On the difficulty (and success) of correlating empirical data and (extended) topological measures in power grid networks

Abstract. Power grids have entered the complex networks realm for quite a long time now. Their structure (i.e., topology) and dynamics have been thoroughly studied and many topological measures have been used in order to classify them, evaluate their behavior in terms of robustness or model their dynamic response to malfunctions. Generally speaking, results have been mainly theoretical and sound correlations between real grid's dynamical behavior (i.e., malfunctions and major events) and any of the mentioned before measures have not yet been found. In recent years, though, new extended topological measures have been used to quantify the ability of a network in sustaining its basic functions. In this paper we present a first attempt to correlate these new measures with real malfunction data for some major European power transmission grids. Similar behavior is found, in terms of robustness to selected attacks to buses, between different networks. This is measured by means of extended topological indexes electrically better defined. These behaviors can be (weakly) correlated with similar probability distributions of major events, identifying similar dynamical response among topologically similar grids. This would raise hopes in finding a more meaningful and significant linkage between structural measures and the real dynamical output (i.e., major events) of a grid.

Keywords: electrical betweenness, entropy degree, topological measures, fat-tailed distribution, maximal information-based statistics, KS test.

1 Introduction

During the last years, complex networks (CN) have been considered a framework for a new kind of approach to complex systems [1]. New topological measures, algorisms and models have been widely used in networks from different fields such as biology, chemistry, social sciences, computer networks, etc., in order to classify their structure, dynamics and evolving patterns. A considerable amount of studies have been performed on a remarkable technological network such as the power grid, where buses and transmission lines are considered nodes and links respectively, in order to define a graph. As far as the structure is concerned, power grids, at least at the transmission level, have been thoroughly studied and different aspects, such as basic topological characteristics and statistical global graph properties have been performed on many grids around the world [2]. Among the latter, static robustness (or vulnerability) analysis based on evaluating the variation in global connectivity due to random failure (i.e., random bus deletion) or selective attack (i.e., in decreasing order of some bus topological feature) of nodes has been mostly used. For most grids, global connectivity decreases exponentially, with a higher variability when buses are “attacked” in decreasing order of degree (i.e., the number of links a node is attached to) [3].

On the other hand, power grids are complex multilayered networks where many decision processes, involving different objectives, are at play. The global behavior of the grid is thus mainly driven by the complex interaction between its structure, its dynamical processes (i.e., power fluxes) and economic and environmental constraints. Since this complex interaction is difficult to unveil at a global level, research has been focused on detecting whether malfunctions, turned into emergent outcomes such as blackouts, can be related to topological constraints, the rationale behind this procedure being that structure affects dynamics somehow. Until now, most of the literature has been concerned on relating purely topological measures, such as analytical results coming from the aforementioned static vulnerability analysis, with aggregated malfunctions outcome (i.e., total loss of power, energy not supplied or restoration time) [4]. But this approach has failed when it has been applied to power systems with different topological characteristics [5], mainly due to the poor definition of purely topological measures, away from the real physical and electrical definition of the system. In order to overcome this limitation, more specific topological measures have been defined [6,7]. Among these better suited to electrical systems extended measures, entropy degree (ED) and electrical betweenness (EB) have been presented as useful means to characterize the topology of the nodes of a power network [8].

In this paper, ED and EB are used in order to characterize the buses of the four biggest transport networks in Europe (i.e., France, Germany, Italy and Spain) and a static robustness analysis is performed. Similar statistical behaviour is observed between Germany and Italy (GI networks), and Spain and France (SF networks), with respect to attacks performed in decreasing order of ED and EB. This behaviour can be correlated with disaggregated cumulative probability distributions of major events. Results show statistically meaningful (although weak) correlations among similar topologically characterized networks, which could finally help in defining a linkage between topological measures (i.e., structure) and malfunctions (i.e., dynamics) on power grids.

The paper is organized as follows. In section 2, the extended topological measures are introduced and used to analyse the robustness of four major European power grids. In section 3, statistical analysis of correlations with cumulative probability distributions is presented and the problem of correlating major malfunctions with theoretical distributions is discussed. Conclusions are summarized in section 4.

