THEORICAL AND DESCRIPTIVE ANALYSIS OF THE WAVE ENERGY IN THE "BARCELONA WORLD RACE"

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Abstract

The main objective of this paper is the presentation of a theoretical study of the wave energy in some zones of the Barcelona World Race (BWR) considering different theories, how much of this energy is transferred to the boat and if it is possible to use that energy, as propulsion energy.

Nowadays, the absorption of wave energy (offshore, inshore or on board) is low utilized and has important costs. That makes this subject become more interesting. The first section of this paper is a study of the movement of the waves with different descriptions and of the energy potential depending on characteristic's value. Secondly, a categorization of the wave's characteristics is done of a part of BWR with the objective to be able to do an energy atlas of different areas. After that, a model of the vertical motion (simplified motion) is done in order to express the energy which is in the kinetic phase difference between the wave action and the boat reaction. Finally, all factors in the synchronism motion between wave and boat have been evaluated (in surf navigation) in order to analyze the results of equilibrium and the result force with different power propulsion.

Keywords
Wave Potential, Wave energy use, Physic principles energy transfer

1. INTRODUCTION. INTEREST OF NO-PROPULSION IN YACHTS

In view of the high demand to find new methods and resources of obtaining energy, maritime competition sector is going for the fact that the participant in the races vessels leading devices or systems onboard to generate "clean energy", with the goal that some soon day, the competition vessels be able to be 0 emissions boat.

For this reason, the technical teams of the participating vessels on high competition racing, design and make new efficient devices to introduce in their vessels.

Barcelona World Race (BWR) is a race around the world with only two crew members per boat. It is a nonstop race in which external assistance is allowed but subject to some rules (penalties). The boats must to be open class with specific restrictions *[17,19]*.

In this race there is another objective: to be 100% sustainable. That is the interesting point of this competition and that type of boats. To be able to have a self-sufficient ship (a ship capable of needing no fossil fuels of any kind throughout its entire useful life) it will be necessary to combine and connect some different systems from sustainable types of energy such as solar, wind and hydrodynamic energy, using them to cover all the mechanical and electronic needs of the boat and its skipper. In addition, an automatic system is needed to connect all that generators with the batteries and with energy consumers. Moreover, they will need to be switched on/off all for them only to be used, when they are be required.

Imoca 60 *[19]* is the vessel that races in the BWR, open class. This boat has a great hydrodynamic design and the different teams introduce them the suitable systems.

A new resource is needed to explore the potential of these vessels on movement or external agents, thus, it is important to know the behavior of the environment in which they move: wind and waves. It is absolutely essential to be able to give a reliable answer.

The problem of that subject is that neither the development nor the resolution of the equations that govern it has been achieved. I.e., nowadays the response of the particles behavior in water in a free movement (influenced and so overlapped with others such as it is wave motion) does not exist.

In consequence of this, there has been an effort to build wave models and wave profiles for different areas, different seas and different weather conditions. This also allows to design computational dynamic behaviors from a lot of types waves against boats, with relevant results.

It is interesting to know how the waves behave and the profiles that they describe, in order to know the potential they carry.

Once it is known the energetic potential, the navigation type in which it can be applied and the systems to design have to be studied, i.e. on which areas, under what conditions and whether it is possible to absorb wave energy.

This is an important point to see the feasibility of designing a device or not, what type, as well as their working conditions.

2. THEORY

To know the magnitude of useable wave's energy, it is interesting to know the average energy flux per unit width that crosses a vertical plane perpendicular to the direction of

propagation of the wave, i.e. the speed at which energy is transmitted through a surface overtime. To understand the energy transmitted by the wave, it is necessary to study the profile of the surface from the oceans and seas, and the nature of the submerged part. The sea and ocean are a big energy reserve, a part of this energy is moving in the waves, and these waves are irregular and with an important form complexity. In addition, because of the great expanse, there are a lot of different types of irregular waves. In each area, the performance of the water is particular, depending of the size zone, salinity, temperature, etc. *[9]*.

Nowadays there are some different models which describe, in a more or less approximate form, profiles of various kind of waves.

Firstly, there are the regular wave theories, the most significant three classified according to their application areas depending on the depth of the seabed:

- Airy Theory, linear description, for deep water representing the profile of wave as a sinusoid that is described by a cosine progressive function with amplitude equal to half the height of the wave *[6]*;

- Stokes theory of second order, nonlinear description, for midwater. It is a profile with higher and thinner ridges and more plains and broad sinus. In this case, the orbital trajectories to which the particles are subjected are considered open, thereby taking into account the net transport of molecules in the direction of wave propagation. For the calculation of potential Airy theory is used, with a correction factor depending on the depth *[6]*, and;

- Solitary Wave Theory, nonlinear description, for coastal waters. The surface of these waves is always above the sea level *[6]*.

And secondly, irregular wave descriptions are classified into two types:

- Geometric-statistic description, which is the characterization of the waves from a set of variables known as "statistical parameters". To obtain these parameters a record of successive waves in individual waves is decomposed, which can be defined with the criterion of "step up to scratch". For individual waves governed by Airy theory, the result of the sum from the Fourier series is the resulting wave, regular and with the same than irregular registration *[4, 17]*.

