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Constraining the neutrino magnetic dipole moment from white dwarf pulsations

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Abstract. Pulsating white dwarf stars can be used as astrophysical laboratories to constrain the properties of weakly interacting particles. Comparing the cooling rates of these stars with the expected values from theoretical models allows us to search for additional sources of cooling due to the emission of axions, neutralinos, or neutrinos with magnetic dipole moment. In this work, we derive an upper bound to the neutrino magnetic dipole moment (μ_ν) using an estimate of the rate of period change of the pulsating DB white dwarf star PG 1351+489. We employ state-of-the-art evolutionary and pulsational codes which allow us to perform a detailed asteroseismological period fit based on fully DB white dwarf evolutionary sequences. Plasmon neutrino emission is the dominant cooling mechanism for this class of hot pulsating white dwarfs, and so it is the main contributor to the rate of change of period with time ($\dot{\Pi}$) for the DBV class. Thus, the inclusion of an anomalous neutrino emission through a non-vanishing magnetic dipole moment in these sequences notably influences the evolutionary timescales, and also the expected pulsational properties of the DBV stars. By comparing the theoretical $\dot{\Pi}$ value with the rate of change of period with time of PG 1351+489, we assess the possible existence of additional cooling by neutrinos with magnetic dipole moment. Our models suggest the existence of some additional cooling in this pulsating DB white dwarf, consistent with a non-zero magnetic dipole moment with an upper limit of $\mu_\nu \lesssim 10^{-11}\mu_{\rm B}$. This bound is somewhat less restrictive than, but still compatible with, other limits inferred from the white dwarf luminosity function or from the color-magnitude diagram of the Globular cluster M5. Further improvements of the measurement of the rate of period change of the dominant pulsation mode of PG 1351+489 will be necessary to confirm our bound.

Keywords: Stars: white dwarfs, stars: oscillations, stars: asteroseismology, stars: evolution, astroparticle physics, neutrinos

1 Introduction

The existence of neutrinos was first postulated by W. Pauli in 1930 to explain the conservation of energy, momentum, and angular momentum in β-decays [1]. According to the Standard Model of particle physics, neutrinos are massless, electrically neutral, have zero dipole moment, and zero decay rate. In order to explain the observed neutrino mixing beyond the Standard Model, it was necessary to extend this simple picture, allowing for non-vanishing neutrino masses and mixings, electromagnetic couplings, neutrino decays and other effects [1].

Neutrino emission takes place in a number of astrophysical contexts and constitutes an energy-loss channel in a variety of stellar configurations, from low-mass red giants and horizontal branch stars to white dwarfs, neutron stars and core-collapse supernovae. Our Sun also emits neutrinos, but in this case the neutrino emission is a by-product of nuclear fusion. For advanced stages of stellar evolution, like red giants and white dwarfs, neutrinos are produced by thermal effects, without nuclear reactions being involved[1]. Specifically, neutrino emission in pre-white dwarf and hot white dwarf stars is the result of different scattering processes, with plasmon decay process $[\gamma \rightarrow \overline{\nu}\nu]$ and bremsstrahlung $[e^-(Ze) \rightarrow (Ze)e^- \, \overline{\nu}\nu]$ the most relevant ones. Plasmon decay process is an important neutrino emission

[1]For the Sun, thermal neutrino emission is found to be irrelevant [1].

mechanism for a broad range of temperatures and densities, even though neutrinos do not couple directly to photons[2]. It has been speculated, however, that neutrinos could have non-trivial electromagnetic properties, in particular a non-zero magnetic dipole moment (μ_ν), allowing the plasmon emission process to be much more efficient. In this case, the non-vanishing magnetic dipole moment results in a *direct* coupling between neutrinos and the electromagnetic field (photons). It has been shown [2] that it is possible to use the observed properties of stars to constrain the possible amount of anomalous energy loss and thus the neutrino electromagnetic properties [3]. The neutrino magnetic dipole moment was first calculated in 1980 [4].

For both pre-white dwarf and hot white dwarf stars, neutrino emission is more important than surface photon cooling, and hence, neutrino losses essentially control the evolutionary timescale [1, 5, 6]. In particular, plasmon neutrino emission is the dominant cooling mechanism in hot white dwarfs down to an effective temperature between $\sim 31\,000$ K and $\sim 23\,000$ K, depending on the value of the stellar mass. Since the existence of a neutrino magnetic dipole moment enhances the plasmon neutrino losses, it is interesting to investigate the effects that a non-zero μ_ν have on the evolution of white dwarfs. In principle, this can be done by employing the white dwarf luminosity function (WDLF). Neutrino cooling causes a depression (the "neutrino dip") of the WDLF at the bright end, which would be enhanced by additional cooling caused by a magnetic dipole moment, allowing to derive a limit on μ_ν. Employing this approach (see ref. [7] for the original idea), it was found $\mu_\nu \lesssim 10^{-11}\mu_{\rm B}$ [8], where $\mu_{\rm B} = e\hbar/(2m_e c)$ is the Bohr magneton. The most recent estimate of a limit for the neutrino magnetic dipole moment derived from the WDLF is $\mu_\nu \lesssim 5 \times 10^{-12}\mu_{\rm B}$ [9].

An alternative way to constrain the neutrino magnetic dipole moment from white dwarfs is provided by asteroseismology. Indeed, DB (pure He atmosphere) white dwarfs go across a g-mode pulsation instability phase at effective temperatures in the range $29\,000 \text{ K} \gtrsim T_{\rm eff} \gtrsim 22\,000$ K (the DBV or V777 Her instability strip [10–12]), Their evolutionary timescale can be inferred, in principle, by measuring the temporal rates of period changes ($\dot{\Pi} \equiv d\Pi/dt$, Π being the pulsation period). Interestingly enough, the range of effective temperatures in which the evolution of DB white dwarfs is dominated by neutrino emission partly overlaps with the DBV instability strip, and thus, the magnitude of the rates of period change of DBV stars is set by plasmon neutrino emission. Therefore, if a non-zero neutrino magnetic dipole moment exists, the value of $\dot{\Pi}$ should be enhanced in comparison with the case in which $\mu_\nu = 0$, and this provides a bound to μ_ν. A similar approach has been used in the context of pulsating DA (H-rich atmospheres) white dwarfs to derive bounds on the axion mass [13–17], and also to place constraints on the secular rate of variation of the gravitational constant G [18]. For DBV stars, no accurate measurement of the rate of period change has been yet derived for any pulsating star. However, it has been possible to determine a preliminary value of $\dot{\Pi} = (2.0 \pm 0.9) \times 10^{-13}$ s/s for the largest amplitude mode with period at ~ 489 s of the pulsating star PG 1351+489, the first such estimate for a DBV [19]. According to its spectroscopic effective temperature (22 000 K −26 000 K), this star could be too cool to allow a measurement of the *normal* plasmon emission rate. However, it can be used for detecting *anomalous* neutrino loss.

In this paper, we employ the above mentioned estimate of the rate of period change for PG 1351+489 to derive an upper limit to the neutrino magnetic dipole moment. The paper is organized as follows. In section 2 we describe the input physics employed in the

[2] Plasmon decay has place owing to an *indirect* coupling between neutrinos and photons through electrons in a plasma [1].

