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A COMPUTATIONAL FRAMEWORK FOR THE ESTIMATION OF DYNAMIC OD TRIP MATRICES

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A COMPUTATIONAL FRAMEWORK FOR THE ESTIMATION
OF DYNAMIC OD TRIP MATRICES

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ABSTRACT

Origin-Destination (OD) trip matrices describe traffic behavior patterns across the network and play a key role as primary data input to many traffic models. OD matrices are a critical requirement, in traffic assignment models, static or dynamic. However, OD matrices are not yet directly observable; thus, the current practice consists of adjusting an initial a priori matrix from link flow counts, speeds, travel times and other aggregate demand data, supplied by a layout of traffic counting stations. The availability of new traffic measurements from ICT applications offers the possibility to formulate and develop more efficient algorithms, especially suited for real-time applications; whose efficiency depends, among other factors, on the quality of the seed matrix. This paper proposes an integrated computational framework in which an off-line procedure generates the time-sliced OD matrices, which are the input to an on-line estimator, whose sensitivity with respect to the available traffic measurements is analyzed.

Keywords: Dynamic OD Matrices, Matrix Estimation, Bi-level Optimization, Kalman filtering, ICT data

1.   INTRODUCTION

In the context of estimating passenger-car transport demand, Origin-to-Destination (OD) trip matrices describe the number of trips between each origin-destination pair of transportation zones in a study area. For private vehicles, route choice models describe how drivers select the available paths between origins and destinations and, as a consequence, the number of trips using a given path (or path flow proportions). The route choice proportions can vary depending on the time-interval in dynamic models, since they depend on traffic states changing over time.

Formulations of static traffic or transit assignment models (Florian and Hearn 1995), as well as dynamic models involved in ATIS (Advanced Transport Information Systems) (Ashok and Ben-Akiva 2000), assume usually that a reliable estimate of an OD matrix is available, and constitutes an essential input for describing the demand to estimate network traffic states and short term predict their evolution. Since OD trips are not yet directly observable, indirect estimation methods have then been proposed. These are the so-called matrix adjustment methods, whose main modeling hypothesis, in the static case, can be stated as follows (Cascetta 2001): if the assignment of an OD matrix to a network defines the number of trips in all network links, then the same OD matrix could be estimated, as the inverse of the assignment problem, as a function of the flows observed on the links of the network. However, since the resulting problem is highly undetermined, additional information is necessary to find suitable solutions, and since the seminal work of Van Zuylen and Willumsen (1980), this has been a fertile domain of research (Lundgen and Peterson 2008, Bullejos et al.2014).

The estimation of time-dependent OD matrices has been usually based on space-state formulations using Kalman Filtering approaches (Ashok and Ben-Akiva 2000), as the most suitable to model dynamic phenomena. These approaches share with the static ones the requirement of an assignment matrix, whose entries determine the proportion of trips between an OD pair, using a link at a given time interval, with a relevant role for those links where traffic detection stations are located. Chang and Wu (1994), proposed a model for freeways that, for each OD pair that estimates time-varying travel times, uses time dependent traffic measures and implicit traffic flow models to account for flow propagation. The state variables are the time-varying OD proportions and the fractions of OD trips that arrive at each off-ramp m time intervals after their entrance from on-ramps at interval k. The observation variables are main section and off-ramp counts for each interval. An Extended Kalman-filter approach is needed to deal with the nonlinear relationship between the state variables and the observations. Variants of this approach have been explored, using variants of Extended Kalman Filters to deal with the time dependencies of model parameters, which are usually included as state variables in the model formulation. Hu et al. (2001) explicitly take into account temporal issues of traffic dispersion, and Lin and Chang (2007) assume that travel time information is available in order to deal with traffic dynamics. Dixon and Rilett (2002), Antoniou et al. (2001) or Work et al. (2008), also include other measurements as those supplied by GPS tracking of equipped vehicles or Automatic Vehicle Identification in the model formulation.

However, when real-time measurements from Information and Communications Technologies (ICT) are available, e.g. those supplied by Bluetooth/Wi-Fi devices, hypothesis on non-linear traffic flow propagation to estimate travel-times between pairs of points in the network are no longer necessary, since they can be measured by these technologies, and then state variables can be replaced by measurements and the Extended Kalman formulation can be successfully replaced by an ad hoc linear formulation, Barceló et al. (2013b), which uses deviations of OD path flows as state variables, as in Ashok and Ben-Akiva (200), and does not require an assignment matrix but instead a subset of the most likely OD path flows identified from a Dynamic User Equilibrium (DUE) assignment. A relevant finding of this approach to estimate dynamic OD matrices exploiting ICT data, Barceló et al. (2013a), is that the three key design factors that determine the quality of the estimate are, respectively, the quality of the detection layout, the quality of the historic dynamic OD matrix used as a priori initial estimate, and the percentage of penetration of the technology. This influence can be clearly seen in the graphics in Figure 1, Barceló et al. (2013a), where the results from a series of computational experiments are represented in terms of response surfaces, each one corresponding to a level of quality of the OD seed used.

Clearly the two first design factors are controllable, while the third is not. Assuming that some of the ICT applications (e.g. Bluetooth/Wi-Fi antennas, Licence Plate Recognition or Electronic ID identifiers, etc.) require fixed locations in the network, the quality of the sensor layout can be guaranteed using an optimized detection layout, purposely suited to the OD estimation objectives (Barceló et al. 2012). Also the quality of a priory initial OD matrices can be ensured, by specific off-line bi-level optimization procedures, Barceló et al. 2014 and Bullejos et al. 2014. The approach has proved to be very efficient when, further than the usual link flow counts, the model also includes travel time measurements between pairs of sensors (e.g. Bluetooth/Wi-Fi detection antennas when Bluetooth/Wi.Fi devices are set to discoverable mode) and the identification of the most likely used partial paths between them (e.g. as resulting from a DUE).

The research reported in this paper is a direct consequence of the previous results, if the quality of the time-dependent OD estimates strongly depends on the controllable design factors and, if given a purposely designed detection layout and an associated traffic data collection procedure, the determinant factor is the quality of the input OD seed, which can be acceptable estimated off-line then, on one hand it is natural to integrate both procedures in a unified computational framework and on the other hand determine the robustness of the real-time estimation procedure when, in the given conditions the integrity of the detection is affected by detector malfunctions which perturb the quality of the expected input.
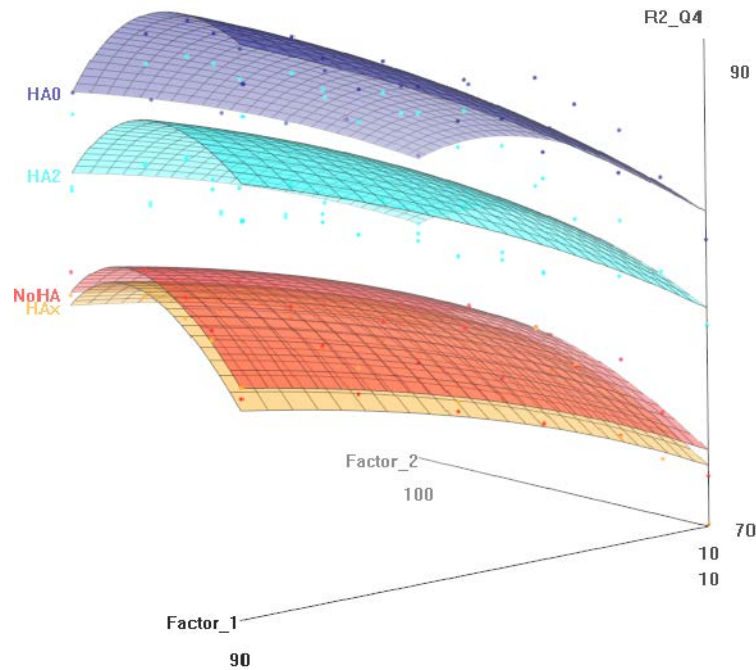
Figure 1 R^2 Fitted vs Target OD flows (1h 15min) for OD pairs in the 4th quartile (R**2_Q4**) according to #BT Inner Sensors (Factor 1, Detection Layout), %BT Equipped Vehicles (Factor 2, Technology Penetration) and Quality of the OD seed (Factor 3).

The paper is structured as follows, Section 2 introduces, discusses and, summarily describes the architecture of the integrated computational framework, Section 3 presents the off-line component, Section 4 the modification required by the Kalman Filter approach for the on-line component, Section 5 discusses a simple case used for testing the computational consistency, and Section 6 presents the conclusions and identifies the future work.

