which both work well in our case of low number of discrete states.

When the set of probability points is selected, the optimal α-vectors are calculated for each probability point. The set of α-vectors can be pruned by taking away the identical models. Thus, the number of the α-vectors is at most the number of probability points. As a result the recursion becomes simpler and faster. The method is presented more in detail in [2].

3.3 Linear-quadratic-Gaussian case

The transition dynamics and measurement description of linear Gaussian case with continuous valued state vector can be written as:

$$\begin{aligned} f^{(dyn)}(x_{t+1} \mid x_t; u) &= N_d\left(A_t x_t + B_t u_t; \Sigma_t^{(dyn)}\right) \\ f^{(meas,m)}(z_t^{(m)} \mid x_t) &= N_{d'}\left(C_t x_t; \Sigma_t^{(meas,m)}\right) \end{aligned} \quad (7),$$

where $N_d(\mu, \Sigma)$ is a d-dimensional Gaussian distribution with mean μ and covariance matrix Σ.

Let us further assume that state, action and measurement costs are additive and that state and action costs are quadratic:

$$h(x_{t+1}, u_t, m_{t+1}) = (x_{t+1} - \hat{x}_{t+1})^T H_X (x_{t+1} - \hat{x}_{t+1}) + (u_t - \hat{u}_t)^T H_U (u_t - \hat{u}_t) + H_M(m_{t+1}) \quad (8).$$

If the initial state information is Gaussian, $p^{(+)}(x_t) = N_d(\mu, \Sigma)$, then by induction with Eq. (3) one can show that

$$V_{T-1}(\mu,\Sigma) = G_{T-1}(\Sigma) + (\mu - \hat{\mu}_{\Gamma-1})^T H_{T-1}^{(V)}(\mu - \hat{\mu}_{\Gamma-1}) \quad (9).$$

Furthermore, when applying this in Eq.(3), one finds that the cost inside the minimization in Eq. (3) is a sum of two terms: one depending on present state covariance matrix and measurement choice, and another one depending on present state expectation value and control action. Thus the optimal choice of measurement depends only on which measurements have been made in past but not on which measurement values were obtained. As a result the choice of measurement schedule for the linear-quadratic-Gaussian case is a policy: as the system runs and measurement values are obtained, no additional information assisting in the choice of future measurements is obtained. Correspondingly, the optimal action depends only on the present state estimate, not through which measurement choices the estimate has been obtained.

The separability of measurement scheduling and control was already proven in [1]. However, we included this short discussion from within the formalism of Section 2, because of the practical importance for the measurement community.

4. MEASUREMENT SYSTEM DESIGN PROBLEM

The optimal performance achievable depends on the measurement system M. The selection of the measurement system defines the accuracy of the information achieved through measuring for operative decision making.

The task in measurement system design is to decide which of the possible measurement system options to choose to maximize the performance of the process. The key points when choosing the measurement system are the uncertainty of the measurements and the number of the possible observations outcomes as well as the price of the system and measuring. Important question are: what is the utility of a better measurement system and how much we are willing to pay for more accurate measurement information.

The basic idea in measurement system design is to define the possible measurement system options and to solve the optimization problem of operational level Eq. (1) for all of the options. By simulating the performance of the system over a certain time horizon and calculating the expected costs during simulation, the optimal measurement design can be defined as minimization of operative and investment costs.

If we assume that the cost $h(x_{t+1},u_t,m_{t+1})$ is scaled to real monetary costs, the cost per time unit, when system is operated optimally with a measurement system M, is given as

$$W(M) \approx \frac{1}{T}\sum_{t=0}^{T-1} h\left(x_{t+1}, u_t{*}, m_{t+1}{*}\right) \mapsto \frac{1-\alpha}{1-\alpha^{T+1}} \int_{domain(p^{(+)}(x))} f\left(p^{(+)}(x)\right) V_T\left(p^{(+)}(x)\right) d\left(p^{(+)}(x)\right) \tag{10.}$$

Here the asterisk denotes that at each time instant the control and measurement action optimal in time horizon T is applied. Function $f(p^{(+)}(x_0))$ allocates a probability density to facing a control problem with initial information $p^{(+)}(x_0)$; i.e. it is a probability density on state probabilities. The prefactor in Eq. (10) normalizes the sum of discount weighting factors in Eq. (1) to one so that comparing formulations with different time horizon can be directly compared.

The probability $f(p^{(+)}(x_0))$ may be thought of arising from a component of regular behavior of the system and/or of abnormal behavior. Regular behavior means that the system behaves according to the system model $p_{ij}(u)$. This component of $f(p^{(+)}(x_0))$ can simply be obtained simulating the system with optimal control actions and measurement choices according to solution of Eq. (1), and then estimating the probability density with observations of $p^{(+)}(x_0)$ during the simulation. Abnormal component arises due to that the system is subject to unforeseeable disturbances: the system model $p_{ij}(u)$ is not valid at such instants. It is the designers' task to specify the abnormal scenarios and their probability of occurrence. The corresponding component in $W(M)$, Eq.(10), describes the system performance when recovering from abnormality.

In the design the operational performance is weighed against investment cost $C(M)$. If the measurement investment is to be uniformly depreciated in time T_d (in units of time steps in operation), the design problem is

$$\min_M \left[T_d \cdot W(M) + C(M)\right] \tag{11.}$$

Similarly, a discounted depreciation problem may be formulated as a Net Present Value problem, see e.g. [9].

