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Dual Commutative Hyper K-Ideals of Type 1 in Hyper K-algebras of Order 3

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Abstract

In this note we classify the bounded hyper K-algebras of order 3, which have $D_1 = \{1\}$, $D_2 = \{1, 2\}$ and $D_3 = \{0, 1\}$ as a dual commutative hyper K-ideal of type 1. In this regard we show that there are such non-isomorphic bounded hyper K-algebras.

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1 Introduction

The hyperalgebraic structure theory was introduced by F. Marty [5] in 1934. Imai and Iseki [3] in 1966 introduced the notion of a BCK-algebra. Borzooei, Jun and Zahedi et.al. [2,8] applied the hyperstructure to BCK-algebras and introduced the concept of hyper K-algebra which is a generalization of BCK-algebra. In [7]we defined the notions of dual commutative hyper K-ideals of type 1 and type 2 (Briefly $DCHKI - T1, T2$). Now we follow it and determine all bounded hyper K-algebras of order 3 which have $DCIHKI - T1$.

2 Preliminaries

Definition 2.1. [2] Let H be a nonempty set and " $\circ$ " be a *hyperoperation* on H, that is " $\circ$ " is a function from $H \times H$ to $\mathcal{P}^*(H) = \mathcal{P}(H)\backslash\{\emptyset\}$. Then H is called a hyper K-algebra if it contains a constant "0" and satisfies the following axioms:
(HK1) $(x \circ z) \circ (y \circ z) < x \circ y$
(HK2) $(x \circ y) \circ z = (x \circ z) \circ y$
(HK3) $x < x$

(HK4) $x < y,\ y < x \Rightarrow x = y$
(HK5) $0 < x$,
for all $x, y, z \in H$, where $x < y$ is defined by $0 \in x \circ y$ and for every $A, B \subseteq H$, $A < B$ is defined by $\exists a \in A$, $\exists b \in B$ such that $a < b$.
Note that if $A, B \subseteq H$, then by $A \circ B$ we mean the subset $\bigcup_{\substack{a \in A \\ b \in B}} a \circ b$ of H.

Theorem 2.2. [2] Let $(H, \circ, 0)$ be a hyper K-algebra. Then for all $x, y, z \in H$ and for all non-empty subsets A, B and C of H the following statements hold:

(i) $x \circ y < z \Leftrightarrow x \circ z < y$,
(ii) $(x \circ z) \circ (x \circ y) < y \circ z$,
(iii) $x \circ (x \circ y) < y$,
(iv) $x \circ y < x$,
(v) $(A \circ C) \circ B = (A \circ B) \circ C$,
(vi) $x \in x \circ 0$,
(vii) $(A \circ C) \circ (A \circ B) < B \circ C$,
(viii) $A \subseteq B$ implies $A < B$,
(ix) $A \circ B < C \Leftrightarrow A \circ C < B$,
(x) $A \circ B < A$.

Definition 2.3. [7] Let D be a non-empty subset of H and $1 \in D$. Then D is called a dual commutative hyper K-ideal of
(i) type 1, if for all $x, y, z \in H$, $N((Nx \circ Ny) \circ Nz) \subseteq D$ and $z \in D$ imply that $N(Nx \circ (Ny \circ (Ny \circ Nx))) \subseteq D$,
(ii) type 2, if for all $x, y, z \in H$, $N((Nx \circ Ny) \circ Nz) < D$ and $z \in D$ imply that $N(Nx \circ (Ny \circ (Ny \circ Nx))) \subseteq D$.
Note that for simplicity of notation we write $DCHKI - T1(T2)$ instead of dual commutative hyper K-ideal of types 1(2).

Theorem 2.4. [7] Let $1 \circ y = \{1\}$; $\forall y \in H - \{1\}$ and $1 \circ 1 = \{0\}$. Then $D = \{1\}$ is a $DCHKI - T1$.

Theorem 2.5. [7] Let $1 \in 1 \circ x$, for all $x \in H$ and $1 \in D \subseteq H$. If $0 \notin D$, then D is a $DCHKI - T1$.

Theorem 2.6. [7] Let $1 \circ y = \{1\}$; $\forall y \in H - \{1\}$, $1 \circ 1 = \{0\}$, $1 \in D \subseteq H$ and $D \neq \{1\}$. Then the following statements are equivalent:
(i) $0 \in D$,
(ii) D is a $DCHKI - T1$,

Theorem 2.7. [8] There are 220 non-isomorphic bounded hyper K-algebras of order 3, to have $D = \{0, 1\}$ as a $DPIHKI - T3$.

Definition 2.8. [2] Let H_1 and H_2 be two hyper K-algebras. A mapping $f : H_1 \longrightarrow H_2$ is said to be a homomorphism if:
(i) $f(0) = 0$,
(ii) $f(x \circ y) = f(x) \circ f(y)$, $\forall x, y \in H_1$.
if f is both 1-1 and onto, we say that f is an isomorphism.

3 $DCHKI-T1$ in Hyper K-Algebras of Order 3

Henceforth we let $H=\{0,1,2\}$ be a bounded hyper K-algebra of order 3 with unit 1 and $D_1=\{1\}$, $D_2=\{1,2\}$ and $D_3=\{0,1\}$ be subsets of H.

Theorem 3.1. Let $1\circ 1=\{0\}$ and $1\circ 2=\{1\}$. Then
(i) D_1 and D_3 are $DCHKI-T1$.
(ii) D_2 is not a $DCHKI-T1$.

Proof. The proofs of (i) and (ii) follow from Theorems 2.4 and 2.6.

Theorem 3.2. Let $1\circ 1=\{0\}$ and $1\circ 2=\{1\}$. Then there are exactly 40 non-isomorphic bounded hyper K-algebras of order 3 which have D_1 and D_3 as a $DCHKI-T1$.

Proof. The proof follows from the proof of Theorem 2.7.

Theorem 3.3. Let $1\circ 1=\{0\}$ and $1\circ 2=\{2\}$. Then
(i) $2\circ 0=\{2\}$, $0\circ 0=\{0\}$ and $2\circ 1=0\circ 2$,
(ii) D_1 is a $DCHKI-T1$ if and only if $2\circ 1\neq\{0\}$ or $2\circ 2=\{0\}$,
(iii) D_2 is not a $DCHKI-T1$,
(iv) D_3 is a $DCHKI-T1$ if and only if $2\in 2\circ 1$ and $2\notin 0\circ 1$.

Proof. (i) By $(HK2)$ and hypothesis we conclude that $2\circ 0=(1\circ 2)\circ 0=(1\circ 0)\circ 2=1\circ 2=\{2\}$, $0\circ 0=(1\circ 1)\circ 0=(1\circ 0)\circ 1=1\circ 1=\{0\}$ and $2\circ 1=(1\circ 2)\circ 1=(1\circ 1)\circ 2=0\circ 2$.

(ii) Let D_1 be a $DCHKI-T1$. Then we prove that $2\circ 1\neq\{0\}$ or $2\circ 2=\{0\}$. On the contrary, let $2\circ 1=\{0\}$ and $2\circ 2\neq\{0\}$. Then $1\circ(((1\circ 2)\circ(1\circ 0))\circ(1\circ 1))=1\circ((2\circ 1)\circ 0)=1\circ(0\circ 0)=1\circ 0=\{1\}=D_1$ and $1\in D_1$, while $1((1\circ 2)\circ((1\circ 0)\circ((1\circ 0)\circ(1\circ 2))))\nsubseteq D_1$. So D_1 is not a $DCHKI-T1$, which is a contradiction.

Conversely, let $2\circ 1\neq\{0\}$ or $2\circ 2=\{0\}$. Then by (i), hypothesis and some manipulations we get that D_1 is a $DCHKI-T1$.

(iii) By (i) and hypothesis we have $1\circ(((1\circ 0)\circ(1\circ 1))\circ(1\circ 2))=1\circ((1\circ 0)\circ 2)=1\circ 2=\{2\}\subseteq D_2$ and $2\in D_2$, while $0\in 1\circ((1\circ 0)\circ((1\circ 1)\circ((1\circ 1)\circ(1\circ 0))))$ and so $1\circ((1\circ 0)\circ((1\circ 1)\circ((1\circ 1)\circ(1\circ 0))))\nsubseteq D_2$. Therefore D_2 is not a $DCHKI-T1$.
(v) The proof is similar to (ii).