- Spectral description, which explains the waves as a complex signal. The energy contained in each wave is proportional to the square of their height and their period, and its distribution on wave frequencies can be represented in the form of a power spectrum. The representation of the spectrum, besides allowing to see how energy is distributed, represents existing types of waves (swell and or wind sea), and also the values of the peak period *[4, 17]*. From spectral density functions spectral parameters can be obtained, which give the information of the characteristics of the recording analyzed. This spectrum is obtained from the coefficients of the Fourier series. Depending on the domain where it is realized they might have a scalar spectral density function, or a directional spectral density function. Nowadays, there are different spectrums depending on the area being analyzed, the type of wave (background or wind), etc. The four spectrums more useful are *[4, 6, 7, 10]*:

1. Spectrum Pierson Moskowitz (1964)

Pierson and Moscowitz developed a spectrum from a North Atlantic wave spectrum study to represent fully developed sea states generated by wind. Fetch and duration are considered infinite. To apply the spectrum it is necessary that the wind blows for several hours over a large area with a relatively constant speed before the signal wave is registered. An example of this spectrum is shown below, in Figure 1.

Figure 1 - Pierson Moskowitz Spectrum
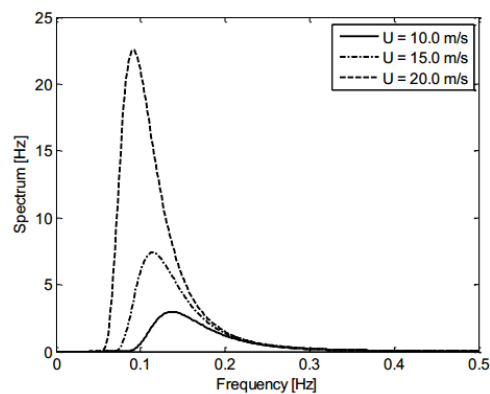
Source: Hong Kwon, S., Jan Yang, Y., Myung Choi, Y. Validity of Ocean Wave Spectrum Using Rayleigh Probability Density Functio. International Journal of Ocean System Engineering 2012; 2(4) 250-258. U= wind velocity [m/s]

2. Spectrum ISSC(1964) International Ship Structure Committee

Spectrum from a minor modification of the spectrum proposed by Bretchneider *[3]*. It is applicable for fully developed sea states on open sea. It has the same shape as Pierson Moskowitz spectrum *[11]*.

There are examples of ISSC spectrum with different periods in the Figure 2, the discontinuous line. These Spectrums come from ISSC (1964) but they have some actual modifications from ITTC (2002) *[16]*. Pierson Moskowitz Spectrum, continuous line, appears to compare the differences of both of them. Figure 2. ISSC and Pierson Moskowitz Spectrum.

Figure 2 - ISSC and Pierson Moskowitz Spectrum
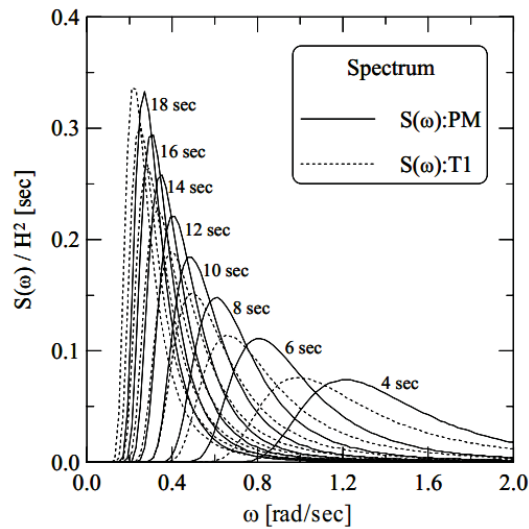

Source: 23rd International Towing Tank Conference.
The Specialist Committee on Waves. Venice, Italy: 2002
$S(\omega)$= Spectrum value ω=frequency [s^{-1}]

3. Spectrum Bretchneider-Mitsuyasu (1970)

Bretchneider proposed a formulation of narrow-band spectrum whose heights and periods follow the Rayleight distribution. Mitsuyasu later made a correction of the coefficients, using the wave generated by the wind of a laboratory and from a bay to represent a spectrum of limited Fetch. This spectrum is represented below, in Figure 3.