5. CASE STUDIES ON OPERATION OF MEASUREMENT SYSTEM

This section presents three simple case studies of joint dynamic optimization of control and measurement actions. First a two-state system is briefly analyzed through the exact solution, and then the exact and point-based approximate solutions of a three state system are compared. Finally a short simulation study of a five-state system corresponding to a simple quality management case is discussed.

5.1 Two-state system

The simplest case to illustrate the method is a two-state system with two action alternatives and two measurement alternatives. Two-state control problem was addressed already in [10].

In our example the states are “good” and “poor”. Being in the “poor” state incurs an additional cost of 0.9 units. The actions are “run as usual”, and “make a correction”. Under the first action the transition probability from “good” to “poor” is 0.3 and that of the opposite transition 0.2. Under the corrective action the “good” state remains with probability 0.9 and the “poor” state turns into “good” with probability 0.8. The additional cost of corrective action is 0.5 units. Furthermore, the state can be measured at an additional cost of 0.04 units, and the probability of measurement giving the erroneous state is 0.05. The discount factor is 0.95.

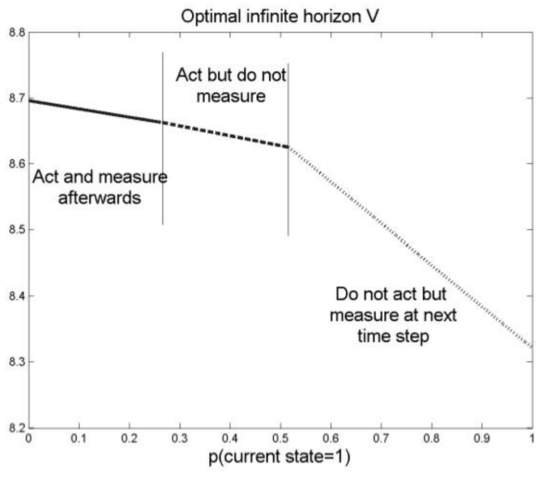

Figure 1. Value function of a two-state system.

Figure 1 presents the value function V and decisions of the infinite horizon case, Eq. (4) as the function of the probability of present state being “good”. The infinite horizon solution was obtained by iteration of finite horizon case, Eq. (3) with the exact piecewise linear methods of section 3.1, Eq. (6). If the state is “poor” with high probability, corrective action is made and verified with followup measurement. If there is high uncertainty about the state, corrective action is made, but as a result the probability of “good” state is so high that no followup measurement is needed during the next step but only on further steps. When there is high certainty about state being “good”, the state is monitored with measurement, but no corrective actions are needed. However, simpler policies throughout low and medium probability of state being “good” of either both making the corrective action and measurement or only making corrective action are only slightly suboptimal.

In finite horizon case, the horizon affects strongly the policy. With horizons 1 and 2, no measurements are ever made. With horizon 3, measurements are always made. From horizon 4 on, the decision policy is as in infinite horizon case with decision limits converged down to 0.001 accuracy in *p(state=1)* at horizon 7. The average level of *V*, not relevant to decisions, converges only after horizon 40, due to discount factor being close to 1.

The exact solution in this case is simple as the number of piecewise linear components considered in *V* remains small, at most 24 if models are pruned at each time horizon. In hindsight, the identical result would have been obtained with the approximate method of Section 3.2 minimally with three points (e.g. 0, 0.4 and 1). If points are chosen at random, or uniformly, the probability of obtaining the exact solution for the binary system with the approximate method is high with only 10...20 points.

5.2 Three-state system

Our second example is a system with three states: “good”, “acceptable” and “poor” with costs 0, 0.5 and 1 unit, respectively. The system is controlled by three actions. The first action, the cheapest one, has a transition probability from “good” to “acceptable” and “poor”, and from “acceptable” to “poor”, but “poor” will always stay “poor”. The second control option turns the system into a better state and the third one turns the system surely into “good”. The costs for these control actions are 0, 1 and 2 units. The dynamics of the system is defined as:

$$p(x_{t+1} \mid x_t, u_t = 1) = \begin{bmatrix} 0.81 & 0.18 & 0.01 \\ 0 & 0.9 & 0.1 \\ 0 & 0 & 1 \end{bmatrix}$$
$$p(x_{t+1} \mid x_t, u_t = 2) = \begin{bmatrix} 1 & 0 & 0 \\ 0.7 & 0.2 & 0.1 \\ 0.2 & 0.7 & 0.1 \end{bmatrix} \qquad (12).$$
$$p(x_{t+1} \mid x_t, u_t = 3) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

To support the decision making, two-valued measurement with cost h_m = 0.2 units can be made. The task in measurement scheduling is to choose whether to measure or not. The measurement description is defined as:

$$q^{(m=1)}(z_t \mid x_t) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$
$$q^{(m=2)}(z_t \mid x_t) = \begin{bmatrix} 1 & 0.5 & 0.25 \\ 0 & 0.5 & 0.75 \end{bmatrix} \qquad (13),$$

where the first matrix refers to not make a measurement (m=1) and the second to make a measurement (m=2). The discount factor α in Eq.(1) is set to 1.

Similar, but not the same, problem with three states was presented by Smallwood and Sondik in 1973 [8].

The problem is solved using point-based solution, in which the points are selected to form a regular grid. The results for optimization horizon T = 2, T = 3 and T = 5 are presented in Figures 2, 3 and 4. Grid points are shown with dots and decision borders by lines. In the figure legends the first value refers to optimal control action and the second value to optimal measurement choice, 1 referring to no measurement and 2 to make a measurement.