2. AN INTEGRATED FRAMEWORK

The experience gained in Barceló et al. (2013a), (2013b), (2014) and Bullejos et al. (2014), lead to propose an integrated architecture, which combines off-line time sliced OD estimation procedures, with on-line time dependent OD estimation procedures that use the off-line as OD seeds, in the real-time estimation process. The resulting architecture for the integrated computational framework is depicted in Figure 2, and can be described as follows:

A. Off-line time-sliced OD estimation, corresponds to the upper box in Figure 2, it assumes that:

   a. Traffic data from the available data sources (e.g. inductive loop detectors, magnetometers, License Plate Recognition (LPR) devices, Bluetooth/Wi-Fi antennas, etc.) have been collected for a long period of time, along with other data (e.g. weather data, calendar events, etc.) which can influence traffic behavior and determine different behavioral patterns. An appropriate data analysis, filtering, fusing and clustering the available traffic data, can identify the traffic profiles corresponding to each relevant behavioral pattern. The available data and associated profiles are stored in an ad hoc Historical Traffic data Base.
   b. Profiles and associated data can be selected to generate primary estimates of time-sliced OD matrices that will be used as target OD matrices by the off-line matrix adjustment procedure. Heuristic procedures can be used to generate these initial estimates from the available traffic data can be found in Spiess and Sutter (1990), Barceló et al. (2014 and Bullejos et al. (2014).

c. The process assumes that a suitable network model is available from which a Dynamic User Equilibrium (DUE) can be conducted. The results of the DUE allow the identification of the Most Likely Used Paths (MLU) between origins and destinations in the network, and a primary estimate of the expected path flow proportions and path travel times in equilibrium.

d. The traffic data associated to the currently selected time-slice, let's say the k-th, its corresponding initial OD matrix, and the MLU and associated path travel times, and path flow proportions, are the main input to the "Static Bi-level OD Adjustment Process", based on the approach described in Barceló et al. (2014) and Bullejos et al. (2014).

e. The iterative application of the procedure to the time-slices in which has been split a time horizon and its profiles allows to generate a Database of adjusted time-sliced which will be used as OD seeds in the on-line adjustment process.

B. On-line real-time OD estimation, corresponds to the lower box in Figure 2, it assumes that:

a. The detection layout, which is fixed, the active detectors at the current time slice k, which can be variable, depending on the incidences, along with the structure of the most likely used paths, and the location of fixed detectors in this structure, are the main input to a procedure that generates the data structures of the Kalman model for the estimation and short term prediction of the OD matrix for time slice k+1.
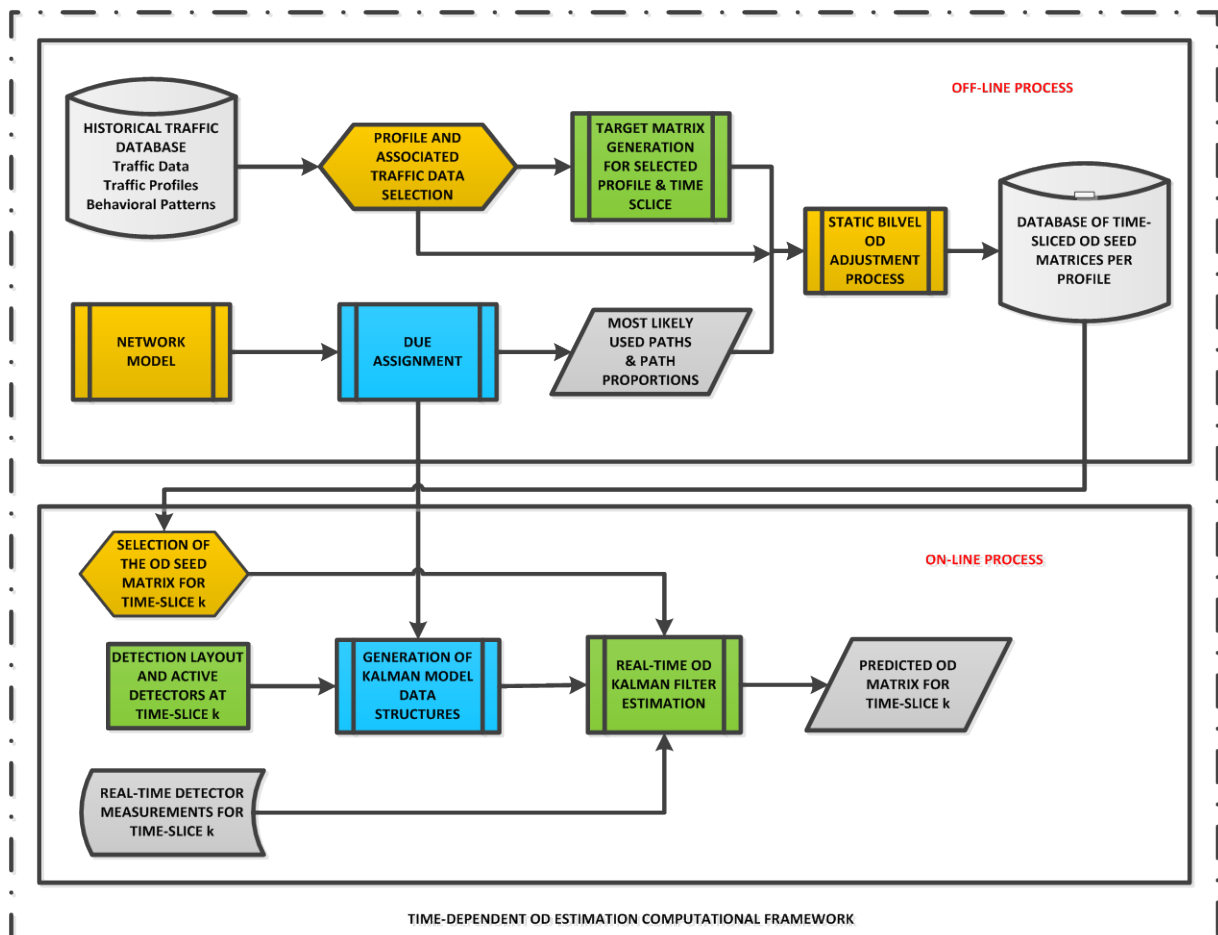

Figure 2: Conceptual architecture of the computational framework for the estimation of time-dependent OD matrices

b. Two other key inputs to the Kalman Filter are the OD seed for the current time slice k, selected from the off-line Database of time-sliced OD matrices, which depends on the current

traffic data profile, and the traffic data measurements supplied in real-time by the available traffic detectors during that time period.

c. The Kalman Filter proposed in this paper that will be described next, is a variant of the versions in Barceló et al. (2013a) and (2013b) purposely adapted to deal with variable configurations of the detection layout.

3. OVERVIEW OF THE OFF-LINE COMPONENT

Static off-line procedures to adjust OD matrices to traffic measurements are usually based on mathematical programming approaches, as has been mentioned in Section 1, and among them the bilevel optimization methods provide the most consistent results (Lundgren and Peterson 2008) since for each time-slice k, at the upper level solves the nonlinear optimization problem

$$\underset{g}{MIN} \, F\left(\widetilde{g}^{\,k}, \widetilde{v}^{\,k}\right) = \gamma_1 F_1\left(g^k, \widetilde{g}^{\,k}\right) + \gamma_2 F_2\left(v^k, \widetilde{v}_k\right) \qquad (1)$$
$$\text{s.t. } \widetilde{g}^{\,k} \geq 0$$

including nonnegative constraints and, depending on the formulation other type of constraints on the adjusted trip variables to preserve the structure and consistency of the adjusted OD matrix. The objective function $F\left(\widetilde{g}^{\,k}, \widetilde{v}^{\,k}\right)$ in this example accounts for the minimization of distance functions F_1 and F_2 between the seed matrix g^k and the adjusted matrix $\widetilde{g}^{\,k}$, and the measured link flows v^k and the estimated link flows $\widetilde{v}^{\,k}$ respectively, with weighting coefficients γ_1 and γ_2 that express the analyst's confidence in the related observations. The estimated link flows $\widetilde{v}^{\,k}$ generated by the corresponding adjusted matrix $\widetilde{g}^{\,k}$ are the solution to the lower level optimization problem:

$$\widetilde{v}^{\,k} = assignmt\left(\widetilde{g}^{\,k}\right) \qquad (2)$$

An user equilibrium traffic assignment problem. This corresponds to the computational scheme depicted in Fig. 3, which ensures consistency at each iteration between the estimated OD matrix $\widetilde{g}^{\,k}$ and the corresponding link flows $\widetilde{v}^{\,k}$, since it accounts for the variability of link and path usage when congestion grows.

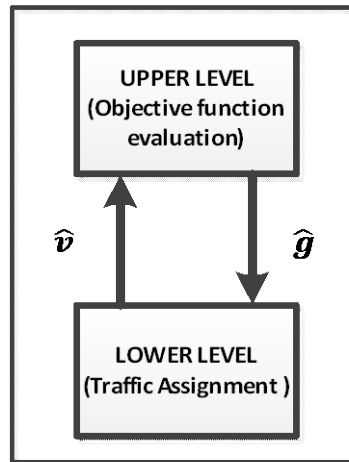
Fig 1. Bi-level Optimization Computational Scheme

This formulation works properly from a nonlinear optimization approach when the weighting functions F_1 and F_2 in the objective function have analytical forms, and the assignment problem at the lower level is a convex optimization problem. However, this is not the case when a richer set of traffic measurements, as for example when travel times from ICT measurements are included in the model formulation. This is the case analyzed in Barceló et al. (2014) and Bullejos et al. (2014) which corresponds to the off-line component of the proposed computational framework. In this case the objective function to be optimized becomes:

$$F\left(\widetilde{g}^{\,k},\widetilde{v}^{\,k},\widetilde{tt}^{\,k}\right)=\gamma_1 F_1\left(g^k,\widetilde{g}^{\,k}\right)+\gamma_2 F_2\left(v^k,\widetilde{v}^{\,k}\right)+\gamma_3 F_3\left(tt^k,\widetilde{tt}^{\,k}\right) \qquad (3)$$

The objective function is non differentiable, therefore the computational scheme of the bi-level cannot be applied in a straightforward manner. In Cipriani et al. (2011) an approach based on the Stochastic Perturbation Stochastic Approximation (SPSA), Spall (1998) is proposed, and improved in Cantelmo et al. (2014).