Figure 3 - Bretchneider-Mitsuyasu Spectrum
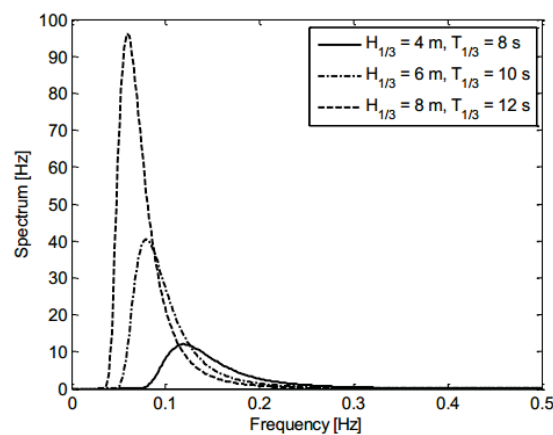
Source: Hong Kwon, S., Jan Yang, Y., Myung Choi, Y.
Validity of Ocean Wave Spectrum Using Rayleigh Probability Density Function.
International Journal of Ocean System Engineering 2012; 2(4) 250-258.
$H_{1/3}$= third biggest value of wave height [m]
$T_{1/3}$= Period of waves with the third biggest value of wave height [s]

4. <u>Spectrum de Jonswap (1973)</u>

Spectrum characterized by sharp peaks, representing waves developed in a limited fletch under strong winds. The Figure 4 shows an example of a Jonswap spectrum.
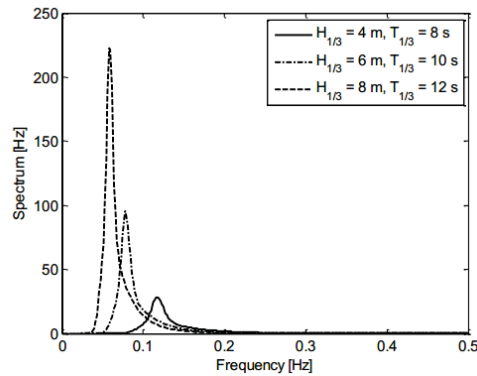
Figure 4 - Jonswap Spectrum

Source: Hong Kwon, S., Jan Yang, Y., Myung Choi, Y.
Validity of Ocean Wave Spectrum Using Rayleigh Probability Density Function.
International Journal of Ocean System Engineering 2012; 2(4) 250-258.

In order to select the most suitable theory, the area to study should be known (Ocean, sea, depth, weather, etc.), what kind of information it is, and what type of parameters gives the source in question.

3. ENERGETIC POTENTIAL CALCULATION AND CHARACTERITZATION OF AREAS

In this section the waves and wind of some BWR's zones are categorized using the wave's descriptions and various different data from a statistical study.

To do that, it is important to know the data used and the information that they give *[18, 20, 21, 22]*. Using different sources and different forms of energy calculation will permit us to make some comparison and to have more proven results.

METHOD

The area of study is separated into two main areas:

- The North Atlantic Area: latitude 10ºN to latitude 40ºN and longitude 40ºW to 10ºW, discarding two major areas where the race fails,
- The Mediterranean-Canary Area, from Barcelona to the Canary Islands. In this area there are two very different sectors, either because of the type of waves, as well as the public information at the time.

In each zone 23 characteristic maps of 23 different days between February and May have been arranged *[18, 20, 21]*. Percentage of each range of heights, periods and speeds of each zone, and the wave direction has been extracted from the maps. Also data of wind speed and direction have been collected. Once the frequency table was

performed, it could verify the significance of the results by statistics from a continuous variable, and give a sampling error to account for the final results *[14, 15]*.

The North Atlantic Area is divided into seven zones of characterization. The study is based on data from weather forecasting by simulating in characteristic maps *[12, 18, 20, 21]*, and a simple sampling of seven regions will be obtained. The results are showed in the Figure 5 with a confidence interval of 0.6m as much *[14, 15]*.
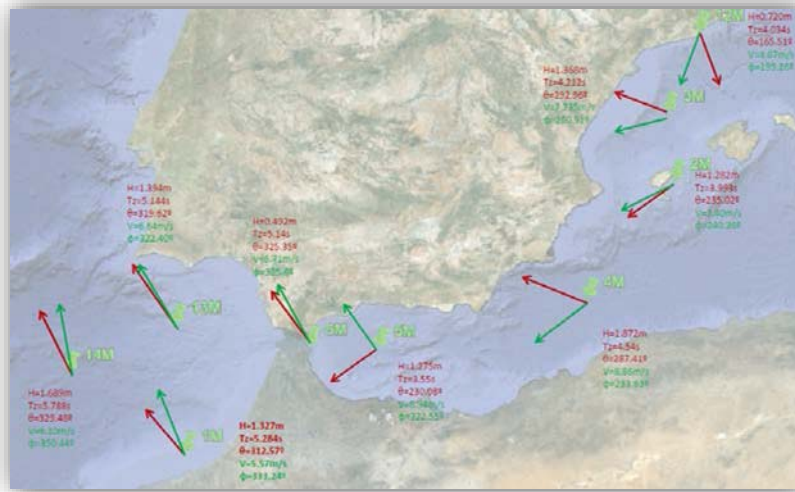
Figure 5 - Characteristic map with the illustrate results of the seven analyzed zones from North Atlantic Ocean
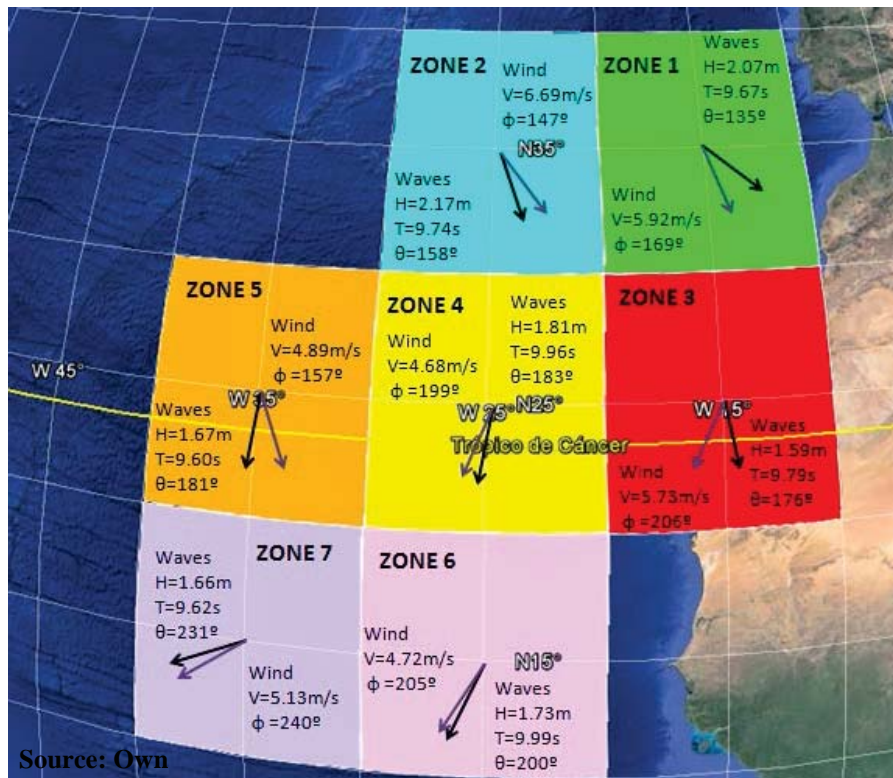
V=wind velocity [m/s]
ϕ=wind direction [degrees]
H= wave height [m]
T= wave period [s]
θ= wave direction [degrees]

