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An Application of the Theory of Intutionistic Fuzzy Multigraphs

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Abstract

In a recent paper by one of the authors it has been shown that there is a relationship between algebraic structures and labeled transition systems. Indeed, it has been shown that an algebraic structures can be viewed as labeled transition systems, which can also be viewed as multigraphs. In this paper, we extend this work by providing an estimation of the transition possibilities between vertices that are connected with multiarcs.

1 Introduction

In a recent paper [Syr03] by the third author of this paper, it has been shown that there is a relationship between algebraic structures and labeled transition systems (or LTS, for short). Labeled transition system can be depicted with multigraphs and they are a frequently used model of concurrency [Mil99]. They consist of a set of states and set of transitions from one state to another.

Intuitionistic fuzzy sets have been introduced by K. Atanassov [Ata86] as an extension of fuzzy sets. The idea behind this extension of fuzzy sets is based on the observation, that, in general, given a fuzzy set (M, μ), where $\mu : M \to \mathrm{I}$ and $\mathrm{I} = [0, 1]$, its complement is the fuzzy set $(M, 1 - \mu)$, which in turn isn't at all justified by real world observations. So, one must provide both the membership function and the non-membership function in order to give the complete picture.

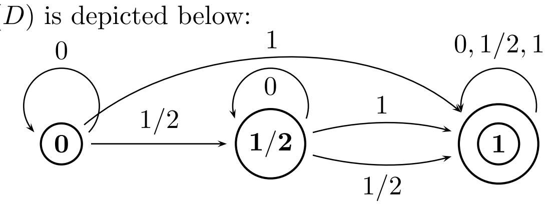
We start by briefly presenting the results of [Syr03] and then we present transition estimations for any multiarc. This of course can have certain consequences, but this is a subject of further research.

2 Algebraic Structures as Labeled Transition Systems

In this section, we brief description of the way one can get a labeled transition system from an algebraic structrure. For reasons of simplicity, we consider algebraic structures of the form $\langle A, \boxplus, \mathbf{0}\rangle$, where $\boxplus$ is binary operation defined over A and for all $a \in A$ it holds that $a \boxplus \mathbf{0} = \mathbf{0} \boxplus a = a$.

In general, if we have an algebraic structure $\langle A, \boxplus, \mathbf{0}\rangle$, then we can generate the following labeled transition system: $(A, \mathbf{0}, A, A_{\boxplus})$, where $(s, a, s') \in A_{\boxplus}$ iff $s \boxplus a = s'$

Example 2.1 If we consider the set $D = \{0, 1/2, 1\}$ and the operations $a \boxplus b = \min(1, a + b)$ and $\neg a = 1 - a$, then $\langle D, \boxplus, \neg, 0\rangle$ is an MV-algebra. The LTS system generated by $\Psi(D)$ is depicted below:



In the above (non-deterministic) automaton we have chosen $\mathbf{1}$ to be the accepting state. Note, that for any MV-algebra the generated LTS can have $\neg\mathbf{0}$ as its accepting state. Of course, we can choose any other node to be the accepting state, but $\neg\mathbf{0}$ is the only one to which there transitions from any other node, while there no transitions from this node to any other node. It is interesting to see what is the accepting language of this automaton. For reasons of clarity, we set $a = 0$, $b = 1/2$ and $c = 1$, then the language generated is $a^* \cdot (c \cdot ((a+b+c)^* \cdot \epsilon) + b \cdot (b^* \cdot (a \cdot ((a+b+c)^* \cdot \epsilon) + b \cdot ((a+b+c)^* \cdot \epsilon))))$.

3 Transition Estimations

It is a fact that most LTSs generated by algebraic structures can be depicted by transition multigraphs. This implies that we actually have a non-deterministic transition system for which it is useful to have a transition estimation. In what follows, we present various transition estimations and define a comparison method.

Let us have a labeled multiarc between vertices v_i and v_j with set of labels $\langle \mu_1^{i,j}, \nu_1^{i,j}\rangle$, $\langle \mu_2^{i,j}, \nu_2^{i,j}\rangle$,..., $\langle \mu_s^{i,j}, \nu_s^{i,j}\rangle$. Then, we can obtain aggregating estimations in the following forms:

i) Optimistic estimation

$$\langle \mu_C^{i,j}, \nu_C^{i,j}\rangle = \langle \max_{1\le k\le s} \mu_k^{i,j}, \min_{1\le k\le s} \nu_k^{i,j}\rangle$$