The variant of this approach, implemented and tested in Barceló et al. (2014) and Bullejos et al. (2014), is the one implemented in the off-line component of the computational framework in this paper. The computational scheme in Figure 4, can be considered a hybrid of a bi-level optimization and an simulation-optimization approach (Fu et al. 2005, Osorio and Bierlaire 2013) , since it solves at the lower level a simulation based user equilibrium traffic assignment while, at the upper level, the SPSA optimization procedure uses the simulation based dynamic user equilibrium assignment for the function evaluations resulting from the perturbation.
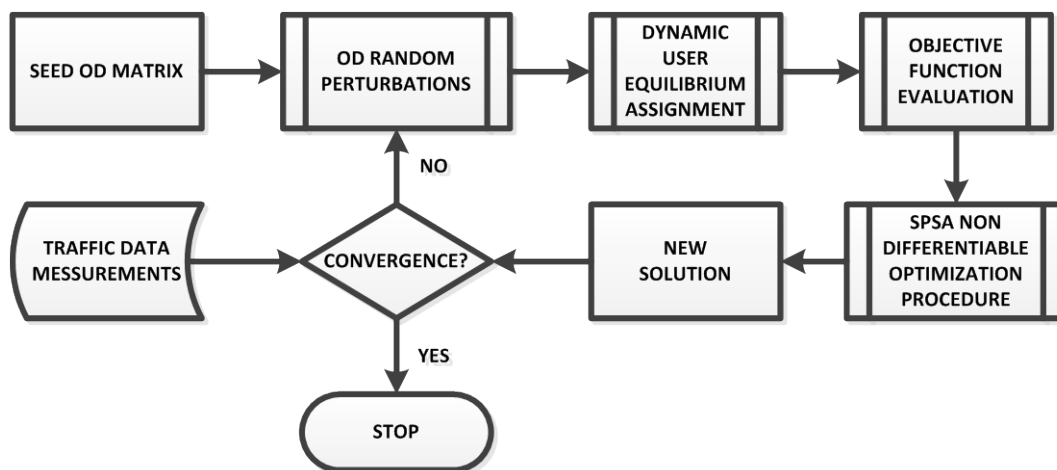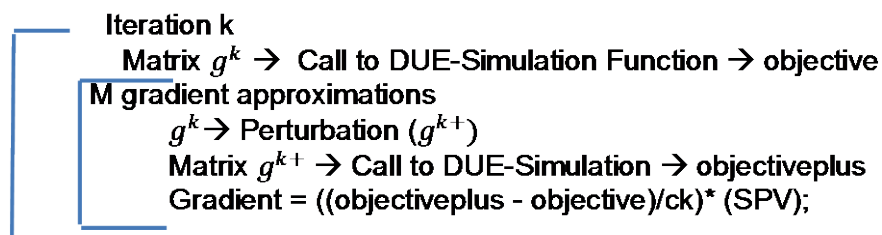

Figure 4: Computational scheme of the Bilevel Optimization-Simulation SPSA based off-line OD adjustment

The procedure starts conducting a simulation (Due-Simulation function call) using the iteration's initial matrix, from which it gets the value of the objective function. The objective value remains unchanged throughout the iteration, but the *objectiveplus* value varies in each of the "M" gradient approximations done at the inner loop, where "M" perturbations of the initial matrix are created. The procedure consists schematically of the following inner and outer loops:

Iteration k
  Matrix g^k → Call to DUE-Simulation Function → objective
  M gradient approximations
    g^k → Perturbation (g^{k+})
    Matrix g^{k+} → Call to DUE-Simulation → objectiveplus
    Gradient = ((objectiveplus - objective)/ck)* (SPV);

Where SPV is the Simultaneous Perturbation Vector, Spall (1998). Two major changes in the implementation of the SPSA with respect to the original Spall's proposal have been the replacement of the gradient procedure by a projected gradient, and the use of a trust region method, Conn et al. (2000), in the optimization process. These two changes improve substantially the computational performance of the process both, in terms of convergence speed and numerical quality Bullejos et al. (2014).

3.1 Conjugate Gradient

The average gradient is used for updating the current time-sliced OD at outer iteration k, with which the next iteration begins. Reducing the number of replications is a key factor, since replications are time-consuming for large networks. In Cantelmo et al. (2014) is suggested that the computation of the approximated average gradient could be enhanced using a conjugate gradient strategy. It is known that the generated conjugate directions permit to reach faster the solution than using the basic gradient method. When we calculate the new solution (see Fig.4), the formulation can be replaced by:

$$x_{k+1} = x_k - a_k(\bar{\hat{g}}(x_k) + \beta\bar{\hat{g}}(x_{k-1})) \tag{4}$$

with

$$\beta = \frac{\|\bar{\hat{g}}(x_k)\|^2}{\|\bar{\hat{g}}(x_{k-1})\|^2}$$

This modification does not require additional computational time per iteration, since previous average approximated gradients can be stored and their values will be available to calculate β.

3.2 Trust Region

The computational costs required by the former implementations of the Bilevel-DUE procedure were very high; therefore it was relevant to look for more efficient solutions. Osorio et al. (2010), describes a simulation-based optimization (SO) method for complex problems with very tight computational budgets for which they report very promising computational performance. The efficiency and suitability of the methods are enhanced by the use of a trust region scheme. The main idea of trust region is to set implicitly at each iteration, a neighborhood around the current solution, where perturbations need to be contained. Following this approach, the perturbations are used to approximate a gradient for the outer level function. Hence, firstly we have analyzed the size and the number of trips of the seed matrix. Based on this analysis, a neighborhood is established for each time slice in particular and for the number of trips of the entire matrix in general. Let be n_s the number of time slices. Then, the trust region can be formulated as follows:

$$(1-\gamma_1)*TS_j(\hat{d}) < TS_j(X^+) < (1+\gamma_1)*TS_j(\hat{d}) \quad \gamma_1 \in (0.01, 0.1)$$
$$(1-\gamma_2)*\hat{d} < X^+ < (1+\gamma_2)*\hat{d} \quad\quad 0 < \gamma_2 \le \gamma_1 \tag{5}$$

where:

$TS_j(\hat{d})$ number of trips in the time slice j of the seed matrix, with $j = 1 \dots n_s$
$TS_j(X^+)$ number of trips in the time slice j of the perturbed matrix, with $j = 1 \dots n_s$
$\hat{d}$ number of trips of the seed matrix
X^+ number of trips of the perturbed matrix

One should take into account that for each perturbed matrix a DUE has to be performed. The trust region ensures in this way that the matrices generated by the SPSA algorithm remain in the right direction towards the real OD matrix. Avoiding replications of matrices outside of the trust region is essential to reduce the computational burden.

Figure 5.a and 5.b, represent fifty perturbed matrices from the seed matrix used in our experiments. Any of them could have been chosen by our method to evaluate the O.F. and update the solution. However, with the trust region we ensure that the chosen matrix is near to the previous one. If we do not move far away from the current matrix, a more reliable convergence process to the real matrix is achieved. The x axis of the Figure 5 shows the number of trips of the matrix and the y axis the number of trips of the first time slice (in this computational experiment the matrix has five time slices whose components are simultaneously perturbed).

If the starting point is an obsolete matrix obtained using census and sample information, the trust region parameters can be set to find perturbed matrices with a higher number of trips than the preexisting origin-destination matrix (see Figure 5.b). Otherwise, if no information is available or for testing purposes of our algorithm with different demands, the trust region will consider matrices either with less or greater number of trips than the seed matrix (see Figure 5.a).
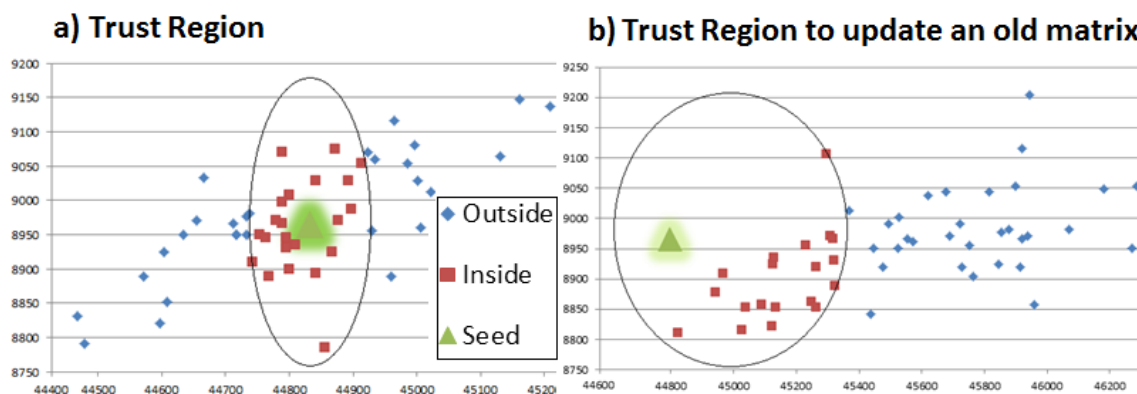