At the end of the horizontal axis the probability vector is [1 0 0], the system is certainly at “good” state. At the end of the vertical axis probability vector is [0 1 0], the system is certainly at “acceptable” state, and in the origin [0 0 1], the system is certainly at “poor” state.

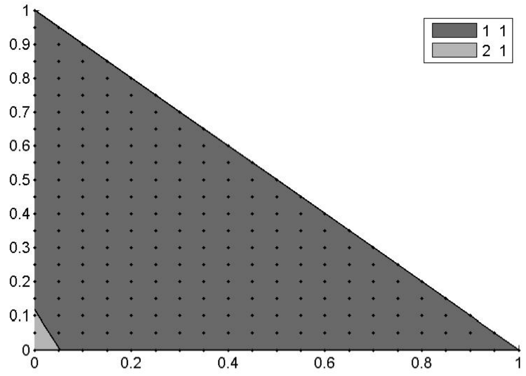

Figure 2. Decision planes when optimization horizon *T* = 2. Grid points are shown with dots.

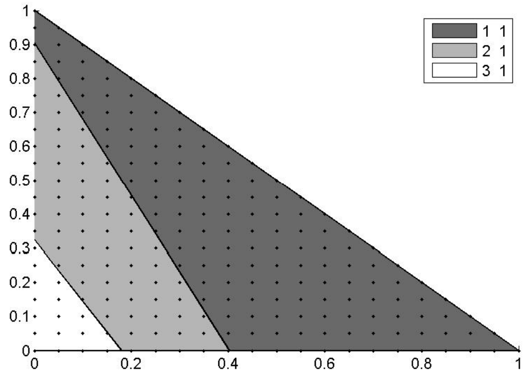

Figure 3. Decision planes when optimization horizon *T* = 3.

For optimization horizon T = 2 only two decisions exists: to control with action 1 or 2 and never to measure. For optimization horizon T = 3, the number of decisions is increased to three, but measuring is still never an optimal option. That is logical as measuring costs, and for short decision horizons the benefit from measurement information cannot be realized within the time horizon considered.

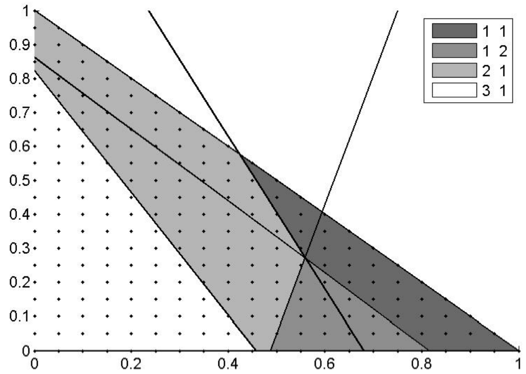

Figure 4. Decision planes when optimization horizon *T* = 5.

For optimization horizon T = 5 the number of optimal decision alternatives is increased to four, with "control 1 and measure" as a one option as shown in Figure 4. Precisely the same result is obtained with the exact solution method.

As the time horizon grows the area of optimal control action being 3 increases and the area of optimal control action being 1 decreases. That is logical as in the long term it becomes more important to ensure the better result and with control action 3 the system is surely turned into the "good" state. Also the importance of measuring grows with the longer time horizon.

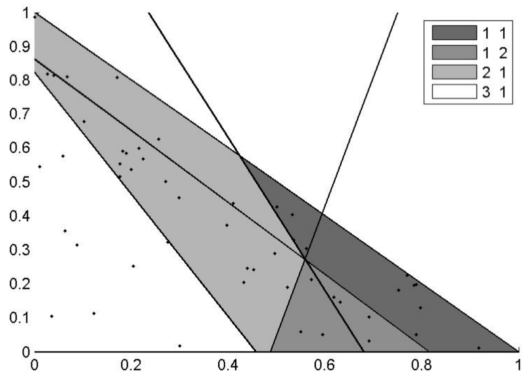

Figure 5. Decision planes when optimization horizon *T* = 5 and the points are selected randomly (50 points).

The same result, i.e. the same α-vectors and decision areas, is achieved also using 50 uniformly distributed random points in the probability triangle as shown in Figure 5. As only four points - one from each decision area - is needed to obtain the optimal result, the number of random points could be radically decreased and still the same exact optimal solution would be obtained with high probability.

As it can be seen from the figures, the optimization horizon affects strongly the decision areas. With higher optimization horizons ($T>6$), the number of decision borders i.e. α-vectors further increases as can be seen in Figure 6, where the optimization horizon T=8. Even though the number of decision areas is five, the number of α-models is increased to 11.

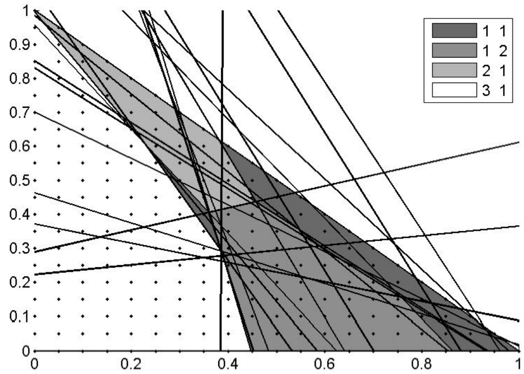

Figure 6. Decision borders and planes for optimization horizon *T* = 8 using point-based approximate method.

The solution obtained by the point-based method with T = 8 differs slightly from the solution obtained using the exact method which consists 13 α-vectors (Figure 7). However, as the differences in decision areas are rather small and as the calculation time using the exact method is 400-fold the point-based approximate method appears a good alternative.