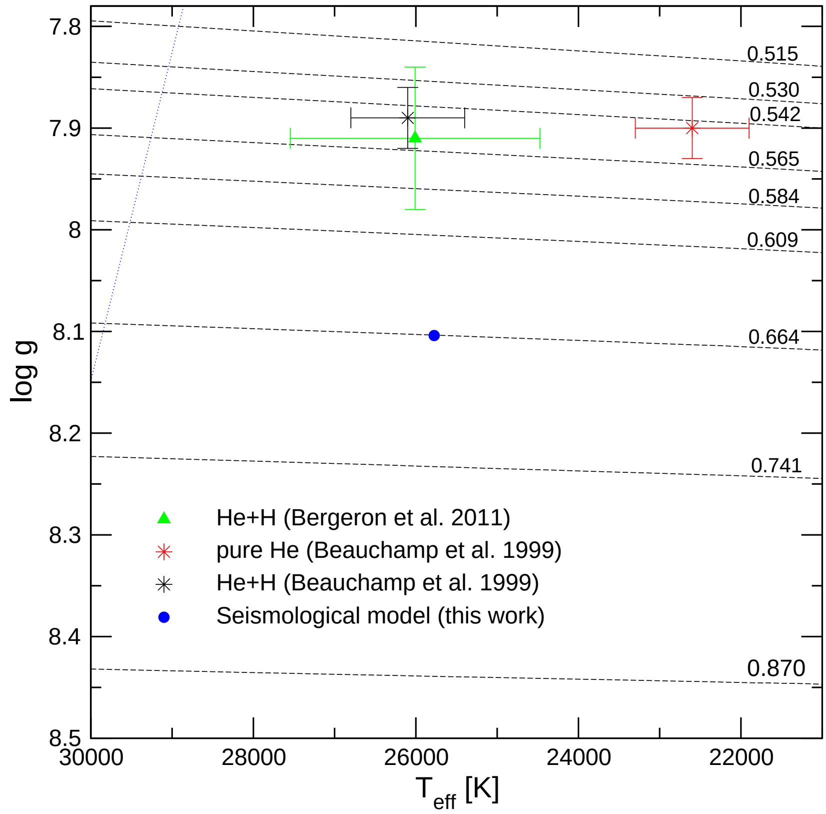

Figure 1. Our set of DB white dwarf evolutionary tracks on the $T_{\rm eff} - \log g$ plane, labeled with the values of the stellar mass. The location of PG 1351+489 [20] is emphasized with a red symbol (a black symbol) assuming a pure He atmosphere (considering a He rich atmosphere with impurities of H). The location of the star as given by Ref. [21] (atmosphere rich in He plus traces of H) is shown with a green triangle. Finally, the location of the star as predicted by our asteroseismological model (section 5) is marked with a blue circle. The theoretical blue edge of the DBV instability strip is depicted with a blue dashed line [22]. See the o-line edition of the journal for a colo version of this figure.

computation of our evolutionary DB white dwarf models, and the effects that a non-zero μ_ν have on the DB white dwarf evolution. Section 3 is devoted to describe the impact of μ_ν on the pulsation properties of DB white dwarfs. In section 4 we derive constraints on the neutrino magnetic dipole moment by considering the rate of period change estimate for the $\sim$ 489 s period of the star PG 1351+489 by relying on their spectroscopically determined effective temperature alone. In section 5 we obtain an upper limit on μ_ν by employing an asteroseismological model for PG 1351+489. Finally, in section 6, we compare our results with current astrophysical bounds on μ_ν, and present our concluding remarks.

2 DB white dwarf models

The evolutionary DB white dwarf models employed in this work were computed using the `LPCODE` stellar evolutionary code [23, 24]. All our white dwarf sequences were computed in a consistent way with the evolution of the chemical abundance distribution due to element diffusion along the entire cooling phase. Neutrino emission rates for pair, photo, and bremsstrahlung processes are taken into account [25]. For plasma processes, we used the treatment presented in Ref. [26]. Of particular interest in this paper is the anomalous energy loss due to the existence of a magnetic dipole moment, $\epsilon_\nu^{\rm mdm}$. It was computed from the plasmon neutrino emission, $\epsilon_\nu^{\rm p}$, using the scaling relation [26]:

$$\epsilon_\nu^{\rm mdm} = 0.318 \; \mu_{12}^2 \left(\frac{10 \text{ keV}}{\hbar\omega_{\rm p}}\right)^2 \frac{Q_2}{Q_3} \; \epsilon_\nu^p \tag{2.1}$$

Here, $\mu_{12} = \mu_\nu / \left(10^{12}\mu_{\rm B}\right)$, $Q_2/Q_3 \sim 1$ [26], and $\omega_{\rm p}$, the plasma frequency, was computed as in [1] (see Ref. [9] for details).

The initial models for our DB white dwarf sequences correspond to realistic H-defficient PG 1159 stars (the precursors of H-defficient white dwarfs) derived from the full evolutionary calculations of their progenitor stars [23, 27]. All the sequences were computed from the Zero Age Main Sequence through the thermally-pulsing and mass-loss phases on the Asymptotic Giant Branch and finally to the born-again stage where the remaining H is violently burned. After the born again episode, the H-deficient, quiescent He-burning remnants evolve at constant luminosity to the domain of PG 1159 stars with a surface chemical composition rich in He, C and O [27] and eventually to the DB white dwarf stage. This set of PG 1159 evolutionary models has succeeded in explaining both the spread in surface chemical composition observed in most PG 1159 stars and the location of the GW Vir instability strip in the $\log T_{\rm eff} - \log g$ plane [28]. Also, these PG 1159 models have been employed in detailed asteroseismological studies of seven pulsating PG 1159 stars [29–33]. It is worth remarking that realistic initial white dwarf stuctures are needed in order to accurately predict the evolutionary properties of DBVs. In fact, it has been shown [34] that differences larger than 10% in the cooling times may arise from assuming different thermal structures of the first white dwarf converged models at the beginning of the cooling sequences.

Specifically, we considered nine DB white dwarf sequences with stellar masses: 0.515, $0.530, 0.542, 0.565, 0.584, 0.609, 0.664, 0.741,$ and $0.870 M_\odot$. These DB sequences are characterized by the maximum He-rich envelope that can be left by the previous evolution if it is assumed that they are the result of a born-again episode. The value of the envelope mass ranges from $M_{\rm He}/M_* \sim 2 \times 10^{-2}$ ($M_* = 0.515 M_\odot$) to $M_{\rm He}/M_* \sim 1 \times 10^{-3}$ ($M_* = 0.870 M_\odot$). The complete set of DB white-dwarf evolutionary sequences (computed neglecting a neutrino magnetic dipole moment) is displayed in the $T_{\rm eff} - \log g$ diagram of Fig. 1, where g is the surface gravity defined as $g = GM_*/R_*^2$, G being the gravitational constant and R_* the stellar radius. Also shown is the spectroscopic location of PG 1351+489 and the location of our asteroseismological model for this star (section 5).