Theorem 3.4. Let $1\circ 1=\{0\}$ and $1\circ 2=\{2\}$. Then:
(i) There are exactly 20 non-isomorphic bounded hyper K-algebras to have D_1 as a $DCHKI-T1$.
(ii) There are exactly 5 non-isomorphic bounded hyper K-algebras to have D_3 as a $DCHKI-T1$.

Proof. (i) By some manipulations and Theorem 3.3(i) and (ii) we conclude that there are 20 bounded hyper K-algebras of order 3 which have D_1 as a $DCHKI-T1$. These hyper K-algebras are

H_1	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1\}$
1	$\{1\}$	$\{0\}$	$\{2\}$
2	$\{2\}$	$\{0,1\}$	$\{0\}$

H_2	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1\}$
1	$\{1\}$	$\{0\}$	$\{2\}$
2	$\{2\}$	$\{0,1\}$	$\{0,1\}$

H_3	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1\}$
1	$\{1\}$	$\{0\}$	$\{2\}$
2	$\{2\}$	$\{0,1\}$	$\{0,2\}$

H_4	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1\}$
1	$\{1\}$	$\{0\}$	$\{2\}$
2	$\{2\}$	$\{0,1\}$	$\{0,1,2\}$

H_5	0	1	2
0	$\{0\}$	$\{0\}$	$\{0,2\}$
1	$\{1\}$	$\{0,1\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,2\}$

H_6	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,1\}$

H_7	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,1,2\}$

H_8	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0\}$

H_9	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,2\}$

H_{10}	0	1	2
0	$\{0\}$	$\{0,1\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{11}	0	1	2
0	$\{0\}$	$\{0,1\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{12}	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{13}	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{14}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0\}$

H_{15}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1\}$

H_{16}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{17}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{18}	0	1	2
0	$\{0\}$	$\{0\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{19}	0	1	2
0	$\{0\}$	$\{0\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{20}	0	1	2
0	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0\}$	$\{2\}$
2	$\{2\}$	$\{0\}$	$\{0\}$

Now we show that non of each pair of the above 20 hyper K-algebras is isomorphic together. On the contrary let there exists an isomorphism $f : H_i \rightarrow H_j$, for $i \neq j$. So $f(x \circ y) = f(x) \circ f(y)$, for all $x, y \in H$. Clearly f is not identity, thus we have $f(0) = 0$, $f(1) = 2$, $f(2) = 1$. But $f(1 \circ 2) = f(\{2\}) = \{1\}$ and $f(1) \circ f(2) = 2 \circ 1 \supseteq \{0\}$, which is a contradiction, since $0 \notin f(1 \circ 2) = \{1\}$.
(ii) We can see that H_5, H_{10}, H_{11}, H_{18} and H_{19} satisfy the conditions of Theorem 3.3(i) and (iv) and so there are exactly 5 non-isomorphism bounded hyper

K-algebras of order 3 which have D_3 as a $DCHKI-T1$.

Theorem 3.5. Let $1 \circ 1 = \{0\}$ and $1 \circ 2 = \{1, 2\}$. Then
(i) $0 \circ 0 = \{0\}$ and $0 \circ 2 = 2 \circ 1$,
(ii) D_1 is a $DCHKI-T1$ if and only if $2 \circ 1 \neq \{0\}$,
(iii) D_2 is a $DCHKI-T1$ if and only if $2 \circ 1 \neq \{0\}$ and ($2 \circ 1 \neq \{0, 2\}$ or $1 \in 2 \circ 2$),
(iv) D_3 is a $DCHKI-T1$ if and only if $2 \in 2 \circ 1$ and $2 \notin 0 \circ 1$.

Proof. (i) By $(HK2)$ and hypothesis we have $0 \circ 2 = (1 \circ 1) \circ 2 = (1 \circ 2) \circ 1 = \{1, 2\} \circ 1 = (1 \circ 1) \bigcup (2 \circ 1) = \{0\} \bigcup 2 \circ 1 = 2 \circ 1$ and $0 \circ 0 = (1 \circ 1) \circ 0 = (1 \circ 0) \circ 1 = 1 \circ 1 = \{0\}$.
(ii) $\Rightarrow$ Let D_1 be a $DCHKI-T1$ and on the contrary, let $2 \circ 1 = \{0\}$. Then $1 \circ (((1 \circ 2) \circ (1 \circ 0)) \circ (1 \circ 1)) = 1 \circ ((\{1, 2\} \circ 1) \circ 0) = 1 \circ 0 = \{1\}$ and $1 \in D_1$, while $0 \in 1 \circ ((1 \circ 2) \circ ((1 \circ 0) \circ ((1 \circ 0) \circ (1 \circ 2))))$. So D_1 is not a $DCHKI-T1$, which is a contradiction.
$\Leftarrow$ Conversely, let $2 \circ 1 \neq \{0\}$. By (i), hypothesis and some manipulations we get that $N(N0 \circ (N0 \circ (N0 \circ N0))) = N(N1 \circ (N0 \circ (N0 \circ N1))) = N(N1 \circ (N1 \circ (N1 \circ N1))) = \{1\}$. Also $N((Nx \circ Ny) \circ N1) \not\subseteq \{1\}$, while $x = 0$ and $y \in \{1, 2\}$, $x = 1$ and $y = 2$ or $x = 2$ and $y \in \{0, 1, 2\}$. Therefore D_1 is a $DCHKI-T1$.
(iii) $\Rightarrow$ Let D_2 be a $DCHKI-T1$. Then we prove that $2 \circ 1 \neq \{0\}$ and ($2 \circ 1 \neq \{0, 2\}$ or $1 \in 2 \circ 2$). On the contrary, let $2 \circ 1 = \{0\}$ or ($2 \circ 1 = \{0, 2\}$ and $1 \notin 2 \circ 2$). If $2 \circ 1 = \{0\}$, then $1 \circ (((1 \circ 2) \circ (1 \circ 0)) \circ (1 \circ 1)) = \{1\} \subseteq D_2$ and $1 \in D_2$, while $0 \in 1 \circ ((1 \circ 2) \circ ((1 \circ 0) \circ ((1 \circ 0) \circ (1 \circ 2))))$. Thus D_2 is not a $DCHKI-T1$, which is a contradiction.
If $2 \circ 1 = \{0, 2\}$ and $1 \notin 2 \circ 2$, then by $(HK2)$ and (i) we have $0 \circ 1 \subseteq (2 \circ 2) \circ 1 = (2 \circ 1) \circ 2 = (0 \circ 2) \bigcup (2 \circ 2) \subseteq \{0, 2\}$. So $1 \circ (((1 \circ 2) \circ (1 \circ 0)) \circ (1 \circ 2)) = \{1, 2\} = D_2$ and $2 \in D_2$, while $0 \in 1 \circ ((1 \circ 2) \circ ((1 \circ 0) \circ ((1 \circ 0) \circ (1 \circ 2))))$. Thus D_2 is not a $DCHKI-T1$, which is a contradiction.
$\Leftarrow$ Conversely, let $2 \circ 1 \neq \{0\}$ and ($2 \circ 1 \neq \{0, 2\}$ or $1 \in 2 \circ 2$). Then by some manipulations we get that D_2 is a $DCHKI-T1$.
(iv) The proof is similar to (ii) and (iii).

Theorem 3.6. Let $1 \circ 1 = \{0\}$ and $1 \circ 2 = \{1, 2\}$. Then:
(i) There are exactly 30 non-isomorphic bounded hyper K-algebras to have D_1 as a $DCHKI-T1$.
(ii) There are exactly 27 non-isomorphic bounded hyper K-algebras to have D_2 as a $DCHKI-T1$.
(iii) There are exactly 10 non-isomorphic bounded hyper K-algebras to have D_3 as a $DCHKI-T1$.