Fig. 5 a) Trust region b) Trust region to update an old matrix

Figure 6 summarizes the conceptual scheme of the modified SPSA implemented in the Bi-level DUE.
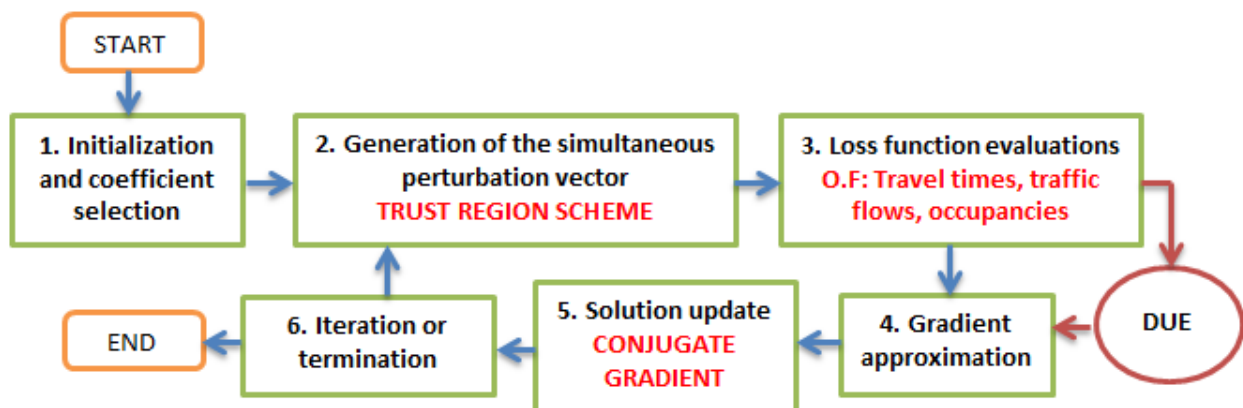

Figure 6: Bi-level DUE – Modified SPSA algorithm

3.3 Computational results

For the travel times term in the O.F., the sensor layout and the most likely used paths between them have been calculated on basis of the procedures already used in Barceló et al. (2012). Figure 7 depicts the optimized layout used with the models of Barcelona's Eixample (7a) and Vitoria (7b) respectively, highlighting the paths for which the travel times measured by Bluetooth antennas (blue circles) are available. Traffic data were collected from the 116 Traffic Detection Stations available in the scenario and 50 ICT sensors (Bluetooth antennas in this case)

Barcelona's Central Business District (CBD), Eixample, is a network consisting of 2111 sections, 1227 nodes (grouped in intersections), 120 generation centroids, 130 destination centroids and a total of 877 non-zero OD pairs. Vitoria's network, in the Basc Country, was made available by TSS to MULTITUDE project, consists of 57 centroids and 2800 intersections. Traffic data were collected from 389 loop detectors and 50 ICT sensors (Bluetooth antennas).

Models of the selected scenarios were built with Aimsun, TSS (2013), simulation software, and Aimsun APIs were used to emulate the ICT traffic detection. Almost 90% of the trips were collected twice at least in the peak-afternoon demand scenario of 1 hour of length, which accounted for 95% of the number of OD pairs and 86% of the most likely used paths identified in a DUE assignment with the 'true' historic OD matrix. The most likely routes between pairs of Bluetooth antenna were also identified and the measured and simulated travel times along these partial paths were used in the objective function.
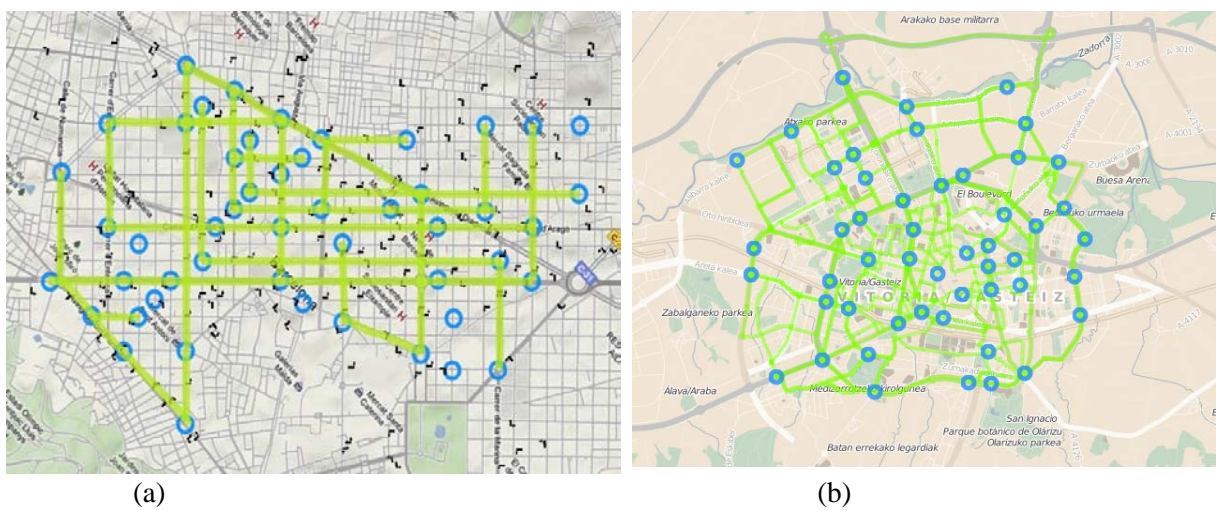

(a)　　　　　　　　　　　　　　　　　　(b)

Fig. 7: Bluetooth layout and available paths (a) Barcelona's CBD (Eixample); (b) Vitoria
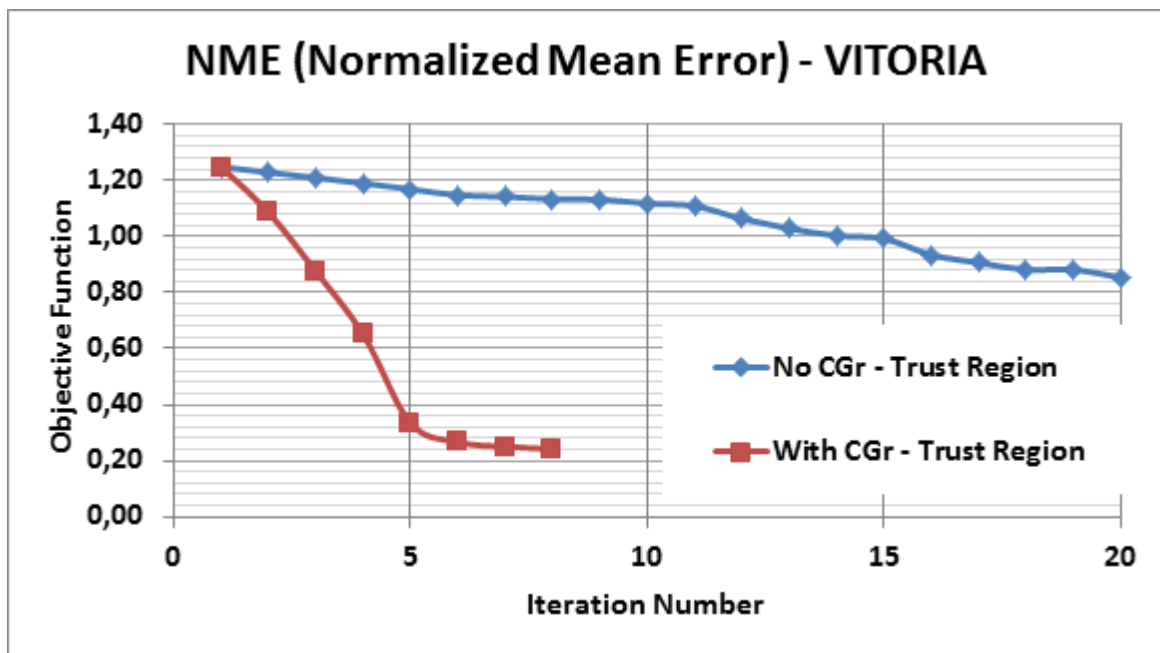

Figure 8: Convergence trend of the O.F. without and with Conjugated Gradient and Trust Region.
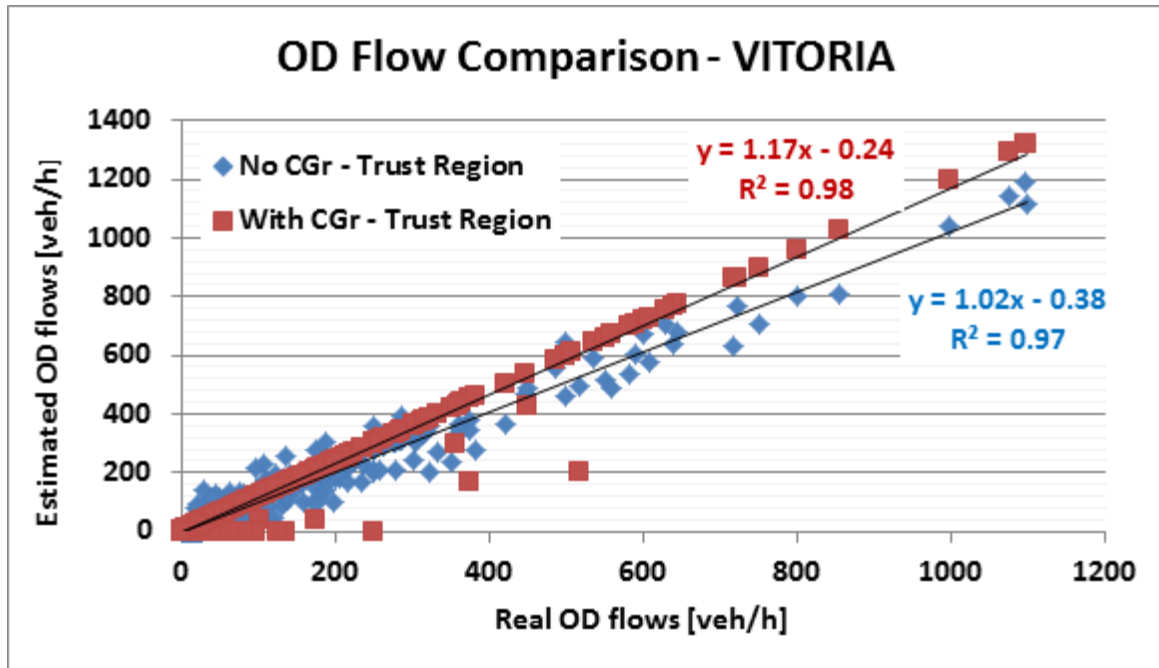
Vitoria's case.