For the Mediterranean-Canary Area instrumental data from oceanographic buoys was used *[22]*. These buoys provide a record published every three days, hour by hour, of the sea state: significant height, average period, peak period, direction, wind speed and direction. They have taken four records between February and May of 2012, i.e., 576 data for each feature of each point where a buoy is found. With all of that, 14 zones are characterized and showed in the following maps (Figure 6 and Figure 7)

Figure 6 - Characteristic map with the illustrate results of the different analyzed points considering Mediterranean and Gibraltar Strait Area

Source: Own

Figure 7 - Characteristic map with the illustrate results of the different analyzed of different points considering Canary Area

Source: Own

4. RESULTS

Potential energy of each area defined in Table 1 and Table 2, energy (kJ/m2) and potential (kW/m) are calculated considering description in Figure 5, Figure 6 and Figure 7.

Due to the fact that interesting areas for browsing of the BWR are in deep water, theories about midwater or coastal waters will not be used.

To calculate the average of wave energy potential by Airy's linear theory, listed in Table 1, the result of the integration of the product between dynamic pressure and celerity is used. This result is the same that the product between the energy and the celerity *[4, 6]*:

$$P = \frac{\rho g^2 H^2 T}{32\pi} \tag{1}$$

P= Potential [kW/m]

g = gravity [m/s^2]

ρ = sea water density [kg/m^3]

To find the value by geometric-statistical description, which are shown in table 1, it has to be used the root mean square wave height and the period of all the waves in the record *[4, 6]*:

$$P = \frac{\rho g^2}{32\pi} \frac{\sum_{i=1}^{N} T_i}{N} \sqrt{\frac{1}{N} \sum_{i=1}^{N} H_i^2} = \frac{\rho g^2 H_{rms}^2 T_z}{32\pi} \tag{2}$$

N=number of wave in the register

H_{rms} = root mean square wave height [m]

T_z= Statistical Period [s]

Table 1 - Energy and Potential of different zones with different theories I

AIRY's THEORY	ENERGY	POTENTIAL	STATISTIC-GEOMETRIC	ENERGY	POTENTIAL
North Atlantic	(J/m^2)	(kW/m)	**North Atlantic**	(J/m^2)	(kW/m)
1	5,381	40,643	**1**	6,207	46,884
2	5,913	44,967	**2**	6,6157	50,310
3	3,206	24,497	**3**	3,576	27,331
4	4,131	32,133	**4**	4,494	34,951
5	3,526	26,443	**5**	3,962	29,707
6	3,788	29,539	**6**	3,986	31,084
7	3,464	26,018	**7**	3,627	27,248
Mediterranean - Canary Area			**Mediterranean - Canary Area**		
1M	2,213	9,130	**1M**	2,354	9,709
2M	2,066	6,440	**2M**	2,499	7,792
3M	2,352	7,735	**3M**	2,687	8,837
4M	4,405	15,615	**4M**	6,279	22,259
5M	2,043	5,659	**5M**	2,912	8,065
6M	1,665	6,980	**6M**	0,114	0,480
7M	3,102	13,319	**7M**	3,275	14,060
8M	3,916	18,173	**8M**	4,082	18,944
9M	2,698	11,135	**9M**	2,823	11,655
10M	3,455	14,724	**10M**	3,588	15,289
11M	3,451	15,615	**11M**	3,581	16,204
12M	0,652	2,052	**12M**	0,705	2,220
13M	2,446	9,822	**13M**	2,668	10,713
14M	3,586	16,200	**14M**	3,745	16,921

