Mathware & Soft Computing 11 (2004) 13-30

The Estimation of Electric Power Losses in Electrical Networks by Fuzzy Regression Model Using Genetic Algorithm

A.V. Mogilenko and D.A. Pavlyuchenko
Department of Power Engineering,
Novosibirsk State Technical University,
20, Karl Marx prospect, Novosibirsk, 630092, Russia
egorushka@risp.ru, pavluc@merlin.power.nstu.ru

Abstract

This paper presents the comparative study for fuzzy regression model using linear programming, fuzzy regression model using genetic algorithms and standard regression model. The fuzzy and standard models were developed for estimation of electric power losses in electrical networks. Simulation was carried out with a tool developed in MATLAB.

Keywords: Fuzzy regression, membership function, genetic algorithms, electrical networks, electric power losses

1 Introduction

The problem of estimating of electric power losses in electrical networks is a major aspect of power systems research.

The main tasks in analyzing electric power losses are the following:

- Detecting and estimating the reserves of the power system divisions for decreasing the power losses.
- Ranking the main factors affecting the level of power losses.
- Developing measures for decreasing power losses and determining the effectiveness of these measures.
- Determining the main causes of commercial losses of electric power.
- Assessing the operation of the power system as a whole and its divisions with respect to power loss.
- Preparation and basis for decisions for the development of electric power systems and for the implementation of measures for decreasing power losses.

A series of methods are now used which allow an exact account of electric power losses (deterministic methods), and an estimation of losses (probabilistic-statistical methods).

Deterministic methods are based on replacing the real process of load modification in an electrical network with the most typical calculated state. The process of load modification is actually a set of realizations of casual processes, making it practically impossible to receive full information about state parameters. Therefore, it is necessary to use estimated statistical methods in accounting for electric power losses. They allow the determination of losses with a defined confidence interval.

The most widespread probabilistic methods of electric power loss estimation in electrical networks are regression analysis, factor analysis, the sampling method, experimental design, and other methods. Regression analysis, based on the method of least-squares, is a very convenient method to develop models for determining the dependence of a parameter on certain factors. However, exact numerical statistical information is necessary for deriving regression models.

In order to determine how power losses depend on the factors that affect them, information must be used which is intended for different purposes, such as operational control, equipment overload control, etc. For power loss estimation, this data is biased and incomplete, which necessitates various assumptions in order to formalize the decision algorithm.

For this reason, and also as a result of the absence of a reliable probabilistic-statistical description of the initial information, it frequently becomes difficult to use probabilistic-statistical methods to analyze power losses. Since a significant part of the initial information can have substantial uncertainty (first of all, this applies to mode parameters), it is necessary to use new methods in loss analysis. These methods combine the merits of probabilistic-statistical methods of loss analysis, and also formalize fuzzy initial information with the help of fuzzy sets. Lotfi Zadeh suggested this theory, whose main idea is the linguistic variable.

For the development of models with fuzzy initial information, Tanaka, Chang and others suggested and developed fuzzy regression analysis. In usual regression analysis, an error between values obtained by the regression model and observable data is considered an error of observation, which is a random variable (with a normal distribution and an average distribution which equal to zero). In fuzzy regression analysis the same errors are considered caused by the fuzziness of the model.

2 Models of fuzzy regression analysis

2.1. Symmetrical triangular membership function of the LR-type

Since the most common type of membership function in fuzzy regression models is the triangular type, it makes sense to examine it more closely. Triangular membership functions are divided into symmetrical and asymmetrical. Here, the symmetrical triangular function is the most interesting.

With an increase in x, a triangular function grows evenly, reaches its maximum, and then evenly falls in the same way [1].

This membership function has the following main features, graphically represented on Fig. 1.

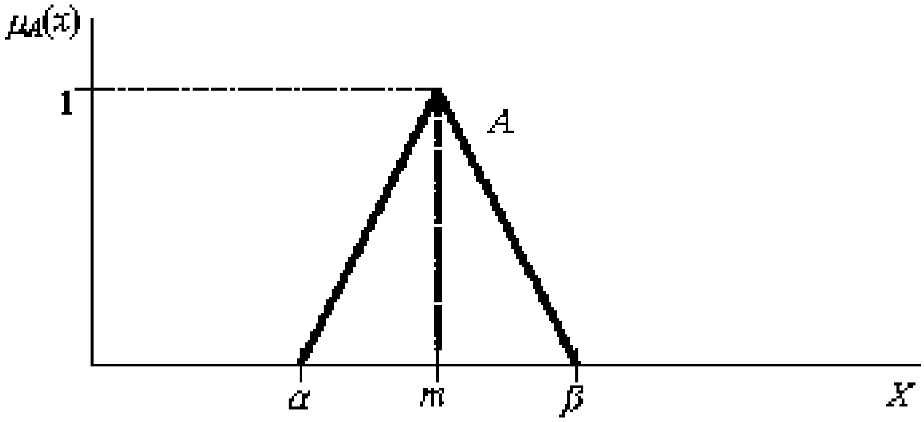

Fig. 1. Membership function for the symmetric triangular fuzzy set A.

As stated earlier, it is worth noting that this membership function has a symmetrical appearance, since the deviation along the X-axis to the left (L) from the centre m is equal to the deviation to the right (R). As a whole, this type of membership function is called an LR-image.

The value of the membership function of fuzzy set A in the main interval [α, β] can be determined using Eq. (2). For this, it is necessary to first determine the value of the centre m of the main interval using Eq. (1).

$$m = \alpha + \frac{\beta - \alpha}{2} \quad (1)$$

$$\mu_A(x) = \begin{cases} L\,(m - x)\,\_for\_x \in [\alpha, m] \\ R\,(x - m)\,\_for\_x \in [m, \beta] \end{cases} \quad (2)$$

In addition, it is clear that the value of $L(x)$ and $R(x)$at the point m is equal to zero.

