Chance-Constrained Model Predictive Control Applied to Inventory Management in Hospitalary Pharmacy

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Abstract. This extended abstract addresses the preliminary results of applying uncertainty handling strategies and advanced control techniques to the inventary management of hospitality pharmacy. Inventory management is one of the main tasks that a pharmacy department has to carry out in a hospital. It is a complex problem because it requires to establish a tradeoff between contradictory optimization criteria. The final goal of the proposed research is to update the inventory management system of hospitals such that it is possible to reduce the average inventory while maintaining preestablished clinical guarantees.

Keywords. pharmacy inventary management, uncertainty handling, model predictive control, optimization

Inventory control is a classical problem that arises in many fields. Wherever there is an organization that provides a certain good or service, there is a need of controlling the items that are bought to this end. Ideally, the organization would know exactly when these items will be needed, and hence they could be ordered to arrive and be used just in time. Unfortunately, this is not realistic due to the existing uncertainties with respect to the demand and material or information delays. As a consequence, some conservatism in the control policy used is necessary in order to avoid stockouts, specially because the consequences of such event can be fatal.

Failures in the stock management in a hospital pharmacy may have catastrophic social and economical consequences. On the one hand, the clinical needs of the hospital have to be satisfied; the social cost of the unavailability of medicines may be enormous as it may lead to the loss of human lives. On the other hand, it is not possible to raise the average stock levels too much. Hospitals have tight budgets that impose constraints on the stock management. In [1] it is estimated that about 35% of hospital expenses on services and goods are due to the pharmacy department. In European countries, where the health care system is public, these expenses are millionaire. Therefore, inventory management is one of the main tasks that a pharmacy department has to carry out in a hospital. It is a complex problem because it requires to establish a tradeoff between

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contradictory optimization criteria. In addition, other factors that typically complicate inventory management problems have also to be taken into account in this context. For example, there are constraints on the placement of stocked drugs. Room is not endless, specially for those drugs that have to be preserved at low temperature, and thus have to be stored in a fridge. Delays on drug deliveries and non deterministic demands are also major issues in this context.

Typically, the pharmacy managers apply very simple inventory control policies. In particular, an (s,S) policy is usually used, which means that when inventory drops below level s an order is placed to raise it back to S. Alternatively, a fixed size Q can be assigned to the orders and then s is defined as the reorder point. Note that other periodic review inventory control are possible as well, see for example [12] or [2]. Nevertheless, these policies lack of enough flexibility to take into account all the factors involved in this optimization problem in a systematic manner. For this reason, in this work we propose to apply model predictive control (MPC) to the pharmacy department inventory management problem. MPC is a popular control strategy for the design of high performance model-based process control systems because of its ability to handle multi-variable interactions, constraints on control (manipulated) inputs and system states, and optimization requirements in a systematic manner. MPC takes advantage of a system model to predict its future evolution starting from the current system state along a given prediction horizon. Due to its high versatility, MPC has become one of the most popular control techniques in industrial applications [3]. In fact, similar problems such as supply chain management have also benefited from the application of MPC. For example, [13] and [14] applied MPC to supply chain management in semiconductor manufacturing. In [7] a popular supply chain benchmark, the MIT Beer Game, is used to test a distributed MPC algorithm with low communicational burden. Likewise, in [11] robust MPC is applied to production-inventory system. Finally, in [9] a variation of MPC is used to reduce the number of tuning parameters when managing inventories and supply chains.

In the design of predictive controllers for dynamical systems subject to disturbances and/or uncertainty, it is well known that even if the controller finds a feasible solution, there is a certain probability that real outputs may violate the system constraints. Therefore, it would be suitable to replace and/or reformulate the original constraints involving random variables by probabilistic statements, allowing not only the treatment of the uncertainty but also avoiding possible unfeasibility of the optimization problem behind the predictive controller. Probabilistic or chance constraints, which have been treated and developed within the stochastic programming framework [6], were firstly introduced during the 60s - 70s [8]. Combined with the standard MPC theory, they allow the designer to arise with a stochastic optimization problem behind the controller by replacing hard constraints (either of states or inputs) with probabilistic constraints and by replacing the nominal cost function with its expected value in the MPC formulation [4]. This stochastic approach, known as Chance-Constrained MPC (CC-MPC) demonstrates to be suitable for large-scale complex systems due to its inherent features such as robustness, flexibility, low computational requirements, and ability to include the level of reliability (or risk) associated with the constraints (which implies its a priori knowledge) [10,5]. Thus, CC-MPC avoids the conservative nature of other MPC approaches taking into account the expected performance of the closed loop with proper constraint handling instead of directly trying to assure robust stability.

In this work, which has been performed in collaboration with two hospitals in Spain, we assess the use of CC-MPC to inventory management in Hospitalary Pharmacy, and it is a preliminary work of the project *Pharmacontrol*. The goal of this collaboration is to update the inventory management system of these hospitals so it is possible to reduce the average inventory while maintaining the same clinical guarantees. In order to illustrate the size of the problem, we will say that the biggest hospital that participates in this work has a total capacity of one thousand and two hundred beds for the inpatients. Besides these inpatients, the pharmacy department provides monthly more than five thousand drug dispensations for external patients. In this hospital the expenses on drugs exceed the amount of fifty millions of euros per year.

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