Proof. (i) By some manipulations and Theorem 3.5 (i) and (ii) we conclude that there are exactly 30 bounded hyper K-algebras of order 3 which satisfy the above conditions and each of them has D_1 as a $DCHKI-T1$. Also similar to the proof of Theorem 3.4 we can prove that they are non-isomorphic hyper K-algebras. These hyper K-algebras are

H_1	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1\}$
1	$\{1\}$	$\{0\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1\}$	$\{0\}$

H_2	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1\}$
1	$\{1\}$	$\{0\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1\}$	$\{0,1\}$

H_3	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1\}$
1	$\{1\}$	$\{0\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1\}$	$\{0,2\}$

H_4	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1\}$
1	$\{1\}$	$\{0\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1\}$	$\{0,1,2\}$

H_5	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1\}$
1	$\{1\}$	$\{0\}$	$\{1,2\}$
2	$\{1,2\}$	$\{0,1\}$	$\{0,2\}$

H_6	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1\}$
1	$\{1\}$	$\{0\}$	$\{1,2\}$
2	$\{1,2\}$	$\{0,1\}$	$\{0,1,2\}$

H_7	0	1	2
0	$\{0\}$	$\{0\}$	$\{0,2\}$
1	$\{1\}$	$\{0\}$	$\{1,2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,2\}$

H_8	0	1	2
0	$\{0\}$	$\{0,1\}$	$\{0,2\}$
1	$\{1\}$	$\{0\}$	$\{1,2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,1,2\}$

H_9	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0\}$	$\{1,2\}$
2	$\{2\}$	$\{0,2\}$	$\{0\}$

H_{10}	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0\}$	$\{1,2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,2\}$

H_{11}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0\}$	$\{1,2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,1,2\}$

H_{12}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0\}$	$\{1,2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,1\}$

H_{13}	0	1	2
0	$\{0\}$	$\{0\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{14}	0	1	2
0	$\{0\}$	$\{0\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{15}	0	1	2
0	$\{0\}$	$\{0\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0\}$	$\{1,2\}$
2	$\{1,2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{16}	0	1	2
0	$\{0\}$	$\{0\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0\}$	$\{1,2\}$
2	$\{1,2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{17}	0	1	2
0	$\{0\}$	$\{0,1\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{18}	0	1	2
0	$\{0\}$	$\{0,1\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{19}	0	1	2
0	$\{0\}$	$\{0,1\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0\}$	$\{1,2\}$
2	$\{1,2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{20}	0	1	2
0	$\{0\}$	$\{0,1\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0\}$	$\{1,2\}$
2	$\{1,2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{21}	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{22}	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{23}	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0\}$	$\{1,2\}$
2	$\{1,2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{24}	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0\}$	$\{1,2\}$
2	$\{1,2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{25}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0\}$

H_{26}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1\}$

H_{27}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{28}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{29}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0\}$	$\{1,2\}$
2	$\{1,2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{30}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0\}$	$\{1,2\}$
2	$\{1,2\}$	$\{0,1,2\}$	$\{0,1,2\}$

(ii) it is easy to see that all of the above hyper K-algebras satisfy the conditions of Theorem 3.5(iii) except of H_7, H_9 and H_{10}. Thus there are exactly 27 non-isomorphic bounded hyper K-algebras to have D_2 as a $DCHKI-T1$.
(iii) We can check that H_7, H_8, H_{13}, H_{14}, H_{15}, H_{16}, H_{17}, H_{18}, H_{19} and H_{20} satisfy the conditions of Theorem 3.5(iv) and so there are exactly 10 non-isomorphic bounded hyper K-algebras to have D_3 as a $DCHKI-T1$.

Theorem 3.7. Let $1\circ 1=\{0,1\}$ and $1\circ 2=\{2\}$. Then
(i) $2\circ 0=\{2\}$, $2\circ 1=(0\circ 2)\bigcup\{2\}$ and $0\circ 0\subseteq\{0,1\}$,
(ii) D_1 is a $DCHKI-T1$,
(iii) D_2 is a $DCHKI-T1$ if and if $1\in(2\circ 2)\bigcap(0\circ 2)$,
(iv) D_3 is a $DCHKI-T1$ if and if $(2\circ 2)\neq\{0,1\}$ or $2\in 0\circ 1$.

Proof. (i) By hypothesis and $(HK2)$ we have $2\circ 0=(1\circ 2)\circ 0=(1\circ 0)\circ 2=1\circ 2=\{2\}$, $2\circ 1=(1\circ 2)\circ 1=(1\circ 1)\circ 2=\{0,1\}\circ 2=(0\circ 2)\bigcup\{2\}$ and $0\circ 0\subseteq(1\circ 1)\circ 0=(1\circ 0)\circ 1=1\circ 1=\{0,1\}$.
(ii) By (i) and some manipulations we get that $N((Nx\circ Ny)\circ N1)\not\subseteq D_1$, for all $x,y\in H$ except $x=y=2$. But for $x=y=2$, consider two cases:
(a) $2\circ 2=\{0\}$, (b) $2\circ 2\neq\{0\}$.
(a) Let $2\circ 2=\{0\}$. Then $N(N2\circ(N2\circ(N2\circ N2)))\subseteq D_1$.
(b) Let $2\circ 2\neq\{0\}$. Then $N((N2\circ N2)\circ N1)\not\subseteq D_1$.
Therefore D_1 is a $DCHKI-T1$.
(iii) $\Rightarrow$ Let D_2 be a $DCHKI-T1$. We prove that $1\in(2\circ 2)\bigcap(0\circ 2)$. On the contrary, let $1\notin 2\circ 2$ or $1\notin 0\circ 2$. If $1\notin 2\circ 2$, then $1\circ(((1\circ 0)\circ(1\circ 2))\circ(1\circ 2))=1\circ(2\circ 2)\subseteq 1\circ\{0,2\}=\{1,2\}=D_2$ and $2\in D_2$, while $0\in N(N0\circ(N2\circ(N2\circ N0)))$. Hence D_2 is not a $DCHKI-T1$, which is a contradiction.
If $1\notin 0\circ 2$, then $1\circ(((1\circ 0)\circ(1\circ 0))\circ(1\circ 2))=1\circ(\{0,1\}\circ 2)=1\circ\{0,2\}=\{1,2\}=D_2$. Now similar to above, we conclude that D_2 is not a $DCHKI-T1$, which is a contradiction.
$\Leftarrow$ Conversely, let $1\in(2\circ 2)\bigcap(0\circ 2)$. Then by some manipulations we get that

$N((Nx \circ Ny) \circ Nz) \nsubseteq D_2$, for all $x, y \in H$ and $z \in D_2$.
(iv) The proof is similar to (iii).

Theorem 3.8. Let $1 \circ 1 = \{0, 1\}$ and $1 \circ 2 = \{2\}$. Then:
(i) There are exactly 37 non-isomorphic bounded hyper K-algebras to have D_1 as a $DCHKI - T1$.
(ii) There are exactly 16 non-isomorphic bounded hyper K-algebras to have D_1 as a $DCHKI - T1$.
(iii) There are exactly 36 non-isomorphic bounded hyper K-algebras to have D_3 as a $DCHKI - T1$.

Proof. (i) By some manipulations and Theorem 3.7 (i) and (ii) we conclude that there are 37 bounded hyper K-algebras of order 3 which satisfy the above conditions and each of them has D_1 as a $DCHKI - T1$. Also similar to the proof of Theorem 3.4, we can see that they are non-isomorphic hyper K-algebras. These hyper K-algebras are

H_1	0	1	2
0	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0,1\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0\}$

H_2	0	1	2
0	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0,1\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,1\}$

H_3	0	1	2
0	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0,1\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,2\}$

H_4	0	1	2
0	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0,1\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,1,2\}$

H_5	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0\}$
1	$\{1\}$	$\{0,1\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,2\}$

H_6	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0\}$
1	$\{1\}$	$\{0,1\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,1,2\}$

H_7	0	1	2
0	$\{0\}$	$\{0\}$	$\{0,2\}$
1	$\{1\}$	$\{0,1\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,1,2\}$

H_8	0	1	2
0	$\{0\}$	$\{0\}$	$\{0,2\}$
1	$\{1\}$	$\{0,1\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,2\}$

H_9	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,1\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0\}$

H_{10}	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,1\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,2\}$

H_{11}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,1\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,1\}$

H_{12}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,1\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,1,2\}$

