

**18<sup>th</sup> International Research/Expert Conference  
"Trends in the Development of Machinery and Associated Technology"  
TMT 2014, Budapest, Hungary 10-12 September, 2014**

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**STUDY OF ALTERNATIVES FOR OBTAINING STATISTICAL  
CORRELATION MODELS FOR MODELLING THE ROUGH HONING  
PROCESS**

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**ABSTRACT**

*In the present paper, in order to model rough honing operations, statistical regression correlation models are presented for surface roughness and material removal rate as a function of process variables. Several different models were studied: linear, quadratic, exponential, etc. In addition, Box-Cox transformations were performed to the models so as to improve their fit. Models were compared taking into account two different criteria:  $R^2(\text{Adjusted})$  coefficient and  $R^2(\text{Predicted})$  coefficient. Reduced second order models with Box-Cox transformations were found to be appropriate for modeling average roughness and reduced first order models with Box-Cox transformations for modeling material removal rate.*

**Keywords:** roughness, material removal rate, honing, regression models

**1. INTRODUCTION**

In honing processes, both surface roughness and material removal rate depend on many variables. In the present work mathematical models are presented for average roughness and material removal rate as a function of five process variables, namely linear and tangential speed of honing stones, pressure of honing stones on the workpiece's surface, grain size of abrasive and density of abrasive stone. Both first and second order models were obtained, as well as exponential ones and models obtained with Box-Cox transformations.

**2. EXPERIMENTAL METHOD**

Since 5 variables were considered, honing experiments were performed according to fractional  $2^{5-1}$  design of experiments (16 experiments), with 5 central points. In addition, in order to obtain second order models, 10 face centered axial points were considered [1]. A total amount of 31 experiments were performed, with two replicates (62 runs).

Experiments were performed in a horizontal honing test machine. The honing head is provided with 3 abrasive stones. Abrasive employed was cubic boron nitride (CBN) and bond was bronze. Grain size ranged between 91 and 181 (FEPA) [2], and density of abrasive between 30 and 60 (superabrasives). Linear speed ranged between 20 and 40  $\text{m}\cdot\text{min}^{-1}$  while tangential speed ranged between 30 and 50  $\text{m}\cdot\text{min}^{-1}$ . Pressure of abrasive stones on the cylinder's surface was varied between 400 and 700  $\text{N}\cdot\text{cm}^{-2}$ .

2D roughness was measured with a portable contact roughness meter Hommel-Etamic W5, in 9 points along a diametral circumference [3]. 3D roughness was measured by means of a Taylor Hobson Talysurf series 2 contact roughness meter.

Material removal rate  $Q_m$  was calculated according to equations 1 and 2.

$$V = \pi(R_f^2 - R_i^2)L \quad (1)$$

Where V is removed volume (mm<sup>3</sup>), R<sub>f</sub> is final radius of the cylinder after honing operation (mm), R<sub>i</sub> is initial radius of the cylinder (mm) and L is total length of the cylinder (mm).

Once volumen is calculated, material removal rate is obtained from equation 2.

$$Q_m = \frac{V}{S \cdot t} \quad (2)$$

Q<sub>m</sub> is material removal rate (cm/min), V is removed volume (calculated with equation 1), S is total surface of abrasive stones (cm<sup>2</sup>) and t is test time (min).

In order to measure diameter of the cylinder an internal measuring gage Mitutoyo 511-723 was employed.

### 3. METHODOLOGY FOR OBTAINING THE MODELS

Regression models were found for all responses, by means of Minitab16 and DesignExpert. First, models with all terms were obtained. Then, terms with highest p-value (ANOVA table) were removed and the model was calculated again. Those steps are repeated until all terms in the model are significant.

When considering only fractional design of experiments, linear models were obtained. In order to improve fit, Box-Cox transformations were added to linear models. After checking for curvature, quadratic models were found. In addition, quadratic models with quadratic interactions were tested, as well as Box-Cox transformations. Exponential models were also obtained, which would be similar to the Taylor equation, which models tool life as a function of process parameters in machining processes [4]. Finally, reduced models were obtained in which number of terms is lower than for the rest of the models considered, although fit is also slightly lower.

In order to compare the models, two different fit indicators were used: R<sup>2</sup>(Adjusted) and R<sup>2</sup>(Predicted).

R<sup>2</sup>(Adjusted) is a basic measure to determine variation with respect to the average explained by the model. It takes into account the number of terms in the model (Equation 3).

$$R^2(Adj) = 1 - \frac{SS_{residual}/n - p}{SS_{total}/n - 1} \quad (3)$$

Where SS<sub>residual</sub> is residual sum of squares,  
 SS total is total sum of squares,  
 n is number of terms in the model,  
 and p is number of factors in the model.

