Damage detection using robust PCA

Fahit Gharibnezhad[1], Luis Eduardo Mujica[2], Jose Rodellar[3]

Abstract —Contam nated data out ers can affect the resu ts of stat st ca methods such as Pr nc pa Component Ana ys s An out y ng observat on or out er s one that appears to dev ate marked y from other members of the samp e n wh ch t occurs Out ers usua y appear due to exper menta errors and cou d have undesr ab e nf uence on the resu t of the method An effect ve way to dea w th th s prob em s to app y a robust e not sens t ve to out ers var ant of PCA In th s rev ew d fferent robust PCA methods are summer zed br ef y; then the most two mportant robust PCA methods are used nstead of c ass ca PCA The resu ts from both methodo og es robust and c ass c are compared The compar sons of the resu ts shows that the use of the ment oned ndexes based on the robust mode s d st ngu sh the damages much better than us ng c ass ca one and even n many cases a ows the detect on where c ass c PCA s not ab e to d scern between damaged and non-damaged structure As we as compar son between c ass ca and robust methods two robust methods are a so compared w th each other and the r features are d scussed Requ red data are co ected us ng an a rcraft turb ne b ade ut z ng p ezoe ectr c transducers as sensors and actuators and d fferent s mu ated damages

Index Terms — SHM, PCA, Robust PCA

1 INTRODUCTION

Billions of dollars are spent every year maintaining and repairing buildings, bridges, aircraft, railroads, and other infrastructure. In aerospace industry the health of the structure plays a vital role. Research performed by Pyles (2003) and Dixon (2006) have shown that the maintenance costs of military and commercial aircraft increases with the age of the aircraft. With high costs and long down times associated with inspection of both military and commercial aircraft, the frequency at which aircraft are inspected is limited, and there exists a greater possibility of not detecting faults, potentially resulting in aircraft malfunction. According to the latest National Transportation Safety Board data regarding aviation accidents, there were two fatal flight accidents of commercial aircraft in 2006 that resulted in a total of 50 deaths. In an effort to improve the safety and reduce the maintenance costs of our aerospace infrastructure, novel inspection and damage detection techniques need to be developed, tested, and implemented on various structures including buildings, bridges, aircraft, and railroads. One such technique that has gained much attention in the research and industrial communities over the past two decades is structural health monitoring [1].

Structural health monitoring (SHM) is a damage detection technique that involves placing intelligent sensors on a structure, periodically recording data from the sensors, and using statistical methods to analyze the data in order to assess the condition of the structure. The field of SHM developed through the combination of nondestructive evaluation (NDE) methods and novel sensing and actuation techniques to create intelligent monitoring systems permanently installed on structures, in other words, SHM is a non-destructive method aims to give, at every moment during the life of a structure, a diagnosis of the "state" of the constituent materials, of the different parts, and of the full assembly of these parts constituting the structure as a whole. It involves the integration of sensors, possibly smart materials, data transmission, computational power, and processing ability inside the structures. It makes it possible to reconsider the design of the structure and the full management of the structure itself and of the structure considered as a part of wider systems. Structural health monitoring offers a powerful method of monitoring structures with a promise of shifting time-based maintenance schedules, which can be costly, to condition- based schedules, thus significantly reducing maintenance and repair costs. Additionally, SHM has the potential to reduce the human error involved in monitoring structures and to improve the effectiveness of monitoring systems and the overall safety of structures. To achieve this aim there are several potentially useful techniques, and their applicability to a particular situation depends on the size of critical damage admissible in the structure. The development of practical monitoring systems that can be implemented on real world structures faces several design challenges