H_{13}	0	1	2
0	$\{0\}$	$\{0\}$	$\{0,1\}$
1	$\{1\}$	$\{0,1\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{14}	0	1	2
0	$\{0\}$	$\{0\}$	$\{0,1\}$
1	$\{1\}$	$\{0,1\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{15}	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,1\}$
1	$\{1\}$	$\{0,1\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{16}	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,1\}$
1	$\{1\}$	$\{0,1\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{17}	0	1	2
0	{0}	{0, 1, 2}	{0, 1}
1	{1}	{0, 1}	{2}
2	{2}	{0, 1, 2}	{0}

H_{18}	0	1	2
0	{0}	{0, 1, 2}	{0, 1}
1	{1}	{0, 1}	{2}
2	{2}	{0, 1, 2}	{0, 1}

H_{19}	0	1	2
0	{0}	{0, 1, 2}	{0, 1}
1	{1}	{0, 1}	{2}
2	{2}	{0, 1, 2}	{0, 2}

H_{20}	0	1	2
0	{0}	{0, 1, 2}	{0, 1}
1	{1}	{0, 1}	{2}
2	{2}	{0, 1, 2}	{0, 1, 2}

H_{21}	0	1	2
0	{0}	{0}	{0, 1, 2}
1	{1}	{0, 1}	{2}
2	{2}	{0, 1, 2}	{0, 2}

H_{22}	0	1	2
0	{0}	{0}	{0, 1, 2}
1	{1}	{0, 1}	{2}
2	{2}	{0, 1, 2}	{0, 1, 2}

H_{23}	0	1	2
0	{0}	{0, 1}	{0, 1, 2}
1	{1}	{0, 1}	{2}
2	{2}	{0, 1, 2}	{0, 2}

H_{24}	0	1	2
0	{0}	{0, 1}	{0, 1, 2}
1	{1}	{0, 1}	{2}
2	{2}	{0, 1, 2}	{0, 1, 2}

H_{25}	0	1	2
0	{0}	{0, 2}	{0, 1, 2}
1	{1}	{0, 1}	{2}
2	{2}	{0, 1, 2}	{0, 1}

H_{26}	0	1	2
0	{0}	{0, 2}	{0, 1, 2}
1	{1}	{0, 1}	{2}
2	{2}	{0, 1, 2}	{0, 2}

H_{27}	0	1	2
0	{0}	{0, 2}	{0, 1, 2}
1	{1}	{0, 1}	{2}
2	{2}	{0, 1, 2}	{0, 1, 2}

H_{28}	0	1	2
0	{0}	{0, 1, 2}	{0, 1, 2}
1	{1}	{0, 1}	{2}
2	{2}	{0, 1, 2}	{0}

H_{29}	0	1	2
0	{0}	{0, 1, 2}	{0, 1, 2}
1	{1}	{0, 1}	{2}
2	{2}	{0, 1, 2}	{0, 1}

H_{30}	0	1	2
0	{0}	{0, 1, 2}	{0, 1, 2}
1	{1}	{0, 1}	{2}
2	{2}	{0, 1, 2}	{0, 2}

H_{31}	0	1	2
0	{0}	{0, 1, 2}	{0, 1, 2}
1	{1}	{0, 1}	{2}
2	{2}	{0, 1, 2}	{0, 1, 2}

H_{32}	0	1	2
0	{0, 1}	{0}	{0, 1, 2}
1	{1}	{0, 1}	{2}
2	{2}	{0, 1, 2}	{0, 1, 2}

H_{33}	0	1	2
0	{0, 1}	{0, 1}	{0, 1, 2}
1	{1}	{0, 1}	{2}
2	{2}	{0, 1, 2}	{0, 1, 2}

H_{34}	0	1	2
0	{0, 1}	{0, 2}	{0, 1, 2}
1	{1}	{0, 1}	{2}
2	{2}	{0, 1, 2}	{0, 1}

H_{35}	0	1	2
0	{0, 1}	{0, 2}	{0, 1, 2}
1	{1}	{0, 1}	{2}
2	{2}	{0, 1, 2}	{0, 1, 2}

H_{36}	0	1	2
0	{0, 1}	{0, 1, 2}	{0, 1, 2}
1	{1}	{0, 1}	{2}
2	{2}	{0, 1, 2}	{0, 1}

H_{37}	0	1	2
0	{0}	{0, 1, 2}	{0, 1, 2}
1	{1}	{0, 1}	{2}
2	{2}	{0, 1, 2}	{0, 1, 2}

(ii) We can see that H_{14}, H_{16}, H_{18}, H_{20}, H_{22}, H_{24}, H_{25}, H_{27}, H_{29}, H_{31}, H_{32}, H_{33}, H_{34}, H_{35}, H_{36} and H_{37} satisfy the conditions of Theorem 3.7(iii). Thus there are exactly 16 non-isomorphic bounded hyper K-algebras to have D_2 as a

$DCHKI-T1$.
(iii) It is seen that except of H_2 all of the above hyper K-algebras satisfy the conditions of Theorem 3.7(iv). Thus there are exactly 36 non-isomorphic bounded hyper K-algebras to have D_3 as a $DCHKI-T1$.

Theorem 3.9. Let $1\circ 1=\{0,1,2\}$ and $1\circ 2=\{2\}$. Then
(i) $2\circ 0=\{2\}$ and $2\circ 1=(0\circ 2)\bigcup(\{2\})\bigcup(2\circ 2)$,
(ii) D_1 and D_3 are $DCHKI-T1$.
(iii) D_2 is a $DCHKI-T1$ if and only if $2\circ 2=\{0,1,2\}$ or $(2\circ 2=\{0,1\}$ and $1\in 0\circ 2)$.

Proof. (i) By $(HK2)$ we get that $2\circ 0=(1\circ 2)\circ 0=(1\circ 0)\circ 2=1\circ 2=\{2\}$ and $2\circ 1=(1\circ 2)\circ 1=(1\circ 1)\circ 2=(0\circ 2)\bigcup(1\circ 2)\bigcup(2\circ 2)=(0\circ 2)\bigcup(\{2\})\bigcup(2\circ 2)$.
(ii) The proof is similar to the proof of Theorem 3.7(ii).
(iii) $\Rightarrow$ Let D_2 be a $DCHKI-T1$. Then we prove that $2\circ 2=\{0,1,2\}$ or $(2\circ 2=\{0,1\}$ and $1\in 0\circ 2)$. On the contrary, let $2\circ 2\neq\{0,1,2\}$ and $(2\circ 2\neq\{0,1\}$ or $1\notin 0\circ 2)$. Let $2\circ 2=\{0,1\}$ and $0\circ 2\subseteq\{0,2\}$. Then $N((N2\circ N2)\circ N2)\subseteq 1\circ(\{0,2\})=\{1,2\}=D_2$ and $2\in D_2$, while $0\in N(N2\circ(N2\circ(N2\circ N2)))$. Thus D_2 is not a $DCHKI-T1$, which is a contradiction.
Let $2\circ 2\subseteq\{0,2\}$ and $1\in 0\circ 2$. Then $N((N0\circ N2)\circ N2)\subseteq D_2$ and $2\in D_2$, while $0\in N(N0\circ(N2\circ(N2\circ N0)))$. So D_2 is not a $DCHKI-T1$, which is a contradiction.
Let $(2\circ 2)\bigcup(0\circ 2)\subseteq\{0,2\}$. Then $N((N0\circ N0)\circ N2)\subseteq D_2$ and $2\in D_2$, while $0\in N(N0\circ(N0\circ(N0\circ N0)))$. So D_2 is not a $DCHKI-T1$, which is a contradiction. Therefore D_2 is a $DCHKI-T1$.
$\Leftarrow$ Conversely, let $2\circ 2=\{0,1,2\}$ or $(2\circ 2=\{0,1\}$ and $1\in 0\circ 2)$. Then by some manipulations we get that $N((Nx\circ Ny)\circ Nz)\nsubseteq D_2$, for all $x,y\in H$ and $z\in D_2$. Therefore D_2 is a $DCHKI-T1$.

Theorem 3.10. Let $1\circ 1=\{0,1,2\}$ and $1\circ 2=\{2\}$. Then:
(i) There are exactly 63 non-isomorphic bounded hyper K-algebras to have both D_1 and D_3 as $DCHKI-T1$.
(ii) There are exactly 36 non-isomorphic bounded hyper K-algebras to have D_2 as a $DCHKI-T1$.