Figure 9: Real vs Estimated OD flows without and with Conjugated Gradient and Trust Region. Vitoria's case.
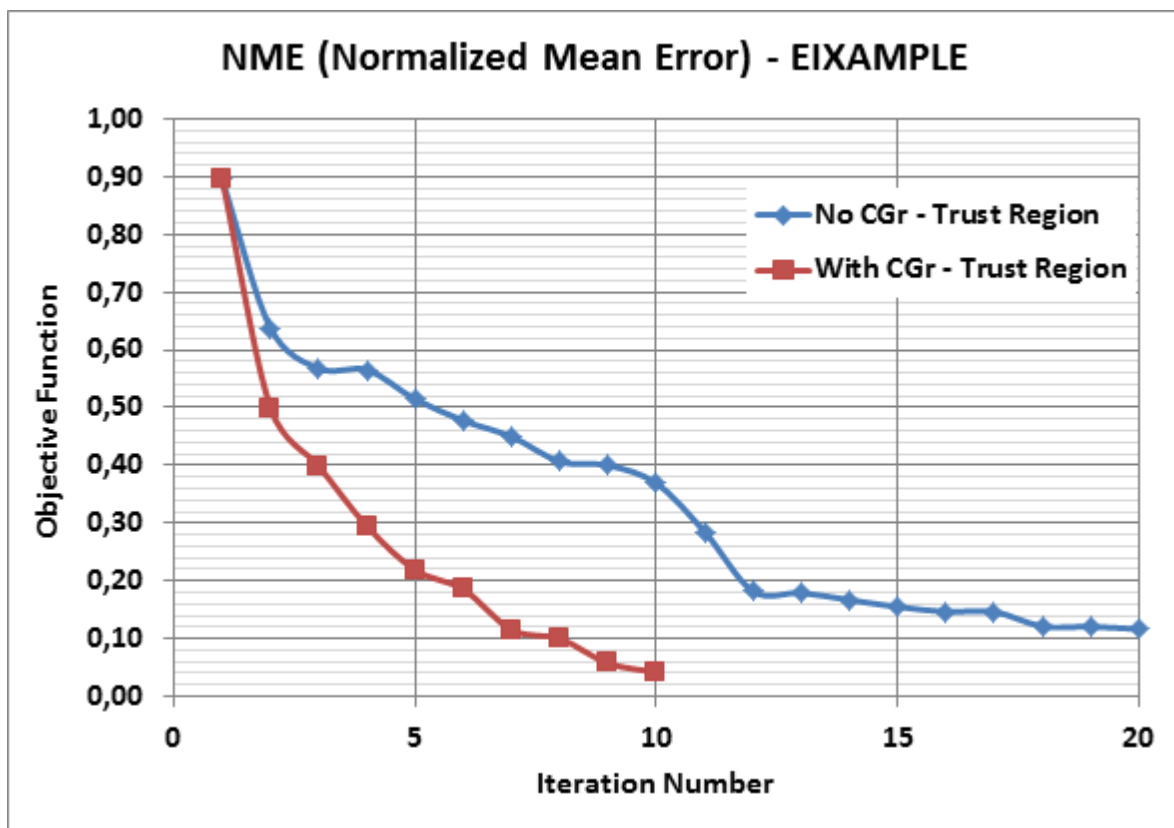

Figure 10: Convergence trend of the O.F. without and with Conjugated Gradient and Trust Region. Eixample's case.
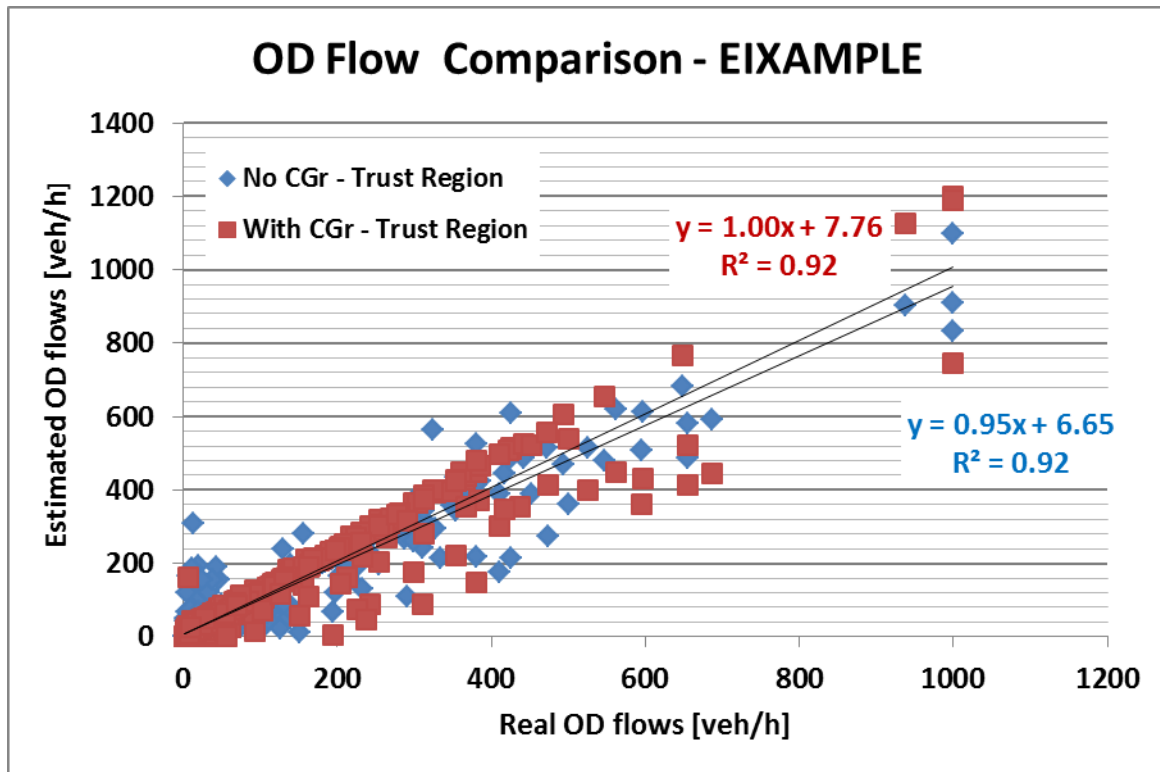
OD Flow Comparison - EIXAMPLE

Figure 11: Real vs Estimated OD flows without and with Conjugated Gradient and Trust Region. Eixample's case.

Figures 8 and 9 display graphically the results for Vitoria's network and Figures 10 and 11 those for Eixample's network. In both cases the O.F. Figures 8 and 10 show about a 95% reduction in the value of the O.F. but this requires around 20 iteration in the general SPSA gradient approach to be achieved, while only 10 or less than 10 are required in the improved version when conjugate gradient and trust region are used, exhibiting always a faster convergence ratio. But, in both cases, the quality of the solution in terms of the coefficient of determination is similar as shown in Figures 9 and 11. Further numerical details can be found in Bullejos et al. (2014) and Djukic et al. (2015),

4. OVERVIEW OF THE ON-LINE COMPONENT: STATE-SPACE FORMULATION REVISITED

The space-state formulation used in the on-line component is the recursive linear Kalman-Filter for state variable estimation discussed in Barceló et al. (2013a) and Barceló et al. (2013b), adapted to exploit the travel times and traffic counts collected respectively by tracking Bluetooth equipped vehicles and conventional detection technologies.

The proposed approach initially assumes flow counting detectors and ICT sensors located in a cordon at each possible flow entrance (*centroids* of the study area), and ICT sensors located at intersections in urban networks covering links to/from the intersection. Flows and travel times are available from ICT sensors for any time interval length higher than 1 second. Trip travel times from origin entry points to sensor locations are measures provided by the detection layout. Therefore, they are no longer state variables but measurements, which simplify the model and make it more reliable.

A basic hypothesis is that equipped and non-equipped vehicles follow common OD patterns. We assume that this holds true in what follows and that it requires a statistical contrast for practical applications. Expansion factors from equipped vehicles to total vehicles, in a given interval, can be

estimated by using the inverse of the proportion of ICT counts to total counts at centroids; expansion factors are assumed to be shared by all OD paths and pairs with a common origin centroid and initial interval.

The proposed linear formulation of the Kalman Filtering approach uses deviations of OD path flows as state variables, calculated with respect to DUE-based Historic OD path flows. A subset of the most likely OD path flows identified from a DUE assignment is used. The DUE is conducted with the historic OD flows, and the number of paths to take into account is a design parameter. A list of paths going through the sensor is automatically built for each ICT sensor from the OD path description, ICT sensor location and the network topology.

The time-varying dependencies between measurements (sensor counts of equipped vehicles) and state variables (deviates of equipped OD path flows), are used for estimating discrete approximations to travel time distributions. Since the approach uses the travel ICT time measurements from equipped vehicles, the nonlinear approximations can be replaced by estimates from a sample of vehicles, and then no extra state variables for modeling travel times and traffic dynamics are needed, since sampled travel times are used to estimate discrete travel time distribution, see Barceló et al. 2014b for details, splitting the time horizon into H subintervals. The demand matrix for the period of study is divided into several time-slices, accounting for different proportions of the total number of trips in the time horizon. The approach assumes an extended state variable for M+1 sequential time intervals of equal length Δt.