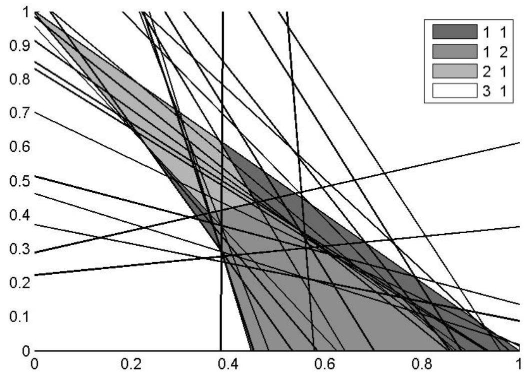

Figure 7. Decision borders and planes for optimization horizon *T* = 8 using exact solution method.

5.3 Paper machine quality control case

As a five-state model we present an example which is inspired by a quality management problem in papermaking. The target is to control a quality variable, the tear strength, optimally. Five states have been defined for tear strength, 1 referring to “poor” state and 5 to “good” state. The costs of quality at states 1 to 5 are 10, 5, 2, 1 and 0 units, respectively.

Tear strength is controlled by fiber furnish fraction ratio which is discretized into four values. The costs of control actions from 1 to 4 are 1, 2, 3 and 4 units. Measurement options are again to measure or not to measure, with costs 0.3 and 0 units. When measurement is made, the result may have 20 different values. Thus, 8 action alternatives exist.

The case is studied by running two simulators in parallel. The first simulator optimizes the control and measurement actions and acts according to them. The second simulator is the so called “true state simulator” which calculates the evolution of the target system. The true state is not known by the first simulator, as the state is known only through uncertain measurement on the second simulator.

The information about the state of the quality, the probability vector, is updated after each control and measurement action. The control and measurement actions are optimized based on the information about the current quality state.

As the probability “triangle” space in a five-state system is four dimensional and thus large to cover by points, it is difficult to assess the sufficient number of points and the degree of suboptimality of the solution is hard to assess. Using 100 points, the number of α-vectors is around 30. Using 1000 points the number of planes is around 100 and using 10000 points around 230. However, even though the solution differs with different number of points, the quality is manageable with fewer points also. Good results can be achieved by using only 100 points.

An example of the results is shown in Table 1. The problem is solved with 500 points. The corner points are selected beforehand and other points are chosen randomly from a uniform distribution. The optimization time horizon in this simulation is 4 time steps. At the beginning the quality is at a state 2, but recovers to good quality quite quickly and stays in good quality.

Table 1. Example of simulated result from the paper quality case. The control and measurement actions are optimized with time horizon of *T*=4. The first line shows the measurement decision with measured value or '-' denoting not a measurement. The second line gives the optimal control action. The third line gives the true process state that is not observable to optimizer. Note the different scale of measurements and states.

	t=0	t=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=9	t=10	t=11	t=12	t=13	t=14
Measured value	-	-	14	16	-	16	-	13	11	-	-	13	11	-	-
Optimal control	4	3	3	1	3	1	3	3	4	1	3	3	4	1	3
True value	2	4	5	5	5	5	4	3	4	5	4	4	4	5	5

6. CASE STUDIES ON SYSTEM DESIGN – THE THREE-STATE SYSTEM

This section presents a design case of a three-state system with measurement accuracy as a design parameter, i.e. we shall calculate W(M) in Eq. (11). Again, two measurement options exist: not to measure (m=1) or to measure (m=2). The measurement description is defined as

$$
\begin{aligned}
q^{(m=1)}\left(z_t \mid x_t\right) &= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \\
q^{(m=2)}\left(z_t \mid x_t\right) &= \begin{bmatrix} 1-d & 0.5 & d \\ d & 0.5 & 1-d \end{bmatrix}
\end{aligned}
\qquad (14),
$$

where d is the design parameter having values from 0 to 0.5. Note, that the measurement description for making a measurement is now symmetric, rather than non-symmetric as in Eq. (13). For small values of d the measurement system distinguish states 1 and 3 quite accurately, but both states 1 and 3 can be confused with state 2. The larger the d is the harder it is to distinguish the states 1 and 3.

Four design cases are presented. In Section 6.1 the case serving as the basis for further comparisons is presented. In Section 6.2 the effect of increased operational measurement cost is analyzed. In section 6.3 the effect of performance of controller is studied by allowing the controller effects to be both more and less predictable. In Section 6.4 the ideal measurement is analyzed in order to provide reference for the benefits from non-ideal measurements.

In all of the cases three control options exist with costs 0, 1 and 2. The costs for states are 0, 0.5 and 1. The discount factor α is set to 1.

To compare the costs, the cases are simulated 1000 time steps ahead using optimization horizon $T = 8$. The true state is simulated in parallel. The cost is calculated as a true state cost over simulation horizon as a mean of 100 times:

$$C = \frac{1}{100}\sum_{k-1}^{100}\sum_{j-1}^{1000} h\left(x_{j,k}, m_{j,k}, u_{j-1,k}\right) \tag{15}$$

The initial information state is chosen randomly from the probability triangle. As was discussed in Chapter 4, this means that we assume the system to be abnormally initialized and then run regularly over 1000 time steps while being operated optimally. After the 1000 time steps, the system is again abnormally reinitialized.

6.1 Case 1: The base-line case

The control actions of the base-line case are the same as the ones studied in Chapter 5 for operational optimization, see Eq. (12). However, the measurement cost $h_m = 0.1$ and the measurement properties given by, Eq. (14).