Proof. (i) By some manipulations and Theorem 3.9 (i) and (ii) we conclude that there are 63 bounded hyper K-algebras of order 3 which satisfy the above conditions and each of them has D_1 and D_3 as $DCHKI-T1$. Also similar to the proof of Theorem 3.4 we can prove that they are non-isomorphic hyper K-algebras. These hyper K-algebras are

H_1	0	1	2
0	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0\}$

H_2	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,2\}$

H_3	0	1	2
0	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,2\}$

H_4	0	1	2
0	$\{0,2\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,2\}$

H_5	0	1	2
0	$\{0,2\}$	$\{0,2\}$	$\{0\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,2\}$

H_6	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0\}$

H_7	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,2\}$

H_8	0	1	2
0	$\{0\}$	$\{0\}$	$\{0,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,2\}$

H_9	0	1	2
0	$\{0,2\}$	$\{0\}$	$\{0,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,2\}$

H_{10}	0	1	2
0	$\{0,2\}$	$\{0,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,2\}$

H_{11}	0	1	2
0	$\{0\}$	$\{0,1\}$	$\{0,1\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1\}$

H_{12}	0	1	2
0	$\{0\}$	$\{0,1\}$	$\{0,1\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{13}	0	1	2
0	$\{0,2\}$	$\{0,1\}$	$\{0,1\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{14}	0	1	2
0	$\{0\}$	$\{0,1\}$	$\{0,1\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{15}	0	1	2
0	$\{0,2\}$	$\{0,1\}$	$\{0,1\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{16}	0	1	2
0	$\{0,1,2\}$	$\{0,1\}$	$\{0,1\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{17}	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,1\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{18}	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,1\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{19}	0	1	2
0	$\{0,1,2\}$	$\{0,2\}$	$\{0,1\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{20}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0\}$

H_{21}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1\}$

H_{22}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{23}	0	1	2
0	$\{0,2\}$	$\{0,1,2\}$	$\{0,1\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{24}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{25}	0	1	2
0	$\{0,2\}$	$\{0,1,2\}$	$\{0,1\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{26}	0	1	2
0	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{27}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0\}$

H_{28}	0	1	2
0	$\{0\}$	$\{0\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1\}$

H_{29}	0	1	2
0	$\{0\}$	$\{0,1\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1\}$

H_{30}	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1\}$

H_{31}	0	1	2
0	$\{0,1\}$	$\{0,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1\}$

H_{32}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1\}$

H_{33}	0	1	2
0	$\{0,1\}$	$\{0,1,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1\}$

H_{34}	0	1	2
0	$\{0\}$	$\{0\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{35}	0	1	2
0	$\{0\}$	$\{0,1\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{36}	0	1	2
0	$\{0,2\}$	$\{0,1\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{37}	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{38}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{39}	0	1	2
0	$\{0,2\}$	$\{0,1,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{40}	0	1	2
0	$\{0\}$	$\{0\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{41}	0	1	2
0	$\{0,1,2\}$	$\{0\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{42}	0	1	2
0	$\{0\}$	$\{0,1\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{43}	0	1	2
0	$\{0,2\}$	$\{0,1\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{44}	0	1	2
0	$\{0,1,2\}$	$\{0,1\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{45}	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{46}	0	1	2
0	$\{0,1,2\}$	$\{0,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{47}	0	1	2
0	$\{0,1\}$	$\{0,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{48}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{49}	0	1	2
0	$\{0,2\}$	$\{0,1,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{50}	0	1	2
0	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{51}	0	1	2
0	$\{0,1\}$	$\{0,1,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{52}	0	1	2
0	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1\}$

H_{53}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1\}$

H_{54}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1\}$

H_{55}	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1\}$

H_{56}	0	1	2
0	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{57}	0	1	2
0	$\{0,1,2\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{58}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{59}	0	1	2
0	$\{0,1,2\}$	$\{0,1,2\}$	$\{0\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{60}	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{61}	0	1	2
0	$\{0,1,2\}$	$\{0,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{62}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{63}	0	1	2
0	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,1,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

(ii) We can see that H_{11}, H_{14}, H_{15}, H_{16}, H_{18}, H_{19}, H_{21}, H_{24}, H_{25}, H_{26}, H_{28}, H_{29} H_{30}, H_{31}, H_{32}, H_{33}, H_{40}, H_{41}, H_{42}, H_{43}, H_{44}, H_{45}, H_{46}, H_{47}, H_{48}, H_{49}, H_{50}, H_{51}, H_{56}, H_{57}, H_{58}, H_{59}, H_{60}, H_{61}, H_{62} and H_{63} satisfy the conditions of Theorem 3.9(iii) and so there are 36 non-isomorphic bounded hyper K-algebras of order 3 which have D_2 as a $DCHKI-T1$.

Theorem 3.11. Let $1\circ 1=\{0,2\}$ and $1\circ 2=\{1\}$. Then
(i) $(0\circ 2)\bigcup(2\circ 2)$=$\{0,2\}$, $0\circ 0\subseteq\{0,2\}$ and $2\circ 0=\{2\}$,
(ii) D_1 is a $DCHKI-T1$,
(iii) D_2 and D_3 are not $DCHKI-T1$.

Proof. (i) By $(HK2)$ and hypothesis we get that $\{0,2\}=(1\circ 2)\circ 1=(1\circ 1)\circ 2=(0\circ 2)\bigcup(2\circ 2)$ and $(0\circ 0)\bigcup(2\circ 0)=(1\circ 1)\circ 0=(1\circ 0)\circ 1=1\circ 1=\{0,2\}$. Thus $(2\circ 0)=\{2\}$ and $(0\circ 0)\subseteq\{0,2\}$.
(ii) By some manipulations we get that $N(Nx\circ(Ny\circ(Ny\circ Nx)))=\{1\}$, in each of the following cases:
(a) $x,y\in\{0,2\}$, (b) $x=1$ and $y=2$, (c) $x=1$ and $y\in\{0,1\}$.
Also $N((N0\circ N1)\circ N1)=N((N2\circ N1)\circ N1)=\{0,2\}\not\subseteq D_1$. Therefore D_1 is a $DCHKI-T1$.
(iii) We prove theorem for D_3, the proof of D_2 is similar to D_3.
Since $N((N0\circ N1)\circ N0)=\{1\}\subseteq D_3$, $0\in D_3$ and $\{0,2\}\subseteq N(N0\circ(N1\circ(N1\circ N0)))$, then D_3 is not a $DCHKI-T1$.

Theorem 3.12. Let $1 \circ 1 = \{0, 2\}$ and $1 \circ 2 = \{1\}$. Then there are exactly 26 non-isomorphic bounded hyper K-algebras to have D_1 as a $DCHKI - T1$.

Proof. By some manipulations and Theorem 3.11 (i) and (ii) we conclude that there are 26 bounded hyper K-algebras of order 3 which satisfy the above conditions and each of them has D_1 as a $DCHKI - T1$. Also similar to the proof of Theorem 3.4 we can prove that they are non-isomorphic bounded hyper K-algebras. These hyper K-algebras are

H_1	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1\}$
2	$\{2\}$	$\{0\}$	$\{0\}$

H_2	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1\}$
2	$\{2\}$	$\{0,2\}$	$\{0\}$

H_3	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1\}$
2	$\{2\}$	$\{0,1\}$	$\{0\}$

H_4	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0\}$

H_5	0	1	2
0	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0,2\}$	$\{1\}$
2	$\{2\}$	$\{0\}$	$\{0,2\}$

H_6	0	1	2
0	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0,2\}$	$\{1\}$
2	$\{2\}$	$\{0,1\}$	$\{0,2\}$

H_7	0	1	2
0	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0,2\}$	$\{1\}$
2	$\{2\}$	$\{0,2\}$	$\{0,2\}$

H_8	0	1	2
0	$\{0,2\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0,2\}$	$\{1\}$
2	$\{2\}$	$\{0,2\}$	$\{0,2\}$

H_9	0	1	2
0	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0,2\}$	$\{1\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{10}	0	1	2
0	$\{0\}$	$\{0,1\}$	$\{0\}$
1	$\{1\}$	$\{0,2\}$	$\{1\}$
2	$\{2\}$	$\{0,1\}$	$\{0,2\}$