Having found the value of the deviation, and then by placing it on the membership function graph, it is possible to find the value of membership at the exact point x. The same can be done directly, that is, without first finding the deviation, by using Eq. (3).

$$\mu_A(x) = \begin{cases} \frac{x-\alpha}{m-\alpha}\_for\_x \in [\alpha, m] \\ \frac{\beta-x}{\beta-m}\_for\_x \in [m, \beta] \end{cases} \quad (3)$$

The types of membership functions can differ depending on the initial information that the researcher has at his disposal, that is, depending on their uncertainty and the way they are formalized.

2.2. Fuzzy regression analysis using the criterion of minimal fuzziness

In fuzzy regression analysis the deviation between observable values and estimated values is due to the fuzziness of the system or the fuzziness of the coefficients of regression [2]. This assumption is shared by the methods of fuzzy regression which are described in this work. The goal of fuzzy regression analysis is to find a regression model that satisfies all observable fuzzy data within the indicated criterion of optimality. Various fuzzy regression models differ depending precisely on the optimality criterion used.

Tanaka first proposed fuzzy linear regression analysis [3, 4]. According to this method, coefficients of regression are fuzzy numbers that can be expressed as interval numbers with values of membership. Since coefficients of regression are fuzzy numbers, the estimated output variable Y is also a fuzzy number. Fuzzy regression analysis with only one independent variable$X$ has the following two-coefficient regression model:

$$\widehat{Y} = \tilde{B}_0 + \tilde{B}_1 X, \tag{4}$$

where $\tilde{B}_0$ is the fuzzy coefficient of intersection, and $\tilde{B}_1$is a fuzzy number that characterizes the slope of the line of regression. The fuzzy parameter $\tilde{B}_j = (m_j, c_j)$ is expressed as a symmetrical triangular membership function, which consists of a fuzzy centre m_j and half of the fuzzy base c_j. In these models, other membership function forms can also be used such as asymmetrical and trapezoidal.

According to this approach, the fuzzy coefficients $\tilde{B}_j$(j=0,1) are determined in such a way that the estimated fuzzy output Y has a minimal fuzzy deviation, while satisfying the assigned degree of reliability h. The term h is a measure of compatibility between the initial data and the regression model. Each of the observable samples of data, which can have a fuzzy value $\tilde{Y}_i$, as well as an exact value Y_i, must fall within the estimated $\widehat{Y}$ at level h.

Determining the fuzzy coefficients$\tilde{B}_j = (m_j, c_j)$, Tanaka and others formulated the fuzzy criterion function of regression as the following linear programming problem [2, 5, 6]:

Minimize

$$S = nc_0 + c_1 \sum_{i=1}^{n} |X_i| \tag{5}$$

With limits $c_0 \geq 0$, $c_1 \geq$0,

$$\sum_{j=0}^{1} m_j X_{ij} + (1-h) \sum_{j=0}^{1} c_j |X_{ij}| \geq Y_i + (1-h) e_i \text{ with } i = 1 \ldots n, \tag{6}$$

$$\sum_{j=0}^{1} m_j X_{ij} - (1-h)\sum_{j=0}^{1} c_j|X_{ij}| \leq Y_i - (1-h)e_i \text{ with } i = 1\ldots n, \tag{7}$$

where S is the level of fuzziness of the regression model. Eq. (6) and (7) can use, as initial data, the fuzzy observable value$\tilde{Y_i} = (Y_i, e_i)$, where Y_i is the fuzzy centre, and e_i is half of the fuzzy base. If the observable value is given exactly, then e is zero. Therefore, a usual exact number is a particular case of a fuzzy number.

Eq. (5) and (7) can also be applied to the model of plural regression. If it is necessary to build a fuzzy regression model of the dependence of the output parameter on k of the influencing factors $_{ik}$, then the criterion function of the linear programming problem will have the following form:

$$S = nc_0 + \sum_{i=1}^{n} (c_1|X_{i1}| + ...+c_k|X_{ik}|) \rightarrow \min. \tag{8}$$

With limits $_0$ ≥ 0, $_1$ ≥ 0, ..., $_\text{k}$ ≥ 0,

$$\sum_{j=0}^{k} m_j X_{ij} + (1-h)\sum_{j=0}^{k} c_j|X_{ij}| \geq Y_i + (1-h)e_i \text{ with } i = 1\ldots n, \tag{9}$$

$$\sum_{j=0}^{k} m_j X_{ij} - (1-h)\sum_{j=0}^{k} c_j|X_{ij}| \leq Y_i - (1-h)e_i \text{ with } i = 1\ldots n, \tag{10}$$

where $j = 0\ldots k$ is the number of the coefficient of regression, and n is the volume of the sample of initial data.

For varying degrees of reliability, the fuzzy centre values remain unchanged, while the fuzziness of the fuzzy regression model grows with increases in the degree of reliability. Therefore, when h is equal to 0, the fuzzy regression model has the narrowest fuzzy deviation among all the values of h between 0.0 and 1.0.

2.3. Fuzzy regression analysis using the genetic algorithm

As mentioned above, the determining of fuzzy coefficients consist in solution of linear programming problem.

Linear programming is used for optimization problems when the objective function and constraints are linear. In this paper the linear programming was implemented using the OPTIMIZATION toolbox of MATLAB which contains the variation of well known simplex method.

Authors propose to use the genetic algorithms as an alternative approach for optimization problem Eq. (5) – (7).

The genetic algorithms are the powerful stochastic optimization technique which determines the global optimal solution and it is not sensitive to the convexity of the solution surface [7, 8].