Figures 8, 9 and 10 present the decisions with d equal to 0, 0.1 and 0.2. In the figure legends the first number refers to the optimal control action, and the second number to the optimal measurement choice.

The behavior of the system is simulated over a time horizon of 1000 time steps. Black crosses (x) in the figures presents the probability states attained in the simulation.

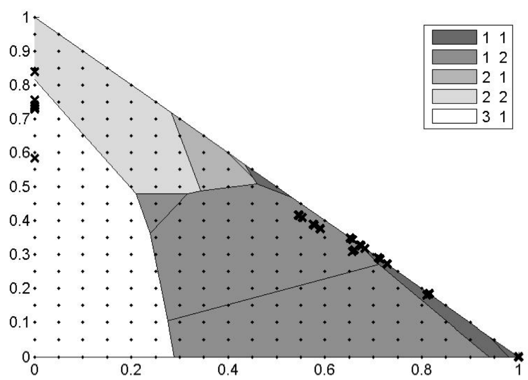

Figure 8. Decision planes for the case 1 with *d* = 0. The expected operational cost over 1000 time steps *C*= 485 units.

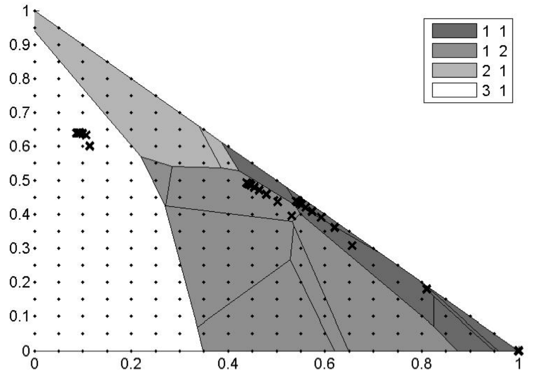

Figure 9. Decision planes for the case 1 with *d* = 0.1. The expected operational cost over 1000 time steps *C*= 514 units.

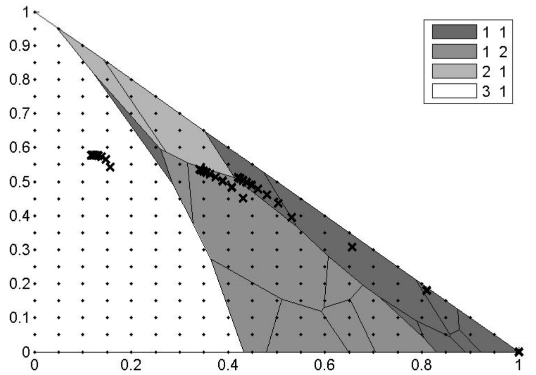

Figure 10. Decision planes for the case 1 with *d* = 0.2. The expected operational cost over 1000 time steps C= 530 units.

The expected operational costs over the time horizon are calculated in simulation according to Eq. (15). For d=0 the operational costs are C=485 units, for d=0.1 the costs are 514 units and for d=0.2 the operational costs are 530 units. Thus, the costs increase as the measurement deteriorates, as expected. Based on that, the investments cost for measurement system according to d=0 could be in maximum $45T_d/1000$ units higher than for system according to d=0.2 to achieve the same performance, where T_d is the life span of the system in units of time steps and measurement benefits are turned into present value without discounting. Figure 11 presents the growth of costs as a function of d.

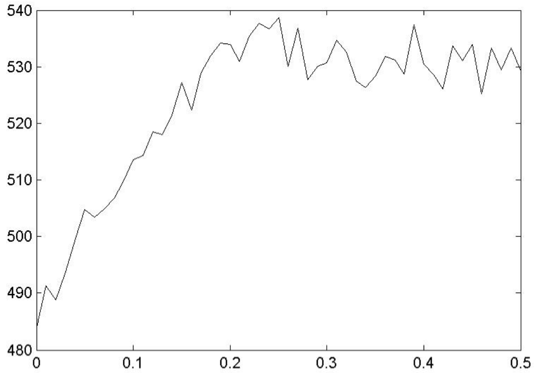

Figure 11. The growth of the operational costs as a function of design parameter *d*.

As can be seen in the figure, the costs increase quite linearly from 485 to 530 as d increases from 0 to 0.2, but stay quite steady after that. That is due the fact that in optimal operation no measurement is done if d >0.27 and then obviously measurement system not used does not affect the operational performance. Figure 12 presents the number of measurement done during 1000 step simulation as a function of d.

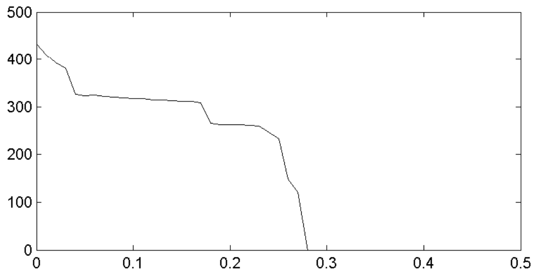

Figure 12. The number of measurements done during 1000 step simulation as a function of design parameter *d*.

6.2 Case 2: Effect of higher measurement cost

The second case studies the effect of measurement cost to the measurement system design. In this case the dynamics of system and the measurement description are as in case 1, but the cost of measuring is increased from 0.1 to 0.2. Figures 13 and 14 represent the decision areas with h_m=0.2 and d =0 and d=0.1.