H_{11}	0	1	2
0	$\{0\}$	$\{0,1\}$	$\{0\}$
1	$\{1\}$	$\{0,2\}$	$\{1\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{12}	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0\}$
1	$\{1\}$	$\{0,2\}$	$\{1\}$
2	$\{2\}$	$\{0,2\}$	$\{0,2\}$

H_{13}	0	1	2
0	$\{0,2\}$	$\{0,2\}$	$\{0\}$
1	$\{1\}$	$\{0,2\}$	$\{1\}$
2	$\{2\}$	$\{0,2\}$	$\{0,2\}$

H_{14}	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0\}$
1	$\{1\}$	$\{0,2\}$	$\{1\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{15}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0\}$
1	$\{1\}$	$\{0,2\}$	$\{1\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{16}	0	1	2
0	$\{0,2\}$	$\{0,1,2\}$	$\{0\}$
1	$\{1\}$	$\{0,2\}$	$\{1\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{17}	0	1	2
0	$\{0\}$	$\{0\}$	$\{0,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1\}$
2	$\{2\}$	$\{0,2\}$	$\{0,2\}$

H_{18}	0	1	2
0	$\{0,2\}$	$\{0\}$	$\{0,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1\}$
2	$\{2\}$	$\{0,2\}$	$\{0,2\}$

H_{19}	0	1	2
0	$\{0\}$	$\{0,1\}$	$\{0,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{20}	0	1	2
0	$\{0,2\}$	$\{0,1\}$	$\{0,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{21}	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1\}$
2	$\{2\}$	$\{0\}$	$\{0,2\}$

H_{22}	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1\}$
2	$\{2\}$	$\{0,2\}$	$\{0,2\}$

H_{23}	0	1	2
0	$\{0,2\}$	$\{0,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1\}$
2	$\{2\}$	$\{0,2\}$	$\{0,2\}$

H_{24}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1\}$
2	$\{2\}$	$\{0,1\}$	$\{0,2\}$

H_{25}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{26}	0	1	2
0	$\{0,2\}$	$\{0,1,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

Theorem 3.13. Let $1 \circ 1 = \{0,2\}$ and $1 \circ 2 = \{2\}$. Then
(i) $2 \circ 0 = \{2\}$, $0 \circ 0 \subseteq \{0,2\}$ and $2 \circ 1 = (0 \circ 2) \bigcup (2 \circ 2)$,
(ii) D_1 (D_3) is a $DCHKI - T1$ if and only if $2 \circ 1 \neq \{0\}$,
(iii) If $1 \notin 2 \circ 2$, then D_2 is not a $DCHKI - T1$,
(iv) If $2 \circ 2 = \{0,1,2\}$, then D_2 is a $DCHKI - T1$,
(v) If $2 \circ 2 = \{0,1\}$, then D_2 is a $DCHKI - T1$ if and only if $1 \in 0 \circ 2$.

Proof. (i) By $(HK2)$ and hypothesis we get that $2 \circ 0 = (1 \circ 2) \circ 0 = (1 \circ 0) \circ 2 = 1 \circ 2 = \{2\}$, $\{0,2\} = 1 \circ 1 = (1 \circ 0) \circ 1 = (1 \circ 1) \circ 0 = (2 \circ 0) \bigcup (0 \circ 0)$ and so $0 \circ 0 \subseteq \{0,2\}$. Also $2 \circ 1 = (1 \circ 2) \circ 1 = (1 \circ 1) \circ 2 = (0 \circ 2) \bigcup (2 \circ 2)$.
(ii) $\Rightarrow$ Let D_1 be a $DCHKI - T1$. Then we prove that $2 \circ 1 \neq \{0\}$. On the contrary, let $2 \circ 1 = \{0\}$. Then by (i) we get that $0 \circ 2 = 2 \circ 2 = \{0\}$ and $0 \circ 0 = (2 \circ 2) \circ 0 = (2 \circ 0) \circ 2 = 2 \circ 2 = \{0\}$. Also $0 \circ 1 \subseteq (2 \circ 2) \circ 1 = (2 \circ 1) \circ 2 = 0 \circ 2 = \{0\}$. Thus $N((N1 \circ N0) \circ N1) = \{1\} = D_1$, while $2 \in N(N1 \circ (N0 \circ (N0 \circ N1)))$. Hence D_1 is not a $DCHKI - T1$, which is a contradiction.
$\Leftarrow$ Conversely, let $2 \circ 1 \neq \{0\}$. Then $2 \in N((Nx \circ Ny) \circ N1)$, for all $x, y \in H$ and so $N((Nx \circ Ny) \circ N1) \nsubseteq D_1$, for all $x, y \in H$. Therefore D_1 is a $DCHKI - T1$. The proof of D_3 is similar to above.
(iii) Let $1 \notin 2 \circ 2$. Then $1 \circ (((1 \circ 0) \circ (1 \circ 1)) \circ (1 \circ 2)) = 1 \circ ((1 \circ \{0,2\}) \circ 2) = \{1,2\} = D_2$ and $2 \in D_2$, while $0 \in N(N0 \circ (N1 \circ (N1 \circ N0)))$. Thus D_2 is not a $DCHKI - T1$.
(iv) Let $2 \circ 2 = \{0,1,2\}$. Then by some calculations and (i) we have $N((Nx \circ Ny) \circ Nz) \nsubseteq D_2$, for all $x, y \in H$ and $z \in D_2$. Therefore D_2 is a $DCHKI - T1$.
(v) $\Rightarrow$ Let $2 \circ 2 = \{0,1\}$ and D_2 be a $DCHKI - T1$. Then we show that $1 \in 0 \circ 2$. On the contrary, let $1 \notin 0 \circ 2$. Then $0 \circ 2 \subseteq \{0,2\}$. Thus $N((N2 \circ N2) \circ N2) = D_2$ and $2 \in D_2$, while $0 \in N(N2 \circ (N2 \circ (N2 \circ N2)))$. Hence D_2 is not a $DCHKI - T1$, which is a contradiction.
$\Leftarrow$ Conversely, let $1 \in 0 \circ 2$. Then by some manipulations and (i) we conclude that $N((Nx \circ Ny) \circ Nz) \nsubseteq D_2$, for all $x, y \in H$ and $z \in D_2$. Therefore D_2 is a $DCHKI - T1$.

Theorem 3.14. Let $1 \circ 1 = \{0,2\}$ and $1 \circ 2 = \{2\}$. Then:
(i) There are exactly 43 non-isomorphic bounded hyper K-algebras to have both D_1 and D_3 as $DCHKI - T1$.
(ii) There are exactly 20 non-isomorphic bounded hyper K-algebras to have D_2 as a $DCHKI - T1$.

Proof. (i) By some manipulations and Theorem 3.13 (i) and (ii) we conclude that there are 43 bounded hyper K-algebras of order 3 which satisfy the above conditions and each of them has D_1 and D_3 as $DCHKI-T1$. Also similar to the proof of Theorem 3.4 we can prove that they are non-isomorphic bounded hyper K-algebras. These hyper K-algebras are

H_1	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1\}$	$\{0\}$

H_2	0	1	2
0	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1\}$	$\{0,1\}$

H_3	0	1	2
0	$\{0\}$	$\{0,1\}$	$\{0,1\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1\}$	$\{0,1\}$

H_4	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1\}$	$\{0,1\}$

H_5	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0\}$

H_6	0	1	2
0	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,2\}$

H_7	0	1	2
0	$\{0,2\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,2\}$

H_8	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,2\}$

H_9	0	1	2
0	$\{0,2\}$	$\{0,2\}$	$\{0\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,2\}$

H_{10}	0	1	2
0	$\{0\}$	$\{0\}$	$\{0,2\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,2\}$

H_{11}	0	1	2
0	$\{0,2\}$	$\{0\}$	$\{0,2\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,2\}$

H_{12}	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,2\}$

H_{13}	0	1	2
0	$\{0,2\}$	$\{0,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,2\}$

H_{14}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0\}$

H_{15}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1\}$

H_{16}	0	1	2
0	$\{0\}$	$\{0,1\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1\}$