The solution provides estimations of the OD matrices for each time interval up to the k-*th* interval. State variables $\Delta g_{ijc}(k)$ are deviations of OD path flows $g_{ijc}(k)$ relative to historic OD path flows $\tilde{g}_{ijc}(k)$ for equipped vehicles. A MatLab prototype algorithm has been implemented to test the approach (named KFX2).

Let $Q_i(k)$ and $q_i(k)$ be respectively the number of vehicles and equipped vehicles entering from centroid *i* at time interval k. Conservation equations from entry points (centroids) are explicitly considered. Without $Q_i(k)$, a generic expansion factor has to be applied. The state equations are formulated as follows.

Let $\Delta\mathbf{g}(k)$ be the column vector of the state variables $\Delta g_{ijc}(k)$ for each time interval k for all *most likely* OD paths (i,j,c). The state variables $\Delta g_{ijc}(k)$ are assumed to be stochastic in nature, and OD path flow deviates at current time k are related to the OD path flow deviates of previous time intervals by an autoregressive model of order r <<M; the state equations are:

$$\Delta\mathbf{g}(k+1) = \sum_{w=1}^{r} \mathbf{D}(w)\Delta\mathbf{g}(k-w+1) + \mathbf{w}(k) \qquad (6)$$

Where $\mathbf{w}(k)$ are zero mean with diagonal covariance matrix $\mathbf{W_k}$, and $\mathbf{D}(w)$ are transition matrices which describe the effects of previous OD path flow deviates $\Delta g_{ijc}(k-w+1)$ on current flows $\Delta \mathbf{g_{ijc}}(k+1)$ for w = 1,…,r . In the implementation tested we assume simple random walks to provide the most flexible framework for state variables, since no convergence problems are detected. Thus r=1 and matrix $\mathbf{D}(w)$ is the identity matrix.

The relationship between the state variables and the observations involves time-varying model parameters (congestion–dependent, since they are updated from sample travel times provided by equipped vehicles) in a linear transformation that considers:

- The number of equipped vehicles entering from each entry centroid during time intervals k,.., k-M, $q_i(k)$.
- H<M *time*-varying *model parameters* in form of *fraction matrices,* $\left[u_{ijcq}^{h}(k)\right]$.

Where $u_{iq}^h(k)$ are the fraction of vehicles that require h time intervals to reach sensor q at time interval k that entered the system from centroid i (during time interval $[(k-h-1)\Delta t \quad , \quad (k-h)\Delta t]$); and $u_{ijcq}^h(k)$ represent the fraction of equipped vehicles detected at interval k whose trip from centroid i to sensor q might use OD path (i,j,c) lasting h time intervals of length Δt to arrive from centroid i to sensor q, where i=1,…,I , j=1,…, J, $h=1\dots M$, $q=1\dots Q$, and I, J and Q are, respectively, the number of origin centroids, the number of destination centroids and the number of ICT sensors. Direct samples of travel times from ICT sensors allow the updating of the H adaptive fractions u_{iq}^h and u_{ijcq}^h, making unnecessary to incorporate models for traffic dynamics. *Time-varying model parameters u_{iq}^h and u_{ijcq}^h* account for temporal traffic dispersion in affected paths and have to satisfy structural constraints, where H<M:

$$u_{ijcq}^h(k) \geq 0 \quad i=1....I, \quad j=1....J, \quad c=1....K_{ij}^{max}, \quad q=1,...Q, \quad h=1...H$$
$$\sum_{h=1}^{H} u_{ijcq}^h(k) = 1 \quad i=1....I, \quad j=1....J, \quad c=1....K_{ij}^{max}, \quad q=1,...Q \quad (7)$$

At time interval *k*, the values of the observations are determined by those of the state variables at time intervals *k, k-1, …k-M.*

$$(8)$$
$$\Delta z(k) = \begin{pmatrix} \mathbf{AU(k)^T} \\ \mathbf{E(k)} \end{pmatrix} \Delta \mathbf{g(k)} + \begin{pmatrix} v_1(k) \\ v_2(k) \end{pmatrix} = \mathbf{F(k)} \Delta \mathbf{g(k)} + \mathbf{v(k)}$$

Where $\mathbf{v}(k)$ are white Gaussian noises with covariance matrices $\mathbf{R_k}$. $\mathbf{U(k)}$ consists of diagonal matrices $U(k),…,U(k-M)$ containing $u_{ijcq}^h(k)$. For $U(k-h)$ is a matrix with the estimated proportion of equipped vehicles whose travel time from the access point to the network takes h intervals and goes through the q sensor at interval k. $\mathbf{E(k)}$ is a matrix with I rows and non-zero columns only for the current time-interval k and defining conservation of flows for each entry at k. And $\mathbf{A}$ is a matrix that adds up sensor traffic flows from any possible entry, given time-varying model parameters at interval k. $\mathbf{F}(k)$ maps the state vector $\Delta \mathbf{g}(k)$ onto the current blocks of measurements at k: deviate counts of equipped vehicles by sensors and entries at centroids, accounting for time lags and congestion effects according to *time-varying model parameters.*

This paper reports on the revisited implementation KFX2, called KFX3, which extends the former in the following way:

- It is a multiclass approach where classes are defined in terms of the available technologies, e.g. non equipped vehicles, Bluetooth/Wi-Fi equipped vehicles, and GPS equipped vehicles and so on.
- Various types of fixed location sensors for traffic data collection are considered, either located at links (e.g. loop detectors, magnetometers…) or at intersections (e.g. Bluetooth/Wi-Fi antennas), either counting vehicles, or identifying the electronic footprint.
- The OD estimation procedure doesn't explicitly use vehicle trajectories, but samples of travel times *between any pair of ICT sensors* to account for traffic congestion.
- No sensors are requested at entry centroids, but they could be located in a subset of the centroids (this is realistic in cordon areas). Since travel time distributions from centroids to ICT sensors have to be provided to the on-line procedure, then two possible sources for approximating them are considered: the first one consists on using the historical travel time average value for each time slice (this needs to define a non-active ICT sensor at the entry centroid i) and the second possibility relies on defining the closest ICT sensor to the entry centroid i , let us assume is r and

thus travel time distributions from the selected centroid *i* to any ICT sensor *s* will be approximated by travel time distribution between *(r,s)* ICT sensors.

- From a design point of view, a critical attribute of a sensor is its state: active/non active. Non active sensors do not capture any data, but the on-line procedure is designed in a flexible way that automatically adapts to the identified situation. The process is designed to work in two modes to deal with non-active sensors: **historically-oriented and non-historical oriented.** In the historically-oriented behavior, once a sensor is labeled as non-active, it is assumed that its historical data are available, e.g. flow counts or travel times from other ICT sensors, and thus used in the model. For non-historical oriented behavior, any sensor count or travel time related to a non-active sensor is removed from the Kalman equations.
- Traditional and some new ICT sensors are assumed to have a fix location currently, but GPS equipped vehicles are mobile sources, modeled by virtual link detectors whose granularity/precision for the emulated GPS data has to be parameterized, currently is assumed a rough precision at link level (so, for each link segment a virtual ICT sensor with a detection range equal to the link length has been defined). More elaborated possibilities will be considered as future developments.

DUE OD path *proportions* for MLU paths and interval are not an input to the Kalman Filter (KFX3) approach, only the description of the most likely OD paths. The KFX3 formulation uses travel times between any pair of regular ICT sensors, as time-varying model parameters playing the role of discretization of travel time distributions. A data model dealing either with fixed link-based sensors or node-based sensors has been considered. New ICT sensors considered as mobile sources (GPS data) have also be included as a basic prototype through the definition of virtual ICT sensors link-level based.

The length of each interval (time-slice) has to be defined as a multiple of the subinterval length Δt, that it is assumed constant during the horizon of study.

The time-varying dependencies between measurements (sensor counts of equipped vehicles) and state variables use the estimation of discrete approximations to travel time distributions between pairs of *(r,s)* ICT sensors in KFX3.

Case (a)

h	1	2	3	4	5
p	1/15	4/15	1/3	1/5	2/15

Case (b)

h	3	4	5	6	7
p	1/15	4/15	1/3	1/5	2/15

Case (c)

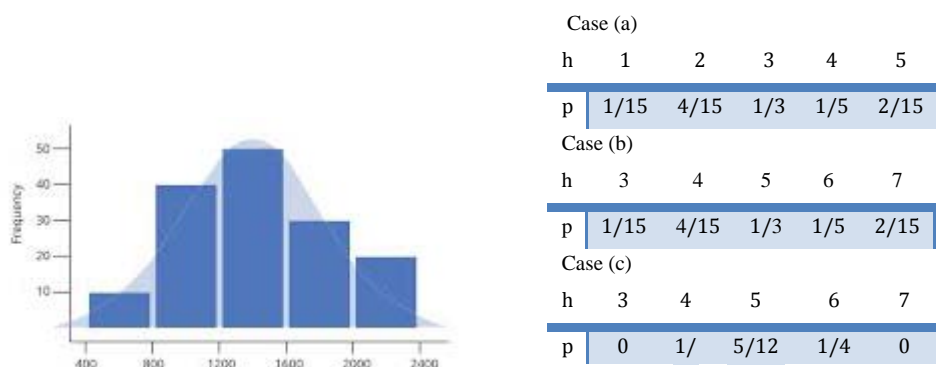| h	3	4	5	6	7
p	0	1/	5/12	1/4	0

Fig. 12: Discrete travel time distribution approximation – Histogram representing Case (a)

Sampled travel times from ICT equipped vehicles are used in KFX3 to approximate travel time distributions between pairs of *(r,s)* ICT sensors by discrete travel time distributions adapting the process described in Barceló et al. (2013b) according to H non null bins. For example, let's assume that travel times between a pair of ICT sensors lie between 400 and 2400 seconds with a 0.9 probability (see 12a) and a subinterval length of 400 seconds to simplify the figures and H=5 bins, a sample of 150 equipped vehicles provides travel time data allowing to approximate the distribution using H= 5 bins, considering non null probability bins from one to five Δt, each with the probability described in Figure 12a.