The impact of a non-zero neutrino magnetic dipole moment on the evolution of white dwarfs has been explored in detail [9]. Here we briefly show how our DB white dwarf evolutionary sequences are affected when a non-vanishing neutrino magnetic dipole moment is taken into account. The initial DB models of our sequences were computed under the standard assumptions, that is, neglecting anomalous neutrino losses. We swiched on the anomalous neutrino losses at very high effective temperatures ($T_{\rm eff} \sim 70\,000$ K) in such a way that the unphysical transitory associated with this artificial procedure finishes long before the models reach the instability strip of DBV stars ($29\,000 \text{ K} \gtrsim T_{\rm eff} \gtrsim 22\,000$ K). Apart from the standard sequences (that assume $\mu_\nu = 0$), we computed additional white dwarf sequences considering a non-zero magnetic dipole moment. The effect of adopting $\mu_\nu \neq 0$ is depicted in Fig. 2, in which we show the luminosity contributions for the case of a DB white-dwarf star with $M_* = 0.664 M_\odot$ in terms of the effective temperature. In panel (a)

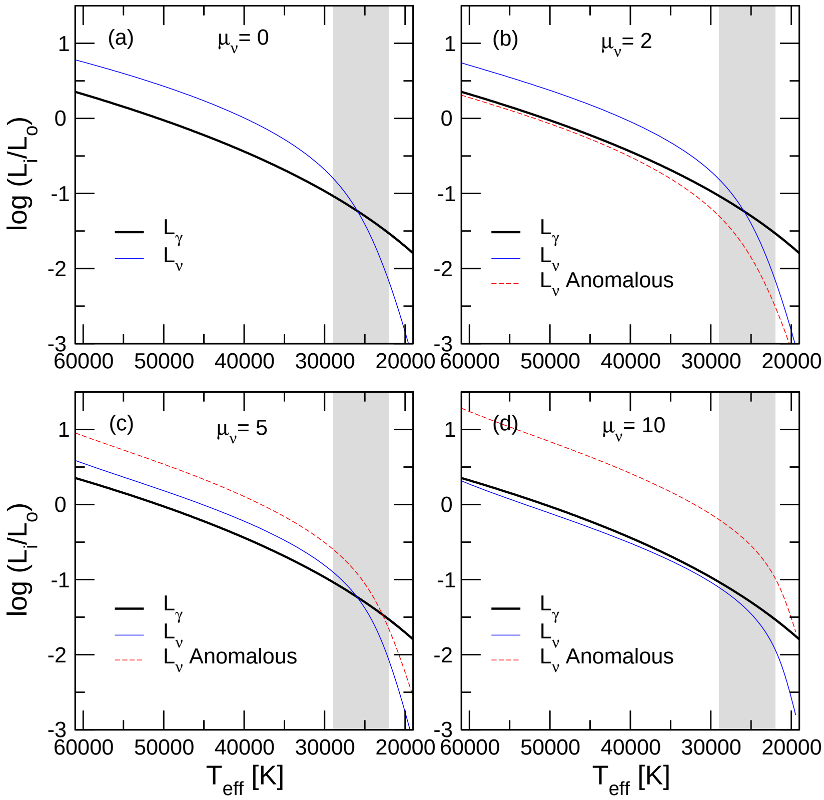

Figure 2. Luminosity contributions for a $0.664 M_\odot$ DB white-dwarf star in terms of the effective temperature. Panel (a): photon luminosity (L_γ), and neutrino losses without a magnetic dipole moment (L_ν). Panels (b), (c), and (d): neutrino emission due to standard model processes (L_ν) but assuming the existence of a neutrino magnetic dipole moment μ_ν (in units of $10^{-12}\mu_\mathrm{B}$), and anomalous neutrino emission (L_ν Anomalous). The gray area indicates the domain of DBV pulsating stars.

we show the standard case in which anomalous neutrino emission is neglected. Photon luminosity (L_γ) and neutrino losses (L_ν) without a magnetic dipole moment ($\mu_\nu = 0$) are displayed with a thick black curve and a thin blue curve, respectively. In panels (b), (c), and (d), we show the cases in which we allow neutrino emission due to standard model processes (again, L_ν) but considering the existence of a neutrino magnetic dipole moment μ_ν. In these panels, we include also anomalous neutrino emission (L_ν Anomalous), depicted with thin dashed red curves. The assumed values of $\mu_\nu/\left(10^{12}\mu_\mathrm{B}\right)$ are 2, 5, and 10. The gray area indicates the domain of DBV pulsating stars. Fig. 2 shows that as the anomalous neutrino emission is increased by increasing μ_ν, the feedback on the thermal structure of the white dwarf forces neutrino emission through the standard channels to be lower. In particular, for $\mu_\nu/\left(10^{12}\mu_\mathrm{B}\right) \gtrsim 3$ (not shown), the anomalous neutrino losses overcome neutrino emission due to standard model processes. The net effect on the evolution is that the higher the values of μ_ν, the faster the cooling of the white dwarf [9]. In summary, the inclusion of a neutrino magnetic dipole moment in the evolution of white dwarfs gives rise to an additional cooling mechanism. This effect is identical to that found when axion emission is considered in the evolution of cooler white dwarfs [13].

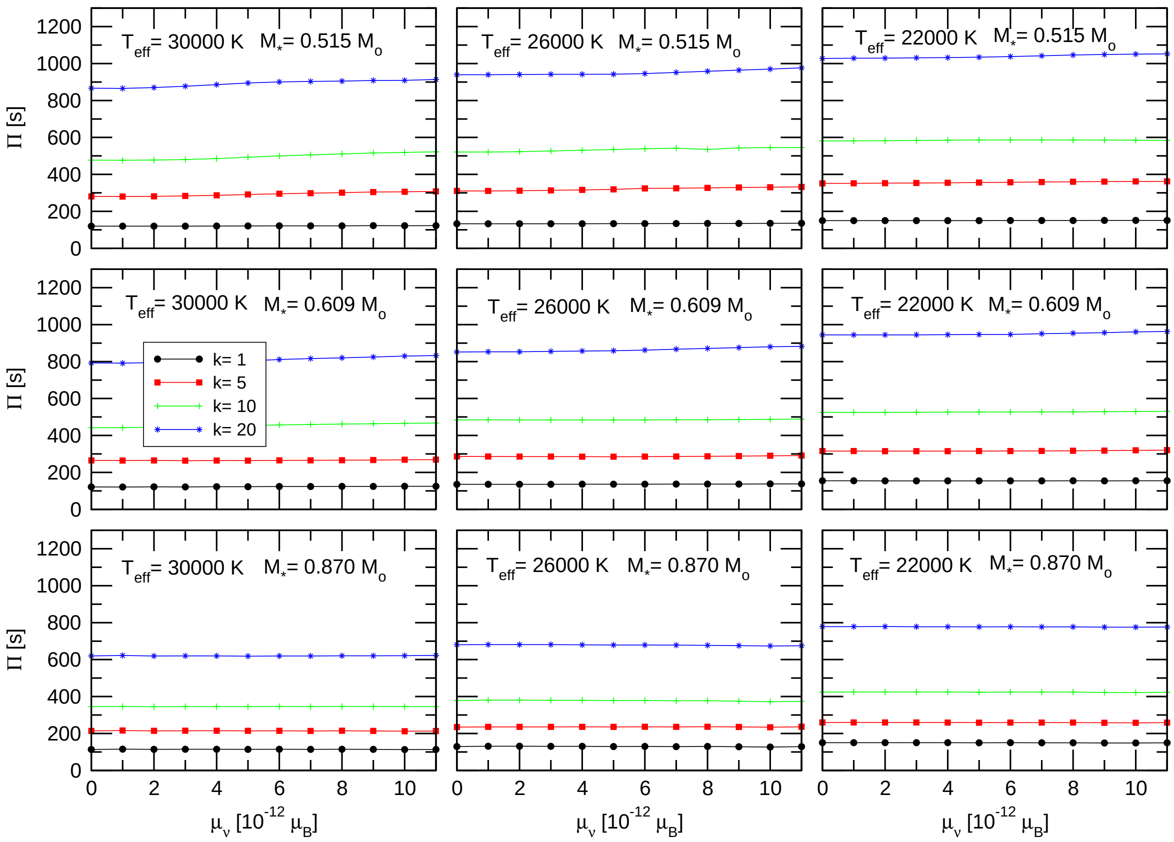

Figure 3. The pulsation periods of $\ell = 1$ g-modes with radial order $k = 1, 5, 10$ and 20 for DB white dwarf models with masses $M_*/M_\odot = 0.515, 0.609$ and 0.870, and effective temperatures $T_{\rm eff} \sim$ 30 000, 26 000 and 22 000 K, in terms of the neutrino magnetic dipole moment.