The genetic algorithms search for an optimal solution using the principles of evolution and heredity. They operate on population which consists of a number of individuals, each representing a particular selection of the values of the variables.

Such population is able to evolve in a given environment by application of the processes of selection, crossover and mutation. As an evolutionary theory, genetic algorithms perform the mechanisms of crossover and mutation to recombine the genetic material.

The first step is to generate the initial population. Then genetic algorithms perform the mechanisms of crossover and mutation as genetic operators to recombine the genetic material. This process that starts from the present population and leads to the new population is named a generation. The degree of optimality of each of the candidate solutions or individuals is measured by their fitness, which can be defined as the objective function of the problem. The individuals compete with each other. The winning ones form the next generation. Normally the average quality of the next population is greater than the previous one. The genetic algorithms are iterative and the process is terminated by stopping rule. The rule widely used is stop after specified number of generations.

Some alternative fussy regression models using genetic algorithms were considered in [9].

3 Applied power engineering problem

The problem of estimation and prediction of electric power losses in electrical networks is used to illustrate considered methods.

It is well known, that total power flows are depend on system load. But load is one of the major factors affecting the level of power losses in electrical networks. Therefore the determining of the functional dependence between power losses and total power generation is actual.

Thus, in this case the independent variable X is the total power flows, thousand kw-hr. The output parameter Y is the total power losses, thousand kw-hr. The sampling is 40 points.

3.1. Results

Two fuzzy regression models were developed to illustrate the characteristics of the proposed methods. The fuzzy coefficients were determined by linear programming (simplex method) and by genetic algorithm. The level of compatibility h for considered fuzzy models (see 2.2) was zero, because there were no sufficient reasons for its definition. Also the standard regression model with confidence interval with probability 0.95 was developed for the results comparison.

Fuzzy regression model using simplex method have the following form:

$$\widehat{Y} = (m_0; c_0) + (m_1; c_1)X = (4,33; 4,19) + (0,06; 0,01)X. \quad (11)$$

Fig. 2 shows the distribution of deviations at sampling points by fuzzy regression model. The fuzzy regression between power losses and total flows is shown in Fig. 3.

Table 1 demonstrates the absolute values of deviations of centre line for fuzzy regression model. The total value is 315.2.

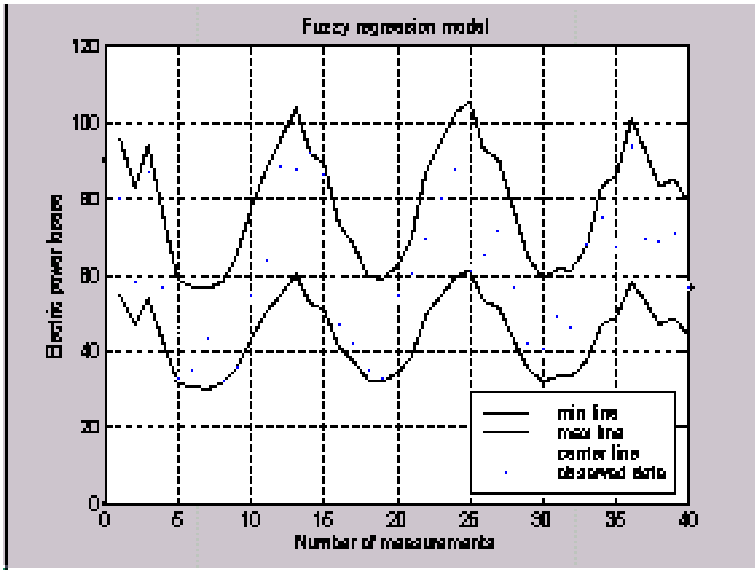

Fig. 2. The distribution of deviations at sampling points by fuzzy regression model.

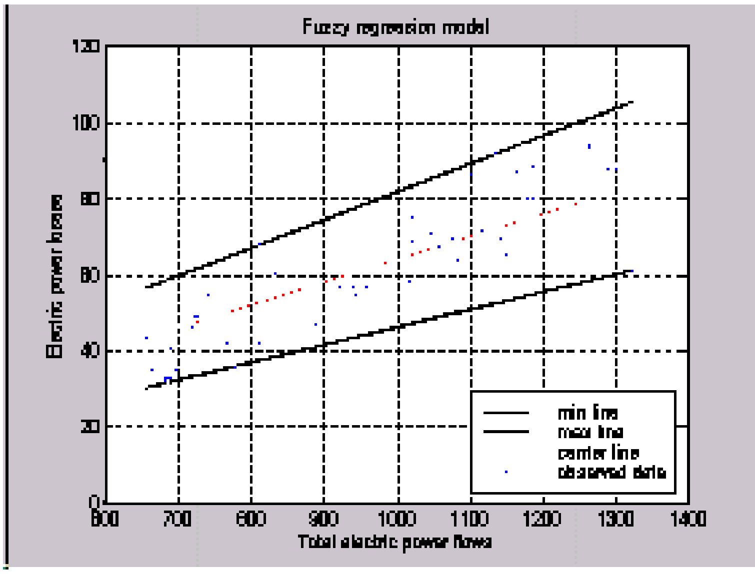

Fig. 3. Fuzzy regression model

Fuzzy regression model using genetic algorithm have the following form:

$$\widehat{Y} = (m_0; c_0) + (m_1; c_1)X = (3,05; 3,20) + (0,06; 0,01)X. \tag{12}$$

It is shown in Fig. 4 the distribution of deviations at sampling points by fuzzy regression model using genetic algorithm. The fuzzy regression between power losses and total flows is shown in Fig. 5. Table 1 shows the absolute values of deviations of center line for fuzzy regression model using genetic algorithm. The total value is 310.8.