Source: Own

Finally the wave energy potential is calculated with the spectral description (Table 2). This potential is a product of a coefficient and the integral of the spectrum over all frequencies and that integrate over all directions *[4, 6]*.

$$P = \gamma \int_0^{2\pi} \int_0^{\infty} \frac{g}{4\pi\omega} S(\omega, \theta) d\omega d\theta = \alpha H_s^2 T_p \tag{3}$$

γ = parameter

H_s = significant wave height [m]

T_p = Peak wave period [s]

When a different spectrum is used, the changing value is "α". And the significant wave height is used, H_s, and statistic period, T_z.

$$Pierson\ Moscowitz\ (1964)\ \rightarrow P = 0.549 H_s^2 T_z \quad \left[\frac{kW}{m}\right] \tag{4}$$

$$ISSC\ (1964) \rightarrow P = 0.595 H_s^2 T_z \quad \left[\frac{kW}{m}\right] \tag{5}$$

$$Bretchneider - Mitsuyasu \rightarrow P = 0.441 H_s^2 T_z \quad \left[\frac{kW}{m}\right] \tag{6}$$

$$Jonswap\ (1973) \rightarrow P = 0.458 H_s^2 T_z \quad \left[\frac{kW}{m}\right] \tag{7}$$

Table 2 - Energy and Potential of different zones with different theories II

Spectral	Pierson Moscoeitz	ISSC	Bretchneider-Mitsuyasu	Jonswap	Pierson Moscoeitz	ISSC	Bretchneider-Mitsuyasu	Jonswap
	kJ/m²	kJ/m²	kJ/m²	kJ/m²	kW/m	kW/m	kW/m	kW/m
	E PM	E ISSC	E B-M	E J	Pot PM	Pot ISSC	Pot B-M	Pot J
North Atlantic								
1	6,522	7,068	5,239	5,441	22,740	24,646	18,267	18,971
2	7,167	7,768	5,757	5,979	25,160	27,268	20,210	20,989
3	3,886	4,211	3,121	3,242	13,706	14,855	11,010	11,435
4	5,008	5,427	4,023	4,178	17,979	19,485	14,442	14,999
5	4,274	4,633	3,434	3,566	14,795	16,035	11,885	12,343
6	4,591	4,976	3,688	3,830	16,527	17,912	13,276	13,788
7	4,198	4,550	3,372	3,502	14,558	15,777	11,694	12,145
Mediterranean - Canary Area								
1M	1,579	1,711	1,268	1,317	5,108	5,536	4,103	4,261
2M	1,787	1,936	1,435	1,490	3,603	3,905	2,894	3,006
3M	2,136	2,315	1,716	1,782	4,328	4,690	3,476	3,610
4M	3,669	3,977	2,947	3,061	8,737	9,469	7,018	7,288
5M	1,906	2,065	1,531	1,590	3,166	3,432	2,544	2,642
6M	1,476	1,599	1,185	1,231	3,906	4,233	3,137	3,258
7M	2,451	2,656	1,968	2,044	7,452	8,077	5,986	6,217
8M	3,142	3,405	2,524	2,621	10,168	11,020	8,168	8,482
9M	1,984	2,151	1,594	1,655	6,230	6,752	5,005	5,198
10M	2,472	2,680	1,986	2,063	8,238	8,929	6,618	6,873
11M	2,480	2,688	1,992	2,069	8,737	9,469	7,018	7,289
12M	0,540	0,586	0,434	0,451	1,148	1,244	0,922	0,958
13M	1,750	1,897	1,406	1,460	5,496	5,956	4,414	4,585
14M	2,781	3,015	2,234	2,320	9,064	9,824	7,281	7,562

Source: Own

4.1 COMMENTS

As Table 1 and Table 2 shown, here are differences between the results based on the theory used. This is because of the different approximation of each theory, since one is only an approximation, and / or that there are descriptions or theories more appropriate to different types of waves than others, thus, giving results that differ between them.

To choose the best one it will be necessary a specific study of comparison, but it should be said that the Airy's theory is only an approximation, and between geometric-statistic or spectral description it would be better one which offers less generous results, to work on a safe path.

There is a characteristic point, 6M, which has a low result in statistic geometric calculation, in Table 1. This point or zone is the Gibraltar's strait zone, where there are different currents, and different waters are mixed. Spectrum description and Airy's theory calculate this energy with defined waves created by the data, and geometric-statistic description works with real data. That means that this description cannot consider neither the different directions, the deformation of the wave profile nor the energy's wave crashes. It makes the H_{rms} become small, then the energy result will be also extremely minor, more than the real one. To conclude, these results cannot be used as easily as others.