3 The effects of μ_ν on the pulsations of white dwarfs

Here we explore the effects that a neutrino magnetic dipole moment μ_ν has on the pulsation periods and rates of period change of white dwarf stars. The pulsation periods employed in the present analysis were computed with the pulsational code `LP-PUL` [35]. This code has been employed in numerous pulsational studies of white dwarfs (see Ref. [12] and references therein). Pulsations in white dwarfs are associated to non-radial g(gravity)-modes which are a sub-class of spheroidal modes[3] whose main restoring force is gravity. These modes are characterized by low oscillation frequencies (long periods) and by a displacement of the stellar fluid essentially in the horizontal direction. g-modes are labelled with the harmonic degree $\ell = 0, 1, 2, \cdots, \infty$ (the number of nodal lines in the stellar surface), the azimuthal order $m = 0, \pm 1, \cdots, \pm \ell$ (the number of such nodal lines in longitude), and the radial order $k = 0, 1, 2, \cdots, \infty$ (the number of nodes in the radial component of the eigenfunction) [10, 12, 36].

We have computed dipole and quadrupole ($\ell = 1$ and $\ell = 2$) g-modes with pulsation periods in the range 100 s $\lesssim \Pi \lesssim$ 1200 s, thus covering the range of observed periods in DBV white dwarfs. We considered DB white dwarf models with masses in the range $0.515 \lesssim M_*/M_\odot \lesssim 0.870$ and effective temperatures in the interval 30 000 K $\gtrsim T_{\rm eff} \gtrsim$ 20 000

[3]Spheroidal modes are characterized by $(\vec{\nabla} \times \vec{\xi})_r = 0$ and $\sigma \neq 0$, where $\vec{\xi}$ is the vector Lagrangian displacement and σ the pulsation frequency [36].

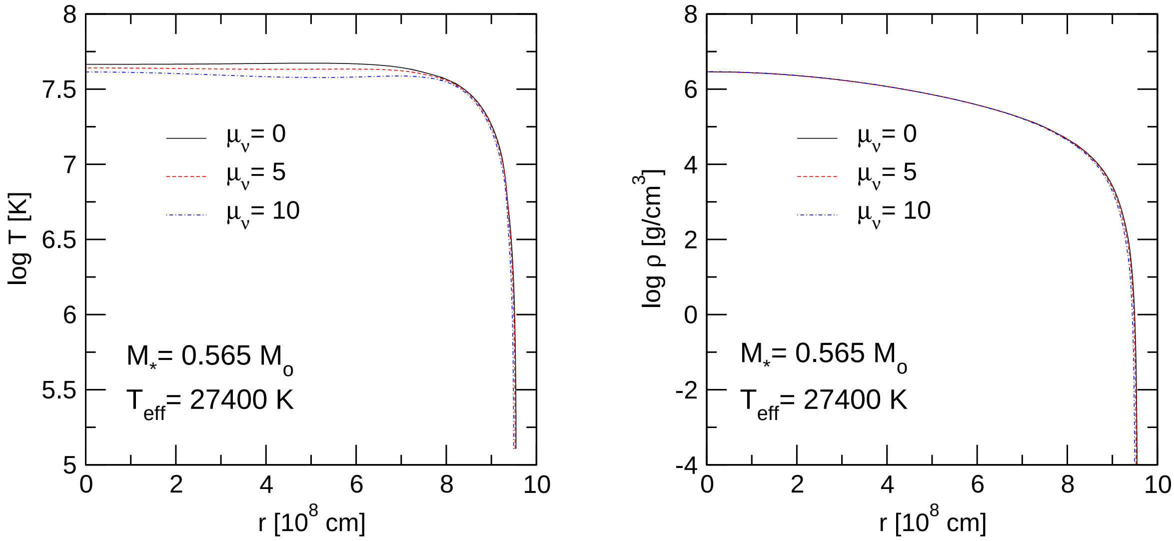

Figure 4. The logarithm of the temperature (left panel) and density (right panel) profiles corresponding to a DB white dwarf model with $M_* = 0.565 M_\odot$ and $T_{\rm eff} = 27\,400$ K for the cases of $\mu_\nu = 0, 5$ and 10 (in units of $10^{-12}\mu_{\rm B}$).

K, thus embracing the mass and effective temperature ranges in which DBV white dwarfs are observed. In order to assess the impact of a non-zero neutrino magnetic dipole moment on the periods and temporal rates of period changes, we carried out these computations for model sequences in which we vary μ_ν in the range $0 \leq \mu_\nu / \left(10^{12}\mu_{\rm B}\right) \leq 11$.

In Fig. 3 we show the dipole pulsation periods with $k = 1, 5, 10$ and 20 ($100\ {\rm s} \lesssim \Pi \lesssim 1000$ s) for DB white dwarf models with masses $M_*/M_\odot = 0.515, 0.609$ and 0.870, and effective temperatures $T_{\rm eff} \sim 30\,000, 26\,000$ and $22\,000$ K, in terms of the neutrino magnetic dipole moment (μ_ν). The variation in the periods is very small, in spite of the rather wide range of μ_ν considered. This result, which was first found in the case of axion emission [14, 16, 17], implies that due to the additional cooling mechanism induced by the existence of a neutrino magnetic dipole moment, the structure of the white dwarf models themselves are slightly affected. Indeed, in Fig. 4 we show the logarithm of the temperature and density profiles of a DB white dwarf model with $M_* = 0.565 M_\odot$ and $T_{\rm eff} = 27\,400$ K for different values of μ_ν, including the case $\mu_\nu = 0$. The effects of a neutrino magnetic dipole moment on these structural quantities are barely seen in the plots. As a result of this, for a fixed value of the effective temperature, the pulsation periods are largely independent of the adopted value for μ_ν. The impact of a non-zero neutrino magnetic dipole moment on the rates of period changes is depicted in Fig. 5, where we show $\dot{\Pi}$ for the same pulsation modes and the same white dwarf models shown in Fig. 3 in terms of μ_ν. At variance with what happens with the pulsation periods in most cases, the values of $\dot{\Pi}$ are strongly affected by the additional cooling source, substantially increasing for increasing values of μ_ν. The effect is stronger for high effective temperatures and low stellar masses ($M_* = 0.515 M_\odot$, $T_{\rm eff} = 30\,000$ K), and is barely noticeable for low effective temperatures and high masses ($M_* = 0.870 M_\odot$, $T_{\rm eff} = 22\,000$ K).