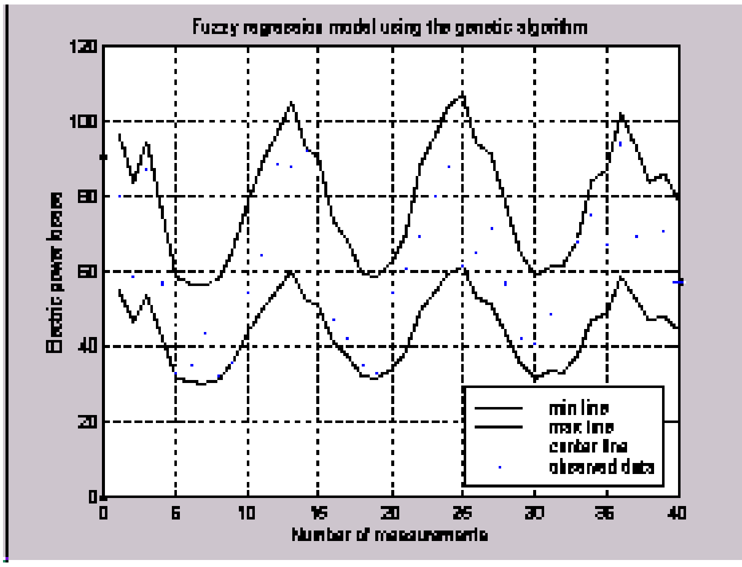

Fig. 4. The distribution of deviations at sampling points by FRUGA model.

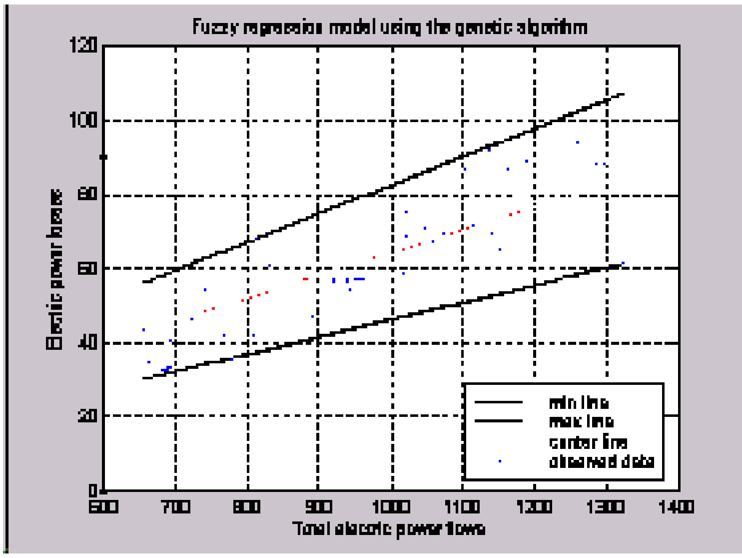

Fig. 5. Fuzzy regression model using the genetic algorithm.

Standard regression model using least squares have the following form:

$$\widehat{Y} = B_0 + B_1X = -14,35 + 0,08X. \quad (13)$$

Fig. 6 illustrates the distribution of deviations at sampling points by standard regression model. The standard regression between power losses and total flows is shown in Fig. 7.

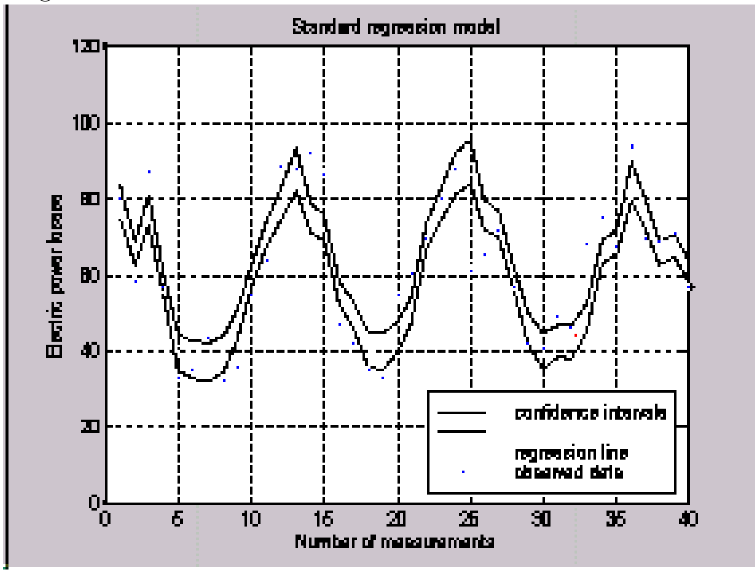

Fig. 6. The distribution of deviations at sampling points by standard regression model.

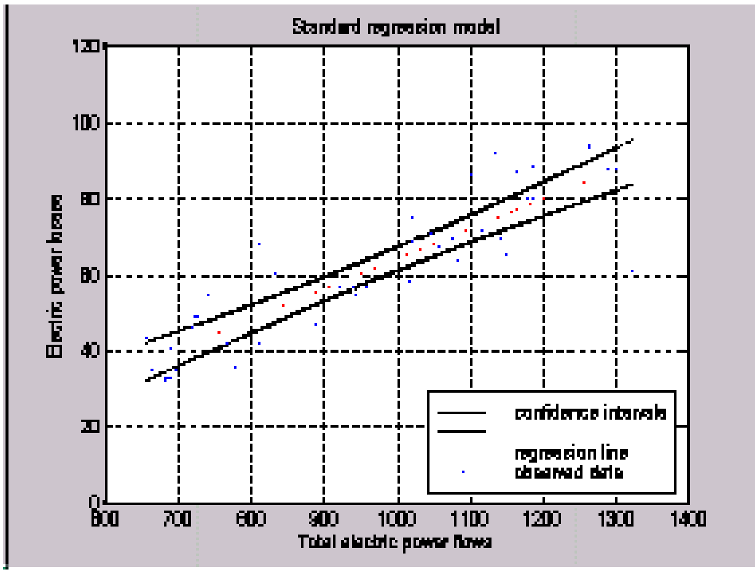

Fig. 7. Standard regression model.