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including the development of stand alone vibration sensors/actuators, effective damage detection algorithms, and low power wireless transmission systems. Some SHM systems currently being developed include vibration actuation, data acquisition, data processing, and data transmission capabilities all in stand alone wireless sensor units attached at various locations along a structure. All of these techniques follow the same general procedure: the pristine structure is excited using appropriate actuators and the dynamical response is sensed at different locations throughout the structure. Any damage will change this vibrational response, as well as the transient by a wave that is spreading through the structure. Several methods have been used to obtain this vibrational response, for instance: using fiber-optic or piezoelectric transducers. In the next step, necessary data is collected and then, the state of the structure is diagnosed by means of the processing of these data. Correlating the signals to detect, locate and quantify these changes is a very complex problem, but very significant progresses have been recently reported in conferences new scientific journals and books. Among these methods, developing a model using Principal Component Analysis (PCA) for feature discrimination has been considered recently [2]. PCA is the most useful tool in dimensional reduction. The central idea of PCA is to reduce the dimensionality of a data set consisting of a large number of interrelated variables, while retaining as much as possible of the variation present in the data set. This is achieved by transforming to a new set of variables, the principal components (PCs), which are uncorrelated, and which are ordered so that the first few retain most of the variation present in all of the original variables. If the data compression is sufficient, the large number of variables is substituted by a small number of uncorrelated latent factors which can explain sufficiently the data structure. The new latent factors, also called principal components (PCs) are obtained by maximizing the variance of projected data. In the classical approach, the first component corresponds to the direction in which the projected observations have the largest variance. The second component is then orthogonal to the first and again maximizes the variance of the data points projected on it. Continuing in this way produces all the principal components, which correspond to the eigenvectors of the empirical covariance matrix. However, despite of these features, PCA is known to possess some shortcomings. One of them is that both the classical variance (which is being maximized) and the classical covariance matrix (which is being decomposed) are very sensitive to anomalous observations. Consequently, the first components are often attracted towards outlying points, and may not capture the variation of the regular observations. Therefore, data reduction and modeling based on classical PCA (CPCA) becomes unreliable if outliers are present in the data. A way to deal with this problem is to remove the outlying objects observed on the score plots and to repeat the PCA analysis again. Another, more efficient way is to apply a robust, i.e. not sensitive to outliers, variant of PCA. In this review, different robust PCA methods are briefly summerized and then two important robust PCA methods [3, 4] are implemented instead of classical PCA to detect damage on a structure. Experimental results have been compared and it has been proved that using robust PCA is prior to using classical PCA in presence of contaminated data.

2. PCA and Damage Detection Indices (T^2 and Q-statistics) Definition

Principal Components Analysis and its specification are discussed in many articles and books [5,6]. PCA model is calculated using the collected data in a matrix form of $\mathbf{X}$ ($n \times m$) containing information from n experimental trials and m sensors. Since physical variables have different magnitudes and scales, each data-point is scaled using the mean of all measurements of the sensor at the same time and the standard deviation of all measurements of the sensor. Once the variables are normalized the covariance matrix $\mathbf{C_X}$ calculated as follows:

$$\mathbf{C}_X \equiv \frac{1}{n-1}\mathbf{X}^T\mathbf{X} \tag{1}$$

$\mathbf{C_X}$ is a square symmetric matrix ($n \times n$) that measures the degree of linear relationship within the data set between all possible pairs of variables (sensors). The subspaces in PCA are defined by the eigenvectors and eigenvalues of the covariance matrix as follow:

$$\mathbf{C_X P} = \mathbf{P}\,\Lambda \tag{2}$$

Where the eigenvectors of $\mathbf{C_X}$ are the columns of $\mathbf{P}$ and the eigenvalues are the diagonal terms of Λ (the off-diagonal terms are zero). Columns of matrix $\mathbf{P}$ are sorted according to the eigenvalues by descending order and they are called the Principal Components (PCs) of the data set. The eigenvector with the highest eigenvalue represents the most important pattern in the data with the largest quantity of information. Choosing only a reduced number r of principal components, those corresponding to the first eigenvalues, the reduced transformation matrix could be imagined as a model for the structure. Geometrically, the transformed data matrix $\mathbf{T}$ **(**score matrix**)** is the projection of the original data over the direction of the principal components $\mathbf{P}$.