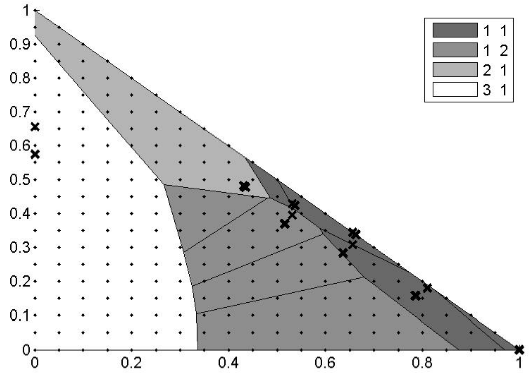

Figure 13. Decision planes for the case 2 with *d* = 0. The expected operational cost over 1000 time steps *C*= 515 units.

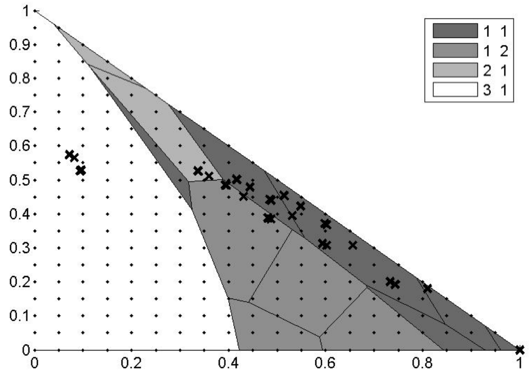

Figure 14. Decision planes for the case 2 with *d* = 0.1. The expected operational cost over 1000 time steps *C*= 534 units.

As it can be seen when comparing Figures 8 and 13, and Figures 9 and 14, the area in which a measurement is made is decreased as operational measurement cost is increased. Also the optimal frequency of measurements is decreased and in this case no measurements are done if d >0.14.

As the measuring cost is increased, also the total operative cost is increased. The simulated cost for d=0 is 515 units. Thus, the operative costs with h_m=0.2 are 30 units higher compared to h_m=0.1. For d=0.1 the operative costs are increased 20 units compared to the basic case in the section 6.1.

6.3 Cases 3 and 4: Effect of controller performance on measurement design

These cases present the effect of controller performance on the measurement design. In this Section the measurement cost is set again to h_m=0.1. Let us first (case 3) study controller performance be deteriorated compared to baseline as:

$$
p(x_{t+1} \mid x_t, u_t = 1) = \begin{bmatrix} 0.81 & 0.18 & 0.01 \\ 0 & 0.9 & 0.1 \\ 0 & 0 & 1 \end{bmatrix}
$$

$$
p(x_{t+1} \mid x_t, u_t = 2) = \begin{bmatrix} 0.9 & 0.05 & 0.05 \\ 0.7 & 0.2 & 0.1 \\ 0.2 & 0.7 & 0.1 \end{bmatrix} \qquad (16).
$$

$$
p(x_{t+1} \mid x_t, u_t = 3) = \begin{bmatrix} 0.9 & 0.05 & 0.05 \\ 0.9 & 0.05 & 0.05 \\ 0.9 & 0.05 & 0.05 \end{bmatrix}
$$

The decision areas are shown in the Figures 15, 16 and 17, with *d* equal to 0, 0.1 and 0.2 respectively.

As the control deteriorates, the importance of measuring increases for small values of *d*. That can be seen in Figures 15 and 16 as the decision area for making a measurement has increased compared to Figures 8 and 9, respectively. With poor accuracy of measurement, Figure 17, no measurement is made and the system cycles in four points in the information triangle.

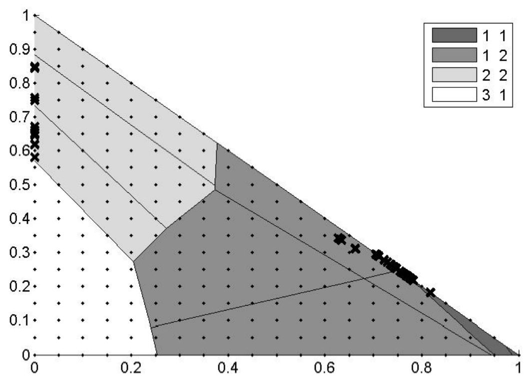

Figure 15. Decision planes for the case 3 with *d* = 0. The expected operational cost over 1000 time steps *C*= 510 units.

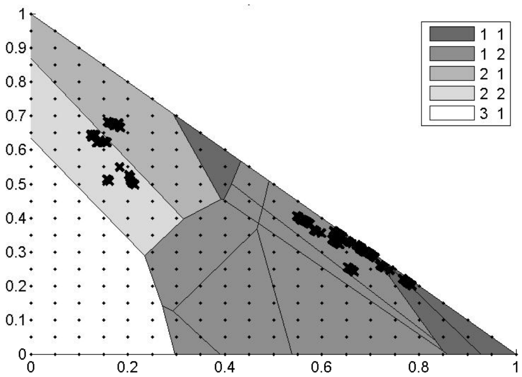

Figure 16. Decision planes for the case 3 with *d* = 0.1. The expected operational cost over 1000 time steps *C*= 543 units.

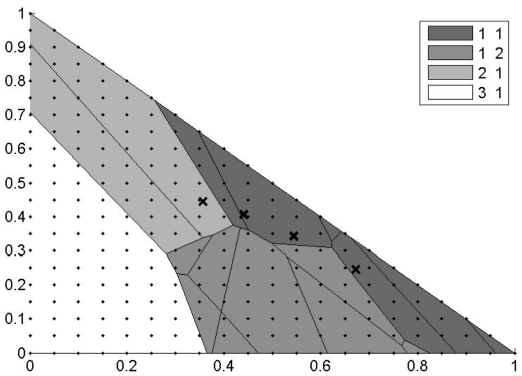

Figure 17. Decision planes for the case 3 with *d* = 0.2. The expected operational cost over 1000 time steps *C*= 566 units.