Table 1 demonstrates the absolute values of deviations of standard regression line. The total value is 267.8.

Table 1. Observed and obtained values of the output variable by fuzzy regression models

No. π/π	Y	$\left\|\widehat{Y}_{SR} - Y\right\|$	$\left\|\widehat{Y}_{FR} - Y\right\|$	$\left\|\widehat{Y}_{FRUGA} - Y\right\|$
1	79.8	0.8	4.5	4.0
2	58.5	7.0	6.5	6.8
3	86.9	9.9	13.1	12.7
4	56.7	1.3	2.6	2.7
5	33.1	6.5	12.3	12.0
6	34.8	2.8	9.1	8.8
7	43.6	6.4	0.1	0.4
8	32.4	6.8	12.6	12.4
9	36.0	10.7	14.7	14.6
10	54.6	5.2	6.1	6.3
11	64.0	6.8	5.1	5.4
12	88.6	9.7	13.3	12.9
13	87.9	0.2	5.9	5.3
14	91.8	16.9	19.6	19.2
15	86.7	14.4	16.5	16.1
16	47.2	8.3	10.3	10.3
17	42.3	6.9	10.3	10.3
18	35.0	5.3	10.8	10.6
19	33.1	6.8	12.5	12.2
20	54.5	10.7	5.9	6.1
21	60.5	9.6	6.6	6.6
22	69.2	1.0	0.6	0.3
23	79.7	1.3	4.9	4.4
24	87.9	1.1	6.6	6.0
25	61.2	28.4	22.2	22.8
26	65.2	11.0	8.0	8.4
27	71.9	1.4	1.0	0.6
28	56.5	2.9	3.9	4.0
29	42.2	3.7	8.0	7.8
30	40.4	0.5	5.2	4.9
31	48.8	6.2	1.2	1.4
32	46.1	3.8	1.3	1.0
33	68.0	18.6	15.2	15.3
34	75.0	9.1	9.7	9.4
35	67.3	1.5	0.2	0.5
36	93.5	8.8	13.8	13.2
37	69.3	6.0	3.2	3.6
38	68.9	3.1	3.6	3.4
39	70.6	2.8	3.8	3.6
40	57.2	3.6	4.3	4.5

The simulation of the considered functions was realized in MATLAB.

4 Classical mathematical problem

Another numeric example is presented to illustrate proposed method. In this case it is the classical mathematical example from [2].

The size of sampling $n = 8$. The values of independent variable are 2, 4, 6, 8, 10, 12, 14 and 16. The values of dependent variable Y are 14, 16, 14, 18, 18, 22, 18 and 22. Furthermore, the case when Y is fuzzy was considered, so the level of fuzziness was 1 at each point of sampling.The fuzzy regression models were developed in the cases with $h = 0.0$ and $h = 0.5$.

In the case of using fuzzy data it is possible to estimate the average value $\bar{\tilde{Y}}$ with the help of fuzzy math as an exact value. And then it is possible to determine the standard deviation $S_{\tilde{y}}$ with the help of Eq. (14).

$$S_{\tilde{y}} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(\tilde{Y}_i - \bar{\tilde{Y}})^2} \tag{14}$$

The value $S_{\tilde{y}}$ is the measure of dispersion or uncertainty of the initial fuzzy data.

As one of the indicators of reliability of fuzzy regression models, the hybrid coefficient of correlation (*HR*) is used to evaluate the assumption of the linearity of the hybrid linear model of regression.

Another indicator of reliability is the hybrid standard error of estimation (*HS*), used to measure the similarity of data obtained with the fuzzy regression model with observable fuzzy data. In determining *HR* and *HS* with the help of fuzzy math, Eq. (15) and (16) can be recorded.

$$(HR)^2 = \frac{\sum_{i=1}^{n}(\widehat{Y}_i - \bar{\tilde{Y}})^2}{\sum_{i=1}^{n}(\tilde{Y}_i - \bar{\tilde{Y}})^2} \tag{15}$$

$$HS_e = \sqrt{\frac{1}{n-p-1}\sum_{i=1}^{n}(\widehat{Y}_i - \tilde{Y}_i)^2}, \tag{16}$$

where $n - p - 1$ is the number of degrees of freedom. For a regression model with one influencing factor X, the value p is equal to one.

The values of *HS* are located in the interval from 0 to $S_{\tilde{y}}$. The lower the value of *HS*, the better the perfection of the data obtained with the model, that is, the better the accuracy of the predictions. If *HS* is near or more than $S_{\tilde{y}}$, then regression analysis did not achieve successful results. In this case, it is necessary to use other methods of regression analysis in order to provide more accurate data obtained using the model.

In that case when $S_{\tilde{y}}$ is constant and independent of the regression method used, the relationship $HS/S_{\tilde{y}}$ is the standardized measure for improving the data obtained with the model. Therefore the values *HS* and $HS/S_{\tilde{y}}$ can be used to assess the effectiveness of various fuzzy regression methods.

4.1. Results *(and Y are crisp)*

The following results were obtained in the case of crisp initial data. Table 2 shows fuzzy regression equations at $h = 0.0$ and $h = 0.5$. Also Table 2 contains values of HR, HS_e, HS_e/S_y, which allows estimating the adequacy of developed models.

Table 2. Reliability measures for regression models.

