

# NUMERICAL SIMULATION OF AN AUTOMATIC DEPTH CONTROLLER FOR AN UNDERWATER VEHICLE

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## Abstract.

In this paper, we analyze two different mathematical strategies for solving the problem which consists in controlling a depth change manoeuvrability for a specific type of submarine. Precisely, we will apply both controllability theory and the more classical linear quadratic optimal control theory to a simplified linear model obtained from the general nonlinear DTNSRDC equations of motion. Finally, numerical results will be contrasted to show the advantages and handicaps of the proposed models. It is also important to emphasize that the results presented in this work are only a first step towards a better understanding of the problem.

## 1. Introduction

In the development of a naval architecture tool for the guidance and autopilot of a submarine is important to choose both an appropriate mathematical model for the equations of motion and a suitable control strategy.

Concerning the underwater vehicle, in this work we are mainly interested in a SSK submarine with parameters: length  $L=60\text{m}$ , diameter  $D=6.4\text{m}$  and displacement  $\Delta=2000\text{t}$ . Taking as a starting point the general nonlinear DTNSRDC submarine equations of motion (see [1,3]) and making the basic assumptions: (a) constant surge velocity, (b) small variations in pitch and yaw Euler angles, and (c) some particular geometrical hypotheses on the submarine, we obtain a

$$(1) \quad x'(t) = Ax(t) + Bu(t)$$

mathematical linear model in the form

where  $t$  is the time variable,  $A$  is a  $7 \times 7$  matrix and  $B$  is a  $7 \times 3$  matrix. The coefficients of these matrices include hydrodynamics derivatives for the SSK model and depend on surge velocity and the properties of the underwater vehicle. We refer to [4] for a detailed description of

$$x = x(t) = (v, r, \psi, w, q, \theta, z)$$

these two matrices. The state variable is

where  $v$  = sway velocity,  $r$  = yaw rate,  $\psi$  = yaw Euler angle,  $w$  = component of velocity in  $z$ -direction,  $q$  = pitch rate,  $\theta$  = pitch angle, and  $z$

$$u = u(t) = (\delta_r, \delta_s, \delta_b)$$

= depth. The control variable is

with  $\delta_r$  = deflection of rudder,  $\delta_s$  = deflection of stern plane, and  $\delta_b$  = deflection of the fwd plane.

The controllability problem we address in this work follows. For a fixed final time  $T$  and given an initial state  $x_0$  and a final state  $x_T$ , we wonder if there exists a control variable  $u = u(t)$ ,  $0 \leq t \leq T$ , such that the solution of (1) is driven from  $x_0$  to  $x_T$  at time  $T$ , i.e.,  $x(0) = x_0$  and  $x(T) = x_T$ .

By using controllability theory one deduces that this problem has a positive answer. Then, the question of numerically computing the control  $u(t)$  is in order. Since the linear model (1) is mainly designed to simulate changes of depth, we will focus only on the control of this manoeuvrability. In the next section, we prove that the control  $u$  can be easily computed from the Gramian matrix that comes from the controllability theory. Then, we show a numerical experiment to

compare these results with the more classical approach based on a Linear Quadratic Regulator (LQR).

## 2. Controllability versus LQR

As is well-known (see [5, p. 737]), the linear system (1) is exactly controllable at time  $T$  if and only if the controllability matrix

$$Q_C = [B \ AB \ A^2B \ \dots \ A^{T-1}B]$$

has maximal range. In our situation, this is so. Moreover, the control  $u(t)$  is explicitly given by

$$u(t) = B^* e^{A^*(T-t)} [P(T)]^{-1} (x_T - e^{AT} x_0),$$

where  $A^*$ ,  $B^*$  are the transpose of  $A$  and  $B$ , respectively,  $e^{AT}$  is the exponential matrix, and  $[P(T)]^{-1}$  is the inverse of the Gramian matrix

$$P(T) = \int_0^T e^{A^*t} B B^* e^{At} dt.$$

Once the control  $u$  is determined, the state  $x(t)$  is obtained in the closed form

$$x(t) = e^{At} x_0 + \int_0^t e^{A(t-s)} B u(s) ds.$$

On another hand, a LQ controller is designed by solving the optimal control problem

$$\text{Minimize in } u: \quad J(u) = \int_0^T [x^T(t) Q x(t) + u^T(t) C u(t)] dt$$

subject to the state equation (1). Here  $Q > 0$  and  $C \geq 0$  are two weighting matrices. The optimal feedback control  $u(t)$  has the form

$$u(t) = -L(x(t) - x_T), \quad 0 \leq t \leq T$$

for an appropriate  $3 \times 7$  diagonal matrix  $L$  and being  $x_T = (0, 0, 0, 0, 0, 0, z_f)$  the final state.

Next, we show some numerical results obtained by implementing both approaches in Matlab for a depth change of 10m in a time  $T=200\text{s}$ .

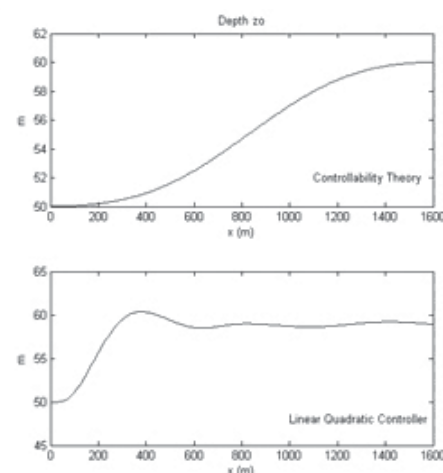


Figure 1: Depth change.