$$\mathbf{T} = \mathbf{XP} \tag{3}$$

In addition to score matrix, other statistical measurements based on PCA model (P matrix) could be used as damage detector indexes. Two well-known are commonly used to this aim: the Q-statistic and the T^2-statistic. The first one is based on analysing the residual data matrix $\tilde{\boldsymbol{X}}$ to represent the variability of the data projection in the residual subspace. The second method is based in analysing the score matrix **T** to check the variability of the projected data in the new space of the principal components. Q and T^2-statistic of the i-th sample (or experiment) are defined as follows:

$$Q_i = \mathrm{x_i}(\mathbf{I} - \mathbf{P}\mathbf{P}^{\mathbf{T}})\,\mathrm{x_i^T} \tag{4}$$

$$T_i^2 = \mathrm{x_i}\,\mathbf{P}\Lambda^{-1}\mathbf{P}^{\mathbf{T}}\,\mathrm{x_i^T} \tag{5}$$

Where, $\boldsymbol{x}_i$ as the m-row vector that represents the measurements from all sensors in the i^{th}-experiment.

1.1. Damage detection using PCA

To generate the PCA model appropriate signal is recorded based on healthy structure. The quantity of experiments can be as much as we want, but the numbers of sensors and collected samples (data-points) must be the same that were used in the modeling phase. Then data from the current structure (damaged or not) are projected on the model (see Figure 1). Projections onto the primary principal components (scores), T^2-statistic and Q-statistic can be used as indexes to compare the structure status, identify and classify the probable damages.

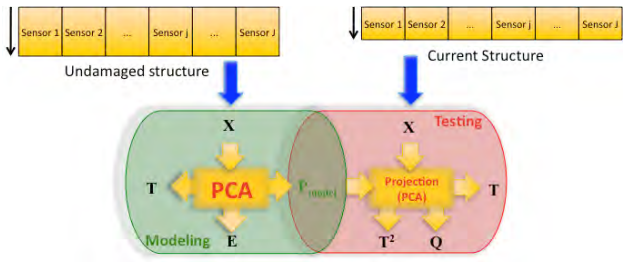

Figure 1: Damage detection using PCA model [2]

Using PCA modeling and mentioned indexes is described in details in [2]. Despite of the fact that PCA model and its derivatives (Q and T) could be used as a damage indicators, they are sensitive to atypical observations in which exist during the real experiments; so appropriate methods are necessary to improve the PCA robustness against outliers.

2. Outliers

It happens quite often that, in the data sets, outliers are present. Outlying observations are observations that lie at a considerable distance from the bulk of the observations or do not conform to the general pattern the observations exhibit. The presence of outliers in the data can be due to two main reasons. One of them is an experimental error; the other reason is the unique character of a few objects. In measurement experiments, sensor inaccuracy or error may result outliers. Outliers are categorized to 3 different types depending to their distance from PCA subspace. First type is good leverage points that lie close to the PCA space but far from the regular observations, such as the observations 1 and 4 in Figure 2. We can also have orthogonal outliers whose orthogonal distance to the PCA space is large but which we cannot see when we only look at their projection on the PCA space, like observation 5. The fourth type of data points are the bad leverage points that have a large orthogonal distance and whose projection on the PCA subspace is remote from the typical projections, such as observations 2 and 3.

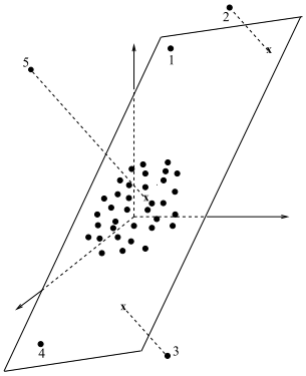

Figure 2: Influence of outliers on PCA modelling, a) without outliers b) with outlier

Regardless of their source and depending on their position, outlying observations may or may not have a large effect on the results of the analysis. For instance, existence of outliers could change the direction of PCA components and result in model inaccuracy, see Figure 3.