Regression method	Regression equation	HR	HS_e	HS_e/S_y	$\sum_{i=1}^{n} \left\lvert \widehat{Y}_i - Y_i \right\rvert$
crisp X – crisp Y					
FR at $h = 0.0$	$\widehat{Y} = (12.00, 1.00) + (0.63, 0.13)?$	1.03	2.13	0.69	10.50
FRUGA at $h = 0.0$	$\widehat{Y} = (13.55, 1.65) + (0.46, 0.11)?$	0.81	2.24	0.72	10.35
FR at $h = 0.5$	$\widehat{Y} = (12.00, 2.00) + (0.63, 0.25)?$	1.16	2.79	0.90	10.50
FRUGA at $h = 0.5$	$\widehat{Y} = (13.55, 2.48) + (0.46, 0.19)?$	0.94	2.72	0.88	10.35
crisp X – fuzzy Y					
FR at $h = 0.0$	$\widehat{Y} = (12.00, 2.00) + (0.63, 0.13)?$	1.08	2.37	0.76	10.50
FRUGA at $h = 0.0$	$\widehat{Y} = (13.55, 2.00) + (0.46, 0.14)?$	0.86	2.42	0.78	10.35
FR at $h = 0.5$	$\widehat{Y} = (12.00, 3.00) + (0.63, 0.25)?$	1.24	3.12	1.00	10.50
FRUGA at $h = 0.5$	$\widehat{Y} = (13.55, 2.95) + (0.46, 0.21)?$	1.00	2.97	0.96	10.35

In addition, in order to compare fuzzy regression model with fuzzy regression model based on genetic algorithm the total deviations $\sum_{i=1}^{n} \left\lvert \widehat{Y}_i - Y_i \right\rvert$ were calculated, where $\widehat{Y}_i$ - values of center line of fuzzy regression model, Y_i - initial data. The total deviation by this approach is 10.35 that is smaller (1.4 %) than the deviation for fuzzy regression model based on linear programming.

Fig. 8 shows fuzzy regression at $h = 0.0$, and Fig. 10 – fuzzy regression at $h = 0.5$.

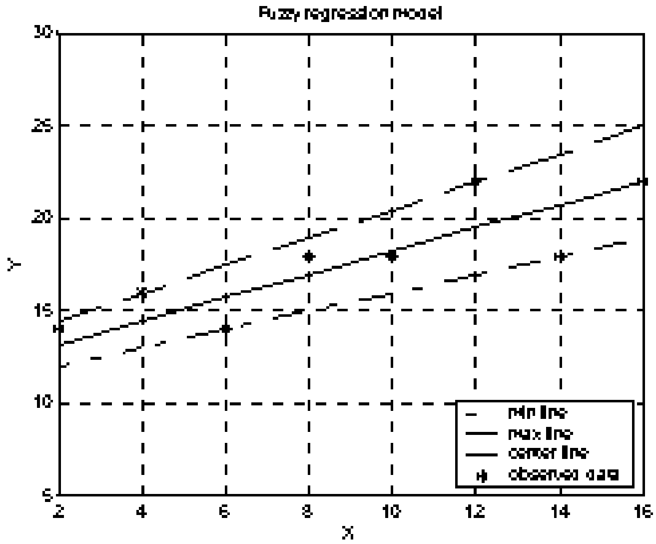

Fig. 8. Fuzzy regression model for crisp X , crisp Y and h = 0.0.

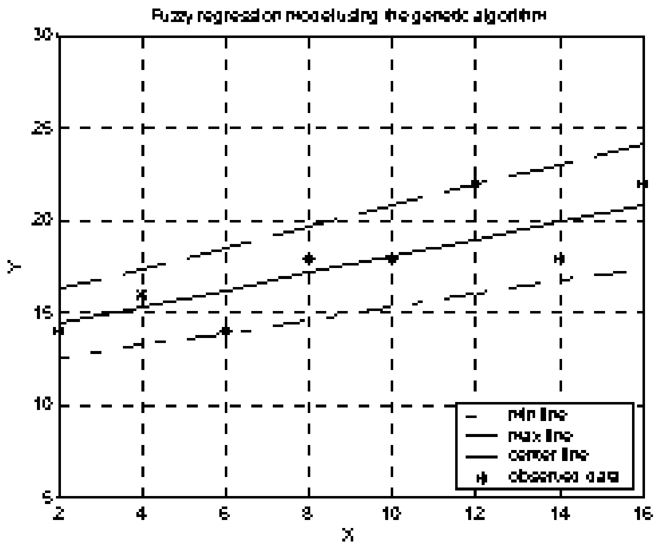

Fig. 9. Fuzzy regression model using the genetic algorithm for crisp X , crisp Y and $h = 0.0$.

Fig. 9 demonstrates fuzzy regression using genetic algorithm at $h = 0.0$, and Fig. 11 – this regression at $h = 0.5$.

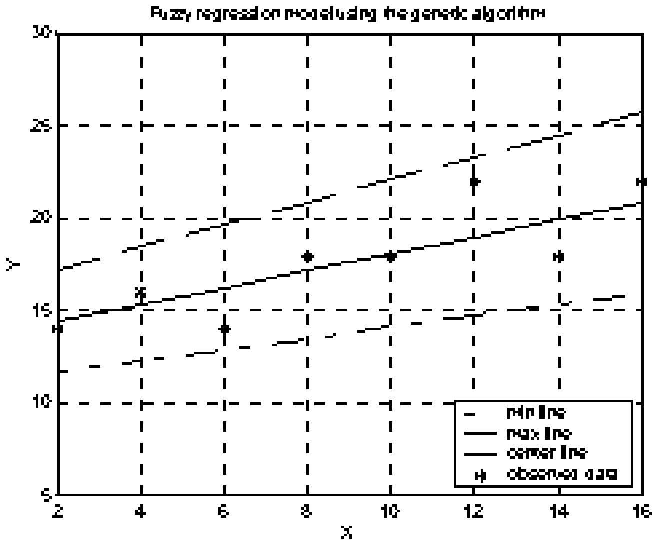

Fig. 10. Fuzzy regression model for crisp X , crisp Y and $h = 0.5$.