H_{17}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1\}$

H_{18}	0	1	2
0	$\{0\}$	$\{0,1\}$	$\{0,1\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{19}	0	1	2
0	$\{0,2\}$	$\{0,1\}$	$\{0,1\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{20}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{21}	0	1	2
0	$\{0,2\}$	$\{0,1,2\}$	$\{0,1\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{22}	0	1	2
0	$\{0\}$	$\{0\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{23}	0	1	2
0	$\{0\}$	$\{0,1\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{24}	0	1	2
0	$\{0,2\}$	$\{0,1\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{25}	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{26}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{27}	0	1	2
0	$\{0,2\}$	$\{0,1,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{28}	0	1	2
0	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{29}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{30}	0	1	2
0	$\{0,2\}$	$\{0,1,2\}$	$\{0\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{31}	0	1	2
0	$\{0\}$	$\{0,1\}$	$\{0,1\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{32}	0	1	2
0	$\{0,2\}$	$\{0,1\}$	$\{0,1\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{33}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{34}	0	1	2
0	$\{0,2\}$	$\{0,1,2\}$	$\{0,1\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{35}	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{36}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{37}	0	1	2
0	$\{0,2\}$	$\{0,1,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{38}	0	1	2
0	$\{0\}$	$\{0\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{39}	0	1	2
0	$\{0\}$	$\{0,1\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{40}	0	1	2
0	$\{0,2\}$	$\{0,1\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{41}	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{42}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{43}	0	1	2
0	$\{0,2\}$	$\{0,1,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,2\}$	$\{2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

(ii) It is easy to check that H_3, H_4, H_{16}, H_{17}, H_{28}, H_{29}, H_{30}, H_{31}, H_{32}, H_{33}, H_{34}, H_{35},H_{36}, H_{37}, H_{38}, H_{39}, H_{40}, H_{41}, H_{42} and H_{43} satisfy the conditions of Theorem 3.13 (iv) and (v) and so there are exactly 20 non-isomorphic bounded hyper K-algebras of order 3.

Theorem 3.15. Let $1 \circ 1 = \{0, 2\}$ and $1 \circ 2 = \{1, 2\}$. Then
(i) $(0 \circ 2) \bigcup (2 \circ 2) = \{0, 2\} \bigcup (2 \circ 1)$, $0 \circ 0 \subseteq \{0, 2\}$ and $2 \circ 0 = \{2\}$,
(ii) D_1 (D_3) is a $DCHKI - T1$ if and only if $2 \circ 1 \neq \{0\}$ or $0 \circ 2 \neq \{0\}$,
(iii) D_2 is a $DCHKI - T1$ if and only if $1 \in 2 \circ 1$.

Proof. (i) By $(HK2)$ we get that $\{0, 2\} \bigcup (2 \circ 1) = (1 \circ 2) \circ 1 = (1 \circ 1) \circ 2 = (0 \circ 2) \bigcup (2 \circ 2)$, $(2 \circ 0) \bigcup (0 \circ 0) = (1 \circ 1) \circ 0 = (1 \circ 0) \circ 1 = 1 \circ 1 = \{0, 2\}$. Thus $2 \circ 0 = \{2\}$ and $0 \circ 0 \subseteq \{0, 2\}$.
(ii) $\Rightarrow$ Let D_1 be a $DCHKI - T1$. Then we show that $2 \circ 1 \neq \{0\}$ or $0 \circ 2 \neq \{0\}$. On the contrary, let $2 \circ 1 = \{0\}$ and $0 \circ 2 = \{0\}$. Then by (i) we get that $2 \circ 2 = \{0, 2\}$. Also by $(HK2)$ and (1) we have $0 \circ 1 \subseteq (2 \circ 2) \circ 1 = (2 \circ 1) \circ 2 = 0 \circ 2 = \{0\}$ and so $0 \circ 1 = \{0\}$. $(0 \circ 0) \circ 1 = (0 \circ 1) \circ 0 = 0 \circ 0$ implies that $0 \circ 0 = \{0\}$. Thus $N((N1 \circ N0) \circ N1) = D_1$, while $2 \in N(N1 \circ (N0 \circ (N0 \circ N1)))$. So D_1 is not a $DCHKI - T1$, which is a contradiction.
$\Leftarrow$ Conversely, let $2 \circ 1 \neq \{0\}$ or $0 \circ 2 \neq \{0\}$. Then by some manipulations and (i) we get that $N((Nx \circ Ny) \circ N1) \not\subseteq D_1$, for all $x, y \in H$.
The proof of D_3 is the same as D_1.
(iii) The proof is similar to (ii).

Theorem 3.16. Let $1 \circ 1 = \{0, 2\}$ and $1 \circ 2 = \{1, 2\}$. Then:
(i) There are exactly 61 non-isomorphic bounded hyper K-algebras to have both D_1 and D_3 as $DCHKI - T1$.
(ii) There are exactly 50 non-isomorphic bounded hyper K-algebras to have D_2 as a $DCHKI - T1$.

Proof. (i) By some manipulations and Theorem 3.15 (i) and (ii) we conclude that there are 61 bounded hyper K-algebras of order 3 which satisfy the above conditions and each of them has D_1 and D_3 as $DCHKI - T1$ and also by the proof of Theorem 3.4 they are non-isomorphic. These hyper K-algebras are

H_1	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0\}$	$\{0\}$

H	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0\}$	$\{0,2\}$

H_3	0	1	2
0	$\{0\}$	$\{0\}$	$\{0,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,2\}$

H_4	0	1	2
0	$\{0,2\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,2\}$	$\{0,2\}$