ii) Pessimistic estimation

$$\langle \mu_I^{i,j}, \nu_I^{i,j}\rangle = \langle \min_{1\le k\le s} \mu_k^{i,j}, \max_{1\le k\le s} \nu_k^{i,j}\rangle$$

iii) Additive estimation

$$\begin{aligned}\langle \mu_{+}^{i,j}, \nu_{+}^{i,j} \rangle &= \langle \sum_{k=1}^{s} \mu_k^{i,j} - \sum_{l=1}^{s-1} \sum_{k=l+1}^{s} \mu_l^{i,j} \mu_k^{i,j} + \\ &+ \sum_{m=1}^{s-2} \sum_{l=m+1}^{s-1} \sum_{k=l+1}^{s} \mu_m^{i,j} \mu_l^{i,j} \mu_k^{i,j}, \\ &\prod_{k=1}^{s} \nu_k^{i,j} \rangle\end{aligned}$$

iv) Multiplicative estimation

$$\begin{aligned}\langle \mu_{\cdot}^{i,j}, \nu_{\cdot}^{i,j} \rangle &= \langle \prod_{k=1}^{s} \mu_k^{i,j}, \sum_{k=1}^{s} \nu_k^{i,j} - \sum_{l=1}^{s-1} \sum_{k=l+1}^{s} \nu_l^{i,j} \nu_k^{i,j} + \\ &+ \sum_{m=1}^{s-2} \sum_{l=m+1}^{s-1} \sum_{k=l+1}^{s} \nu_m^{i,j} \nu_l^{i,j} \nu_k^{i,j} \rangle\end{aligned}$$

Definition 3.1 Suppose that $a, b, c, d \in [0, 1]$ such that $a + b \leq 1$, $c + b \leq 1$. The pair $\langle a, b \rangle$ is *better* than the pair $\langle c, d \rangle$ if and only if $a \geq c$ and $b \leq d$. And we shall write this as $\langle a, b \rangle \geq \langle c, d \rangle$.

Theorem 3.1 *The estimations above can be ordered as follows:*

$$\langle \mu_{+}^{i,j}, \nu_{+}^{i,j} \rangle \geq \langle \mu_{C}^{i,j}, \nu_{C}^{i,j} \rangle \geq \langle \mu_{I}^{i,j}, \nu_{I}^{i,j} \rangle \geq \langle \mu_{\cdot}^{i,j}, \nu_{\cdot}^{i,j} \rangle$$

Proof. The proof follows from the fact that given two number $a, b \in [0, 1]$, then $a \cdot b \leq \min(a, b) \leq \max(a, b) \leq a + b$. □

If the numbers $\alpha, \beta \in [0, 1]$ are fixed and $\alpha + \beta \leq 1$, we can obtain the aggregating (α, β)-estimations in the following forms:

i) Optimistic $C - (\alpha, \beta)$-estimation

$$\langle \max(\alpha, \mu_C^{i,j}), \min(\beta, \nu_C^{i,j}) \rangle = \langle \max(\alpha, \max_{1 \leq k \leq s} \mu_k^{i,j}), \min(\beta, \min_{1 \leq k \leq s} \nu_k^{i,j}) \rangle$$

ii) Pessimistic $C - (\alpha, \beta)$-estimation

$$\langle \min(\alpha, \mu_C^{i,j}), \max(\beta, \nu_C^{i,j}) \rangle = \langle \min(\alpha, \max_{1 \leq k \leq s} \mu_k^{i,j}), \max(\beta, \min_{1 \leq k \leq s} \nu_k^{i,j}) \rangle$$

iii) Optimistic $I - (\alpha, \beta)$-estimation

$$\langle \max(\alpha, \mu_I^{i,j}), \min(\beta, \nu_I^{i,j}) \rangle = \langle \max(\alpha, \min_{1 \leq k \leq s} \mu_k^{i,j}), \min(\beta, \max_{1 \leq k \leq s} \nu_k^{i,j}) \rangle$$

iv) Pessimistic $I-(\alpha,\beta)$-estimation

$$\langle \min(\alpha, \mu_I^{i,j}), \max(\beta, \nu_I^{i,j})\rangle = \langle \min(\alpha, \min_{1\le k\le s} \mu_k^{i,j}), \max(\beta, \max_{1\le k\le s} \nu_k^{i,j})\rangle$$