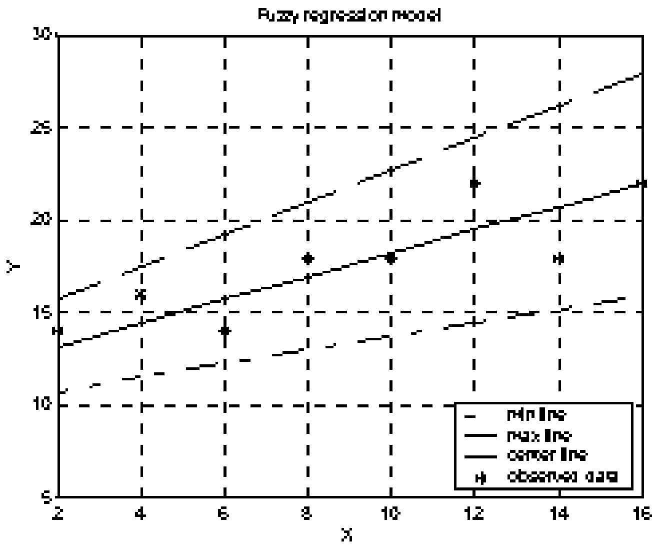

Fig. 11. Fuzzy regression model using the genetic algorithm for crisp X , crisp Y and $h = 0.5$.

Analyzing results, we can draw a conclusion that regressions are similar and differ slightly. It is necessary to note, that fuzzy regression based on genetic algorithm has center line which is more fitting to initial data. It provides more accurate estimations with practically the same bounds of fuzzy models.

As it is shown in Table 2, fuzzy regression models using genetic algorithm are more adequate at h not equal to 0, because then HS_e/S_y smaller, than for fuzzy regression models using simplex-method. Although at h=0 the situation is opposite.

4.2. Results *(is crisp, Y is fuzzy)*

The following results were obtained at fuzzy initial data. Table 2 shows fuzzy regression equations at $h = 0.0$, $h = 0.5$ and values of HR, HS_e, HS_e/S_y, as it was for crisp data, which allows estimating the adequacy of developed models

To compare fuzzy regression model with fuzzy regression model based on genetic algorithm the total deviations $\sum_{i=1}^{n} \left|\widehat{Y}_i - Y_i\right|$ were calculated, where $\widehat{Y}_i$ - values of center line of fuzzy regression model, Y_i - initial data. The total deviation by this approach is 10.35 that is smaller (1.4 %) than the deviation for fuzzy regression model based on linear programming.

In Fig. 12 fuzzy regression at $h = 0.0$ and in Fig. 14 fuzzy regression at $h = 0.5$ are shown.

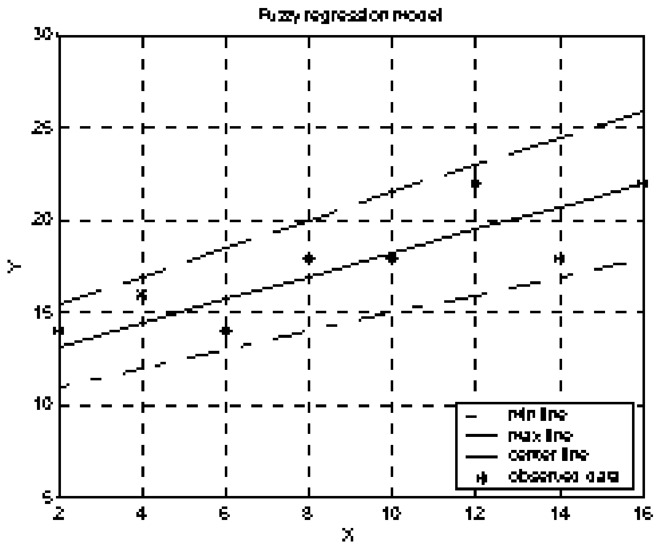

Fig. 12. Fuzzy regression model for crisp X, fuzzy Y and $h = 0.0$.

Fig. 13 and Fig. 15 show fuzzy regression using genetic algorithm at $h = 0.0$, and at $h = 0.5$.

The figures show, that regressions with fuzzy initial data are also very similar. In comparison with crisp data the bounds of regression are extended proportionate coefficient h.

It necessary to point that fuzzy regression using genetic algorithm has better fitting. It provides more accurate estimations with practically the same bounds of fuzzy models.

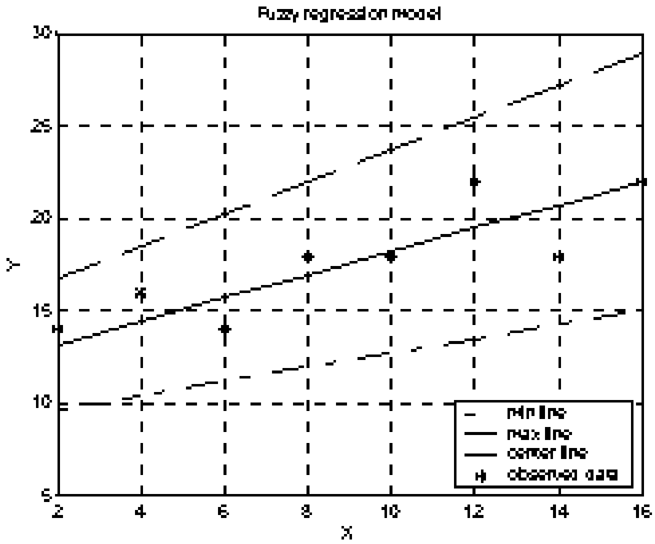

Fig. 13. Fuzzy regression model using the genetic algorithm for crisp X , fuzzy Y and $h = 0.0$.