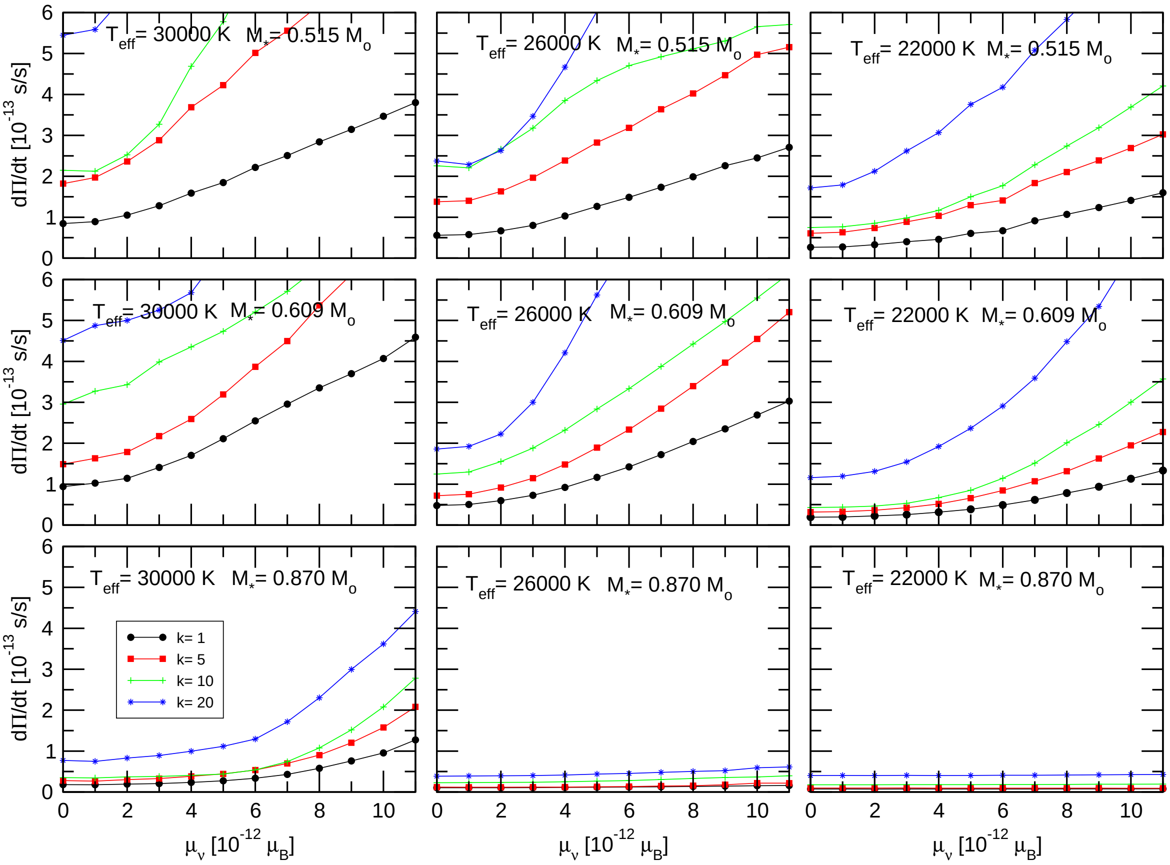

Figure 5. Same as in Fig. 3, but for the rates of period change.

4 Constraints on μ_ν from the DBV star PG 1351+489

The star PG 1351+489 is one of the 21 pulsating DB (He-rich atmosphere) white dwarfs known up to date [37]. Since the discovery of pulsations in this star [38], it was realized that it could be a candidate for the first measurement of a rate of period change in a DBV star, because its power spectrum is dominated by a single high-amplitude pulsation mode with a period at ~ 489 s. In addition, two smaller amplitude modes and several linear combination frequencies were detected [39]. The pulsations in this star were re-analyzed, and the previous results were confirmed, providing more precise periods and also an additional low-amplitude mode [19]. More importantly, it was possible to obtain, for the first time for a DBV star, an estimate of the rate of period change for the period at ~ 489 s of $\dot{\Pi} = (2.0 \pm 0.9) \times 10^{-13}$ s/s [19]. Although not definitive, this estimate constitutes a very valuable opportunity to place constraints on μ_ν *independently* from other techniques. In fact, we can compare the estimated value of the rate of period change for the mode with period ~ 489 s of PG 1351+489 with theoretical values of the rate of period changes computed under the assumption of a non-zero μ_ν. The value of μ_ν for which the theoretical $\dot{\Pi}$ matches the observed rate of period change, may be regarded as an estimation of the magnitude of μ_ν. In order to apply such a procedure, we have to choose a DB white dwarf model with an effective temperature close to the observed one. Also, we have to consider theoretical values of $\dot{\Pi}$ corresponding to modes with periods close to the observed period (~ 489 s). Regarding the effective temperature

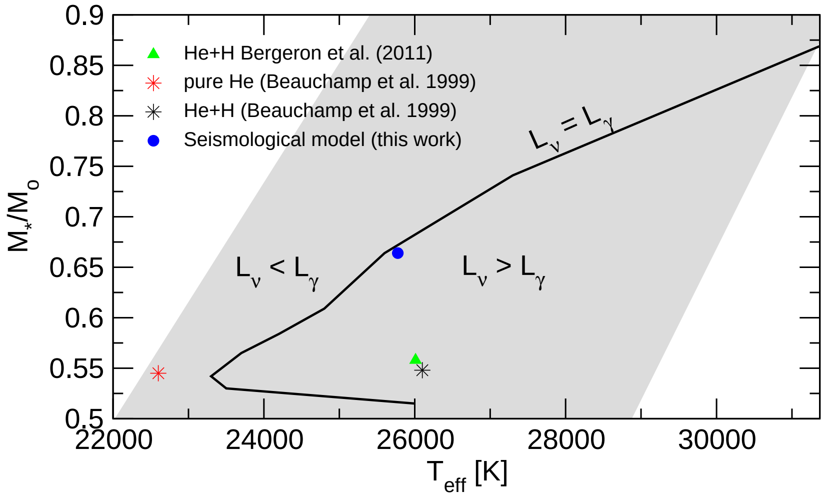

Figure 6. A $M_* - T_{\rm eff}$ diagram showing the regions in which neutrino luminosity dominates over photon luminosity ($L_\nu > L_\gamma$), and the regions in which the opposite holds ($L_\nu < L_\gamma$). The black curve corresponds to the case in which $L_\nu = L_\gamma$. The location of the star PG 1351+489 is displayed according to different spectroscopic determinations and also as predicted by an asteroseismological model (section 5). The gray band indicates the DBV instability strip.

and gravity of PG 1351+489, there are several spectroscopic determinations for this star. The first detailed analysis [20] provided $T_{\rm eff} = 22\,600 \pm 700$ K, $\log g = 7.90 \pm 0.03$ using pure He atmospheres, and $T_{\rm eff} = 26\,100 \pm 700$ K, $\log g = 7.89 \pm 0.03$ employing atmospheres with impurities of H. The most recent analysis [21] indicates that $T_{\rm eff} = 26\,010 \pm 1536$ K, $\log g = 7.91 \pm 0.07$, obtained by assuming H contamination ($\log {\rm H/He} = 4.37 \pm 0.82$).

According to the spectroscopic effective temperature of PG 1351+489 (22 000 K −26 000 K), it could be argued that this star is a bit cool for studying plasmon emission processes [19]. In Fig. 6 we show the regions in which neutrino luminosity dominates over photon luminosity (to the right of the black curve), and the regions in which photon luminosity overwhelms neutrino luminosity (to the left of the black curve). We have included the location of PG 1351+489 on this plane according to different estimations of the $T_{\rm eff}$ and M_*. As can be seen, in all of the cases the star is located in the region in which neutrino emission dominates. From this plot, we conclude that the star is actually an appropriate target to study neutrino emission, and in particular, a hypotetical anomalous neutrino emission.