As the control is deteriorated, the operational costs increase. The increase when compared with the baseline case is 25 units for small values of *d* and the operational cost increases even more at higher values of *d*.

Finally (case 4), the effect of improved control is presented. In this case the control is defined as:

$$p(x_{t+1} \mid x_t, u_t = 1) = \begin{bmatrix} 0.81 & 0.18 & 0.01 \\ 0 & 0.9 & 0.1 \\ 0 & 0 & 1 \end{bmatrix}$$
$$p(x_{t+1} \mid x_t, u_t = 2) = \begin{bmatrix} 1 & 0 & 0 \\ 0.7 & 0.3 & 0 \\ 0.2 & 0.7 & 0.1 \end{bmatrix} \qquad (17).$$
$$p(x_{t+1} \mid x_t, u_t = 3) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

This is rather small change compared to the baseline case. The only difference is that for u=2 and x_t =2 the probability is changed from [0.7 0.2 0.1] to [0.7 0.3 0] thus ruling out state 3 when control action 2 has been chosen. Even though the difference is rather small, the effect to optimal operation and to the costs is significant.

Figures 18 and 19 show the decision areas for the case 4. Similarly as in Figure 17 for d=0.2 also in Figure 19 for d=0.1 no measurement is made and the information of the system state cycles in four points of the information triangle. However, this is for quite different reasons. In Figure 17 no measurements were made, because controller performance was so poor (compared to cost) that the benefits of the measurement information could not be utilized in a way to justify the cost. In Figure 19, the controller performance is so good (compared to cost) that no measurement to verify the effect of the controller action can be justified.

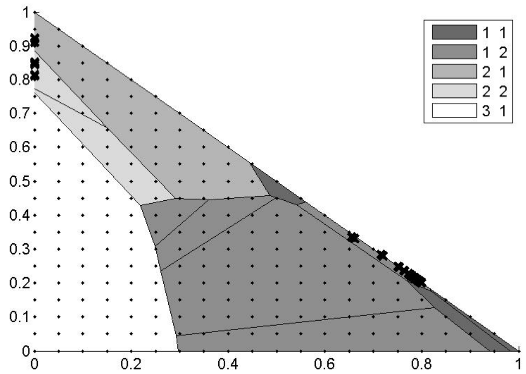

Figure 18. Decision planes for the case 4 with *d* = 0. The expected operational cost over 1000 time steps *C*=480 units.

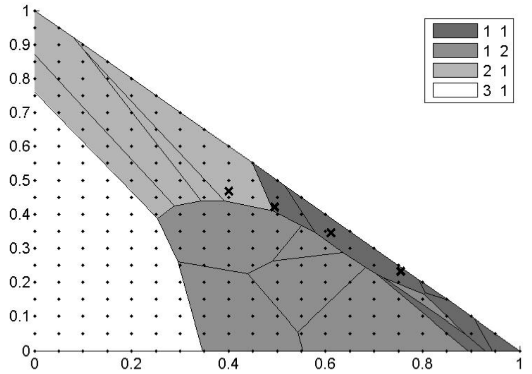

Figure 19. Decision planes for the case 4 with *d* = 0.1. The expected operational cost over 1000 time steps *C*=497 units.

6.4 Comparison of the cases and ideal measurement

In Figure 20 the operational costs as a function of design parameter *d* is presented for all of the four cases.

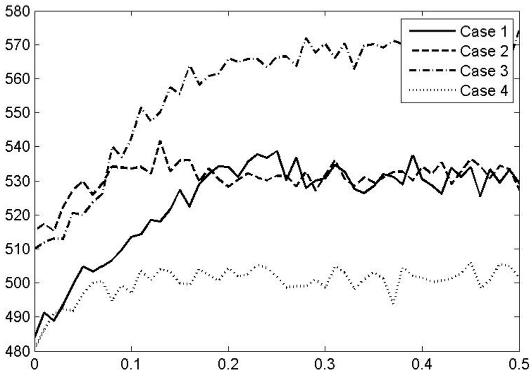

Figure 20. The growth of the operational costs as a function of design parameter *d* for four cases.

As it can be seen, the operational costs are highest in with the poor control (case 3). In the baseline case (case 1) and with higher operational measurement cost (case 2) the costs are equal for $d > 0.2$, which is simply because no measurements are made and in spite of the measurement cost the the systems are identical. With improved control (case 4) the cost difference between $d=0$ and no measurement at all (d=0.5) is only 20 units and hence only the measurement with quite perfect separation of states 1 and 3 adds value. For poor controller (case 3) the cost difference is highest, almost 70 units. It can be concluded that measurement system has largest potential with poor control. However, even in this case the measurement must be accurate enough so that the poor controller may utilize the information with benefits justifying the operational costs of measurement, see Figure 17.

Figure 21 shows the number of measurements made during a 1000 step simulation as a function of design parameter *d*.

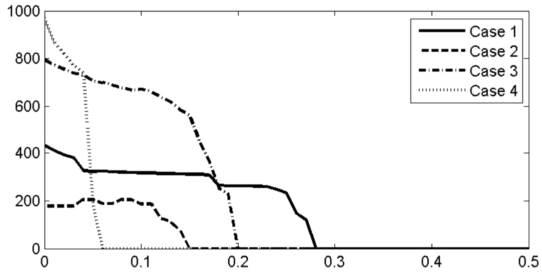

Figure 21. The number of measurements done during 1000 step simulation as a function of design parameter *d*.