H_5	0	1	2
0	{0}	{0, 2}	{0}
1	{1}	{0, 2}	{1, 2}
2	{2}	{0, 2}	{0, 2}

H_6	0	1	2
0	{0, 2}	{0, 2}	{0}
1	{1}	{0, 2}	{1, 2}
2	{2}	{0, 2}	{0, 2}

H_7	0	1	2
0	{0}	{0, 2}	{0, 2}
1	{1}	{0, 2}	{1, 2}
2	{2}	{0, 2}	{0}

H_8	0	1	2
0	{0}	{0}	{0, 2}
1	{1}	{0, 2}	{1, 2}
2	{2}	{0, 2}	{0, 2}

H_9	0	1	2
0	{0, 2}	{0}	{0, 2}
1	{1}	{0, 2}	{1, 2}
2	{2}	{0, 2}	{0, 2}

H_{10}	0	1	2
0	{0}	{0, 2}	{0, 2}
1	{1}	{0, 2}	{1, 2}
2	{2}	{0, 2}	{0, 2}

H_{11}	0	1	2
0	{0, 2}	{0, 2}	{0, 2}
1	{1}	{0, 2}	{1, 2}
2	{2}	{0, 2}	{0, 2}

H_{12}	0	1	2
0	{0}	{0}	{0}
1	{1}	{0, 2}	{1, 2}
2	{2}	{0, 1}	{0, 1, 2}

H_{13}	0	1	2
0	{0}	{0, 1, 2}	{0}
1	{1}	{0, 2}	{1, 2}
2	{2}	{0, 1}	{0, 1, 2}

H_{14}	0	1	2
0	{0}	{0, 2}	{0, 1}
1	{1}	{0, 2}	{1, 2}
2	{2}	{0, 1}	{0, 2}

H_{15}	0	1	2
0	{0}	{0, 1, 2}	{0, 1}
1	{1}	{0, 2}	{1, 2}
2	{2}	{0, 1}	{0, 2}

H_{16}	0	1	2
0	{0}	{0, 1}	{0, 1}
1	{1}	{0, 2}	{1, 2}
2	{2}	{0, 1}	{0, 1, 2}

H_{17}	0	1	2
0	{0}	{0, 1, 2}	{0, 1}
1	{1}	{0, 2}	{1, 2}
2	{2}	{0, 1}	{0, 1, 2}

H_{18}	0	1	2
0	{0}	{0, 1, 2}	{0, 2}
1	{1}	{0, 2}	{1, 2}
2	{2}	{0, 1}	{0, 1}

H_{19}	0	1	2
0	{0}	{0, 2}	{0, 2}
1	{1}	{0, 2}	{1, 2}
2	{2}	{0, 1}	{0, 1, 2}

H_{20}	0	1	2
0	{0}	{0, 1, 2}	{0, 2}
1	{1}	{0, 2}	{1, 2}
2	{2}	{0, 1}	{0, 1, 2}

H_{21}	0	1	2
0	{0}	{0, 1, 2}	{0, 1, 2}
1	{1}	{0, 2}	{1, 2}
2	{2}	{0, 1}	{0}

H_{22}	0	1	2
0	{0}	{0, 1}	{0, 1, 2}
1	{1}	{0, 2}	{1, 2}
2	{2}	{0, 1}	{0, 1}

H_{23}	0	1	2
0	{0}	{0, 1, 2}	{0, 1, 2}
1	{1}	{0, 2}	{1, 2}
2	{2}	{0, 1}	{0, 1}

H_{24}	0	1	2
0	{0}	{0, 2}	{0, 1, 2}
1	{1}	{0, 2}	{1, 2}
2	{2}	{0, 1}	{0, 2}

H_{25}	0	1	2
0	{0}	{0, 1, 2}	{0, 1, 2}
1	{1}	{0, 2}	{1, 2}
2	{2}	{0, 1}	{0, 2}

H_{26}	0	1	2
0	{0}	{0}	{0, 1, 2}
1	{1}	{0, 2}	{1, 2}
2	{2}	{0, 1}	{0, 1, 2}

H_{27}	0	1	2
0	{0}	{0, 1}	{0, 1, 2}
1	{1}	{0, 2}	{1, 2}
2	{2}	{0, 1}	{0, 1, 2}

H_{28}	0	1	2
0	{0}	{0, 2}	{0, 1, 2}
1	{1}	{0, 2}	{1, 2}
2	{2}	{0, 1}	{0, 1, 2}

H_{29}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1\}$	$\{0,1,2\}$

H_{30}	0	1	2
0	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{31}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{32}	0	1	2
0	$\{0\}$	$\{0,1\}$	$\{0,1\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{33}	0	1	2
0	$\{0,2\}$	$\{0,1\}$	$\{0,1\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{34}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{35}	0	1	2
0	$\{0,2\}$	$\{0,1,2\}$	$\{0,1\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{36}	0	1	2
0	$\{0\}$	$\{0,1\}$	$\{0,1\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{37}	0	1	2
0	$\{0,2\}$	$\{0,1\}$	$\{0,1\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{38}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{39}	0	1	2
0	$\{0,2\}$	$\{0,1,2\}$	$\{0,1\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{40}	0	1	2
0	$\{0\}$	$\{0,1\}$	$\{0,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1\}$

H_{41}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1\}$

H_{42}	0	1	2
0	$\{0\}$	$\{0,1\}$	$\{0,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{43}	0	1	2
0	$\{0,2\}$	$\{0,1\}$	$\{0,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{44}	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{45}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{46}	0	1	2
0	$\{0,2\}$	$\{0,1,2\}$	$\{0,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{47}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0\}$

H_{48}	0	1	2
0	$\{0\}$	$\{0,1\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1\}$

H_{49}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1\}$

H_{50}	0	1	2
0	$\{0\}$	$\{0\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{51}	0	1	2
0	$\{0\}$	$\{0,1\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{52}	0	1	2
0	$\{0,2\}$	$\{0,1\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{53}	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{54}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{55}	0	1	2
0	$\{0,2\}$	$\{0,1,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,2\}$

H_{56}	0	1	2
0	$\{0\}$	$\{0\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{57}	0	1	2
0	$\{0\}$	$\{0,1\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{58}	0	1	2
0	$\{0,2\}$	$\{0,1\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{59}	0	1	2
0	$\{0\}$	$\{0,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{60}	0	1	2
0	$\{0\}$	$\{0,1,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

H_{61}	0	1	2
0	$\{0,2\}$	$\{0,1,2\}$	$\{0,1,2\}$
1	$\{1\}$	$\{0,2\}$	$\{1,2\}$
2	$\{2\}$	$\{0,1,2\}$	$\{0,1,2\}$

(ii) We can see that the above hyper K-algebras from H_{12} to H_{61} satisfy the conditions of Theorem 3.15(iii) and so there 50 non-isomorphic bounded hyper K-algebras of order 3 which have D_2 as a $DCHKI-T1$.

Theorem 3.17. Let $1 \in (1 \circ 1) \bigcap (1 \circ 2)$. Then D_1, D_2 and D_3 are $DCHKI-T1$.

Proof. By Theorem 2.5, D_1 and D_2 are $DCHKI-T1$. Now we prove D_3 is a $DCHKI-T1$ too. Consider two cases: (i) $2 \in 1 \circ 1$ (ii) $2 \notin 1 \circ 1$
(i) Let $2 \in 1 \circ 1$. Also $1 \in 1 \circ 1$ implies that $2 \in 1 \circ 1 \subseteq N((Nx \circ Ny) \circ Nz)$, for all $x, y, z \in H$. Thus $N((Nx \circ Ny) \circ Nz) \nsubseteq D_3$, for all $x, y \in H$ and $z \in D_3$. So D_3 is a $DCHKI-T1$.
(ii) Let $2 \notin 1 \circ 1$. Then $1 \circ 1 = \{0, 1\}$. Now consider two cases: (a) $2 \notin 1 \circ 2$ (b) $2 \in 1 \circ 2$
(a) if $2 \notin 1 \circ 2$, then by some manipulations we get that $N(Nx \circ (Ny \circ (Ny \circ Nx))) \subseteq D_3$, for all $x, y \in H$.
(b) If $2 \in 1 \circ 2$, then $1 \circ 2 = \{1, 2\}$ and also by hypothesis and $(HK2)$ we have $(1 \circ 1) \bigcup (2 \circ 1) = (1 \circ 2) \circ 1 = (1 \circ 1) \circ 2 = (0 \circ 2) \bigcup (1 \circ 2) = \{0, 1, 2\}$. Thus $2 \in 2 \circ 1$. Hence we get that $N((N0 \circ N2) \circ Nz)$, $N((N1 \circ N2) \circ Nz)$, $N((N2 \circ N0) \circ Nz)$, $N((N2 \circ N1) \circ Nz)$ and $N((N2 \circ N2) \circ Nz) \nsubseteq D_3$, for all $z \in D_3$. Also $N((Nx \circ Ny) \circ Nz) \subseteq D_3$ implies that $N(Nx \circ (Ny \circ (Ny \circ Nx))) \subseteq D_3$, for all $x, y \in \{0, 1\}$ and $z \in D_3$. Therefore D_3 is a $DCHKI-T1$.

Theorem 3.18. Let $1 \in (1 \circ 1) \bigcap (1 \circ 2)$. Then :
(i) if $1 \circ 1 = \{0, 1\}$ and $1 \circ 2 = \{1\}$, then there are 180 non-isomorphic hyper K-ideals which have D_1, D_2 and D_3 as $DCHKI-T1$.

(ii) if $1 \circ 1 = \{0,1\}$ and $1 \circ 2 = \{1,2\}$, then there are 120 non-isomorphic hyper K-ideals which have D_1, D_2 and D_3 as $DCHKI - T1$.
(iii) if $1 \circ 1 = \{0,1,2\}$ and $1 \circ 2 = \{1\}$, then there are 316 non-isomorphic hyper K-ideals which have D_1, D_2 and D_3 as $DCHKI - T1$.
(iv) if $1 \circ 1 = \{0,1,2\}$ and $1 \circ 2 = \{1,2\}$, then there are 402 non-isomorphic hyper K-ideals which have D_1, D_2 and D_3 as $DCHKI - T1$.

Proof. (i) The proof follows from the proof of Theorem 2.7.
By hypothesis and some manipulations we can see that there are exactly 120 in part (ii), 316 in part (iii) and 402 in part (iv) non-isomorphic hyper K-algebras of order 3 which have D_1, D_2 and D_3 as $DCHKI - T1$.
But to avoid the increasing of the pages of this paper, we refer the readers to http://math.uk.ac.ir/ zahedi.

Conclusion: The above results show that there are exactly 1338(1167, 1276) non-isomorphic bounded hyper K-algebras of order 3, in which they have D_1 (D_2, D_3) as a $DCHKI - T1$.

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