R<sup>2</sup>(Adjusted) decreases as more terms are added to the model (given that SS residual increases).

R<sup>2</sup>(Predicted) is a measure of the goodness of a model for predicting a value. It is defined in Equation 4.

$$R^2(Pred) = 1 - \frac{PRESS}{SS_{total}} \quad (4)$$

Where PRESS is Predicted Residual Sum of Squares (Equation 5).

$$PRESS = \sum_{i=1}^n (y_i - \hat{y}_i)^2 \quad (5)$$

Where  $y_i$  is actual value of a point,  
And  $\hat{y}_i$  is predicted value of a point.

In order to obtain an appropriate fit, difference between  $R^2$ (Adjusted) and  $R^2$ (Predicted) should be < 20 %. Otherwise, there might be any no detected problem, either with data or with designed model.

## 4. MODELS

### 4.1. Models for average roughness $R_a$

The different models obtained for  $R_a$  are presented in Table 1.

Table 1. Mathematical models for average roughness  $R_a$

TYPE OF MODEL	MODEL	$R^2$ (Adj) (%)	$R^2$ (Pre) (%)
First Order	$R_a = 1,35114 - 0,000058988G_s - 0,039716D_e + 0,000857331P_r - 0,00950798V_t + 0,003628V_l - 5,07226 \cdot 10^{-4}G_sD_e + 4,71614 \cdot 10^{-5}D_eP_r - 1,04571 \cdot 10^{-3}D_eV_l - 5,10462 \cdot 10^{-5}P_rV_t + 1,02747 \cdot 10^{-3}V_lV_t$	92.91	91.57
First Order Box-Cox	$R_a^{0.5} = 1,28518 + 0,000906482G_s - 0,00868298D_e + 0,00025435Pr - 0,00900654V_t + 0,00377777V_l + 1,31894 \cdot 10^{-4}G_sD_e + 1,15317 \cdot 10^{-5}D_eP_r - 3,10699 \cdot 10^{-4}D_eV_l + 2,39959 \cdot 10^{-4}V_lV_t$	94.20	93.09
Second Order	$R_a = 6,22950 - 1,99027 \cdot 10^{-3}G_s - 0,22700D_e + 1,7789 \cdot 10^{-2}Pr - 0,036586V_t + 7,79919 \cdot 10^{-3}V_l + 5,07226 \cdot 10^{-4}G_sD_e + 4,7614 \cdot 10^{-5}D_eP_r - 1,04571 \cdot 10^{-3}D_eV_l + 1,02747 \cdot 10^{-3}V_lV_t + 2,07972 \cdot 10^{-3}D_e^2$	90.55	89.18
Second Order Box-Cox	$\ln(R_a) = 0,54381 + 1,92756 \cdot 10^{-3}G_s - 0,048579D_e + 3,37283 \cdot 10^{-4}Pr + 0,018395V_l + 1,38005 \cdot 10^{-4}G_sD_e - 3,86442 \cdot 10^{-4}D_eV_l + 5,36406 \cdot 10^{-4}D_e^2$	89.42	88.20
Second Order Interactions	$R_a = 0,91578 - 0,062868G_s + 0,48049D_e + 8,63952 \cdot 10^{-4}P_r - 8,51047 \cdot 10^{-3}V_t - 0,31009V_l - 4,53418 \cdot 10^{-3}G_sD_e + 5,10402 \cdot 10^{-3}G_sV_l + 4,71614 \cdot 10^{-5}D_eP_r - 1,04571 \cdot 10^{-3}D_eV_l - 5,10402 \cdot 10^{-3}P_rV_t + 1,02747 \cdot 10^{-3}V_lV_t + 5,92871 \cdot 10^{-4}G_s^2 - 5,78128 \cdot 10^{-3}D_e^2 - 1,85386 \cdot 10^{-5}G_s^2V_l + 5,60157 \cdot 10^{-3}G_sD_e^2$	94.60	92.73
Exponential	$\ln(R_a) = -5,18941 + 1,03370\ln G_s + 0,26042\ln D_e$	74.59	72.88
Reduced Second Order Box-Cox	$\ln(R_a) = 1,28116 + 1,92756 \cdot 10^{-3}G_s - 0,060173D_e + 1,38 \cdot 10^{-4}G_sD_e + 5,36406 \cdot 10^{-4}D_e^2$	86.64	85.29

For average roughness  $R_a$ , lowest fit was obtained for exponential models. On the contrary, highest  $R^2$ (Adjusted) value (94.60 %) was obtained with a second order model with second order interactions. However, highest  $R^2$ (Predicted) value (93.09 %) was obtained with a first order model with Box-Cox interactions. A reduced second order model with Box-Cox interactions provided quite high  $R^2$ (Adjusted) and  $R^2$ (Predicted) values of 86.64 % and 85.29 % respectively. In this case, the model is simpler than the rest of the models with only 5 terms.