2 Extended topological measures

Topological measures used to characterize the structure of networks can be the number of links connected to a node (i.e., degree of a node), the number of shortest paths passing through a node or a link (i.e., betweenness) or the amount of nodes tightly connected (i.e., modularity) [1]. In the particular case of power grids, the simplicity embedded in purely topological measures have made them useless for practical purposes. Instead, this approach has been recently extended by considering the following electrical properties [6-8]:

- **Distance**. From the perspective of electrical engineering, distance should have more practical meaning which should be a measure of the "cost". For electrical power grids, the cost of power transmission between two buses can be described from both economic and technological point of view, such as transmission loss or voltage drop.
- **Bus classification.** In traditional complex networks methodology, all elements have been treated identically. Correspondingly, the physical quantity was considered to be transmitted from any vertex to any other. However, the essential function of power grids is to transmit electrical power from any generator bus to any load bus. Generally, the buses in power transmission grids can be classified as generation buses, transmission buses and load buses.
- **Line flow limit.** In a pure topological approach, edges are generally described in an unweighted way. However, in electrical engineering, transmission lines have line flow limits which restrict the ability of one line for power. As this feature is critical for the networks to perform their essential functions, it cannot be neglected in vulnerability assessment.
- **Flow-based network.** The physical quantity to be transmitted between two vertices is always supposed to be through the shortest path, which is the most unrealistic assumption from the point of view of electrical engineering. Power transmission from a generator bus to a load bus will involve most lines or a huge number of paths with contribution to different extent. In a linear model of power flow, the different contributions of lines in power transmission can be described by the Power Transmission Distribution Factors (PTDF).

It is more feasible to model an electrical power grid as a weighted and directed network identified by a set **Y** = {B, L, W} where B (dim{B} = N_B) is the set of vertices (or nodes), L (dim{B} = N_L) is the set of edges (or links) and W is set of line weights. Vertices are identified by index i. Edges are identified by l_{ij}, which represents a connection between vertex i and vertex j. And the weight element w_{ij} in the set W is associated with each line l_{ij}. Based on the consideration mentioned above, two extended metrics, entropy degree (ED) and electrical betweenness (EB) have been proposed as extended measures to characterize the topology of the grid.

- **Entropy degree.** In unweighted networks, the degree k_i of a vertex i is the number of edges attached to it [1]. In a weighted network, it is named the strength s_i of the vertex, and it is the sum of the weights of the edges connecting the node i. Degree (or its associated probability distribution) is a basic metric that can measure the relative importance of a node. However, these two definitions cannot take simultaneously into account (a) the strength of connections in terms of weights of edges, (b) the number of edges connected to that vertex and (c) the distribution of weights among edges. To consider these three factors, the concept of entropy is introduced to redefine the entropic degree k_i^w of a vertex i in the following way [6]:

$$k_i^w = \left[1 - \sum_{j=1}^{N} p_{ij} \cdot log(p_{ij})\right] \sum_{j=1}^{N} w_{ij} \tag{1}$$

where p_{ij} is the normalized weight of edge l_{ij} connecting vertices i and j:

$$p_{ij} = \frac{w_{ij}}{\sum_{j=1}^{N} w_{ij}} \tag{2}$$

- **Electrical betweenness.** The betweenness of a node (or edge) i is defined as the number of geodesic paths connecting whichever pair of vertices, passing through this given vertex (or edge) [1]. Since the definition of betweenness neglects the electrical features of the power system, this can be redefined in the following terms. In the case of a line $l_{ij} \in L$, extended (or electrical) betweennes can be defined as [7,8]:

  $$B_e(l_{ij}) = Max\left(B_e^p(l_{ij}), \left|B_e^n(l_{ij})\right|\right) \tag{3}$$

  where

  $$B_e^p(l_{ij}) = \sum_{g \in G} \sum_{d \in D} C_g^d \, f_{l_{ij}}^{gd} \tag{4}$$

  if $f_{l_{ij}}^{gd} > 0$ and

  $$B_e^n(l_{ij}) = \sum_{g \in G} \sum_{d \in D} C_g^d \, f_{l_{ij}}^{gd} \tag{5}$$

  if $f_{l_{ij}}^{gd} < 0$. C_g^d is the power transmission capacity of the line which is defined as:

  $$C_g^d = \min\left[\frac{P_{l_{ij}}^{max}}{\left|f_{l_{ij}}^{gd}\right|}\right] \tag{6}$$

where $f_{l_{ij}}^{gd}$ is the power on line l_{ij} ($l_{ij} \in L$) for a unit of power injected at generation bus g ($g \in G$) and withdrawal at load bus d ($d \in D$). $f_{l_{ij}}^{gd}$ can be computed as follows:

$$f_{l_{ij}}^{gd} = f_{l_{ij}}^{g} - f_{l_{ij}}^{d} \tag{7}$$

where $f_{l_{ij}}^{g}$ and $f_{l_{ij}}^{d}$ are the l_{ij} –th, row g -th column and the l_{ij} –th, row d -th column of matrix $\boldsymbol{F}$ respectively. Matrix $\boldsymbol{F}$ represents the $N_L \times N_B$ matrix of PTDF in which an element $f_{l_{ij}}^{v}$ represents the change of power on line l_{ij} for a unit of power injected at bus v and withdrawn at the reference bus. If $f_{l_{ij}}^{v}$ is consistent with the reference direction of line l_{ij}, then $f_{l_{ij}}^{v}>0$; otherwise, $f_{l_{ij}}^{v}<0$. The input power of a bus v should be equal to output power of the bus, so the extended betweenness *of a bus v* is the half of sum of power flowing through the lines connecting this bus:

$$B_e(v) = \frac{1}{2}\sum_{g\in G}\sum_{d\in D} C_g^d \sum_{l_{ij}\in L^v} f_{l_{ij}}^{gd} \quad (8)$$

where L^v is the set of lines connecting to a bus v.

2.1 Attacks for major national power grids

The robustness of the power grid is an example of a generalized feature of most complex networks, from the Internet to the genome [9-13]. Specifically, real networks are often characterized by a considerable resilience against random removal or failure of individual units but experience important short-comings when the highly connected elements are the target of the removal. Such directed attacks have dramatic structural effects, typically leading to network fragmentation [14-17]. In this subsection the evolution of this fragmentation is evaluated in the case of four European power grids: France, Germany, Italy and Spain (Table 1).

Number of	**France**	**Germany**	**Italy**	**Spain**
Buses	1401	1197	535	447
Lines	1819	1714	645	644
Generators	136	156	126	100
Loads	881	602	249	349

Table 1. Basic characteristics of the four major national power grids analyzed in this paper.

Entropy degree and electrical betweenness have been used as new metrics to evaluate how differently the power grids behave when random or selective nodes are eliminated and compared to traditional purely topological metrics. Since entropy degree and electrical betweenness implies already an ordered list of nodes, random deletion is neglected and selective attacks are considered instead.

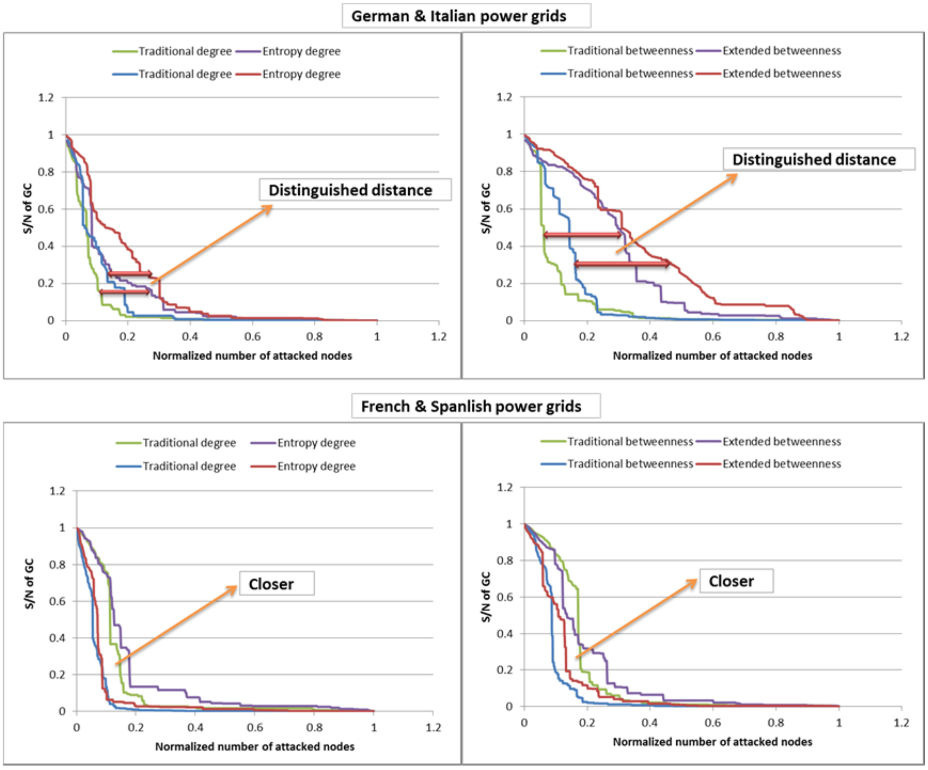

Fig. 1. Effects of attacks on the topology of France, Germany, Italy and Spain power grids. The static tolerance to selective removal of a fraction of nodes, by decreasing order of each particular metric is measured by the relative size *S/N* of the largest connected component.