Following the spectroscopic analysis of Ref. [20], the stellar mass of PG 1351+489 must be $M_* \sim 0.542 M_\odot$ according to our evolutionary tracks (see Fig. 1). Accordingly, we have considered two DB white dwarf models belonging to this sequence, one of them with $T_{\rm eff} \sim 22\,600$ K and the other one with $T_{\rm eff} \sim 26\,100$ K, in order to include the case in which the atmosphere is made of pure He, and also the case in which there are small abundances of H. In Fig. 7 we show the theoretical rates of period changes in terms of μ_ν for dipole modes with periods of $\sim$ 450 s ($k = 8$) and $\sim$ 504 s ($k = 9$) corresponding to the model with $T_{\rm eff} = 22\,600$ K (upper panel) and modes with periods of $\sim$ 468 s ($k = 9$) and 496 s ($k = 10$) corresponding to the model with $T_{\rm eff} = 26\,100$ K (lower panel). Note that in both cases the periods adopted embrace the observed period ($\sim$ 489 s). By comparing the theoretical values of $\dot{\Pi}$ with the estimate of $\dot{\Pi}$ for PG 1351+489 ($\dot{\Pi} = (2.0 \pm 0.9) \times 10^{-13}$ s/s), we infer $3 \lesssim \mu_\nu / \left(10^{12} \mu_{\rm B}\right) \lesssim 9$ in the first case, and $\mu_\nu / \left(10^{12} \mu_{\rm B}\right) \lesssim 4$ in the second case. We

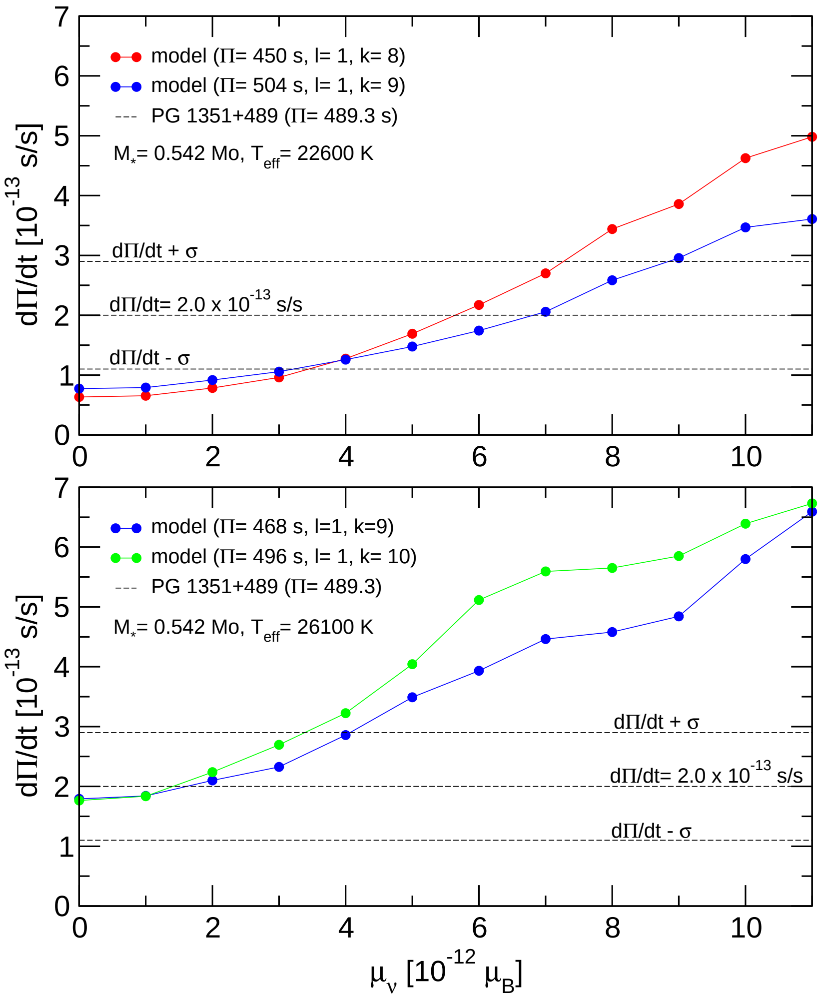

Figure 7. The rates of period change of selected modes in terms of the neutrino magnetic dipole moment, corresponding to models with $M_* = 0.542 M_\odot$ and $T_{\rm eff} = 22\,600$ K (upper panel), and $T_{\rm eff} = 22\,600$ K (lower panel). The estimate of the rate of period change of the 489 s period [19] of PG 1351+489 and its uncertainties are shown using thin dashed horizontal lines.

repeated this analysis by considering the modern estimate for $T_{\rm eff}$ and $\log g$ [21]. In this case, according to our evolutionary tracks, the stellar mass of PG 1351+489 is $M_* \sim 0.565 M_\odot$ (see Fig. 1). The results are shown in Fig. 8. The upper bound in this case is $\mu_\nu / \left(10^{12} \mu_{\rm B}\right) \lesssim 4$, in perfect agreement with the upper limit derived by adopting the high effective temperature determination of [20].

5 Constraints on μ_ν from an asteroseismological model for PG 1351+489

Another way in which we can derive a constraint on the neutrino magnetic dipole moment is by employing a seismological model for PG 1351+489, that is, the DB white dwarf model that best reproduces the observed periods exhibited by the star. In obtaining the asteroseismological model, we neglect the existence of a neutrino magnetic dipole moment, that is, we assume $\mu_\nu = 0$, since we have shown that the period pattern is not affected by the inclusion of μ_ν. In order to search for the asteroseismological model, we employ a quality function:

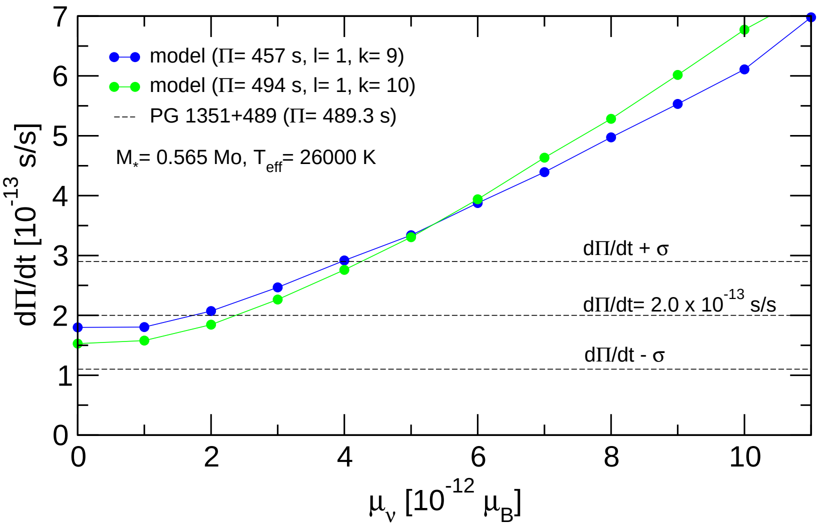

Figure 8. Same as Fig. 7, but for the case of a model with a stellar mass of $M_* = 0.565 M_\odot$ and $T_{\rm eff} = 26\,000$ K.

Table 1. The asteroseismological solutions for PG 1351+489 when we assume that all the periods are associated to $\ell = 1$ modes (first row), and in the case in which we suppose that they are a combination of $\ell = 1$ and $\ell = 2$ modes (second to fourth row).