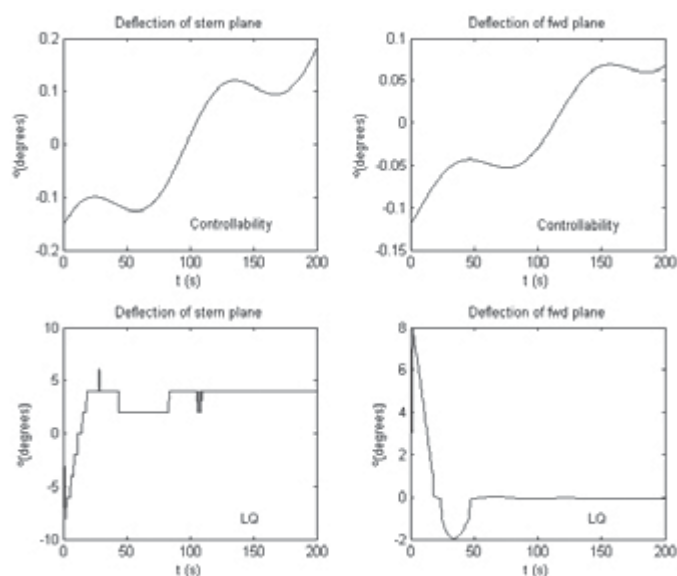


Figure 2: Controls.

### 3. Conclusions

In addition to the classical LQ control strategy, a new approach based on controllability theory has been implemented for the automatic simulation of a depth change manoeuvrability for an SSK submarine. The main differences between both methodologies follow:

(1) The controls obtained from the LQ controller appear in a feedback form. So, from a practical point of view it is necessary to complete the control system with a suitable Kalman filter to correct the data of the state provided by the sensors of the submarine. For the contrary, the controls obtained from the controllability theory do not require the use of those.

(2) No constraints on the controls are imposed in the controllability strategy. This may lead in some cases, for instance for short times, to some unrealistic results with sharp changes of controls and states. With the LQ controller, these sharp changes may be corrected by using appropriate weights in the associated cost. This, however, requires a preprocessing work.

(3) Concerning the accuracy of reaching the final state, it is evident that the best strategy is controllability theory (see Figure 1). Nevertheless, the LQ controller can be also designed to improve this property by choosing an appropriate cost functional.

(4) As for the optimality of controls, we notice that the controls obtained from the controllability theory are optimal in the  $L_2(0,T;)$  norm (see Figure 2).

As indicated in the abstract, the present work is only a preliminary study on this topic. Many interesting open questions could be analyzed.

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### 4. References

- [1] J. Felman, Revised standard submarine equations of motion. Report DTNSRDC/SPD-0393-09, David W. Taylor Naval Ship Research and Development Center, Washington D.C., 1979.
- [2] E. Fernández-Cara and E. Zuazua, Control theory: History, mathematical achievements and perspectives, *Bol. Soc. Esp. Mat. Apl.* 26 (2003), 79-140.
- [3] T. I. Fossen, Guidance and control of ocean vehicles, John Wiley and sons, 1994.
- [4] J. García, J. A. Murillo, I. A. Nieto, D. Pardo and F. Periago, On a linear automatic control model for the manoeuvrability of an underwater vehicle, in preparation.
- [5] K. Ogata, Ingeniería de control moderna, Prentice Hall, 1998.

## UNDERWATER SLAM FOR MANMADE ENVIRONMENTS

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### 1. Introduction

The possibility of having truly autonomous vehicles heavily depends on their ability to build accurate models or maps of the environments they traverse, and to know their location in them. This has made this problem, known as Simultaneous Localization and Mapping (SLAM), the focus of a great deal of attention in recent years [1-2]. Multiple techniques had shown promising results in a variety of different applications and scenarios. Some of them perform SLAM indoor, outdoor, on land and even on air. However, the underwater environment is still one of the most challenging scenarios for SLAM because of the reduced sensorial possibilities. Acoustic devices are the most common choice while the use of cameras and laser sensors is limited to applications where the vehicle navigates very near to the seafloor. Another important issue is the difficulty to find reliable features. There are approaches using clusters of acoustic data as features [3-4], or merging visual and acoustic information in order to improve the reliability [5], while other strategies simply introduce artificial beacons to deal with complex environments [6].

This article focuses on underwater SLAM applied to some particular manmade environments (harbours, marinas, marine platforms, dams, etc.) where structured elements are present and can be used to produce reliable features. Although most of the previous work done

on this field focuses on open sea and coastal applications, obtaining an accurate positioning in such scenarios would notably increment AUVs capabilities. Monitoring, inspection and surveillance of underwater structures are some examples of applications that can benefit from such a system.

### 2. Feature extraction from acoustic images

The algorithm presented in this paper relies in a mechanically scanned imaging sonar (MSIS) to obtain the features that will conform the map. Although these mechanically actuated devices usually have a low scanning rate they are quite popular because of their low cost. This work propose the use of line features in underwater environment as a representation of the cross sections produced when a sonar scan intersects with existing planar structures (see Figure 1). Using this kind of sonar presents some difficulties. First, due to the low scanning rate it is necessary to merge information from dead-reckoning sensors in order to reduce the effects of movement-induced distortion in the resulting data. Second, these devices do not produce instantaneous acoustic snapshots of the surroundings but a constant continuous dataflow. Therefore, the feature extraction algorithm should be able to deal with this continuous stream of data while detecting the line features as soon as they appear. Our approach consists on an adapted version of the Hough transform [7]. This algorithm accumu-