Figure 1 shows the static tolerance to selective removal of a fraction of nodes, by decreasing order of each metric and for the four major national grids studied. Robustness is measured by the relative size *S/N* of the largest connected component. As it is shown, German and Italian power grids present a distinguished distance between traditional and entropy degree static tolerance procedures (especially between traditional and electrical betweenness ones). However, in Spain and France power grids, the curves under different scenarios are much similar and closely depicted. An analysis of the maximal information coefficient (MIC) between all data is shown in Table 2 [18]. As far as the electrical betweenness is concerned, there exists a higher correlation between France and Spain, and Germany and Italy. As far as the entropy degree is concerned, results are less conclusive although Germany and Italy are significantly correlated.

MIC strength		Electrical Betweenness	Entropy Degree
France	**Germany**	0.99624	0.97894
France	**Italy**	0.98761	0.97313
France	**Spain**	**0.99668**	0.951
Germany	**Italy**	**0.99976**	**0.99825**
Germany	**Spain**	0.99639	**0.99825**
Italy	**Spain**	0.98456	**0.99844**

Table 2. Maximal information coefficient (MIC) for electrical betweenness and entropy degree evaluated among France, Germany, Italy and Spain power grids, following their functional behaviour during static tolerance to attacks (Fig. 1).

This fact suggests two significant conclusions. Firstly, the evolution of the largest connected component during the attack is obviously different between G-I power grids and S-F power grids. Secondly, while traditional metrics are not able to detect differences, extended metrics (especially electrical betweenness) have this capability. Furthermore, this dissimilar behaviour coincides with the conclusion published in [4], where G-I networks and S-F networks are segregated in different groups, in this case termed as *robust* ($\gamma < 1.5$) and *fragile* ($\gamma > 1.5$) according to γ, the exponential degree distribution characteristic parameter respectively. In this same reference, the authors provide an evidence for the correlation between topological structure and vulnerability performance in terms of aggregated values of major events. Although our defined extended topological metrics can illustrate the difference between two particular types of network, it is difficult to directly assume that these extended metrics can be correlated with any real dynamic feature of the grid. Therefore, a linkage between structural measures and the real dynamical output (i.e., major events) of a grid is needed.

3 Vulnerability, extended topological measures and probability distributions of major events

Probability distribution analysis is one of the methods to study the statistics and dynamics of series of empirical data with approximate global models. Heavy tailed probability distributions seem to be ubiquitous statistical features of self-organized natural and social complex systems [19], and the appearance of the power law distribution is often thought to be the signature of hierarchy, robustness, criticality and basically, non-random behaviour [20]. In this sense, European power transmission grids major events data [21] provide us with a set of real malfunction data for the vulnerability analysis in power transmission grids. Probability distribution analysis is used in order to detect correlations between real dynamic output and topological measures.

3.1 Probability distributions of major events

European power transmission grids reliability data is given through energy not supplied (ENS), total loss of power (TLP) and restoration time (RT). These can be found in the UCTE/ENTSO webpage and they are publicly available from 2002 onwards [21]. Figure 2 shows the cumulative distribution functions for the aforementioned reliability measures and for the four major power grids. Logarithmic binning has been used in order to diminish the noise associated with statistical fluctuations [22].