Further to set up a reference for these incomplete measurements, let us analyze the ideal measurement:

$$q^{(m=1)}(z_t \mid x_t) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
$$q^{(m=2)}(z_t \mid x_t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (18).$$

Figure 22 shows the decision areas using the ideal measurement without measurement cost (h_m=0). The control transfer matrix is as in the baseline case. Now the optimal measurement action is to make a measurement unless the control action is 3 in which case both measurement options provide identical results: the system is known to be in state 1 even without measurement. The cost over a simulation horizon of 1000 time steps is 346. Because of the ideal measurement without cost, the system information vector is only in one of the three corner point s of the information triangle.

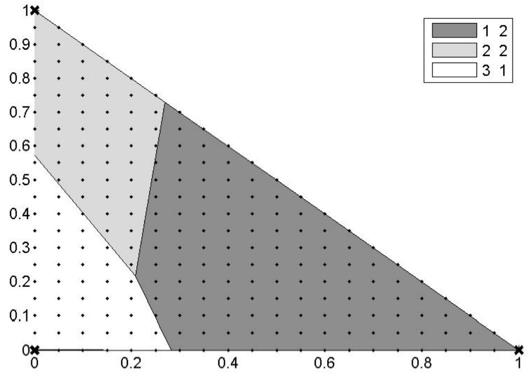

Figure 22. Decision planes using ideal measurement. The expected operational cost after 1000 time steps C = 346 units.

Figure 23 shows the decision areas using the ideal measurement with measurement cost $h_m = 0.1$. The solution is similar to previous one except that in that case for a small area, where the probability of state 1 is close to one, the optimal action is to make control action 1 and not make a measurement. The simulated cost over a time horizon of 1000 time steps is 422.

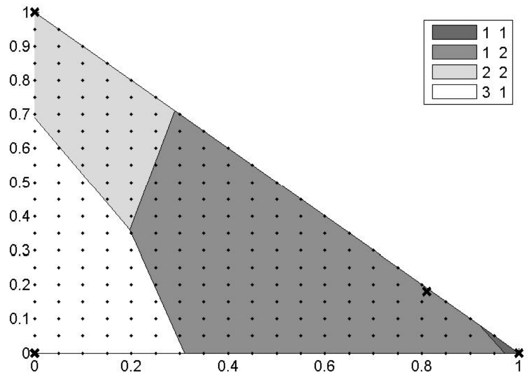

Figure 23. Decision planes using ideal measurement. The expected operational cost after 1000 time steps C=422 units.

To compare the ideal measurement to the case 1 it can be seen that the two-valued measurement in the case 1 achieves over 40 percent of the performance improvement of the ideal three-valued measurement with same operational costs:

$$\frac{530-485}{530-422}\cdot 100 = 42\% \qquad (19).$$

Here 530 is the operational cost without measurement, 485 the operational cost using 2-valued measurement with d=0 and 422 is the operational cost using the ideal measurement.

7. DISCUSSION AND FUTURE WORK

We have presented the measurement scheduling and design problems of a stochastic dynamic system in terms of dynamic programming. We have provided examples in which both exact and point-based approximate methods to solve the dynamic optimization problem have been applied. We have presented the solution of the problem in the simplest of cases, a binary state system, and in the three-state system. The presented five-state system follows an industrial quality management case [3,4].

We have presented an example of the design problem for a three-state system and shown the effects of measurement accuracy, operational measurement cost, and controller performance to the design and optimal system performance. We have simulated the cases and shown the variation of operational costs as a function of design. All the operational optimization solutions in design analysis are calculated with the point-based solution.

An obvious problem of the point-based approximate solution is the selection of points. It cannot be guaranteed that the optimal solution is found using the approximate solution. On the other hand using the exact solution the calculation time grows extremely rapidly as time horizon increases.

The dynamic programming approach relies heavily on the system model $p_{ij}(u)$. This raises – as in all model-based control or optimization problems – the issue of robustness against model inaccuracy. Such robustness problems can be addressed with the well-known method of Q-learning in infinite horizon dynamic programming problems [11, 12]. However, applying Q-learning in the case of uncertain measurements has not been extensively studied and remains as a challenging research task.

We have also studied a practical applications in papermaking process: the overall quality control, in particular managing paper strength and brightness [3]. This control is based on laboratory measurements that are rather uncertain as only few paper sheets are taken to represent the machine reel of 40 metric tons of paper. Furthermore, the effects of the main control actions – furnish component ratio and dosage of bleaching chemicals – are known somewhat vaguely due to nonlinearities, complex interactions and long dead times. Therefore, the present description that discretizes the quality parameters, measurement results and control actions and presents the system dynamics through conditional probabilities appears appropriate.

The decisions about strength/brightness control actions are made by several operators and currently there is little if any communication to make the actions coherent. The optimization approach outlined in this paper may serve in addition to automated quality management tool as a operations' decision support tool for harmonizing the operator actions.

In measurement system design, an obvious difficulty in applying it in practice is defining the disturbance scenarios and their probabilities. If these can be reasonably done, the measurement systems can be simulated and design optimized.

The presented approach puts the process system dynamics design and measurement/control design on an equal footing. Currently we are studying how this approach can be applied to concurrent design of material and information flows of a paper production system.

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