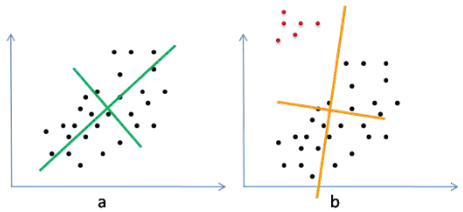

Figure 3: Influence of outliers on PCA modeling, a) without outliers b) with outlier

Given that certain observations are outliers or influential, it may be desirable to adapt the analysis to remove or diminish the effects of such observations; that is, the analysis is made robust.

3. Robust PCA

The goal of robust PCA methods is to obtain principal components that are not much influenced by outliers. To achieve this goal,

many methods have been proposed. These methods are generally based on three different approaches:

- taking the eigenvectors of robust covariance matrix
- projection pursuit
- Combination of both.

Minimum covariance determinant (MCD) [7] and fast version [8] S-estimators [9] and minimum volume ellipsoid (MVE) [10] are some methods in which belong to first approaches. The result of these methods are more robust rather than classical approach but unfortunately limited to small to moderate dimensions where the number of variables ,m, are larger than half the number of observations ,n.

From the Projection Pursuit point of view, PCA uses the variance of the projected data as a projection index. It means that, if the data contain outliers, PCA will find directions (principal components, PCs) mainly influenced by them. However, the main disadvantage of the method is the lack of robustness. The reason is that the variance itself is not robust. When in Projection Pursuit, a robust scale (not sensitive to the outliers' presence) is used as a projection index, robust principal components (RPCs) and robust dispersion matrix can be obtained. The goal of robust PCA methods is to obtain principal components that are not much influenced by outliers. To achieve this goal, many methods have been proposed. These methods are generally based on three different approaches: (i) taking the eigenvectors of robust covariance matrix, (ii) projection pursuit, (iii) combination of both. In [11, 12] a method has proposed belong to second approach. In this method, robust components are getting by calculating the candidate directions for the first component via all directions from each data point through the center of the data cloud using L_1-median estimate. Subsequent components are then estimated in a similar way, but the search process is done in the orthogonal complement of the previously identified components.

Hubert et al [13] has proposed a method belong to third approach in which uses both robust estimation and projection pursuit. Projection pursuit part is used for the initial dimension reduction and then MCD estimator is applied to this lower-dimensional data. It yields more accurate estimates at non-contaminated data and more robust at contaminated data. In addition, this method can handle high dimensional data.

In this work, because of huge dimension of data m>> n, two last methods have been used to estimate the robust PCA model. Then the new robust model is used to calculate T^2 and Q damage indices. In next part, these two methods are described briefly.

3.1. Robust PCA algorithm of Croux and Ruiz-Gazen

In this method, to center data around the mean, L_1-median is used instead of classical mean in PCA. The L_1 median is defined to be any point which minimizes the sum of Euclidean distances to all points in the data set. In literature this estimator is often called "median center". The breakdown point of the L_1 median has been found to be 50%. This is evident by noticing that if we place just over 50% of the data at one point, then the median will always stay there. The breakdown point is the most popular measure of robustness of an estimator. It measures the smallest fraction of outliers in the data that is needed to drive the scale estimators to their extreme values, i.e., zero for breakdown to zero and infinity for explosion breakdown (14). In the next step, instead of variance in classic PCA a robust scale, first quartile of the pairwise differences between all data points (15), is used to find all directions.

$$\boldsymbol{Q_n(x_1,\dots,x_n) = 2.2219 * c_n * \{|x_i - x_j|; i < j\}_{(k)}} \quad (6)$$

Where $k = \binom{h}{2} \approx \frac{\binom{n}{2}}{4}, h = \left[\frac{n}{2}\right] + 1$ and c_n a correction factor which tends to 1 when the number of objects,n, increases. The algorithm can be summarized as follows [4]:

Let X is the data matrix with elements x_{ij}, $i = 1,\dots,n$ (observation) and $j = 1,\dots,m$ (variables)

1. Center $\boldsymbol{X}$ around L_1 -median. It leads to the new centered data matrix $\boldsymbol{X}_c$.
2. For i = 1 to f_n, where f_n is the number of robust principal components to be extracted, construct a data matrix containing normalized rows of $\boldsymbol{X}_c$ (all possible eigenvectors).
3. Project all objects onto the eigenvectors.
4. Calculate the projection index of all eigenvectors.
5. Select the eigenvector with maximal value of the projection index.
6. Update the $\boldsymbol{X}_c$ by its orthogonal complement.
7. Go to step 2 until f_n robust PCs are found. Project all objects onto the eigenvectors found.