The differences between energy and potential values by spectrums descriptions are a product of using different spectrums. Each spectrum has specific conditions of sea where it is employed. In Table 2 four spectrums are used for the same waves, but the most correct spectrum on zones 1 to 7 and canary area is Pierson and Moscowitz, because it refers to Atlantic North typical and developed wave, which are the most interesting in this study. However, on Mediterranean zones, ITTC spectrum has to be used, due to an analysis of the spectrum and wave compatibility.

5. SISTEMS ENERGY USE

This section explains which kind of energy has the boat, how to calculate it, and an approximation to observe if it is interesting to study it considering two different methods: Oscillatory movement method and synchrony method.

It is important to state that this study is theoretical, so results obtained are from an ideal case.

5.1 OSCILLATORY MOVEMENT

This method is based in constructing an approximation of the vertical movement which results from all external forces, and relative acceleration of the boat *[1]*, to a forced dumped harmonic motion. This is done in order to find how much energy is in the kinetic phase difference between the action and the reaction wave of the vessel.

It is normally used to simplify the calculations into an ordinate differential equation.

The characteristic fact of this movement and of this equation is that the energy which wave transports is used to move, more or less in two amplitudes, the boat vertically, and when the wave goes back and the boat is going down the potential energy will be given to the sea again.

That is the energy which could be calculated and which could be used for propulsion.

It is observed in the equation and it is uncomplicated to imagine that the energy (and the transmission) depends on the amplitude of the wave: if the amplitude grows up, also wil the energy; and related to the difference between the frequency of the wave and the frequency of the boat: if there is less difference, the energy is higher.

The hypothesis imposed is summarized in: not to contemplate neither pitching nor roll, to consider the problem in two-dimensions system, the speed of the vessel has the opposite direction of the waves, the wind force is zero and to consider the level raising of the water surface, or draught level, the same for the entire boat (point effect).

Finally, taking into account the delay of the vessel's reaction motion relative to the wave motion, the typical equation of motion of these systems is constructed [5].

$$\ddot{x} + 2\gamma\dot{x} + w_0^2 x = F_{ext} \quad \begin{cases} F_{ext} \neq 0 \\ F_{ext} = Fcos(\omega t) \quad harmonic\ vibration \end{cases} \tag{8}$$

$\ddot{x}$ = acceleration

$\dot{x}$ = velocity

x = position

w_0 = Natural boat frequency

γ = dump parameter

F_{ext} = External Force

F = Module of External Force

t = time

Because the important part, i.e., stable, is the particular solution, that exist along over time and it is different from zero, it will be the solution which define the motion [2].

$$x_p(t) = Acos(\omega t - \theta) \tag{9}$$
$$with\ the\ same\ frecuency\ than\ the\ external\ force$$

x_p = particular solution (position of boat) [m]

θ = Phase difference [rad]

Substituting the external force and the particular solution in the general equation:

$$-\omega^2 \cos(\omega t + \theta) - 2\gamma\omega \sin(\omega t + \theta) + \omega_0^2 \cos(\omega t + \theta) = \frac{F}{A}\cos(\omega t) \tag{10}$$

ω = wave frequency [s^{-1}]

Isolating the amplitude:

$$A = \frac{F}{\left[(\omega_0^2 - \omega^2)^2 + 4\gamma^2\omega^2\right]^{\frac{1}{2}}} \tag{11}$$

And knowing the phase difference value:

$$tan\theta = \frac{2\gamma\omega}{\omega_0^2 - \omega^2} \tag{12}$$

The solution of the description/movement will be (remember that it is considering a steady state):

$$x(t) = \frac{F}{\left[(\omega_0^2 - \omega^2)^2 + 4\gamma^2\omega^2\right]^{\frac{1}{2}}} \cos\left(\omega t - tan^{-1}\frac{2\gamma\omega}{\omega_0^2 - \omega^2}\right) \tag{13}$$

$x(t)$ = Movement solution (position) [m]

Through this equation it is possible to represent the movement that the boat will experience. To adjust this description to the real movement, the sinusoidal force may be replaced by the deduced force from wave Airy's movement or other theory's movement.

Knowing the motion of the boat, we can easily extract the energy that the vessel absorbs from the movement of the sea.

Finally, to determine the usable energy of this transmission, it is necessary to make some tests with models to deduce the characteristics of oscillation or use some specific programs. In this paper, it is not possible to calculate the energy transferred by this method, since experimental tests or simulation and computational fluid dynamics' programs are required. So, the energy that the vessel takes will be obtained through the following method, in section 5.2.

5.2 SYNCHRONY METHOD

This method is based in synchrony machine work form. The objective is to make the vessel to be in synchrony, longitudinally, with the wave, like surfing, to understand what part of propulsion comes from the wave.

The vessel used is one which races in BWR, Imoca 60, a high velocity vessel in sailing competitions. Its great hydrodynamic conditions and low draught make the resistance decrease, and that is an important point in the method studied.

To find out the viability of that method it is necessary to analyze all factors which take part in navigation.

Navigation conditions are specific. The wave and the vessel must have the same direction, not to change the race trajectory. To get the best conditions, the direction of the wind has to be the same as the vessel and waves.