For another OD pair and travel time distribution between 1200 and 3200 seconds (with probability greater than 0.9), the discrete approximation would follow the Case (b) described in Figure 12b. And finally, let us assume OD pair travel time distribution showing less dispersion, since the range between 1600 and 2800 seconds coverts a probability greater than 0.9, and thus, the discrete approximation follows the Case (c) described in Figure 12.

Discrete distributions are updated every time subinterval Δt. Clearly, the number of bins H depends on the dispersion of travel time distributions, in our experience H between 3 and 5 is enough to cope with travel time variability between pairs of points in medium size networks for a proper Δt. The same procedure has been applied in KFX3 for building and updating travel time distributions between OD pairs. There is a relationship between the subinterval length and the suitability of the scheme for approximating travel time distributions: a large Δt diminishes the ability to capture the variability of travel time distributions and thus large H are non-effective and rough approximations are provided. However a short Δt diminishes the ability to capture the central trend in travel time distributions, providing inaccurate discrete travel time approximations unless the number of bins H is increased, and thus increasing the computational burden. A trade-off has to be determined in a tuning process according to network characteristics and congestion issues.

The solution provides estimations of the OD matrices for each time interval up to the k-th interval. State variables $\Delta g_{ijc}(k)$ are deviations of OD path flows $g_{ijc}(k)$ relative to historic OD path flows $\tilde{g}_{ijc}(k)$ for equipped vehicles. State equations are defined as in Eq. (4). The formulation is modified with respect to the time-varying model parameters (in form of fraction matrices, $\left[u_{ijcq}^h(k)\right]$ and $u_{ijc,rs}^h(k)$ for H<M) and the observation equations affecting the relationship between the state variables and the observations in a linear transformation that considers:

1. The number of equipped vehicles entering from each entry centroid during time intervals k,.., k-M, $q_i(k)$ if provided on-line by the active sensor layout or selected according to a priori historic data if not available. Total number of entry centroids is I.
2. The number of equipped vehicles tracked at each ICT sensor during time interval k if provided on-line by the active sensor layout. Total number of ICT sensors is QQ and active are Q.
3. The number of vehicles counted at each traditional sensor during time interval k if provided on-line by the active sensor layout. Total number of traditional sensors is PP and active is P.
4. H<M time-varying model parameters in form of fraction matrices, $\left[u_{ijcq}^h(k)\right]$ and $u_{ijc,rs}^h(k)$.

Since travel times between any pair of (r,s) ICT are available, a more efficient use of the data allows to define $u_{rs}^h(k)$ the fraction vehicles that require h time intervals to reach sensor s at time interval k from sensor r (tracked at sensor r during time interval $[(k-h-1)\Delta t , (k-h)\Delta t]$); and $u_{ijc,rs}^h(k)$ that represent the fraction of equipped vehicles detected at interval k whose trip from sensor r to sensor s might use OD path (i,j,c) lasting h time intervals of length Δt to arrive from sensor r to sensor s, where $i=1,\dots,I$, $j=1,\dots,J$, $h=1\dots M$, $r,s=1\dots Q$, and I, J and Q are, respectively, the number of origin centroids, the number of destination centroids and the number of ICT sensors, and thus (i,j) identifies an OD pair. Previous model parameters, $u_{iq}^h(k)$ and $u_{ijcq}^h(k)$, are kept if no redundancy appears in the formulation.

Direct samples of travel times from ICT sensors allow the updating of the H adaptive fractions u_{iq}^h and u_{ijcq}^h, in particular, and also the general $u_{rs}^h(k)$ and $u_{ijc,rs}^h(k)$. Time-varying model parameters u_{iq}^h, u_{ijcq}^h, $u_{rs}^h(k)$ and $u_{ijc,rs}^h(k)$ are forced to satisfy non-negativity structural constraints and approximations to travel time distributions in H bins must be well-defined probability distributions for H<M and thus:

$$\sum_{h=1}^{H} u_{ijcq}^{h}(k) = 1 \quad i = 1....I, \quad j = 1....J, \quad c = 1....K_{ij}^{\max}, \quad q = 1,...Q$$

$$\sum_{h=1}^{H} u_{ijc,rs}^{h}(k) = 1 \quad i = 1....I, \quad j = 1....J, \quad c = 1....K_{ij}^{\max}, \quad r = 1,...Q, \quad s = 1,...Q$$

(9)

At time interval k, the values of the observations are determined by those of the state variables at time intervals $k, k-1, ...k-M$.

$$\Delta z(k) = \begin{pmatrix} \mathbf{A_1 U_1(k)^T} \\ \mathbf{A_2 U_2(k)^T} \\ \mathbf{E(k)} \end{pmatrix} \Delta \mathbf{g}(k) + \begin{pmatrix} v_1(k) \\ v_2(k) \\ v_3(k) \end{pmatrix} = \mathbf{F(k)} \Delta \mathbf{g(k)} + \mathbf{v(k)}$$

(10)

Where,

- $\mathbf{v}(k)$ are white Gaussian noises with covariance matrices $\mathbf{R_k}$.

- $\mathbf{U_1(k)}$ consists of diagonal matrices $U_1(k),...,U_1(k-M)$ containing $u_{ijcq}^{h}(k)$. $U_1(k-h)$ is a matrix with the estimated proportion of equipped vehicles whose travel time from the access point to the network takes h intervals and goes through the q sensor at interval k.

- $\mathbf{U_2(k)}$ consists of diagonal matrices $U_2(k),...,U_2(k-M)$ containing $u_{ijc,rs}^{h}(k)$. $U_2(k-h)$ is a matrix with the estimated proportion of equipped vehicles whose travel time from sensor r to s takes h intervals and goes through the s sensor at interval k.

- $\mathbf{E(k)}$ is a matrix with I rows and non-zero columns only for the current time-interval k and defining conservation of flows for each entry (I) at k.

- A_1 is a matrix that adds up sensor traffic flows from any possible entry, given time-varying model parameters at interval k and A_2 is a matrix that adds up traffic flows from a pair (r,s) of ICT sensors, given time-varying model parameters at interval k.

- $\mathbf{F}(k)$ maps the state vector $\Delta \mathbf{g}(k)$ onto the current blocks of measurements at k: deviate counts of equipped vehicles by sensors and entries at centroids, accounting for time lags and congestion effects according to time-varying model parameters.

KFX3 observation equations (10) are considered in three blocks according to the selected functional mode (historical-oriented or non-historical oriented):

- In non-historical oriented mode, the dimensionality of the first block is the number of active sensors (P+Q) and the dimensionality of the second block is equal to the number of pairs of active ICT sensors. The third block consists on the balance equations at origin centroids that it is set to I and it does not change dependent of the functional mode.

- In historical oriented mode, the dimensionality of the first block is the total number of sensors (PP+QQ) and the dimensionality of the second block is equal to the total number of pairs of ICT sensors. The third block dimension is I.

- Block 2 is considered optional in the KFX3 setting and thus, can be included or not according to input parameter for customizing the execution setting. This block can be huge for a mid-size network and thus, a flexible configuration that allows splitting the use of counts from pairs of ICT sensors and the use of travel times as a source for time-varying model parameters seems convenient. Currently, block 2 can be fully included or excluded; partial consideration for some pairs of ICT sensors will be developed in the future.

5. COMPUTATIONAL RESULTS

The experimental approach, used in the computational experiments to test KFX3 has been based on the use of synthetic data to perform controlled experiments. This is an approach well suited to verify inverse problems where, typically the inputs to the forward problem are not measurable, as in the case

of the estimation of OD matrices. In this case a synthetic demand is generated and used as input to the forward problem to produce synthetic measurements from which the inverse problem estimates the synthetic input. The values of the selected performance indicators measure the distance between the known synthetic input and the unknown estimated output, to determine the quality of the estimation procedure.

The parallel highway network of Hu *et al.* (2009) in Figure 13 has been used primarily for debugging and verification purposes. The OD matrix consists of four non-zero OD flows (1,8), (1,9), (2,8) and (2,9) (identified as 1 to 4). The set of MLU OD paths is composed of 10 routes, described in Table 1. Traditional and ICT sensors are initially assumed available at entry nodes 1 and 2, and they will be non-active to test the performance of the KFX3.