Solution	M_* $[M_\odot]$	$T_{\rm eff}$ [K]	ℓ	$\overline{\delta}$ [s]	σ [s]	BIC
1	0.870	24 395	1	2.167	2.478	1.089
2	0.870	23 184	1,2	1.210	1.253	0.496
3	0.870	24 395	1,2	1.575	1.964	0.887
4	0.664	25 775	1,2	1.065	1.397	0.591

$$\chi^2(M_*, T_{\rm eff}) = \frac{1}{N} \sum_{i=1}^{N} \min[(\Pi_{{\rm obs},i} - \Pi_k)^2], \tag{5.1}$$

that measures the goodness of the match between the theoretical pulsation periods (Π_k) and the observed individual periods ($\Pi_{{\rm obs},i}$). Here, N $(= 4)$ is the number of observed periods, which are: $\Pi_{{\rm obs},i} = 335.26, 489.33, 584.68$ and 639.63 s [19]. The DB white dwarf model that shows the lowest value of χ^2 is adopted as the "best-fit model". We have considered $\ell = 1$ and $\ell = 2$ modes. The quality of our period fits is assessed by means of the average of the absolute period differences, $\overline{\delta} = (\sum_{i=1}^{N} |\delta_i|)/N$, where $\delta_i = \Pi_{{\rm obs},i} - \Pi_k$, by the root-mean-square residual, $\sigma = \sqrt{(\sum |\delta_i|^2)/N} = \sqrt{\chi^2}$, and by the Bayes Information Criterion BIC $= N_{\rm p} \, (\log N/N) + \log \sigma^2$, where $N_{\rm p}$ is the number of free parameters, and N the number of observed periods [40]. In our case, $N_{\rm p} = 2$ (stellar mass and effective temperature). The smaller the value of BIC, the better the quality of the fit.

The quantity $(\chi^2)^{-1}$ in terms of the effective temperature for different stellar masses is shown in Fig. 9 together with the most recent spectroscopic determination of the effective temperature of PG 1351+489 and its uncertainties (gray strip) [21]. In the upper panel

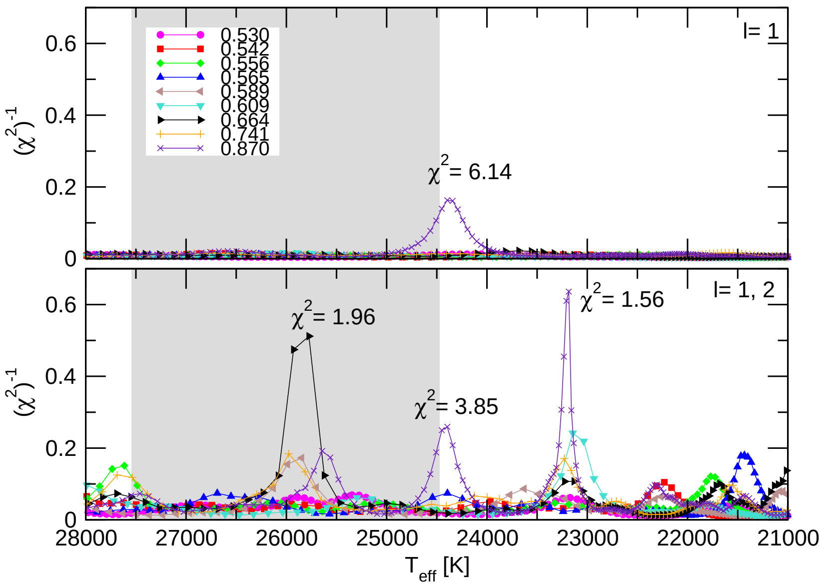

Figure 9. The quantity $(\chi^2)^{-1}$ in terms of the effective temperature for different stellar masses, along with the spectroscopic effective temperature of PG 1351+489 and its uncertainties (gray area) according to Ref. [21]. The upper panel corresponds to the case in which all the periods of PG 1351+489 are assumed to be $\ell = 1$, and the lower panel shows the results for the case in which we allow the observed periods to be a combination of $\ell = 1$ and $\ell = 2$.

we depict the results of the case in which all the periods of PG 1351+489 are supposed to be $\ell = 1$. In this case, we found a single solution corresponding to a best-fit model with $M_* = 0.870 \pm 0.015 M_\odot$ and $T_{\rm eff} = 24\,395 \pm 180$ K. For this model, we obtain $\overline{\delta} = 2.17$ s, $\sigma = 2.48$ s and BIC$= 1.089$. The effective temperature of this model is at the cool boundary of the interval in $T_{\rm eff}$ allowed by spectroscopy for this star. The case in which we allow the modes to be $\ell = 1$ or $\ell = 2$ is shown in the lower panel of Fig. 9. We found three possible seismological solutions. Two of them correspond to models with $M_* = 0.870 M_\odot$ and $T_{\rm eff} = 24\,395$ K and $T_{\rm eff} = 23\,184$ K. The third solution corresponds to a model with $M_* = 0.664 M_\odot$ and $T_{\rm eff} = 25\,775$ K. In Table 1 we summarize the main characteristics of all the solutions with the corresponding $\overline{\delta}$, σ and BIC values. Clearly, when we allow for the observed periods to be a combination of $\ell = 1$ and $\ell = 2$ modes, the seismological solutions are substantially better (solutions 2 to 4) than assuming that all the periods are $\ell = 1$ (solution 1). Among solutions 2, 3 and 4, the worst one is solution 3. Solution 2 has the lower χ^2 value, but 3 of the 4 periods observed in PG 1351+489 are fitted with theoretical periods associated to $\ell = 2$ modes, and it is true in particular for the period at ~ 489 s which has the largest amplitude. In view of geometrical cancellation effects at the stellar surface [41], it is improbable that the largest-amplitude mode with period at ~ 489 s corresponds to $\ell = 2$. Solution 4 has a quality comparable than solution 2, but in this case the period of the highest amplitude mode is reproduced by a theoretical $\ell = 1$ period. In addition, the effective temperature of this solution is in agreement with the spectroscopic derivation for $T_{\rm eff}$. For these reasons, we adopt solution 4 as the asteroseismological model for PG 1351+489.

In Table 2 we show the characteristics of our best-fitting model and the parameters derived from spectroscopy. Note that the effective temperature of the model is in good agreement with that of PG 1351+489 inferred by spectroscopy, but the surface gravity is

Table 2. Characteristics of PG 1351+489 derived from the spectroscopic analysis [21] and results of our best asteroseismological model. $X_{\rm C}$ and $X_{\rm O}$ are the fractional abundances of C and O, respectively, at the stellar center. The quoted uncertainties in the asteroseismological model are the *internal* errors of our period-fitting procedure.

Quantity	Spectroscopy	Asteroseismology
$T_{\rm eff}$ [K]	$26\,010 \pm 1536$	$25\,775 \pm 150$
$M_*/M_\odot$	0.558 ± 0.027	0.664 ± 0.013
$\log g$	7.91 ± 0.07	8.103 ± 0.020
$\log(R_*/R_\odot)$	—	-1.912 ± 0.015
$\log(L_*/L_\odot)$	—	-1.244 ± 0.03
$M_{\rm He}/M_*$	—	3.63×10^{-3}
$X_{\rm C}, X_{\rm O}$ (center)	—	$0.32, 0.65$

Table 3. The observed periods and rates of period change of PG 1351+489 (columns 1 and 5), and the theoretical periods and rates of period changes of the asteroseismological model (columns 2 and 6), along with the corresponding (ℓ, k) mode identification (columns 3 and 4).

$\Pi^{\rm o}$ [s]	$\Pi^{\rm t}$ [s]	ℓ	k	$\dot{\Pi}^{\rm o}$ [10^{-13}s/s]	$\dot{\Pi}^{\rm t}$ [10^{-13}s/s]
335.26	336.81	2	13	—	0.60
489.33	489.47	1	11	2.0 ± 0.9	0.81
584.68	586.99	2	25	—	1.02
639.63	639.37	1	15	—	1.19

substantially larger than the spectroscopic one. We emphasize that the quoted errors in the parameters of the asteroseismological models are just *internal* errors. The assessment of the *true* uncertainties is beyond the scope of the present paper, but we envisage that they could be somewhat larger than those included in Table 2.