Once the parameters are known (H, T, θ, V_{wave}, ϕ and V_{wind} from Figure 5, Figure 6 and Figure 7),the best zone or the best zones can be located to get the synchronism. There are three zones in that part of the race that have de correct conditions and those conditions are specified in Table 3: the same wave and boat direction, and a similar wind direction in order to help the sail propulsion. The velocity of the wave will determine the velocity of the boat in synchronism.

Table 3 - Environment conditions of the three analyzed zones

ZONE	H(m)	T(s)	Θ (º)	V_{wave}(m/s)	φ(º)	V_{wind}(m/s)
Zone 4	1.81	9.96	183	15.555	199	4.68
2M	1.282	3.993	235.02	6.234	240.26	7.4
8M	1.765	5.945	5.24	9.282	41.0	6.61

Source: Own

The force that the boat takes from the wave can be calculated with this method. In this case the horizontal and vertical forces are considered.

5.2.1 NAVIGATION EQUATION

The synchrony condition is based on the equilibrium between wave velocity and vessel velocity. To make that permanent the next equation must be satisfied (x: horizontal/sea plane direction; y: vertical or sea plan normal in 2D analysis).

$$X \rightarrow \sum F_x = ma_x = E_x + F_{p,x} + F_{wave\ part,x} + F_{sail,x} + F_{wind,x} - F_{f,x} \tag{14}$$

$$Y \rightarrow \sum F_y = ma_y$$
$$= F_{wave,k} + F_{f,y} + E_y - F_{wave\ part,y} - F_{p,y} - P - F_{sail,y} - F_{wind,y} \tag{15}$$

F: Result force [N]

E : Buoyancy force [N]

F_p: Propulsive force [N]

$F_{wave\ part}$: Force of the particles of the wave circular movement [N]

F_f: Friction force [N]

$F_{wave,\ k}$: Natural force from oscillatory movements [N]

F_{sail}: Force created by the wind pressure against the sails [N]

F_{wind}: Force created by the wind pressure against hull [N]

P: Weight [N]

Firstly, we need to know the best position of the vessel on the wave in each case.

The chosen position in each zone makes the vessel (longitudinal line) have some angle (β) with the horizontal sea plane. The wave velocity or wave celerity has also to be known. The different velocities come from Figure 5, Figure 6 and Figure 7. In the Table 4 there are the values of these parameters.

$$c = \frac{gT}{2\pi} \tag{16}$$

(Parameters defined in Equation 1 and Figure 5)

Table 4 - Wave Celerity, Vessels Tilt and Significant Wave Height

Zone	H	β (degree)	Celerity (m/s)
Zona 4	1.813	-40.76	15.54
2M	1.282	-31.37	6.24
8M	1.765	-40.01	9.28

Source: Own

After that position is known all the components of the result force, expression 14 and 15, are calculated.

The buoyancy force acts in the surface direction. So, in Table 5 the different forces are shown with the same boat's weigh:

Table 5 - Buoyancy Force

Zone	P (N)	E (N)
Zone 4	81602.4249	61804.55
2M	81602.4249	69675.83
8M	81602.4249	62504.70

Source: Own

The natural force from oscillatory movements is implicit in the buoyancy force.

Friction force is calculated by the sum of the friction resistance and residual resistance by the ITTC form and Delft Systematic Yacht Hull Series (DSYHS) *[8]* and the parameters are obtained by CAD programs or by approximations with the SIMPSON method. The results of the sum are in Table 6.

$$R = R_f(ITTC\ 57) + R_r\ (DSYHS) \tag{17}$$

Table 6 - Advance Resistance

Zone	Resistance (N)
Zone 4	13210.51
2M	2500.94
8M	5064.61

Source: Own

The force created by wave particles in circular movement has been achieved by the Airy's theory *[4, 6]*. In that part the most important point is the position of the Imoca 60. The method is the same as with the resistance: sum of friction resistance and residual resistance with ITTC 57 and DSYHS, although with different velocity.

$$Total\ velocity \begin{cases} u = \dfrac{\pi H}{T} e^{\frac{2\pi z}{\lambda}} cos\theta \\ v = \dfrac{\pi H}{T} e^{\frac{2\pi z}{\lambda}} \sin\theta \end{cases} \tag{18)\ (19}$$

u = horizontal velocity [m/s]

v = vertical velocity [m/s]

z = depth [m]

The parameters to calculate these velocities with the equation 18 and equation 19 are extracted from the data of Figure 5, Figure 6 and Figure 7. And the values of these forces are in the Table 7 with the positive sign going to the front of the vessel and going to the sky.

Table 7 - Wave Particles Resistance

Zone	U (m/s)	W (m/s)	Total velocity (m/s)	Resistance (N)
Zone 4	0.5486	-0.4729	0.7243	308.38
2M	0.9890	-0.6028	1.1583	360.58
8M	0.7716	-0.6475	1.0073	339.91

Source: Own

The weight of the IMOCA 60 can be obtained in the specifications of the vessel *[19]*.