v) Optimistic additive (α,β)-estimation

$$\begin{aligned}\langle \max(\alpha, \mu_+^{i,j}), \min(\beta, \nu_+^{i,j})\rangle \quad = \quad & \langle \max(\alpha, \sum_{k=1}^{s} \mu_k^{i,j} - \\ & -\sum_{n}^{s-1}\sum_{m}^{s} \mu_n^{i,j}\cdot\mu_m^{i,j} + \\ & +\sum_{n}^{s-2}\sum_{m}^{s-1}\sum_{l}^{s} \mu_n^{i,j}\cdot\mu_m^{i,j}\cdot\mu_l^{i,j}), \\ & \min(\beta, \prod_{n=1}^{s} \nu_n^{i,j})\rangle\end{aligned}$$

vi) Pessimistic additive (α,β)-estimation

$$\begin{aligned}\langle \min(\alpha, \mu_+^{i,j}), \max(\beta, \nu_+^{i,j})\rangle \quad = \quad & \langle \min(\alpha, \sum_{n=1}^{s} \mu_n^{i,j} - \\ & -\sum_{n}^{s-1}\sum_{m}^{s} \mu_n^{i,j}\cdot\mu_m^{i,j} - \\ & -\sum_{n}^{s-2}\sum_{m}^{s-1}\sum_{l}^{s} \mu_n^{i,j}\cdot\mu_m^{i,j}\cdot\mu_l^{i,j}), \\ & \max(\beta, \prod_{l=1}^{s} \nu_l^{i,j})\rangle\end{aligned}$$

vii) Optimistic multiplicative (α,β)-estimation

$$\begin{aligned}\langle \max(\alpha, \mu_{\cdot}^{i,j}), \min(\beta, \nu_{\cdot}^{i,j})\rangle \quad = \quad & \langle \max(\alpha, \prod_{m=1}^{s} \mu_m^{i,j}), \\ & \min(\beta, \sum_{m=1}^{s} \nu_m^{i,j} - \\ & -\sum_{n}^{s-1}\sum_{m}^{s} \nu_n^{i,j}\cdot\nu_m^{i,j} - \\ & -\sum_{n}^{s-2}\sum_{m}^{s-1}\sum_{l}^{s} \nu_n^{i,j}\cdot\nu_m^{i,j}\cdot\nu_l^{i,j})\rangle\end{aligned}$$

viii) Pessimistic multiplicative (α, β)-estimation

$$\begin{aligned}\langle \min(\alpha, \mu_{\cdot}^{i,j}), \max(\beta, \nu_{\cdot}^{i,j})\rangle \quad = \quad & \langle \min(\alpha, \prod_{m=1}^{s} \mu_1^{i,j}), \\ & \max(\beta, \sum_{n=1}^{s} \nu_1^{i,j} - \\ & - \sum_{n}^{s-1} \sum_{m}^{s} \nu_n^{i,j} \cdot \nu_m^{i,j} - \\ & - \sum_{n}^{s-2} \sum_{m}^{s-1} \sum_{l}^{s} \nu_n^{i,j} \cdot \nu_m^{i,j} \cdot \nu_l^{i,j})\rangle\end{aligned}$$

Theorem 3.2 *The estimations above can be ordered as follows:*

$$\begin{aligned}\langle \max(\alpha, \mu_{+}^{i,j}), \min(\beta, \nu_{+}^{i,j})\rangle \quad \geq \quad & \langle \max(\alpha, \mu_{C}^{i,j}), \min(\beta, \nu_{C}^{i,j})\rangle \\ \geq \quad & \langle \max(\alpha, \mu_{I}^{i,j}), \min(\beta, \nu_{I}^{i,j})\rangle \\ \geq \quad & \langle \max(\alpha, \mu_{\cdot}^{i,j}), \min(\beta, \nu_{\cdot}^{i,j})\rangle \\ \geq \quad & \langle \min(\alpha, \mu_{+}^{i,j}), \max(\beta, \nu_{+}^{i,j})\rangle \\ \geq \quad & \langle \min(\alpha, \mu_{C}^{i,j}), \max(\beta, \nu_{C}^{i,j})\rangle \\ \geq \quad & \langle \min(\alpha, \mu_{I}^{i,j}), \max(\beta, \nu_{I}^{i,j})\rangle \\ \geq \quad & \langle \min(\alpha, \mu_{\cdot}^{i,j}), \max(\beta, \nu_{\cdot}^{i,j})\rangle\end{aligned}$$

4 Conclusions

In this paper, we have used the theory of Intuitionistic fuzzy sets to provide an estimation of the transition possibilities between vertices that are connected with multigraphs generated by algebraic structures. Since the multigraphs generated are actually models of concurrency, it is possible to apply the theory to models of concurrency.

References

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