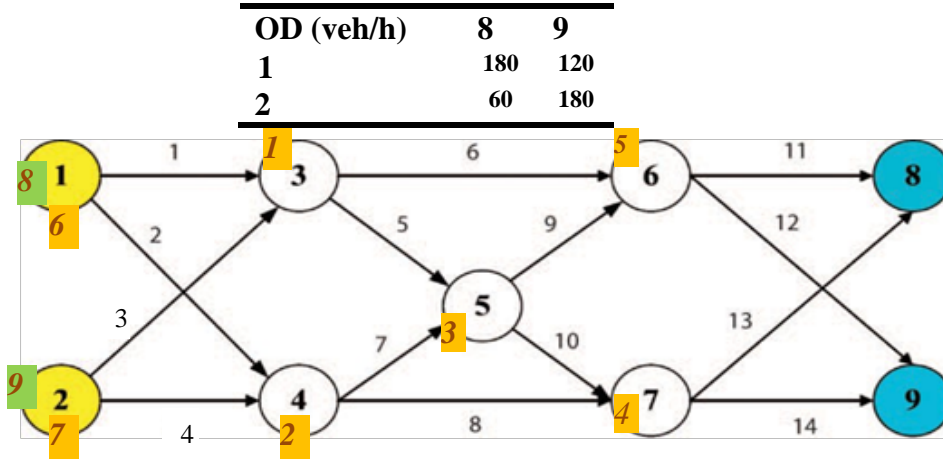| OD (veh/h)	8	9
1	180	120
2	60	180

Fig. 13. Parallel highway network of Hu, Peeta and Chu (2009), ICT sensors (orange box), non-ICT sensors (green box), link and node identifiers

Table 1. Description of MLU OD paths related to DUE assignment

MLU OD Path id	OD path links	OD pair	ICT sensor id	Entry id	MLU OD Path id	OD path links	OD pair	ICT sensor id	Entry id
1	1-6-11	1=(1,8)	6, 1,5	1	6	3-6-11	3=(2,8)	7,1,5	2
2	2-7-9-11	1=(1,8)	6,2,3,5	1	7	4-7-9-11	3=(2,8)	7,2,3,5	2
3	2-8-13	1=(1,8)	6,2,4	1	8	4-8-13	3=(2,8)	7,2,4	2
4	1-5-10-14	2=(1,9)	6,1,3,4	1	9	3-5-10-14	4=(2,9)	7,1,3,4	2
5	2-8-14	2=(1,9)	6,2,4	1	10	4-8-14	4=(2,9)	7,2,4	2

Thus, the number of OD pairs is 4, the number of MLU OD paths is 10 and there are 18 feasible pairs of ICT sensors (r,s). We assume a subinterval of Δt=5 min. Since the time-horizon is one hour, 12 subintervals are defined and 15 min time slices account for (10-20-40-30) percentage of the total demand. No GPS virtual sensors have been initially considered and all vehicles are assumed ICT equipped for checking purposes. The sensor layout is initially oversized, and the availability of sensors is progressively reduced and the corresponding data model for the Kalman filtering formulation in Equation (10) adapted automatically each block with the following dimensions:

- The number of traditional sensors is PP=2 and QQ=7 ICT sensors are initially considered, thus the block 1 has dimensionality PP+QQ=9 when historical-oriented mode is set or all the sensors are active.
- The number of ICT pairs is 18 and block 2 would be 18 rows when historical-oriented mode is set or all the ICT sensors are active. For testing purposes, no observation equations in block 2 have been set.
- The number of entries is 2, being the dimension of block 3.

The quality of the computational results is measured in term of three performance indicators, Theil's

coefficient, NRMSE (Normalized Root Mean Squared error; i.e., sum of squared differences between *target* and estimated OD flows per interval, relative to total *target* flows) and R^2 (coefficient of determination). Robustness of the estimated OD flows is calculated by combining all 4 OD pairs in a Global NRMSE (GNRMSE), defined by:

$$GNRMSE = \frac{\sqrt{4 \cdot 12 \sum_{k=1:12} \sum_{od=1:4} \left(y_{od,k} - \hat{y}_{od,k}\right)^2}}{\sum_{k=1:12} \sum_{od=1:4} y_{od,k}} \tag{11}$$

The integration and validation of the proposed framework for OD estimation with the new on-line estimation tool KFX3 is focused on testing the availability of ICT data. Initially all vehicles are assumed to be ICT equipped and the historical matrix is assumed to be increased by 25% with respect to the known target. Two situations are shown in Figure 14 assuming a non-historical-oriented mode that allows taking benefit of a reduced dimensionality in Equation (10) when non-active sensors are present: on the left, all ICT sensors are active and on the right, entry sensors 7-8 are non-active, but internal sensors 1 to 5 are considered active. The fit for the total period (1h) between estimated and target OD flows is over 95% for both active layout setting. For example, when using internal ICT sensors we are assuming a 'Target Matrix (h)' (in form of OD flow list is 180 120 60 180). The estimated matrix for the one hour horizon is (170 30 54 186). Theil's Coefficient for each OD pair is (0.19 0. 14 0.34 0. 15) and NRMSE is (0.44 0.33 0.72 0.32).
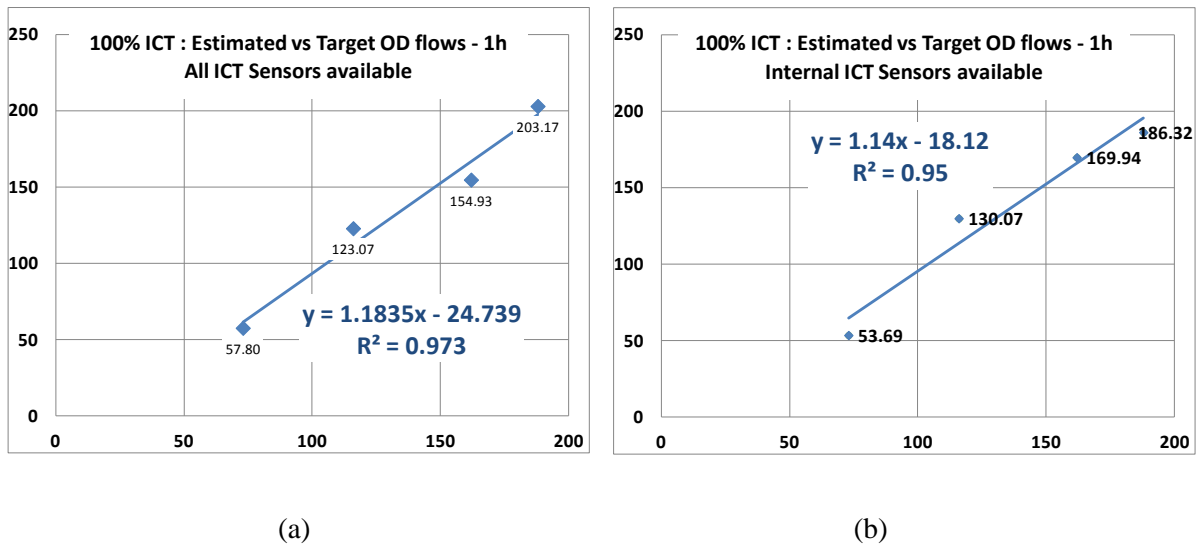

(a)   (b)

Figure.14. R^2 for linear regression of Estimated vs Target OD flows - 1h.

Since the target pattern is assumed to be known, this would be the optimal case when the off-line matrix estimation procedure has produced the best matrix according to the OD pattern. The number of trips (Factor 1) is increased/decreased proportionally for each OD pair and time-slice according to (nu) parameter (50% means half of the ground truth OD trips and 150% means a fifty per cent increase). Table 2 describes the goodness of fit for estimated OD flows according to ICT Sensors availability (Factor 2) and the increase/decreased over the target matrix that it is assumed as input matrix (Factor 1). 100% are ICT equipped vehicles and traditional detectors are set non-active. Non historical-oriented mode is set.

Table 2 Goodness of fit indicators according to ICT Sensors availability (Factor 2) and nu parameter affecting the input matrix (Factor 1)

Factor 2 – Availability ICT Sensors	Factor 1 – A priori total OD flows (target OD Pattern) Global Theil Coefficient (GU) and NRMSE (GNRMSE) and R^2 according to v parameter. (In parenthesis goodness of fit statistics for OD pair 1)											
	50%			100%			150%			0% Without A priori OD		
	GU	GRM SEN	R^2	GU	GRM SEN	R^2	GU	GRMSE N	R^2	GU	GRMS EN	R^2
All: 1 to 7	0.15 (0.12)	0.34 (0.26)	99.1%	0.15 (0.12)	0.34 (0.26)	99.1 %	0.17 (0.14)	0.38 (0.29)	94.7 %	0.14 (0.12)	33% (27%)	95.7%
ICT : 1 to 5	0.19 (0.18)	0.42 (0.42)	94.4%	0.20 (0.19)	0.44 (0.43)	95.8 %	0.21 (0.20)	0.47 (0.45)	94.1 %	0.18 (0.18)	41% (42%)	89.8%
ICT : 1 to 4	0.21 (0.20)	0.48 (0.51)	75.2%	0.20 (0.20)	0.46 (0.50)	81.9 %	0.20 (0.20)	0.46 (0.49)	87.7 %	0.21 (0.21)	50% (53%)	67.9%
ICT : 1 to 3	0.16 (0.15)	0.36 (0.34)	94.6%	0.16 (0.15)	0.36 (0.35)	94.5 %	0.16 (0.15)	0.37 (0.35)	94.5 %	0.16 (0.14)	36% (34%)	94.6%

6. CONCLUSIONS AND FURTHER WORK

The proposed computational framework for the estimation of time-dependent OD integrates two procedures, an off-line, to estimate time-sliced OD seeds, which are the input to an on-line procedure, suitable for the real-time estimation necessary for real-time traffic management. The on-line procedure, based on a revisited Kalman Filtering, exploits the availability of ICT traffic measurements, dealing with variable configurations of detection layouts

The approach has been successfully numerically tested with a small artificial network to verify and check the numerical correctness of the procedure. The future work will apply this modified variant of Kalman Filter to two real urban networks, the City of Vitoria and the CBD of Barcelona already used for testing the off-line component.

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