Wind sail force has been calculated in each case of navigation (Table 8), with the most common sails in that kind of navigation (Area and surface form), and with the proper angle between the direction of the boat and the sails position *[8, 19]*. Also, the wind velocity from each zone is used (obtained from the Figure 5, Figure 6 and Figure 8 information), they are the statistics results from different environment registers.

$$P_r = \frac{C_d R_o V^2}{2} \qquad\qquad (20)$$

C_d = Aerodynamic coefficient

R_0 =Air density [kg/m^3]

P_r = Pressure [N/m^2]

V = wind velocity [m/s]

Table 8 - Wind Sail Force and parameters

Zone	Wind Velocity (m/s)	Ro (kg/m^3)	Cd	Sails	A (m^2)	Wind Force (N)
Zone 4	3.520	1.223	1.50	Main+Gennaker	168+240	4630.27
2M	4.305	1.223	1.50	Main+Gennaker	168+240	6335.74
8M	3.930	1.223	1.50	Main+Gennaker	168+240	4617.38

Source: Own

The hull wind force has been obtained from the pressure of wind on the event area.

$$F = \frac{C_d R_o V^2 A}{2} \qquad\qquad (21)$$

A = area [m^2]

To know the exact wind velocity and direction the difference between vessel aerodynamic resistance and wind resistance has to be studied. That difference is a simple vectorial difference between the two parameters obtained from Figure 5, Figure 6, Figure 7. The aerodynamic coefficient has to be lower because the hull has better aerodynamic surface that the sails. Then it is not difficult to know the projected area of wind result attack. In the following table (Table 9) are the different values.

Table 9 - Hull Wind Force and Parameters

Zone	Resultant Velocity	Incident angle from bow	Projected Area	Cd	Ro (kg/m^3)	Hull Wind Force (N)

	(m/s)	(degrees)	(m²)			
Zone 4	3.52	161.34	8.28	1.17	1.223	73.40
2M	4.305	244.05	7.15	1.17	1.223	94.82
8M	3.93	80.63	19.61	1.17	1.223	216.69

Source: Own

5.3 RESULTS

Considering the values obtained in the above section, the resultant force on the vessel is calculated solving the equation (14 and 15). However the important and propulsive force is the horizontal force. Figure 8 shows results obtained from equation 14:

Figure 8 - Resultant force on the boat depending on the propulsive force.
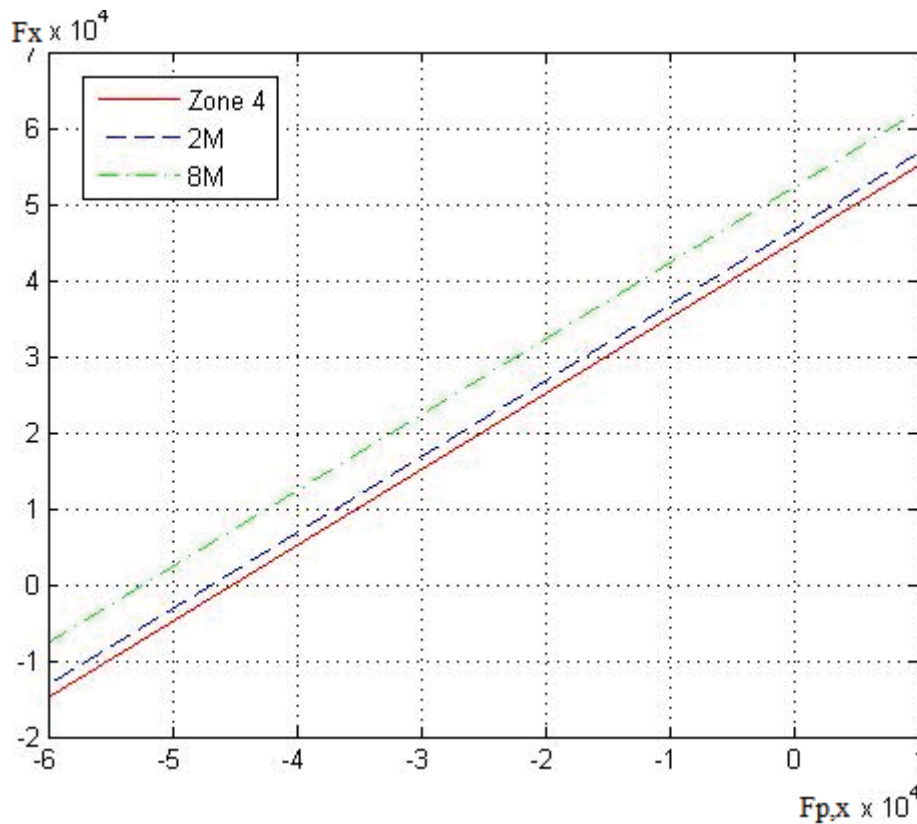
Source: Own

5.4 COMMENTS

As it is seen in Figure 8, there is a positive result of the equilibrium system. A natural propulsion exists in that conditions which make sail the vessel with environment forces, with no propellers or other mechanical propulsion, and also, the systems give to the boat an additional force that makes it go faster than the wave.

Therefore, to get the perfect equilibrium and be propelled continuously, the force's result has to be zero, so the additional propulsion force has to disappear. And that is another positive result, because it indicates that there is some energy that can be used by the boat for internal systems or for charging the batteries.