3.2. ROBPCA:

The ROBPCA method utilizes ideas of both projection pursuit (P.P.) and robust covariance estimation. The P.P. part is used for the initial dimension reduction and MCD estimator is applied to this lower-dimensional data space.

If original data are stored in an $n \times p$ data matrix, $X = X_{n,p}$, the ROBPCA method proceeds in three major steps. The first step of ROBPCA consists of performing a singular value decomposition of the data in order to project the observations on the space spanned by them.
If $m \gg n$ this step yields to huge dimension reduction with not losing information. Next, a preliminary covariance matrix S_0 is constructed that is used for selecting the number of components k that will be retained in the sequel, yielding a k-dimensional subspace that fits the data well. Then the data points are projected on this subspace where their location is robustly estimated and their scatter matrix, of which we compute its k non-zero eigenvalues $l_1, \ldots, l_k$. The corresponding eigenvectors are the k robust principal components [3].

4. Experimental setup

This specimen is a turbine blade of a commercial aircraft. It could be determined that the blade is manufactured by a homogenous material with a similar density like titanium (3.57 g/ml). Seven PZT sensors are distributed over the surface to detect time varying strain response data. Three of the sensors are on one face and four on the other face as can be seen in Figure 4-a.

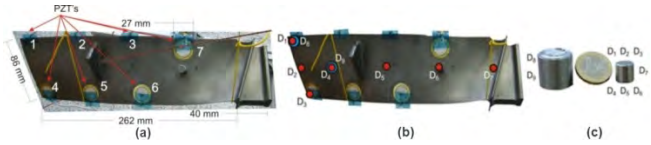

Figure 4: Experimental setup a) PZT's location b) damage location c) simulated damages

Blade is suspended by elastic ropes. The actuators are excited by a burst signal of three peaks and 350 KHz of frequency (see Figure 5-a). A measured signal in one of the PZTs is shown in Figure 5-b.

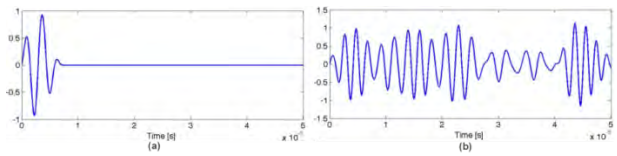

Figure 5 : a) Signal excitation, and b) Dynamical response

Damages are simulated adding masses at several locations as shown in Figure 4-b and Figure 4-c.
Data are arranged in two parts. First training data in which contains signals from non-damaged structure and second, test data that contains signals from non-damaged and damaged structure. 140 experiments were performed and recorded: 50 with the undamaged structure and 10 per each damage. The 80% of the data set collected using the undamaged structure was used for building the baseline. For diagnosis testing, the other 20% of the data set of the undamaged structure and the whole data set of the damaged structure were used.

5. Classic PCA vs. Robust PCA [16]

To build the baseline, when structure is healthy, PCA is applied to the data matrix that contains dynamical responses to a known excitation at different locations. The projection matrix **P**, which offers a better and dimensionally reduced representation of the original data **X**, is calculated. This matrix is used as a model to apply Test data in which contain data from both damaged and non-damaged structure.
Using T and Q index based on PCA model, 10 patterns are presented; One for non-damaged and 9 for different damages. The rest of paper is dedicated to compare how these indexes can distinguish between patterns from damaged and non-damaged structure when they are based on classic PCA or robust PCA. Figure 6 shows Q-T scatter graph illustrating mentioned patterns. As it is shown, robust PCA can eliminate existed outlier, mentioned by arrow, and present more aggregate pattern for non-damaged structure. Moreover, non-damaged pattern keeps more distances from others in which shows that it is distinguished better by robust method. Colors represent different conditions of the specimen (healthy, damages 1, 2,.. etc.). As $Q \gg T$ the