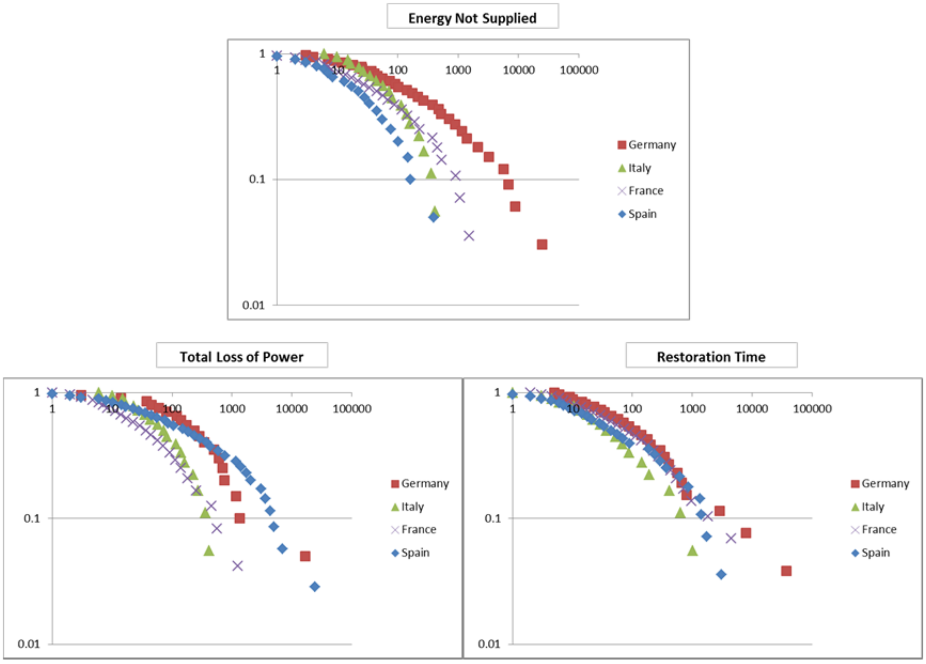

Fig. 2. Cumulative distribution functions for the four major power grids reliability measures energy not supplied, total loss of power and restoration time.

The methodology described in [22] offers the possibility to statistically fitting a function to the tail of the distribution. This methodology has been followed in this section, where a maximum likelihood approach is proposed to estimate the heavy tailed function from the data and a significance test is constructed to evaluate the plausibility of some specific distributions. Table 3 shows likelihood ratios and p-values results with respect to log-normal, exponential, stretched exponential and power law with cut off distributions, all of them with power law function taken as comparative means. Positive likelihood values favour the power law hypothesis and p-values higher than 0.1 imply no significance on the results. As we can see, although power law could be accepted only for the TLP (total loss of power) in Spain, the value of the likelihood ratio does not support this option. In general terms, results are not conclusive and no function can be adjusted with enough statistical significance.

		power law	log-normal		exponential		stretched exp.		power law + cut-off	
		p	LR	p	LR	p	LR	p	LR	p
ENS	France	0.11	-1.26	0.21	0.91	0.36	-1.23	0.22	13.55	1.00
	Germany	0.80	-0.68	0.50	1.04	0.30	-0.63	0.53	122.08	1.00
	Italy	0.14	-0.87	0.39	-0.57	0.57	-0.76	0.45	9.41	1.00
	Spain	0.72	-0.42	0.68	0.30	0.76	-0.57	0.57	37.31	1.00
TLP	France	0.81	-0.34	0.73	0.79	0.43	-0.52	0.61	66.15	1.00
	Germany	0.65	1.03	0.31	-0.42	0.67	0.00	1.00	82.00	1.00
	Italy	0.13	-0.87	0.39	-0.57	0.57	-0.76	0.45	9.41	1.00
	Spain	**0.07**	-1.65	0.10	0.47	0.64	-1.79	**0.07**	68.33	1.00
RT	France	0.86	0.05	0.96	0.91	0.36	-0.18	0.86	114.54	1.00
	Germany	0.91	0.43	0.67	1.58	0.11	0.66	0.51	80.16	1.00
	Italy	0.80	-0.51	0.61	0.89	0.38	-0.47	0.64	26.38	1.00
	Spain	0.28	-1.19	0.23	1.56	0.12	-1.19	0.24	9.03	1.00

Table 3. Test of fat-tailed behavior taking the power law as comparative function for energy not supplied (ENS), total loss of power (TLP) and restoration time (RT). Positive values of the likelihood ratios LR favors the power law model. Values of $p \geq 0.1$ imply, though, that results can not be trusted.