### 4.2. Models for material removal rate $Q_m$

The models for material removal rate are as shown next (Table 2):

Table 2. Mathematical models for material removal rate  $Q_m$

TYPE OF MODEL	MODEL	R2-ADJ (%)	R2-PRED (%)
First Order	$Q_m = 0,65131 - 0,003410249G_s - 0,008689789D_e - 0,000718937Pr - 0,001834882Vt - 0,004004117Vl - 5,44679 \cdot 10^{-5}G_sD_e + 2,11331 \cdot 10^{-6}G_sP_r + 3,14903 \cdot 10^{-5}G_sV_l + 8,30421 \cdot 10^{-6}D_eP_r + 8,30421 \cdot 10^{-6}P_rV_l$	85.82	77.08
First Order Box-Cox	$\ln(Q_m) = -4 - 2,31917 \cdot 10^{-3}G_s - 4,28639 \cdot 10^{-3}D_e + 2,41357 \cdot 10^{-3}Pr + 0,031044Vt + 0,022206Vl + 1,76751 \cdot 10^{-4}G_sD_e - 1,09919 \cdot 10^{-4}G_sV_l + 9,74817 \cdot 10^{-5}G_sV_l - 3,05529 \cdot 10^{-4}D_eV_l + 3,67205 \cdot 10^{-5}P_rV_l$	89.21	81.84
Second Order	$Q_m = 0,98046 - 0,00352751G_s - 0,00880008D_e - 0,0000741105Pr - 0,00189243Vt - 0,02606Vl + 5,44679 \cdot 10^{-5}G_sD_e + 2,11331 \cdot 10^{-6}G_sP_r + 3,14903 \cdot 10^{-5}G_sV_l + 8,30421 \cdot 10^{-6}D_eP_r + 1,05749 \cdot 10^{-5}P_rV_l + 3,64941 \cdot 10^{-4}V_l^2$	81.15	73.36
Second Order Box-Cox	$\ln(Q_m) = -2,6068 - 0,0042G_s - 0,0135D_e + 0,00235P_r + 0,01644V_t - 0,0301V_l + 0,00018G_sD_e - 0,00004P_rV_l + 0,00085V_l^2$	76.10	70.73
Second Order Interactions	$Q_m = 1,05519 + 0,019474G_s + 0,043589D_e + 9,03869 \cdot 10^{-4}Pr - 1,89243 \cdot 10^{-3}Vt - 0,019544Vl - 3,67675 \cdot 10^{-4}G_sD_e - 2,46855 \cdot 10^{-5}G_sP_r + 3,14903 \cdot 10^{-5} + 8,30421 \cdot 10^{-6}D_eP_r + 1,05749 \cdot 10^{-5}P_rV_l - 5,30922 \cdot 10^{-5}G_s^2 - 5,82098 \cdot 10^{-4}D_e^2 + 2,56322 \cdot 10^{-4}V_l^2 + 9,8525 \cdot 10^{-3}G_s^2D_e + 4,69048 \cdot 10^{-6}G_sD_e^2$	82.97	74.33
Exponential	$\ln(Q_m) = -11,59091 + 0,46194\ln(G_s) + 0,42581\ln(D_e) + 0,63704\ln(Pr) + 0,62018\ln(V_t)$	60.75	55.03
Reduced First Order Box-Cox	$\ln(Q_m) = -2,69506 - 4,20750 \cdot 10^{-3}G_s - 0,013498D_e + 1,24726 \cdot 10^{-3}P_r + 0,016445V_l + 1,76751 \cdot 10^{-4}G_sD_e$	73.66	69.58

For material removal rate  $Q_m$ , lowest fit was obtained with the exponential model. Highest  $R^2$ (Adjusted) value and highest  $R^2$ (Predicted) value were obtained with a first order model with Box-Cox interactions: 89.21 % and 81.84 % respectively. If a reduced first order model with Box-Cox interactions is used,  $R^2$ (Adjusted) and  $R^2$ (Predicted) decrease to 73.66 % and 69.58 % respectively. However, model is simpler than the rest of the models, with only 6 terms.

## 5. CONCLUSIONS

Reduced first order models with Box-Cox transformations were selected for both average roughness  $R_a$  and material removal rate  $Q_m$ . Such models provide quite high fit values with a low number of terms. It is possible to obtain various models having different number of terms and different fit. In many cases, it is preferable to choose models with a smaller number of terms but more simple, provided that fit is not excessively reduced.

## 6. REFERENCES

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