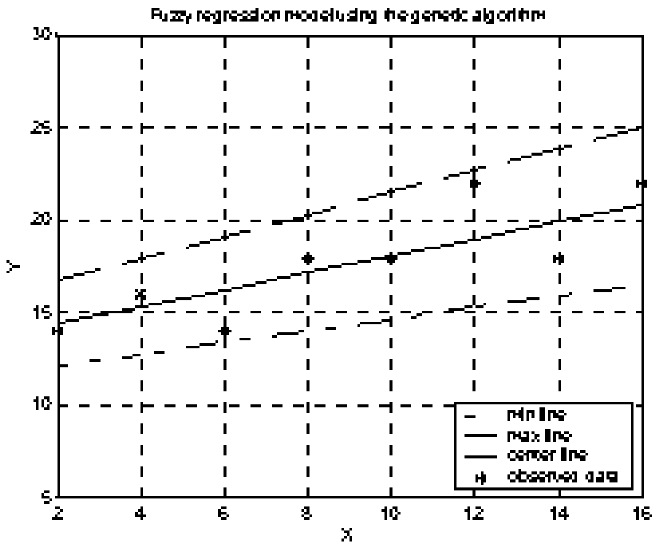

Fig. 14. Fuzzy regression model for crisp X , fuzzy Y and $h = 0.5$.

Also as it is shown in Table 2, fuzzy regression models using genetic algorithm are more adequate at h not equal to 0, because then HS_e/S_y smaller, than for fuzzy regression models using simplex-method. Although at h=0 as it was mentioned the situation is opposite, that is traditional models are more adequate.

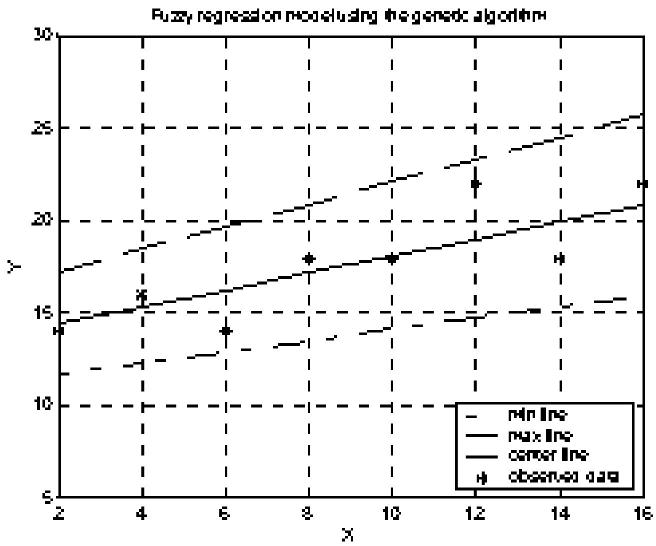

Fig. 15. Fuzzy regression model using the genetic algorithm for crisp X , fuzzy Y and $h = 0.5$.

5 Conclusions

1. All the observed data are within the lines determined by the considered fuzzy regression model but the confidence interval of standard regression model doesn't cover almost the half of observed values of electric power losses.

2. Fuzzy regression analysis based on genetic algorithms determines more accurate center regression line using the same minimum and maximum lines of the model. The total deviation by this approach is smaller (1.4 %) in comparison with fuzzy regression model based on linear programming.

3. Fuzzy regression analysis using genetic algorithm allows to estimate and predict the level of power losses in electrical networks. The minimum and maximum lines of the proposed model can be considered as pessimistic and optimistic plans of events.

4. The observed values of power losses in this paper are crisp. However the proposed method unlike the standard regression can use the fuzzy observed data.

In authors' opinion, these conclusions confirm the possibility and effectiveness of fuzzy regression analysis using genetic algorithms in estimation and prediction of electric power losses in electrical networks and also in any other technical applications.

References

1. Boehme, *Fuzzy-Logik. Einfuehrung in die algebraischen und logischen Grundlagen,* (Berlin, Springer-Verlag, 1993).

2. Yun-Hsi O. Chang, Bilal M. Ayyub, "Fuzzy regression methods - a comparative assessment", *Fuzzy Sets and Systems*, **119** (2001), 187 - 203.

3. H. Tanaka, S. Uejima, K. Asai, "Linear Regression Analysis with Fuzzy Model", *IEEE Transactions on Systems, Man and Cybernetics,* **6** (1982), 903 – 906.

4. H. Tanaka, "Fuzzy data analysis by possibilistic linear models", *Fuzzy Sets and Systems*, **24** (1987), 363 – 375.

5. J. Nazarko, W. Zalewski, "The Fuzzy Regression Approach to Peak Load Estimation in Power Distribution Systems", *IEEE Transactions on Power Systems*, **3** (1999), 809 - 814.

6. S.A. Soliman, Mansour H. Abdel Rahman, M.E. El-Hawary, "Application of fuzzy linear regression algorithm to power system voltage measurements", *Electric Power Systems Research*, **42** (2000), 195 - 200.

7. A.V. Mogilenko, D.A. Pavlyuchenko, V.Z. Manusov, "Development of fuzzy regression models using genetic algorithms", *International journal of uncertainty, fuzzyness and knowledge-based systems*, **4** (2003), 429-444

8. D.E. Goldberg, *Genetic Algorithms in Search, Optimization and Machine Learning.* (Reading, MA, Addison-Wesley, 1989).

9. J. Kacprzyk, M. Fedrizzi, *Fuzzy Regression Analysis.* (Physica-Verlag, Heidelberg, 1992).