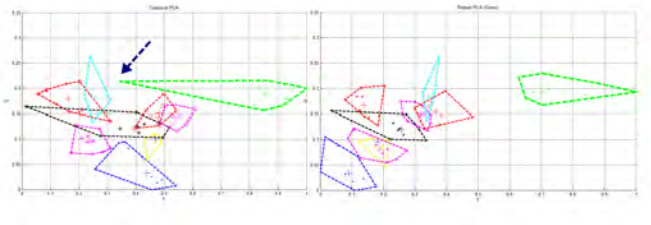

Figure 6:Classic PCA and Robust PCA a) Classic PCA b) Robust PCA

Hausdroff distance is used to compare the performance of PCA methods with robust PCA to distinguish damaged and non-damaged structure [15].

5.1. Hausdorff distance

Hausdorff distance is the maximum distance of a set to the nearest point in the other set. More formally, Hausdorff distance from set A to set B is a maximum function, defined as

$$\boldsymbol{h(A,B) = max_{a \in A}\{min_{b \in B}\{d(a,b)\}\}} \qquad (7)$$

Where a and b are points of sets A and B respectively, and $d(a,b)$ is any metric between these points; for simplicity, the Euclidian distance between a and b is selected as $d(\mathrm{a},\mathrm{b})$. Mentioned index is used in two ways. First Hausdorff distance could be used to show the distance between members of two different patterns and

secondly could be used to show distances between all members of one pattern. Second usage of Hausdorff distance is used to show how much a pattern is united. Figure 7 shows that robust methods propose patterns in which are more united.

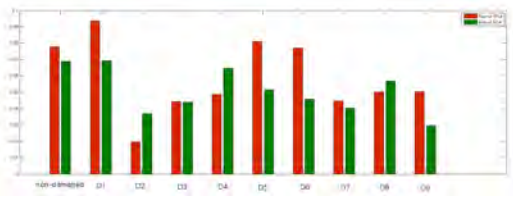

Figure 7: Hausdroff distance between members of one pattern, Classic PCA and Robust PCA

5.2. Adding simulated outliers

Pure data, directly from the experiments, may have natural outlier but to have a more accurate comparison, some artificial outliers are added to training data. Training data are contaminated using data from different damages. Philosophy behind this procedure is that the gathered data from damaged structure does not belong to the baseline so they could behave as outliers. Percentage of outliers is changing to pursue the ability of robust methods to ignore outliers in different percentages.

5.2.1. Comparing robust PCA methods with Classical PCA [15]

Figure 8 shows that when training data are contaminated, classical PCA is not able to distinguish between undamaged and damaged structure. In other words, pattern of non-damaged structure in classical PCA is conflicted with pattern of damaged structure but in ROBPCA (Figure 8-b) undamaged pattern is completely separated. As it is declared in Figure 8-a and b, the nearest pattern to pattern of non-damaged structure is 3 times farther in ROBPCA rather than classic PCA.

The same result could be seen comparing classical PCA with Croux PCA (Figure 8-c). In this case, the minimum distance is 6 times farther. Number written beside each pattern in Figure 8 is Housdroff distance between different patterns to pattern of non-damaged structure. These numbers are shown in Figure 9.

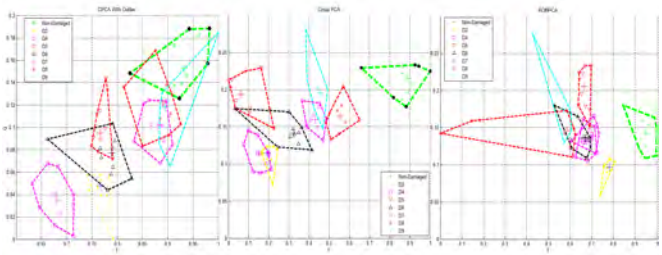

Figure 8: *Q-T* scatter graph, a) Classic PCA b) Croux PCA c) ROBPCA

As could be seen, Croux and ROBPCA have farther distance in majority of patterns. This means that these methods can separate patterns of damages from non-damaged much better. Beside, Croux PCA shows that in almost all patterns it could separate undamaged pattern even better than other robust method.