3.2 Kolmogorov-Smirnov test for aggregated major events

One drawback observed in the previous section is the amount of major events data considered, which might be less than desired when fitting any fat tailed function. In this section aggregated data for all combinations of major events has been considered. On the other hand, although no conclusions can be drawn from the previous probability distribution analysis, cumulative distributions shown in Figure 2 present obvious differences which make them depart from or approach to fitting functions. This can be detected with other statistical tests like the Kolmogorov-Smirnov (KS) test, defined as the maximum distance *D* between the cumulative distribution functions of the data *S(x)* and the fitted model *P(x):*

$$D = max|S(x) - P(x)| \tag{9}$$

KS test is used in order to detect how close a theoretical probability distribution function is from the real one. It is performed with the aim of detecting whole function approximation and not only fitting the tail of the function. Table 4 shows KS test results for the meaningful combination of pairs of grids, which are coincident with the previous selection: Germany and Italy on one side, and France and Spain on the other.

We can see that although log-normal and stretched exponential distributions cannot be ruled out completely, power law with exponential cut-off can be ruled out for energy not supplied, total loss of power and restoration time for Germany and Italy but not for France and Spain combined major events data.

		power law + exp. cutoff	log-norm	stretched exp.	exp.
ENS	**Germany + Italy**	0.096	**0.064**	**0.064**	0.387
	France + Spain	**0.083**	**0.083**	**0.083**	0.250
TLP	**Germany + Italy**	0.107	**0.071**	**0.071**	0.357
	France + Spain	**0.071**	**0.071**	**0.071**	0.321
RT	**Germany + Italy**	0.090	0.121	**0.090**	0.424
	France + Spain	**0.062**	**0.062**	**0.062**	0.375

Table 4. Values of the KS test for different fitting functions to energy not supplied (ENS), total loss of power (TLP) and restoration time (RT) probability distribution functions. Although log-normal and stretched exponential functions are statistically sound, differences arise in the case of the power law with exponential cutoff.

Even though statistically speaking the evidence is somehow weak, these results would favour the existence of a linkage between structure and dynamics. Some grids, in this case France and Spain, can be adjusted by power law with cutoff, lognormal and stretched exponential. Germany and Italy, on the other side, can be adjusted by lognormal and stretched exponential but not by power law with cutoff. Although firm conclusions cannot be drawn, the probability distributions of major events for these networks would suggest a different performance in terms of vulnerability, distinguished by frequency of major events and MW, MWh and minutes (i.e., restoration time) involved in these failures. From the physics point of view, an exponential cutoff could be understood in the following manner:

- For the Energy Not Supplied (ENS), which means the loss of energy from consumption side, it reveals the physical constraints on the maximum energy consumption from consumers (residential, commercial and industrial).
- For Total Loss of Power (TLP), which means the loss of production from the generation side, the fast decaying tail is consistent with the maximum power output of the generator at each vertex.
- For Restoration Time (RT), it is the signature of an obvious upper bound since the power facilities cannot be damaged forever.

The physical meaning described above can help us suggesting the meaning of this dissimilar behaviour. Spain and France grids' dynamic behaviour (i.e., major events) is closer to what would seem the limit of their reality, while Germany and Italy power grids are not, since there is no exponential decay in their probability functions. Back to their topological structure, the metrics (i.e., the extended metrics EB and ED or the exponential degree distribution characteristic parameter γ cited in Ref. [4]) also discriminate the four major power grids in two groups, this is Germany and Italy, and Spain and France. So a direct linkage can be suggested between structural measures and the real dynamical output: on the one hand, the topological structure of Spain and France power grids indicates that the whole networks anearly reach their maximum power transmission ability. In other words, the networks are more *fragile* and, correspondingly, their dynamic output (in terms of major events) shows the existence of maximum constraints. On the other hand, Germany and Italy power grids seem not yet at their maximum capacity, and there is still a margin to reach the upper bound of their dynamic output. Equivalently, they could be considered (for the time being) more *robust*.