A detailed comparison of the observed periods in PG 1351+489 with the theoretical periods of the best-fit asteroseismological model is provided in Table 3. We include also the observed and theoretical rates of period changes for each mode. The model very closely reproduces the observed periods. Interestingly, the observed rate of change of the ~ 489.33 s period is more than two times larger than the theoretically expected value. If we assume that the rate of period change of this mode reflects the evolutionary time-scale of the star, and that the asteroseismological model truly represents the real star, then the disagreement between the observed and theoretical values of $\dot{\Pi}$ would indicate that PG 1351+489 could be cooling faster than that predicted by the standard theory of white dwarf evolution.

Having found that the theoretically expected rate of change of period with time for the $k = 11$ mode is somewhat smaller than the value measured for PG 1351+489, suggesting the existence of some additional cooling mechanism in this star, we now assume that this additional cooling is entirely attributable to the existence of a neutrino magnetic dipole moment[4]. In this way, we can establish an upper bound for the possible value of μ_ν. Specifically, we considered different values of $\mu_\nu/\left(10^{12}\mu_{\rm B}\right)$ in the range $(0 - 11)$ and assumed the same

[4]We neglect the emission of axions or other weakly interacting particles.

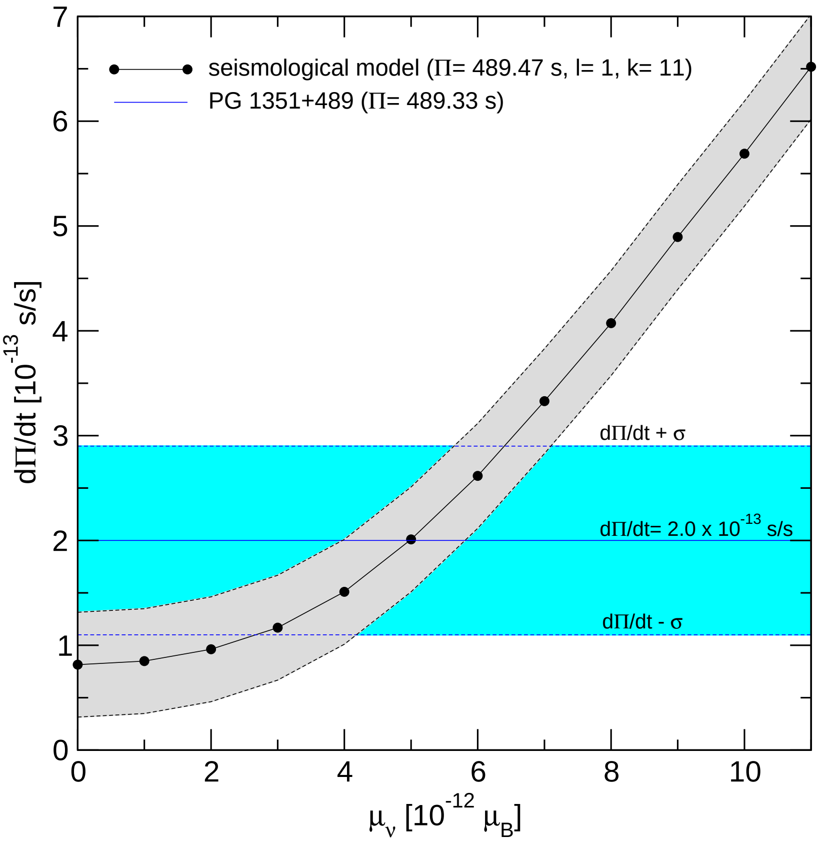

Figure 10. The rate of period change of the $\ell = 1$ and $k = 11$ mode in terms of the neutrino magnetic dipole moment, corresponding to the asteroseismological model for PG 1351+489 ($M_* = 0.664 M_\odot$ and $T_{\rm eff} = 25\,775$ K). The estimate of the rate of period change of the 489 s period [19] of PG 1351+489 and its uncertainties are shown with thin blue horizontal lines.

structural parameters ($T_{\rm eff}, M_*$) as the asteroseismological model. This is a valid procedure because, as shown in Sect. 3, the pulsation periods do not depend on the magnitude of μ_ν, which means that the asteroseismological model for PG 1351+489 in the case of a non-zero neutrino magnetic dipole moment is still valid. In Fig. 10 we display the theoretical value of $\dot{\Pi}$ corresponding to the period $\Pi \sim 489$ s for increasing values of the neutrino magnetic dipole moment (black solid curve and dots). The dashed curves embracing the solid curve represent the uncertainty in the theoretical value of $\dot{\Pi}$, $\varepsilon_{\dot{\Pi}} = 0.5 \times 10^{-13}$ s/s. This value has been roughly estimated for the case in which $\mu_\nu = 0$ by comparing the $\dot{\Pi}$ of the ~ 489 s mode of the asteroseismological model with the value of $\dot{\Pi}$ of the eigenmodes with similar periods belonging to the sequences with $M_* = 0.609 M_\odot$ and $M_* = 0.741 M_\odot$ at similar effective temperature ($T_{\rm eff} \sim 25\,800$ K). Additionally, we assume that the uncertainties do not depend on the value of μ_ν. The horizontal (blue) solid line indicates the observed value of 2.0×10^{-13} s/s, whilst its corresponding 1σ uncertainties ($\pm 0.9 \times 10^{-13}$ s/s) [19] are shown with dashed lines. If one standard deviation from the observational value is considered, we conclude that the neutrino magnetic dipole moment is $\mu_\nu \lesssim 7 \times 10^{-12} \mu_{\rm B}$. This upper limit is compatible with those obtained in the previous section without considering an asteroseismological model ($\mu_\nu \lesssim 9 \times 10^{-12} \mu_{\rm B}$).

6 Summary and conclusions

In this paper we have derived for the first time an upper bound to the neutrino magnetic dipole moment employing the DBV white dwarf pulsator PG 1351+489, for which an estimate of the rate of period change of the dominant pulsation period is available. Our analysis is based on state-of-the-art evolutionary DB white dwarf models consistent with the history of progenitor stars. We obtain a bound of $\mu_\nu \lesssim 9 \times 10^{-12} \mu_{\rm B}$ when we rely on the spectroscopic values of $T_{\rm eff}$ and $\log g$ of this star, and a limit of $\mu_\nu \lesssim 7 \times 10^{-12} \mu_{\rm B}$ if we consider an asteroseismological model that closely reproduces its pulsation periods. We adopt a conservative limit of $\mu_\nu \lesssim 10^{-11} \mu_{\rm B}$ as the main result of our analysis, which reinforces the more restrictive upper bound derived on the basis of the WDLF [9], $\mu_\nu < 5 \times 10^{-12} \mu_{\rm B}$, and that derived from an analysis of red giants from the color-magnitude diagram (CMD) of the Galactic globular cluster M5, $\mu_\nu < 4.5 \times 10^{-12} \mu_{\rm B}$ (95 % CL) [42, 43]. Notwithstanding the rate of change of the dominant period in PG 1351+489 we are using in our analysis is still not a definitive measurement, the derived constraint for μ_ν does not depend on actually measuring the rate of period change. This is because our constraint is based on a measured upper limit to the period drift, not on the period drift itself. Of course, if the uncertainty on the measured rate of period change would improve, this would also improve the limits on μ_ν, or could even lead to a clear requirement for additional cooling effects.

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