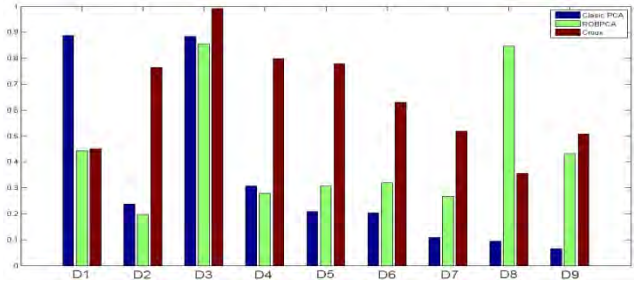

Figure 9: comparing distance of different damages to non-damaged pattern

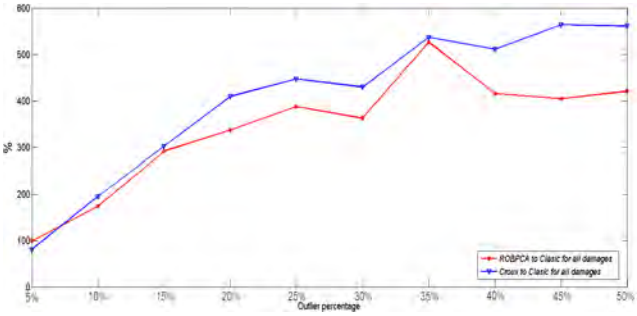

Figure 10: Ratio between robust methods to classic one of the average distance between test patterns to non-damaged pattern

It is expected that by increasing percentage of outlier, robust methods can distinguish pattern of non-damaged structure from patterns of different damages better than classic method. Figure 10 confirm this claim. For each quantity of outlier the average of Hausdorff distance of all patterns from pattern of non-damaged structure is calculated. This average is repeated 100 times. Each repeat means selecting a new group of outliers. Then the ratio of this average for robust methods is computed rather than classic one. As it could be seen, this ratio is increasing rapidly when the percentage of outliers is increased.

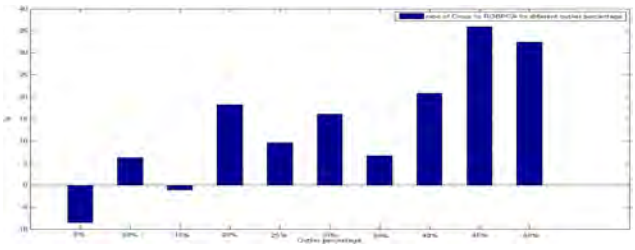

Figure 11: ratio of Croux PCA to ROBPCA in different outlier percentages

Figure 11 compare two robust PCA methods. It could be seen that for instance, in the presence of 30% outliers Croux method is 15% better than ROBPCA and goes on. As it is declared, although there is no specific trend during increasing the outliers, Croux method almost in all percentages has better results rather than ROBPCA.
From speed point of view, Croux PCA is faster than its competitor but both robust methods are much slower than classic one (about 63 % to 80 %). According to Table 1 , not depending on percentage of outliers, Croux PCA is 11 % faster than ROBPCA. All calculations have been done on a PC with 3.5 GHZ CPU and 4 GB ram.

Table 1: Speed comparison of different PCA methods

	second
Classic PCA	22 ms
ROBPCA	41 ms
Croux PCA	35 ms

6. Conclusion

Classical PCA has been used widely in SHM field. In this work, two robust PCA methods have been used instead of classic one to generate damage indices, Q and T2. Using mentioned indices simulated damages are detected in a real part of commercial aircraft. Robust methods are compared with each other and also with classical method. According to the result, although robust methods are slower than classical one, they distinguish damages much better than classic one in presence of outliers.

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