4 Conclusions

Although a contradiction as it seems, complex networks science allows a simplified view of the reality. Algorithms, measures and models involved in studying complex systems as networks, have allowed an understanding of some common features which characterize their topology and, in a lesser extent, their dynamic processes. Power grids have been thoroughly studied as complex networks and many topological measures have been used in order to classify their structure, evaluate their behaviour in terms of robustness or model their dynamic response to malfunctions. Results have been mainly theoretical and no correlation between real grid's dynamical behaviour (i.e., malfunctions and major events) and any structural measure has yet been found. In this paper new extended topological measures have been used in order to quantify the ability of four European power grids (i.e., France, Germany, Italy and Spain) to sustain selective removal of buses. A maximal information coefficient has been used to find a similar robustness behaviour between Spanish and French networks on one side, and German and Italian networks on the other. In order to find a correlation with any dynamical output (i.e., blackouts), binned cumulated probability distributions of majors events in terms of energy not supplied, total loss of power and restoration time have been fitted to some characteristic fat-tailed functions, with no success. This could be probably due to the small amount of major events data actually available for the studied power grids (or simply because real cumulative probability distributions do not follow any of the fat-tailed function used for the fitting). To avoid the first drawback, aggregated data for every two networks has been used to significantly increase the amount of values included in the probability distributions. Although a favourable fitting is not found, the paper shows that a significant (although weak) statistical approximation appears when Germany and Italy on one side and France and Spain on the other are considered in aggregated manner, thus identifying similar dynamical response among topologically similar grids. Although much research must be done, such as extending this methodology to distribution networks or exploring the cascading failure in power grids, combining topological measures that include electrical engineering perspectives, this evidence would raise hopes in finding a more meaningful and significant linkage between structural measures and real dynamical output, in terms of major events, of a power grid.

5 References

1. Newman, M.E.J.: *Networks. An introduction*, Oxford University Press (2010).
2. G.A. Pagani and Aiello, M.: *The Power Grid as a Complex Network: a Survey*, (http://arxiv.org/abs/1105.3338).
3. M. Rosas-Casals, S. Valverde, and R. V. Solé: *Topological vulnerability of the European power grid under errors and attacks*, Int. J. Bifurcation Chaos, 17, 7 (2007).
4. Ricard V. Solé, M. Rosas-Casals, B. Corominas-Murtra and S.Valverde: *Robustness of the European power grids under intentional attack*, Physical Review E, Vol. 77, 026102 (2008).
5. M. Rosas-Casals, personal communication.
6. E. Bompard, R. Napoli and F. Xue: *Analysis of Structural vulnerability in power transmission grids*, International Journal of Critical Infrastructure Protection, 2, 5-12, (2009).
7. E. Bompard, R.Napoli and F.Xue: *Extended topological approach for the assessment of structural vulnerability in transmission networks*, IET Gener. Transm. Distrib., 4, 716-724, (2010).
8. E. Bompard, E. Pons and D. Wu: *Analysis of the structural vulnerability of the interconnected power grid of continental Europe with Integrated Power System and Unified Power System based on extended topological approach*, Euro. Trans. Electr. Power, 10.1002/etep.1618, (2012).
9. R. Albert and A. L. Barabási: *Statistical Mechanics of Complex Networks*, Rev. Mod. Phys. 74, 47 (2002).
10. S. Bornholdt and G. Schuster, ed.: *Handbook of Graphs and Networks*, Wiley-VCH, Berlin (2002).
11. M. E. J. Newman: *The structure and function of complex networks*, SIAM Review 45, 167–256 (2003).
12. S. N. Dorogovtsev and J. F. F. Mendes: *Evolution of Networks: From Biological Nets to the Internet and WWW*, Oxford University Press, New York (2003).
13. S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, and D-U. Hwang: *Complex networks: Structure and dynamics*, Phys. Rep. 424, 175 (2006).
14. R. Albert, I. Albert, and G. L. Nakarado: *Structural vulnerability of the North American power grid*, Phys. Rev. E 69, 025103R (2004).
15. A. E. Motter and Y. C. Lai: *Cascade-based attacks on complex networks*, Phys. Rev. E 66, 065102R (2002).
16. B. A. Carreras et al.: *Critical points and transitions in an electric power transmission model for cascading failure blackouts*, Chaos 12, 985 2002; 14, 643 (2004).
17. A. E. Motter: *Cascade, control and defence in complex networks*, Phys. Rev. Lett. 93, 098701 (2004).
18. D. Reshef, Y. Reshef, H. Finucane, S. Grossman, G. McVean, P. Turnbaugh, E. Lander, M. Mitzenmacher and P. Sabeti: *Detecting novel associations in large datasets*. Science 334, 6062 (2011).
19. M. Buchanan: *Ubiquity. Why catastrophes happen*, Tree Rivers Press, New York (2001).
20. M. E. J. Newman: *Power laws, Pareto distributions and Zipf's law*, Contemporary Physics 46, 323–351 (2005).
21. UCTE/ENTSO (https://www.entsoe.eu/)

22. Aaron Clauset, Cosma Rohilla Shalizi, and M. E. J. Newman: *Power-law distributions in empirical data*, SIAM Review 51, 661-703 (2009).