COMPUTATIONAL VADEMECUMS FOR A FAST AND RELIABLE SIMULATION OF RTM PROCESSES

E. Abisset-Chavanne[1], E. Cueto*[2], A. Huerta[3], F. Chinesta[1]

[1]*GEM, UMR CNRS - Ecole Centrale de Nantes, France*
[2]*Aragon Institute of Engineering Research, Universidad de Zaragoza, Spain*
[3]*Laboratori de Calcul Numeric (LaCaN), Universitat Politecnica de Catalunya. Barcelona, Spain*
* *Corresponding Author: ecueto@unizar.es*

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Abstract

Proper Generalized Decomposition methods allow to obtain an efficient solution for multi-parametric problems without the need for simulating numerous problems to obtain a response surface. Instead, PGD obtains a priori a reduced solution in the form of a finite sum of separable functions, easy to store in memory so as to be evaluated under real-time constraints. The present work proposes to use this tool to optimize the main RTM process parameters, the injection flow rate and the injection/mould temperature, in order to ensure the complete filling of the mould and reasonable fabrication costs (fabrication time, mould heating). To do so, the two process parameters should be introduced in the model as new coordinates, and the Proper Generalized Decomposition method used to solve the multiparametric model then obtained. By using this procedure, we could build computational vademecums*, having the two parameters of interest as coordinates, allowing the fabricant to define the best compromise between injection time and process cost (mould heating) while ensuring the complete filling of the mould. In this work, after revisiting some applications of PGD in RTM processes, the separability of parametric RTM solutions will be evaluated.*

1. Introduction

The reactive resin transfer moulding process has been developed in a view of combining the numerous advantages of thermoplastic materials and the ones of liquid composite molding processes. Its principle is comparable to the classic RTM process, but, because polymerized thermoplastic resins have a high viscosity, the prepolymer are mixed just before the injection nozzle. This technique allows then to ensure a low viscosity for the injected fluid. On the other hand, it implies that the polymerization process occurs in the mould during the filling stage, leading to numerous thermo-chemico-mechanical couplings influencing the final part properties.

In this work we pursue to develop a fast and reliable technique for the in situ optimization of process parameters. To this end, we rely on the concept of computational vademecum, first developed by the authors in [1]. Essentially, a computational vademecum is a response surface obtained in an entirely computational manner. To do so, a multi-parametric problem must be solved, by allowing parameters to take values in a prescribed interval. But if we consider a parametric problem with an important number of parameters (equivalent to state-space dimensions) and try to mesh them to use finite elements/volumes/differences, we will face the so-called *curse of dimensionality* [2], i.e., an exponential growth of the number of degrees of freedom with the number of dimensions (parameters).

To solve this *curse*, Chinesta and coworkers [3] developed a technique coined as Proper Generalized Decomposition (PGD) that allows to overcome these difficulties. See [4] [5] for recent surveys on the field.

Let us model RTM processes as a flow in porous media governed by Darcy's law:

$$\nabla \cdot (\boldsymbol{K} \cdot \nabla P) = 0.$$

Essentially, PGD overcomes the curse of dimensionality by assuming a *separated representation* [6] of the solution (here, pressure field depending on some parameters q_i)

$$P(x, y, z, \boldsymbol{q}) \approx \sum_{i=1}^{n} X_i(x, y, z) \cdot Q_i^1(q_1) \cdot \ldots \cdot Q_i^n(q_n).$$

Functions X_i, Q_i^j are determined by the method by using a greedy algorithm in which one sum is computed at a time. For each sum we seek a product of functions, thus giving a non-linear problem, for which your favorite method can be used (Newton-Raphson, ...), although simple alternating directions, fixed-point algorithms provide excellent results.

With the parametric solution just computed one has access to a completely general solution in the spirit of response surface methodologies, but obtained through a completely computational method and without the need for design-of-experiments techniques. This vademecum is, in addition, very compactly stored in memory in the form of vectors composed by nodal values of the finite element description of the (generally, one-dimensional) functions. The interested reader can refer to [1] and references therein.

2. Shell-like geometries

One of the main difficulties we face when modeling RTM processes is the shell-like geometry of the domains, that forces, if a three-dimensional finite element discretization is employed, to use an abnormally high number of degrees of freedom to adequately reproduce through-the-thickness flow effects. To alleviate this burden, a first attempt has been made by performing an in-plane/out-of-plane decomposition of the solution:

$$P(x, y, z) \approx \sum_{i=1}^{n} X_i(x, y) \cdot Z_i(z),$$

which is sketched in Fig. 1.

This approach presents several advantages, but the main one is that, being the one-dimensional problem (that along the thickness direction) CPU cost negligible, the computational complexity reduces roughly to that of a 2D problem, but providing full 3D details of the solution.

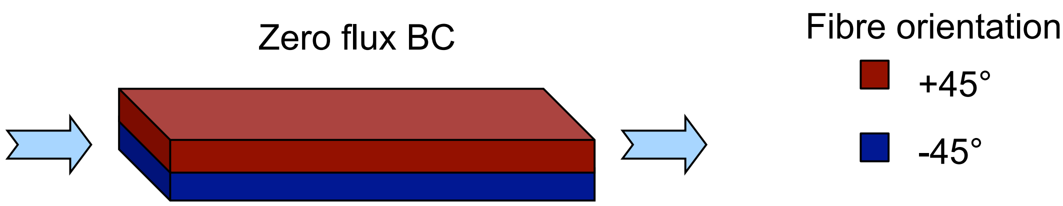

Figure 1. In-plane/out-of-plane decomposition of the pressure field to adequately capture through-the-thickness effects in the solution. This approach allows to efficiently simulate multi-ply composites with different reinforcement angles.

Generalizing this approach for the multi-parametric problem, we arrive at an expression of the form:

$$P(x, y, z, \boldsymbol{q}) \approx \sum_{i=1}^{n} X_i(x, y) \cdot Z_i(z) \cdot Q_i^1(q_1) \cdot \ldots \cdot Q_i^n(q_n), \quad (1)$$

In which the problem is solved as a sequence of two-dimensional problems followed by a series of one-dimensional ones. This same approach was presented in [7] for plate structures.

For composite laminates composed by different plies oriented at different angles, an approach is developed by stating the permeability tensor as

$$\boldsymbol{K}(x, y, z) = \sum_{i=1}^{n_P} \boldsymbol{K}_i(\boldsymbol{x}) \cdot \xi_i(z)$$

where

$$\xi_i(z) = \begin{cases} 1 & z_i \le z \le z_{i+1} \\ 0 & \text{otherwise} \end{cases}$$

In Fig. 2 a streamline plot of the resin flow for a plate reinforced by two plies at ±45° is shown. Clear three-dimensional effects are observed near the wall boundaries as well as a rich evolution in the thickness direction.

In a much more realistic scenario, a workpiece with intricate geometry, see Fig. 3, whose reinforcement is assumed to be composed by 51 plies, has been studied.

In Fig. 5 the complex pattern of streamlines in such a model is represented. It must be highlighted that an equivalent FE model in three dimensions would have been composed by some 2M degrees of freedom.

3. Parametric RTM

The parametric dependence of RTM process on two parameters, namely injection temperature and flow rate, is studied here.

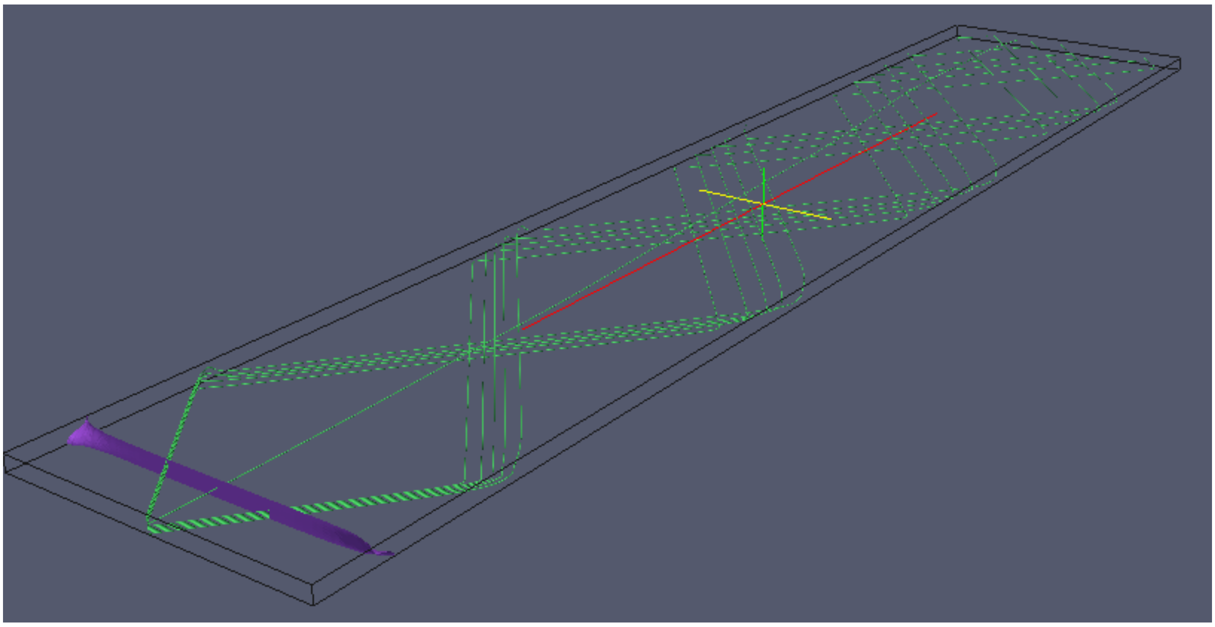

Figure 2. Streamlines for the resin flow in the plate reinforced at ±45° of Fig. 1. Noteworthy, three-dimensional effects are noticed near the lateral walls.

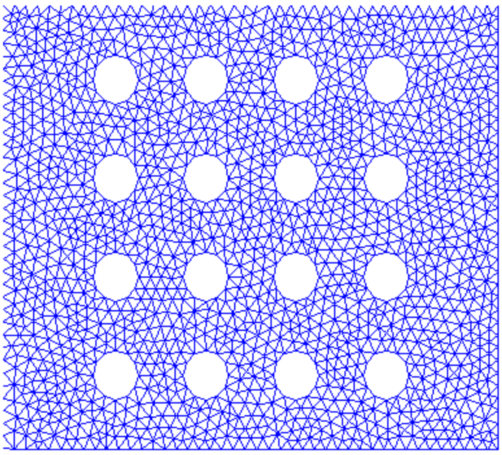

Figure 3. Geometry of the plate with holes.

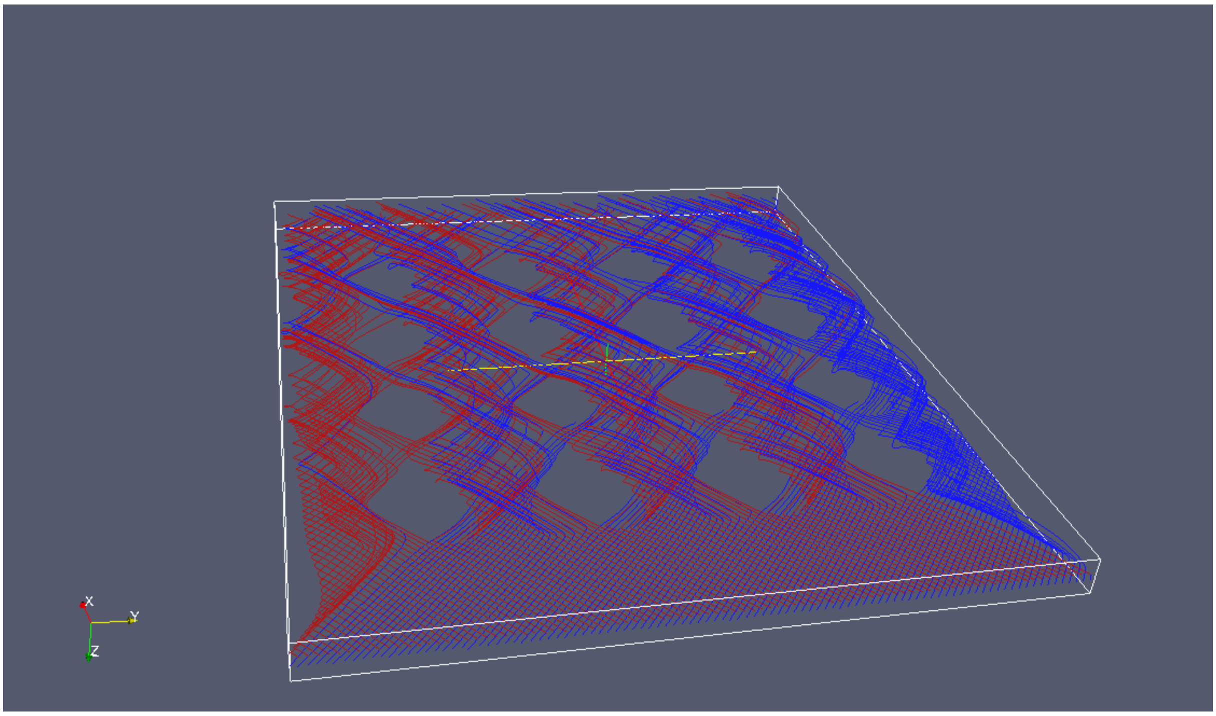

Figure 4. Streamlines for the problem in Fig. 3.

3.1. Temperature-dependent vademecum

A prescribed interval of possible temperatures for injection is considered between 200 and 500°C. Following Eq. (1), a decomposition of the form

$$P(\boldsymbol{x}, t, T_{\text{inj}}) \approx \sum_i F_i(\boldsymbol{x}, t) \cdot G_i(T_{\text{inj}})$$

was obtained by applying SVD on different solutions obtained for different values of T_{inj}. The number of modes required to achieve a certain precision in the result is studied in Fig. 5.

3.2. Flow-rate dependent vademecum

With the same spirit, a test in which flow rate is considered as an additional coordinate in the model was performed:

$$P(\boldsymbol{x}, t, Q_{\text{inj}}) \approx \sum_i F_i(\boldsymbol{x}, t) \cdot G_i(Q_{\text{inj}}).$$

In this case a poor separability of the solution was observed (i.e., a big value of n is found to be necessary to achieve a reasonable accuracy). This must be due, undoubtedly, to the fact that flow rate is too coupled with filling history.

4. Conclusions

The first steps towards the development of numerical vademecums for RTM processes have been developed. This technique allows to obtain a sort of response surface or meta-model in which the parametric RTM problem is solved once for life for any value of the parameters within

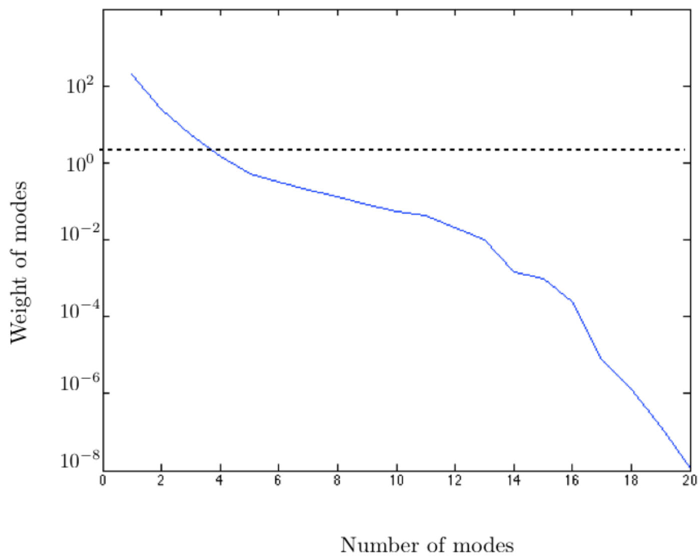

Figure 5. Number of modes, i.e., value of n in Eq. (1), for a prescribed accuracy in the temperature-dependent problem.

a prescribed interval. Three different approaches have been described. Firstly, an in-plane/out-of-plane decomposition for plate or shell geometries has been presented that allows to simulate 3D effects in the filling pattern with roughly 2D computational cost. Then, parametric RTM problems have been addressed.

The first one was a temperature-dependent problem, which has demonstrated to perform well with a minimum number of degrees of freedom. However, for the flow rate-dependent problem, it has been observed that a much higher number of modes seems to be necessary.

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